EE351k: Homework 2

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0.1

a. $\mathbb{P}(two\ kings,\ one\ ace) = \dfrac{\binom{4}{2}\binom{4}{1}11!}{\binom{52}{13}}$

b. $\mathbb{P}(one\ ace|two\ kings) = \dfrac{\binom{4}{2}\binom{4}{1}11!}{\binom{4}{2}12!}$

0.2

Counting problem:

$\#\ of\ 1\ color\ only = 3$
$\#\ of\ 2\ colors = 3 * 2^k$
$\#\ of\ total\ combinations = 3^k$
$\mathbb{P}(not\ all\ colors) = \dfrac{3 + 3 * 2^k}{3^k}$

0.3

a. $\mathbb{P} = \dfrac{\binom{15}{6}}{\binom{25+15+35}{6}}$

b. $\mathbb{P} = \dfrac{\binom{25}{2}\binom{15}{3}\binom{35}{1}}{\binom{25+15+35}{6}}$

0.4

Conditional Probability: $\mathbb{P}(A|B) = \dfrac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|A^c)\mathbb{P}(A^c)}$

$\mathbb{P}(D|T) = \dfrac{0.98 * 0.6}{0.98 * 0.6 + 0.02 * 0.4} = 0.9866$

0.5

$\mathbb{P}(Four\ heads \cap Dice_4) = \mathbb{P}(Four\ heads)\mathbb{P}(Dice_4) = \dfrac{1}{96}$

0.6

$$\mathbb{P}(C|A\cap B) = \mathbb{P}(C|B)$$
$$\frac{\mathbb{P}(C\cap A\cap B)}{\mathbb{P}(A\cap B)} = \frac{\mathbb{P}(C\cap B)}{\mathbb{P}(B)}$$
$$\frac{\mathbb{P}(C\cap A\cap B)}{\mathbb{P}(C\cap B)} = \frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}$$
$$\mathbb{P}(A|B\cap C) = \mathbb{P}(A|B)$$

0.7

Proof by induction:

$n = 0 \implies \mathbb{P} = \frac{1}{2}[1 + \frac{2}{3}^0] = 1$

Assuming n case, attempt n+1 case

If the previous n were even, the probability that n+1 is also even...

$\mathbb{P} = \frac{5}{6}\frac{1}{2}[1 + \frac{2}{3}^n] = \frac{1}{2}[1 + \frac{2}{3}^{n+1}]$

If the previous n were odd, the probability that n+1 is even...

$\mathbb{P} = \frac{1}{6}(1 - \frac{1}{2}[1 + \frac{2}{3}^n]) = \frac{1}{2}[1 + \frac{2}{3}^{n+1}]$

0.8

a. $\frac{1}{2}^n$

b. $\binom{n}{n/2}\frac{1}{2^n}$

c. $\binom{n}{2}\frac{1}{2^n}$

d. $\sum_{i=2}^{n}\binom{n}{i}\frac{1}{2^n}$