Math for Hierarchical Latent Space Models

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General formulation:

$$
\begin{aligned}
\ell &= -\sum_k \sum_{i<j} \left[y_{ijk} \ln \sigma_{ijk} + (1 - y_{ijk}) \ln(1 - \sigma_{ijk}) \right] \\
\sigma_{ijk} &= \left(1 + \exp\left\{ -\alpha_k + \|z_{ik} - z_{jk}\|_2^2 \right\} \right)^{-1} \\
\frac{\partial \ell}{\partial z_{ik}} &= -\sum_k \sum_{ij} \left[\frac{y_{ijk}}{\sigma_{ijk}} \frac{\partial \sigma_{ijk}}{\partial z_{ik}} - \frac{1 - y_{ijk}}{1 - \sigma_{ijk}} \frac{\partial \sigma_{ijk}}{\partial z_{ik}} \right] \\
\frac{\partial \sigma_{ijk}}{\partial z_{ik}} &= \frac{\exp\left\{ \alpha_k - \|z_{ik} - z_{jk}\|_2^2 \right\}}{\left(1 + \exp\left\{ \alpha_k - \|z_{ik} - z_{jk}\|_2^2 \right\} \right)^2} (2 z_{ik} - z_{jk})
\end{aligned}
$$