Hierarchical Latent Space Models for Social Network Analysis

Alex Loewi[1,4], Francisco Ralston[2], Octavio Mesner[2,3,4]

[1]Public Policy, [2]Engineering & Public Policy, [3]Statistics & Data Science, [4]Machine Learning

Carnegie Mellon University

Introduction

Multigraphs: "Parallel" network models sharing identical nodes but different edges connecting nodes, used to model data where actors (nodes) maintain varying types of relationships.
Social Network Analysis (SNA), where nodes represent individual people and edges represent differing relationships, such as friendship, professional colleague, or facebook connected, for example.
Problem: Social science and network analysis rely on intuitive and interpretable models. Visualizing the similarities between multiple related graphs is challenging.

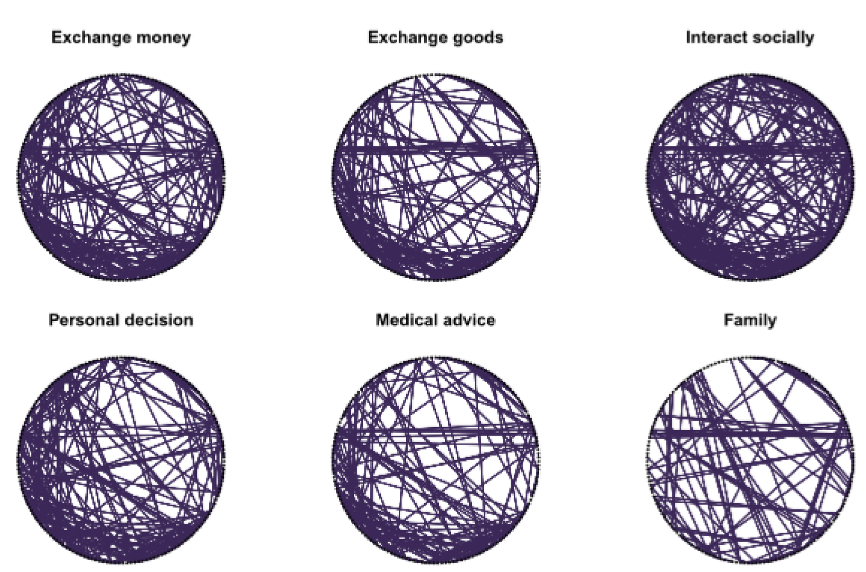

Figure 1: (Source: Salter-Townshend, Michael, and Tyler H. McCormick)

Solution: Using a LASSO penalty, we aim to force similar graphs to be displayed in comparable, visualizable, ways

Employee Relationships Data

A recently conducted study at a charter school in West Baltimore
collected a five-layer valued network between the approximately eighty employees of the school:

- Frequency of interaction
- Discussing Academics
- Discussing Behavior
- Social interaction
- Professionally Helpful Relationship

While empirically these layers are found to have very high correlations, modeling them in a way that demonstrates those correlations is challenging.

Latent Space Projection

- Map nodes, $i \mapsto z_i \in \mathbb{R}^n$ where proximity, $\|z_i - z_j\|_2 = d_{ij} < 1$, indicates nodes are connected (and not connected otherwise)
- Intuitively captures reciprocity ($j \to i \Rightarrow i \to j$) and transitivity ($i \to j, j \to k \Rightarrow i \to k$)
- Edge probability: $\sigma_{ij} = \mathbb{P}(Y_{ij} = 1 | z_i, z_j) = \text{logit}^{-1}\left(\alpha + \|z_i - z_j\|_2^2\right)$ where $Y_{ij} = 1$ indicates i, j are connected in the data, find z
- Likelihood: $\Pi_{i<j}\, \sigma_{ij}^{y_{ij}} (1 - \sigma_{ij})^{1 - y_{ij}}$

Hierarchical Models

Goal: Collapse similar networks indicated by k
We include a LASSO penalty on the log-likelihood:

$$\sum_k \sum_{i<j} \left[y_{ijk} \ln \sigma_{ijk} + (1 - y_{ijk}) \ln(1 - \sigma_{ijk}) \right] + \lambda \sum_i \sum_k \|\epsilon_{ik}\|_1$$

where $z_{ik} = b_i + \epsilon_{ik}$ for initialization points, b_i

Optimization Approaches:

1. Proximal Gradient Descent
2. Coordinate-Wise Optimization
3. Hamiltonian Monte Carlo methods

Model's Advantages

Our model has the additional inclusion of a hierarchy on the latent variables:

$$z_{ik} = b_i + \epsilon_{ik}$$

each node has a separate latent position to explain its ties within a single layer of the multigraph, but these multiple positions are all tied together by a single "base" position for that node b_i, and a layer-specific perturbation, ϵ_{ik}.
This allows two major advantages. One is graph-wise regularization with a group lasso penalty, allowing entire redundant layers to be removed for meaningful dimensionality reduction. Second, regularization allows a manual tuning between layouts that are expressive, and those that are comparable, solving the difficult problem of visual comparability.

Results for Proximal Gradient Descent

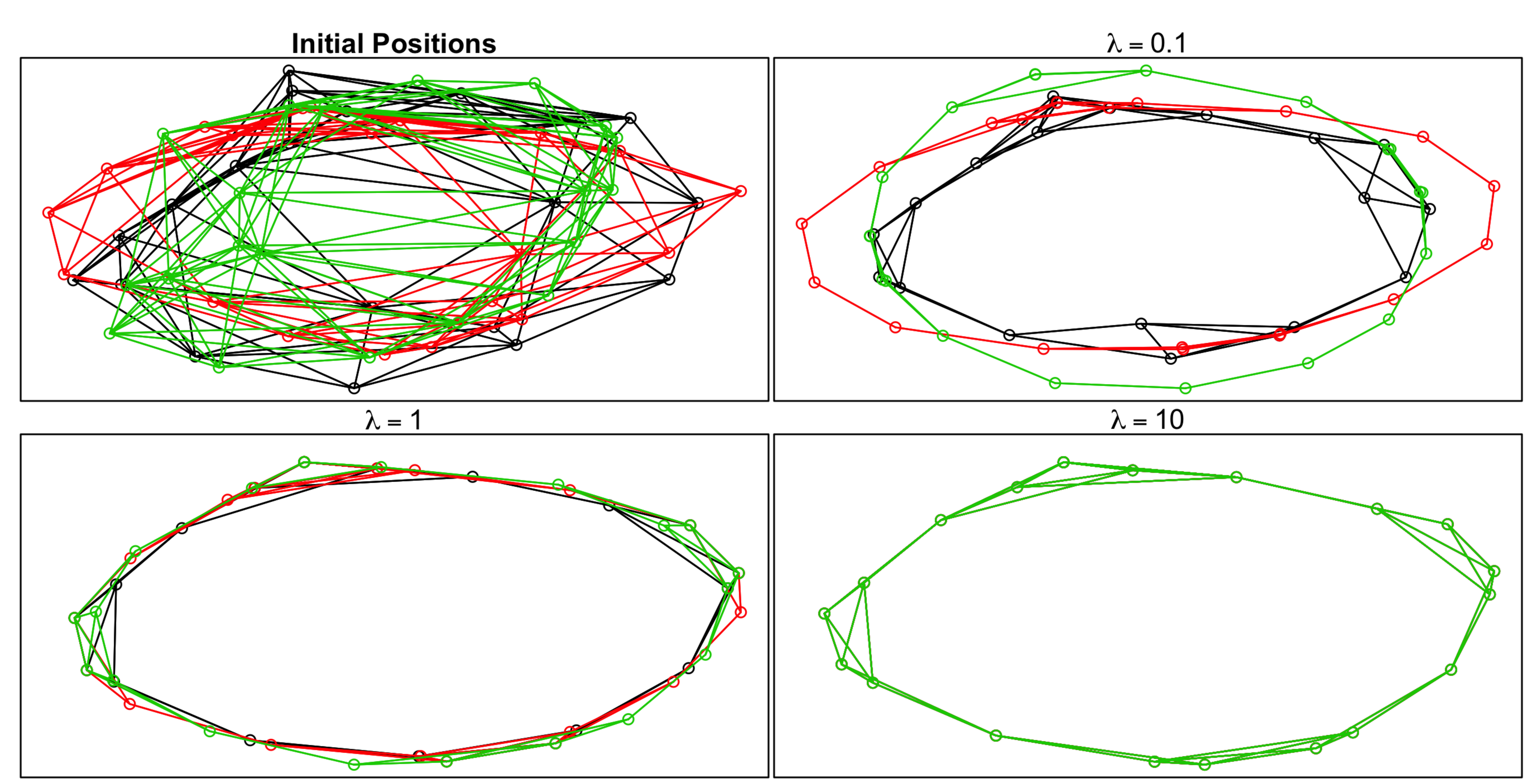

Figure 2: Initial and final positions using $\lambda = 1$

Objective Functions

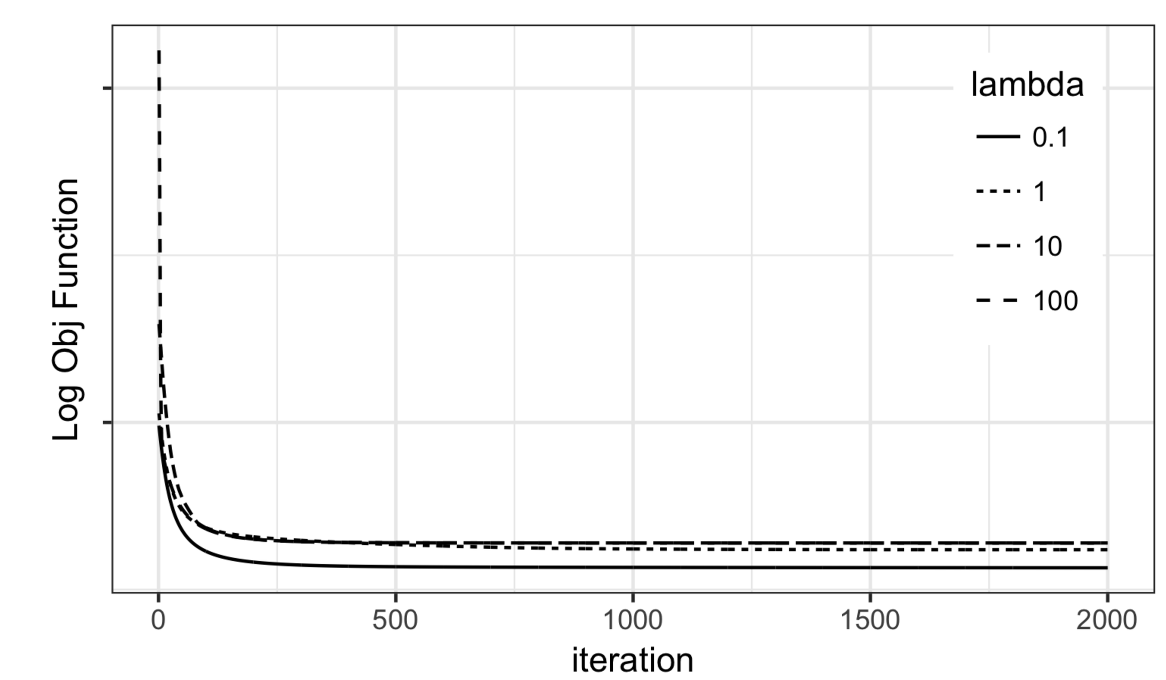

Figure 3: Objective function for different values of λ

Challenges

- Negative log-likelihood is not convex in the positions
  Fix: We started from carefully chosen initializations
- Distances between positions will yield same likelihood under translations and rotation
  Fix: Fit coordinate-wise, not allowing for spin, unlike the standard sampling approaches

Conclusion

We found that proximal gradient methods are an effective tool for refining an HLSM fit, given a good starting point.

References

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