Homework No. 3 (Due on 02/13/14)

Problem 1. Telegraphic signals 'dot' and 'dash' are sent in the proportion 2 : 3. Owing to conditions causing very erratic transmission, a dot becomes a dash with probability $\frac{1}{6}$, whereas a dash becomes a dot with probability $\frac{1}{5}$. If a dot is received what is the probability that it was sent as a dot?

Problem 2. There are n urns of which the r^{th} contains $r-1$ red balls and $n-r$ magenta balls. You pick up an urn at random and remove two balls at random without replacement. Find the probability that:

(a) the second ball is magenta;

(b) the second ball is magenta, given that the first is magenta.

Problem 3. From a lot containing 25 items, 5 of which are defective, 4 are chosen at random. Let X be the number of defectives found. Find the probability mass function of X.

Problem 4. We have three boxes. Box I contains 1,000 components 1 out of 8 of which are defective. Box II contains 800 components 3 out of 8 of which are defective, and Box III contains 3,200 components 1 out of 4 of which are defective. We first select at *random* one of the boxes and then remove at *random* a single component.

(a) What is the probability that the selected component is defective.

(b) We examine the component selected above and we find it defective. On the basis of this evidence, determine the probability that it came from Box II.

Problem 5. Three players A, B and C take turns in tossing (repeatedly) a biased coin having $\mathbb{P}(\text{Heads}) = p$, $p \in (0, 1)$. The first to get heads wins the game. Suppose A goes first, B second and C last. What is the probability that A wins? What is the probability that B wins?

Problem 6. Suppose n balls are distributed at random into r boxes. Find the probability that there are exactly k balls in the first r_1 boxes (here $r_1 < r$).