Stochastic Processes
1401-2
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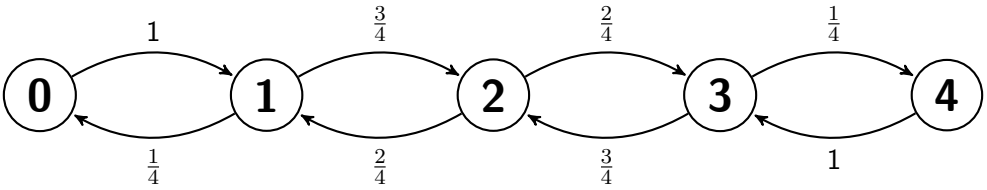
Homework 2

Let X_n be the number of balls in urn A at time n in the Ehrenfest chain model with $N = 4$ balls. The transition matrix P is given by

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{4} & 0 & \frac{2}{4} & 0 \\ 0 & 0 & \frac{3}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$



1. **Show that $\lim_{n\to\infty} P^n$ exists and has the same row values, which are the stationary distribution of the chain.**

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} = SJS^{-1}, J = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -1 & 1/2 & 0 & -1/2 & 1 \\ 1 & 0 & -1/3 & 0 & 1 \\ -1 & -1/2 & 0 & 1/2 & 1 \\ 1 & 1 & 1 & 1 &) \end{bmatrix}, S^{-1} = \begin{bmatrix} 1/16 & -1/4 & 3/8 & -1/4 & 1/16 \\ -1/4 & 1/2 & 0 & -1/2 & 1/4 \\ 3/8 & 0 & -3/4 & 0 & 3/8 \\ -1/4 & -1/2 & 0 & 1/2 & 1/4 \\ 1/16 & 1/4 & 3/8 & 1/4 & 1/16 \end{bmatrix}$$

$$P^n = SJ^nS^{-1} \to \lim_{n\to\infty} P^n = S \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} S^-1 = \begin{bmatrix} 1/16 & 1/4 & 3/8 & 1/4 & 1/16 \\ 1/16 & 1/4 & 3/8 & 1/4 & 1/16 \\ 1/16 & 1/4 & 3/8 & 1/4 & 1/16 \\ 1/16 & 1/4 & 3/8 & 1/4 & 1/16 \\ 1/16 & 1/4 & 3/8 & 1/4 & 1/16 \end{bmatrix}$$