EE351k: Homework 4

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0.1

$$\begin{aligned}
\mathbb{E}[X] = \mathbb{E}[Y] &= 0 \\
\sigma_X^2 = \sigma_Y^2 &= 1 \\
\mathbb{E}[X^2] = \mathbb{E}[X^2] &= 1 \\
\mathbb{E}[XY]^2 &\leq \mathbb{E}[X]^2\mathbb{E}[Y]^2 \\
p^2 = \mathbb{E}[XY]^2 &\leq 0 \\
1 + \sqrt{1-p^2} &\geq 2 \\
\mathbb{E}[max\{X^2, Y^2\}] &= \mathbb{E}[\frac{1}{2}(X^2+Y^2) + \frac{1}{2}\mathbb{E}[\ |X^2 - Y^2|\ ] \\
&= \frac{\mathbb{E}[X^2+Y^2] + \mathbb{E}[\ |X^2 - Y^2|\ ]}{2} \\
&= 1 + \frac{\mathbb{E}[\ |X^2 - Y^2|\ ]}{2} \\
&\leq 2 \leq 1 + \sqrt{1-p^2}
\end{aligned}$$

0.2

4	$\frac{1}{1296}$	0	0	0	0
3	$\frac{1}{81}$	$\frac{1}{324}$	0	0	0
2	$\frac{2}{27}$	$\frac{1}{27}$	$\frac{1}{216}$	0	0
1	$\frac{16}{81}$	$\frac{8}{27}$	$\frac{1}{27}$	$\frac{1}{324}$	0
0	$\frac{8}{243}$	$\frac{16}{81}$	$\frac{2}{27}$	$\frac{1}{81}$	$\frac{1}{1296}$
	0	1	2	3	4

0.3

$p_X(k) = p_Y(k) = \begin{cases} (p-1) & \text{if } k = 0 \\ p & \text{if } k = 1 \end{cases}$

$U = X + Y; \qquad V = |X - Y|$

$E[U] = 0 * 1/4 + 1 * 1/2 + 2 * 1/4 = 1$

$E[V] = 0 * 1/2 + 1 * 1/2 = 1/2$

$E[UV] = 0 * 1/2 + 1 * 1/2 = 1/2$

$cov(U, V) = \mathbb{E}[(U - E[U])(V - E[V])] = \mathbb{E}[UV - 1/2U - V + 1/2]$
$= 1/2 - 1/2 * 1 - 1/2 + 1/2 = 0 \implies U$ and V are uncorrelated

$\mathbb{P}(U = 0|V = 1) = \mathbb{P}(X + Y = 0|abs(X - Y) = 1) = 0$
$\mathbb{P}(U = 0) = 1/4$

Thus U and V are dependent and uncorrelated

0.4

0.5

$Z = X + Y$
$p_X(k) = p_Y(k) = pq^k$
$p_Z(k) = p_X(k) * p_Y(k) = p^2q^{2k}$
$\mathbb{P}(X = k|Z = n) = \frac{p_{X,Z}(x,z)}{p_X(x)}$
$p_{X,Z}(x, y) = p_X(x)p_{Z|X}(z|x) = p_X(x)\mathbb{P}(Z = z|X = x)$
... I ran out of time, sorry.

0.6

Yes.

0.7

$\mathbb{E}[H] = \mathbb{E}[\mathbb{E}[H|D]]$
$var[H] = \mathbb{E}[var(H|D)] + var(\mathbb{E}[H|D])$

$\mathbb{E}[H|D] = np = \mathbb{E}[D]/2 = 1/2 * (1 + 7)/2 = 7/4$
$var(H|D) = np(1 - p) = D/4$

$\mathbb{E}[H] = 7/4$
$var(H) = \mathbb{E}[D/4] + var(D/2) = 7/8 + (1/2)^2(1/12 * (6 - 1)^2) = 67/48$