**Task 1: Quadratic Function Analysis**

**Scenario: Bungee Jumping from a Bridge**

Consider a bungee jumper leaping from a bridge, where the jumper’s height above the river surface is modeled by the function:

h(t)=−0.5t2+v0t+h0h(t) = -0.5t2 + v\_0t + h\_0h(t)=−0.5t2+v0​t+h0​

* **h**: height in meters
* **t**: time in seconds
* **v₀**: jumper’s initial velocity (m/s)
* **h₀**: initial height above the river (meters)

Given that the initial velocity, v0v\_0v0​, is 0 m/s, and the initial height, h0h\_0h0​, is 210 meters, the height function simplifies to:

h(t)=−0.5t2+210h(t) = -0.5t2 + 210h(t)=−0.5t2+210

**(i) Mathematical Analysis of the Bungee Jumping Scenario**

1. **Domain and Range of h(t)h(t)h(t)**
   1. **Domain**: The domain of h(t)h(t)h(t) represents the time during which the jumper is in motion. Since the jumper starts jumping at t=0t=0t=0 seconds and eventually hits the river (height = 0), we need to determine when h(t)=0h(t)=0h(t)=0.

Setting h(t)=0h(t)=0h(t)=0:

−0.5t2+210=0 ⟹ −0.5t2=−210 ⟹ t2=420 ⟹ t≈20.49 seconds-0.5t2 + 210 = 0 \implies -0.5t2 = -210 \implies t2 = 420 \implies t \approx 20.49 \text{ seconds}−0.5t2+210=0⟹−0.5t2=−210⟹t2=420⟹t≈20.49 seconds

Thus, the domain is: t∈[0,20.49]t \in [0, 20.49]t∈[0,20.49].

* + **Range**: The range of h(t)h(t)h(t) is determined by the highest and lowest points the jumper reaches. The initial height is 210 meters, and the lowest height is 0 meters (when the jumper hits the river).

Thus, the range is: h(t)∈[0,210]h(t) \in [0, 210]h(t)∈[0,210].

* + **Physical Significance**:
    1. The **domain** represents the time interval during which the jumper is in the air.
    2. The **range** represents the heights above the river that the jumper reaches, from the initial height of 210 meters to 0 meters (river surface).

1. **Vertex of h(t)h(t)h(t)**

The vertex of the quadratic function h(t)=−0.5t2+210h(t) = -0.5t2 + 210h(t)=−0.5t2+210 can be found using the formula for the time of the vertex:

t=−b2at = -\frac{b}{2a}t=−2ab​

where a=−0.5a = -0.5a=−0.5 and b=0b = 0b=0. Plugging in the values:

t=−02(−0.5)=0t = -\frac{0}{2(-0.5)} = 0t=−2(−0.5)0​=0

The height at t=0t=0t=0 is h(0)=210h(0)=210h(0)=210. Thus, the vertex is (0,210)(0, 210)(0,210).

* + **Physical Interpretation**: The vertex represents the initial position of the jumper at the maximum height above the river. The height decreases as the jumper falls.

1. **Maximum Height and Time of Occurrence**

Since the maximum height occurs at the vertex, the maximum height is 210 meters, occurring at t=0t=0t=0 seconds.

1. **Time When the Jumper Reaches a Height of 11 Meters**

We solve h(t)=11h(t)=11h(t)=11:

−0.5t2+210=11 ⟹ −0.5t2=11−210 ⟹ −0.5t2=−199 ⟹ t2=1990.5=398 ⟹ t≈19.95 seconds-0.5t2 + 210 = 11 \implies -0.5t2 = 11 - 210 \implies -0.5t2 = -199 \implies t2 = \frac{199}{0.5} = 398 \implies t \approx 19.95 \text{ seconds}−0.5t2+210=11⟹−0.5t2=11−210⟹−0.5t2=−199⟹t2=0.5199​=398⟹t≈19.95 seconds

Therefore, the jumper reaches the height of 11 meters at approximately 19.95 seconds.

1. **Height After 20 Seconds**

Substituting t=20t=20t=20 in h(t)h(t)h(t):

h(20)=−0.5(20)2+210=−0.5(400)+210=−200+210=10 metersh(20) = -0.5(20)2 + 210 = -0.5(400) + 210 = -200 + 210 = 10 \text{ meters}h(20)=−0.5(20)2+210=−0.5(400)+210=−200+210=10 meters

After 20 seconds, the jumper is 10 meters above the river. This indicates that the jumper is very close to the river surface.

1. **Time When the Jumper Touches the River**

Solving h(t)=0h(t)=0h(t)=0:

−0.5t2+210=0 ⟹ t2=420 ⟹ t≈20.49 seconds-0.5t2 + 210 = 0 \implies t2 = 420 \implies t \approx 20.49 \text{ seconds}−0.5t2+210=0⟹t2=420⟹t≈20.49 seconds

The jumper touches the river at approximately 20.49 seconds.

**(ii) Graphical Analysis of the Bungee Jumping Scenario**

1. **Graph of h(t)=−0.5t2+210h(t) = -0.5t2 + 210h(t)=−0.5t2+210**

We will use GeoGebra or a graphing tool to plot the quadratic function. The graph should display a parabola opening downwards, starting at (0,210)(0, 210)(0,210) and touching the ttt-axis at t≈20.49t \approx 20.49t≈20.49.

1. **Time Intervals of Increasing and Decreasing Height**
   1. **Increasing Interval**: None, as the initial velocity is zero, and the height decreases immediately.
   2. **Decreasing Interval**: t∈(0,20.49]t \in (0, 20.49]t∈(0,20.49].
2. **Axis of Symmetry**

The axis of symmetry for the quadratic function is t=0t=0t=0. This line represents the point where the height is at its maximum. Beyond this point, the jumper’s height decreases symmetrically as time progresses.

1. **Intercepts**
   1. **ttt-Intercept**: The ttt-intercept occurs when h(t)=0h(t)=0h(t)=0. As calculated, t≈20.49t \approx 20.49t≈20.49 seconds.
   2. **hhh-Intercept**: The hhh-intercept occurs when t=0t=0t=0. At this point, h(0)=210h(0)=210h(0)=210 meters.
   3. **Physical Interpretation**:
      1. The ttt-intercept represents the time when the jumper touches the river.
      2. The hhh-intercept represents the initial height from which the jumper started.

**Task 2: Optimal Route Planning and Traffic Flow Analysis (continued)**

**(v) Visual Infrastructure Mapping**

To create a graphical map of the road alignment, we can use tools like GeoGebra to plot the line defined by the equation:

y=−2x+17y = -2x + 17y=−2x+17

We will also plot the parallel and perpendicular routes, represented by the following equations:

* **Parallel Route**: y=−2x+cy = -2x + cy=−2x+c, where ccc is a chosen intercept to distinguish this route from the original.
* **Perpendicular Route**: y=12x+dy = \frac{1}{2}x + dy=21​x+d, where ddd is selected to position the perpendicular line relative to points A and B.

The graph should include the following elements:

1. **Road Between Points A and B**: Highlighted in a distinct color to signify the main route.
2. **Parallel Routes**: Two parallel lines, one on each side of the main road, spaced evenly for additional lanes or service roads.
3. **Perpendicular Routes**: Perpendicular lines that connect to various parts of the main road to depict cross-sections or intersections.

We can enhance the map by labeling key intersections and adding elements such as signal points or speed breakers for comprehensive analysis.

**Task 3: Mathematical Properties of a Quadratic Function**

Given the general form of a quadratic function:

f(x)=ax2+bx+cf(x) = ax2 + bx + cf(x)=ax2+bx+c

**(i) Determining Key Characteristics**

1. **Vertex**: The vertex of f(x)f(x)f(x) can be calculated as:

x=−b2ax = -\frac{b}{2a}x=−2ab​

Substituting this value back into f(x)f(x)f(x) gives us the yyy-coordinate of the vertex.

1. **Axis of Symmetry**: The line of symmetry is defined by x=−b2ax = -\frac{b}{2a}x=−2ab​.
2. **Maximum/Minimum Value**: If a>0a > 0a>0, the vertex represents the minimum value of f(x)f(x)f(x); if a<0a < 0a<0, it represents the maximum value.
3. **Increasing/Decreasing Intervals**: Depending on aaa, the function is:
   1. **Increasing** for x>−b2ax > -\frac{b}{2a}x>−2ab​.
   2. **Decreasing** for x<−b2ax < -\frac{b}{2a}x<−2ab​.
4. **Intercepts**: Setting f(x)=0f(x) = 0f(x)=0 allows us to find the xxx-intercepts, and evaluating f(0)f(0)f(0) gives the yyy-intercept.

**(ii) Application: Revenue Optimization**

Let’s consider a quadratic revenue function for a company, given by:

R(x)=−5x2+500x−2000R(x) = -5x2 + 500x - 2000R(x)=−5x2+500x−2000

where:

* **R(x)R(x)R(x)**: Revenue in dollars
* **xxx**: Number of units sold

We want to determine the following:

1. **Maximum Revenue and Number of Units Sold**
   1. **Vertex Calculation**: The maximum revenue occurs at the vertex, which we find as:

x=−5002(−5)=50 unitsx = -\frac{500}{2(-5)} = 50 \text{ units}x=−2(−5)500​=50 units

Substituting x=50x = 50x=50 back into R(x)R(x)R(x):

R(50)=−5(50)2+500(50)−2000=−12500+25000−2000=10500 dollarsR(50) = -5(50)2 + 500(50) - 2000 = -12500 + 25000 - 2000 = 10500 \text{ dollars}R(50)=−5(50)2+500(50)−2000=−12500+25000−2000=10500 dollars

Therefore, the maximum revenue is $10,500 when 50 units are sold.

1. **Revenue When 30 Units Are Sold**

R(30)=−5(30)2+500(30)−2000=−4500+15000−2000=8500 dollarsR(30) = -5(30)2 + 500(30) - 2000 = -4500 + 15000 - 2000 = 8500 \text{ dollars}R(30)=−5(30)2+500(30)−2000=−4500+15000−2000=8500 dollars

1. **Break-Even Points (x-Intercepts)**

To find the break-even points, we set R(x)=0R(x) = 0R(x)=0:

−5x2+500x−2000=0-5x2 + 500x - 2000 = 0−5x2+500x−2000=0

Dividing through by -5:

x2−100x+400=0x2 - 100x + 400 = 0x2−100x+400=0

Solving this quadratic equation using the quadratic formula:

x=−(−100)±(−100)2−4(1)(400)2(1)x = \frac{-(-100) \pm \sqrt{(-100)2 - 4(1)(400)}}{2(1)}x=2(1)−(−100)±(−100)2−4(1)(400)​​

Simplifying:

x=100±10000−16002=100±84002≈100±91.652x = \frac{100 \pm \sqrt{10000 - 1600}}{2} = \frac{100 \pm \sqrt{8400}}{2} \approx \frac{100 \pm 91.65}{2}x=2100±10000−1600​​=2100±8400​​≈2100±91.65​

The solutions are:

x≈100+91.652=95.82andx≈100−91.652=4.18x \approx \frac{100 + 91.65}{2} = 95.82 \quad \text{and} \quad x \approx \frac{100 - 91.65}{2} = 4.18x≈2100+91.65​=95.82andx≈2100−91.65​=4.18

Thus, the break-even points are approximately 4.18 units and 95.82 units sold.

**(iii) Interpretation of Results**

* The maximum revenue of $10,500 is achieved when 50 units are sold.
* The revenue at 30 units sold is $8,500, indicating that the company is still profitable at this level.
* The break-even points suggest that the company breaks even when selling between 4.18 and 95.82 units. Outside of this range, the company experiences losses.

**Conclusion**

Through the above analysis, we have:

1. Mathematically and graphically analyzed the bungee jumping scenario using a quadratic function to determine key features such as maximum height and time of descent.
2. Applied linear function analysis for optimal route planning, calculating the slope and creating parallel and perpendicular lines for infrastructure planning.
3. Explored the revenue optimization problem using a quadratic function to determine the maximum revenue and break-even points, providing actionable insights for business strategy.

This approach exemplifies how mathematical models can be effectively used to interpret real-world situations, allowing for data-driven decision-making in various fields, from engineering to business analytics.