Free Groups

For a wedge of n circles,

$$\pi_1\left(\bigvee_{i=1}^{n} S^1\right) \cong F_n,$$

where F_n is the free group on n generators.

TopoPlay implements:

- generators,
- inverses,
- concatenation,
- reduction by cancellation.

The fundamental reduction rule is:

$$gg^{-1} = e.$$

Surfaces

An orientable surface of genus g is modeled using generators

$$a_1, b_1, \ldots, a_g, b_g$$

subject to the defining relation

$$\prod_{i=1}^{g} [a_i, b_i] = e,$$

where the commutator is defined as

$$[a, b] = aba^{-1}b^{-1}.$$

This relation is explicitly constructed and used to test null-homotopy.

Fundamental Group Examples

Circle

$$\pi_1(S^1) \cong \mathbb{Z}.$$

Wedge of two circles

$$\pi_1(S^1 \vee S^1) \cong F_2.$$

Torus

$$\pi_1(T^2) = \langle a, b \mid aba^{-1}b^{-1} = e \rangle.$$