EE351k: Homework 2

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0.1

$$\begin{aligned}\mathbb{P}(S_{dot}|R_{dot}) &= \frac{\mathbb{P}(R_{dot}|S_{dot})\mathbb{P}(S_{dot})}{\mathbb{P}(R_{dot}|S_{dot})\mathbb{P}(S_{dot}) + \mathbb{P}(R_{dot}|S_{dash})\mathbb{P}(S_{dash})} \\ &= \frac{\frac{5}{6} * \frac{2}{3}}{\frac{5}{6} * \frac{2}{3} + \frac{1}{5} * \frac{1}{3}} \\ &= \frac{25}{28}\end{aligned}$$

0.2

Counting problem:

$\# \ of \ 1 \ color \ only = 3$

$\# \ of \ 2 \ colors = 3 * 2^k$

$\# \ of \ total \ combinations = 3^k$

$\mathbb{P}(not \ all \ colors) = \dfrac{3 + 3 * 2^k}{3^k}$

0.3

a. $\mathbb{P} = \dfrac{\binom{15}{6}}{\binom{25+15+35}{6}}$

b. $\mathbb{P} = \dfrac{\binom{25}{2}\binom{15}{3}\binom{35}{1}}{\binom{25+15+35}{6}}$

0.4

Conditional Probability: $\mathbb{P}(A|B) = \dfrac{\mathbb{P}(B|A)\mathbb{P}(A)}{\mathbb{P}(B|A)\mathbb{P}(A) + \mathbb{P}(B|A^c)\mathbb{P}(A^c)}$

$\mathbb{P}(D|T) = \dfrac{0.98 * 0.6}{0.98 * 0.6 + 0.02 * 0.4} = 0.9866$

0.5

$\mathbb{P}(Four \ heads \cap Dice_4) = \mathbb{P}(Four \ heads)\mathbb{P}(Dice_4) = \dfrac{1}{96}$

0.6

$$\mathbb{P}(C|A \cap B) = \mathbb{P}(C|B)$$
$$\frac{\mathbb{P}(C \cap A \cap B)}{\mathbb{P}(A \cap B)} = \frac{\mathbb{P}(C \cap B)}{\mathbb{P}(B)}$$
$$\frac{\mathbb{P}(C \cap A \cap B)}{\mathbb{P}(C \cap B)} = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$
$$\mathbb{P}(A|B \cap C) = \mathbb{P}(A|B)$$

0.7

Proof by induction:

$n = 0 \implies \mathbb{P} = \frac{1}{2}[1 + \frac{2}{3}^0] = 1$

Assuming n case, attempt n+1 case

If the previous n were even, the probability that n+1 is also even...

$\mathbb{P} = \frac{5}{6}\frac{1}{2}[1 + \frac{2}{3}^n] = \frac{1}{2}[1 + \frac{2}{3}^{n+1}]$

If the previous n were odd, the probability that n+1 is even...

$\mathbb{P} = \frac{1}{6}(1 - \frac{1}{2}[1 + \frac{2}{3}^n]) = \frac{1}{2}[1 + \frac{2}{3}^{n+1}]$

0.8

a. $\frac{1}{2}^n$

b. $\binom{n}{n/2}\frac{1}{2^n}$

c. $\binom{n}{2}\frac{1}{2^n}$

d. $\sum_{i=2}^{n}\binom{n}{i}\frac{1}{2^n}$