EE351k: Homework 2

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February 12, 2014

0.1

$$\begin{aligned}
\mathbb{P}(S_{dot}|R_{dot}) &= \frac{\mathbb{P}(R_{dot}|S_{dot}) * \mathbb{P}(S_{dot})}{\mathbb{P}(R_{dot}|S_{dot}) * \mathbb{P}(S_{dot}) + \mathbb{P}(R_{dot}|S_{dash}) * \mathbb{P}(S_{dash})} \\
&= \frac{\frac{5}{6} * \frac{2}{3}}{\frac{5}{6} * \frac{2}{3} + \frac{1}{5} * \frac{1}{3}} = \frac{25}{28}
\end{aligned}$$

0.2

a. Second ball is magenta:
Event $M_r = r^{th}$ urn, second ball magenta

$$\begin{aligned}
\mathbb{P}(M_r) &= \frac{\binom{r-1}{1}}{\binom{n-1}{1}} * \frac{\binom{n-r}{1}}{\binom{n-2}{1}} + \frac{\binom{n-r}{1}}{\binom{n-1}{1}} * \frac{\binom{n-r-1}{1}}{\binom{n-2}{1}} \\
&= \frac{(r-1)(n-r) + (n-r)(n-r-1)}{(n-1)(n-2)} \\
&= \frac{(n-r)((r-1) + (n-r-1))}{(n-1)(n-2)} \\
&= \frac{(n-r)}{(n-1)}
\end{aligned}$$

Event A = random urn, second ball magenta

$$\begin{aligned}
\mathbb{P}(A) &= \frac{1}{n}\sum_{r=1}^{n} \mathbb{P}(M_r) \\
&= \frac{1}{n}\sum_{r=1}^{n} \frac{(n-r)}{(n-1)} \\
&= \frac{1}{n} * \frac{n}{2} \\
&= \frac{1}{2}
\end{aligned}$$

b. Second ball is magenta given first is magenta:
Event $M_r = r^{th}$ urn, second ball magenta

$$\mathbb{P}(M_r) = \frac{(n-r)}{(n-1)}$$

Event $Q_r = r^{th}$ urn, first ball magenta

$$\begin{aligned}\mathbb{P}(Q_r) &= \frac{\binom{n-r}{1}}{\binom{n-1}{1}} \\ &= \frac{(n-r)}{(n-1)}\end{aligned}$$

Event $R_r = r^{th}$ urn, second ball magenta given first ball magenta

$$\begin{aligned}\mathbb{P}(A) = \mathbb{P}(M_r|Q_r) &= \frac{\mathbb{P}(M_r \cap Q_r)}{\mathbb{P}(Q_r)} \\ &= \frac{\frac{n-r}{n-1} * \frac{n-r-1}{n-2}}{\frac{n-r}{n-1}} \\ &= \frac{n-r-1}{n-2}\end{aligned}$$

Event A = random urn, second ball magenta given first ball magenta

$$\begin{aligned}\mathbb{P}(A) &= \frac{1}{n}\sum_{r=1}^{n} \mathbb{P}(R_r) \\ &= \sum_{r=1}^{n} \frac{n-r-1}{n^2-2n} \\ &= \frac{n-3}{2(n-2)}\end{aligned}$$

0.3

$\mathbb{P}(X=0) = \frac{\binom{5}{0}\binom{20}{4}}{\binom{25}{4}} = 0.3830$

$\mathbb{P}(X=1) = \frac{\binom{5}{1}\binom{20}{3}}{\binom{25}{4}} = 0.4506$

$\mathbb{P}(X=2) = \frac{\binom{5}{2}\binom{20}{2}}{\binom{25}{4}} = 0.1502$

$\mathbb{P}(X=3) = \frac{\binom{5}{3}\binom{20}{1}}{\binom{25}{4}} = 0.0158$

$\mathbb{P}(X=4) = \frac{\binom{5}{4}\binom{20}{0}}{\binom{25}{4}} = 0.0004$

0.4

a. $\mathbb{P}(D) = \mathbb{P}(D \cap B_1) + \mathbb{P}(D \cap B_2) + \mathbb{P}(D \cap B_3) = \frac{1}{3} * (\frac{1}{8} + \frac{3}{8} + \frac{2}{8}) = \frac{1}{4}$

b. $\mathbb{P}(B_2|D) = \dfrac{\mathbb{P}(B_2) * \mathbb{P}(D|B_2)}{\mathbb{P}(D)} = \dfrac{\frac{1}{3} * \frac{3}{8}}{\frac{1}{4}} = \frac{1}{2}$

0.5

$q := 1 - p$

a. $\mathbb{P}(A) = \sum\limits_{n=0}^{\infty} p * q^{3n} = p * \sum\limits_{n=0}^{\infty} q^{3^n} = \dfrac{p}{1 - q^3}$

b. $\mathbb{P}(B) = \sum\limits_{n=0}^{\infty} p * q^{3n+1} = p * q * \sum\limits_{n=0}^{\infty} q^{3^n} = \dfrac{p * q}{1 - q^3}$

0.6

X_{r_1} is the number of balls in r_1 boxes
$p_{X_{r_1}}(k) = \binom{n}{k}(\frac{r_1}{n})^k(1 - \frac{r_1}{n})^{n-k}$