EE351k: Homework 2

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0.1

$$
\begin{aligned}
\mathbb{P}(S_{dot}|R_{dot}) &= \frac{\mathbb{P}(R_{dot}|S_{dot}) * \mathbb{P}(S_{dot})}{\mathbb{P}(R_{dot}|S_{dot}) * \mathbb{P}(S_{dot}) + \mathbb{P}(R_{dot}|S_{dash}) * \mathbb{P}(S_{dash})} \\
&= \frac{\frac{5}{6} * \frac{2}{3}}{\frac{5}{6} * \frac{2}{3} + \frac{1}{5} * \frac{1}{3}} \\
&= \frac{25}{28}
\end{aligned}
$$

0.2

a. Second ball is magenta:
   Event $M_r = r^{th}$ urn, second ball magenta

$$
\begin{aligned}
\mathbb{P}(M_r) &= \frac{\binom{r-1}{1}}{\binom{n-1}{1}} * \frac{\binom{n-r}{1}}{\binom{n-2}{1}} + \frac{\binom{n-r}{1}}{\binom{n-1}{1}} * \frac{\binom{n-r-1}{1}}{\binom{n-2}{1}} \\
&= \frac{(r-1)(n-r) + (n-r)(n-r-1)}{(n-1)(n-2)} \\
&= \frac{(n-r)((r-1) + (n-r-1))}{(n-1)(n-2)} \\
&= \frac{(n-r)}{(n-1)}
\end{aligned}
$$

   Event A = random urn, second ball magenta

$$
\begin{aligned}
\mathbb{P}(A) &= \frac{1}{n} \sum_{r=1}^{n} \mathbb{P}(M_r) \\
&= \frac{1}{n} \sum_{r=1}^{n} \frac{(n-r)}{(n-1)} \\
&= \frac{1}{n} * \frac{n}{2} \\
&= \frac{1}{2}
\end{aligned}
$$

b. Second ball is magenta given first is magenta:
Event $M_r = r^{th}$ urn, second ball magenta

$$\mathbb{P}(M_r) = \frac{(n-r)}{(n-1)}$$

Event $Q_r = r^{th}$ urn, first ball magenta

$$\begin{aligned}\mathbb{P}(Q_r) &= \frac{\binom{n-r}{1}}{\binom{n-1}{1}} \\ &= \frac{(n-r)}{(n-1)}\end{aligned}$$

Event $R_r = r^{th}$ urn, second ball magenta given first ball magenta

$$\begin{aligned}\mathbb{P}(A) = \mathbb{P}(M_r|Q_r) &= \frac{\mathbb{P}(M_r \cap Q_r)}{\mathbb{P}(Q_r)} \\ &= \frac{\frac{n-r}{n-1} * \frac{n-r-1}{n-2}}{\frac{n-r}{n-1}} \\ &= \frac{n-r-1}{n-2}\end{aligned}$$

Event A = random urn, second ball magenta given first ball magenta

$$\begin{aligned}\mathbb{P}(A) &= \frac{1}{n}\sum_{r=1}^{n}\mathbb{P}(R_r) \\ &= \sum_{r=1}^{n}\frac{n-r-1}{n^2-2n} \\ &= \frac{n-3}{2(n-2)}\end{aligned}$$

0.3

0.4

0.5

0.6