EE351k: Homework 1

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0.1

a. $A \cup B \cup C$

b. $(A \cup B \cup C) \setminus (A \triangle B \triangle C)$

c. $(A \triangle B \triangle C)^c$

d. $(A \cap B \cap C \cap)^c$

0.2

Model: $\Omega = \{1, 2, 3, 4, 5, 6\}$; $\mathfrak{F} = \{1, 2, 3, 4, 5, 6\}$
$\mathbb{P}(Even) = 2 * \mathbb{P}(Odd)$
$\mathbb{P}(\{1\}) = \mathbb{P}(\{3\}) = \mathbb{P}(\{5\}) = \frac{1}{9}; \mathbb{P}(\{1\}) = \mathbb{P}(\{2\}) = \mathbb{P}(\{4\}) = \frac{2}{9}$

$\mathbb{P}(\{1, 2, 3\}) = \mathbb{P}(\{1\}) + \mathbb{P}(\{2\}) + \mathbb{P}(\{3\}) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$

0.3

a. $\mathbb{P}(A^c \cup B^c) = \mathbb{P}((A \cup B)^c) = 1 - \mathbb{P}((A \cup B)) = z$

b. $\mathbb{P}(A^c \cap B) = \mathbb{P}(A \setminus B) = \mathbb{P}(A) - \mathbb{P}(B) = x - y$

c. $\mathbb{P}(A^c \cup B) = \mathbb{P}((A \cap B^c)^c) = 1 - (\mathbb{P}(B) - \mathbb{P}(A)) = 1 - y + x$

d. $\mathbb{P}(A^c \cap B^c) = \mathbb{P}(A^c) - \mathbb{P}(B) + \mathbb{P}(A \cap B) = x - y + z$

0.4

Probability that at least one of A, B, and C occurs = $\mathbb{P}(A \cup B \cup C)$
$\mathbb{P}(A \cap B) = \mathbb{P}(B \cap C) = \emptyset \implies$ the pairs {A, B} and {B, C} are disjoint
$\mathbb{P}((A \cup C) \cup B) = (\mathbb{P}(A) + \mathbb{P}(C) - \mathbb{P}(A \cap C)) + \mathbb{P}(B) = \frac{5}{8}$

0.5

Conditional probablity $\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$

$Event\ AB = A \leftrightarrow B; Event\ CD = C \leftrightarrow D)$

$\mathbb{P}(AB) = \mathbb{P}(AB_1 \cup AB_2) = \mathbb{P}(AB_1) + \mathbb{P}(AB_2) - \mathbb{P}(AB_1 \cap AB_2) = 2p - p * p$

$AB \perp CD^c \implies \mathbb{P}(AB|CD^c) = \mathbb{P}(AB) = 2p - p^2$

$AB \perp AC^c \implies \mathbb{P}(AB|AC^c) = \mathbb{P}(AB) = 2p - p^2$

0.6

Given universe, $\Omega = \{OO, OE, EE, EO\}$

$A = \{OO, OE\}; B = \{EE, OE\}; C = \{OO, EE\}$

$\mathbb{P}(A) = \mathbb{P}(B) = \mathbb{P}(C) = \frac{1}{2}$

$\mathbb{P}(A \cap B) = \mathbb{P}(\{OE\}) = \frac{1}{4} \equiv \mathbb{P}(A) * \mathbb{P}(B) \implies A \perp B$

$\mathbb{P}(A \cap C) = \mathbb{P}(\{OO\}) = \frac{1}{4} \equiv \mathbb{P}(A) * \mathbb{P}(C) \implies A \perp C$

$\mathbb{P}(B \cap C) = \mathbb{P}(\{EE\}) = \frac{1}{4} \equiv \mathbb{P}(B) * \mathbb{P}(C) \implies B \perp C$

$\mathbb{P}(A \cap B \cap C) = \mathbb{P}(\emptyset) \not\equiv \mathbb{P}(A) * \mathbb{P}(B) * \mathbb{P}(C) \implies$ A, B, C are not independent

0.7

Probability of n keys, $\mathbb{P}(A_0 \cup A_1 \cup ... \cup A_n)$

Using the formula taken for granted from the homework, and the fact that each event is independent...

$$\mathbb{P}(no\ keys) = \sum_{i=1}^{n} (-1)^{(i+1)} * \frac{1}{n} * \binom{n}{i}$$

This value does tend to 0 as $n \to \infty$. It also appears that $\mathbb{P}(no\ keys) \equiv \dfrac{1}{n}$.