Capacitance & Dielectrics

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Applied Physics

Capacitors

Capacitors—devices that store electric charge. Capacitors are commonly used in a variety of electric circuits. For instance, they are used to

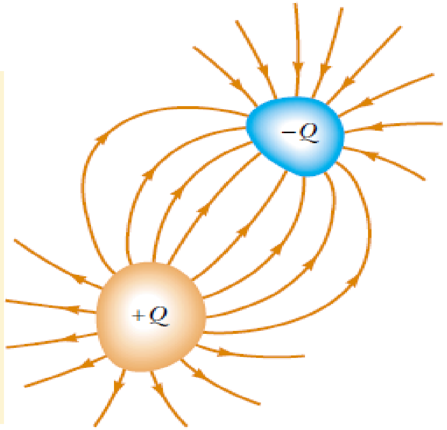
- tune the frequency of radio receivers,
- as filters in power supplies,
- to eliminate sparking in automobile ignition systems, and
- as energy-storing devices in electronic flash units.

Consider two conductors carrying charges of equal magnitude but of opposite sign, as shown in Figure. Such a combination of two conductors is called a capacitor. The conductors are called *plates.*

A ***potential difference*** *V exists between* the conductors due to the presence of the charges.

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$



- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

$$\rho \equiv \frac{Q}{V}$$

where ρ has units of coulombs per cubic meter (C/m^3).

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ (lowercase Greek sigma) is defined by

$$\sigma \equiv \frac{Q}{A}$$

where σ has units of coulombs per square meter (C/m^2).

- If a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density** λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

where λ has units of coulombs per meter (C/m).

CALCULATING CAPACITANCE

Parallel-Plate Capacitors

Two parallel metallic plates of equal area *A are separated by a distance d, as shown* in Fig.

One plate carries a charge -*Q, and the other carries a charge* +*Q.*

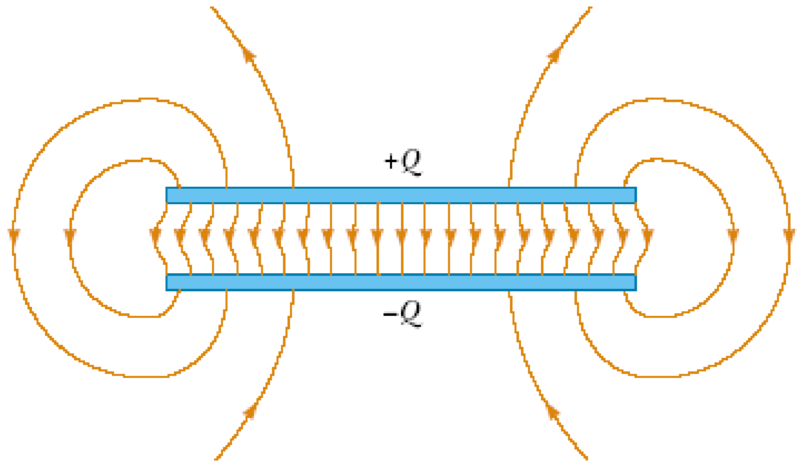
The value of the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}}$$



the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation,

The Cylindrical Capacitor

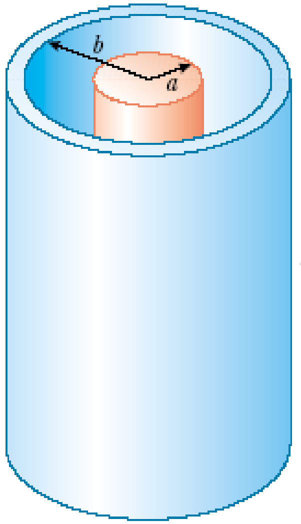
A solid cylindrical conductor of radius a and charge Q is coaxial with a cylindrical shell of negligible thickness, radius $b > a$, and charge $-Q$

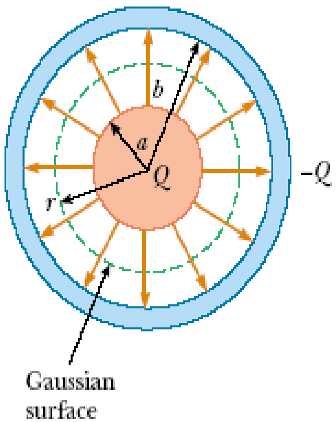
First calculate the potential difference between the two cylinders,

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s} \qquad E_r = 2k_e\lambda / r$$

$$V_b - V_a = -\int_a^b E_r\, dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{2k_e Q}{\ell} \ln\left(\frac{b}{a}\right)} = \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}$$





$$\frac{C}{\ell} = \frac{1}{2k_e \ln\left(\frac{b}{a}\right)}$$

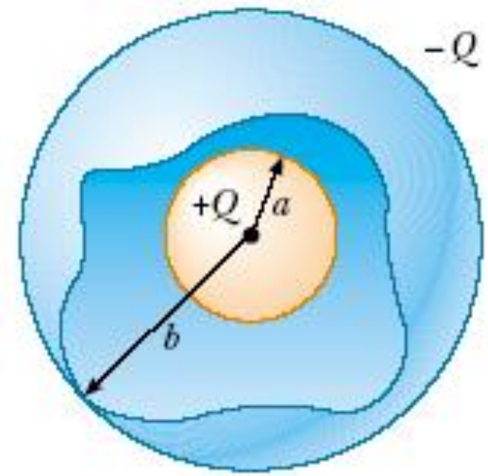
the capacitance per unit length

The Spherical Capacitor

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$

$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$



$$\Delta V = |V_b - V_a| = k_eQ\frac{(b-a)}{ab}$$

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

COMBINATIONS OF CAPACITORS

Parallel Combination

The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.

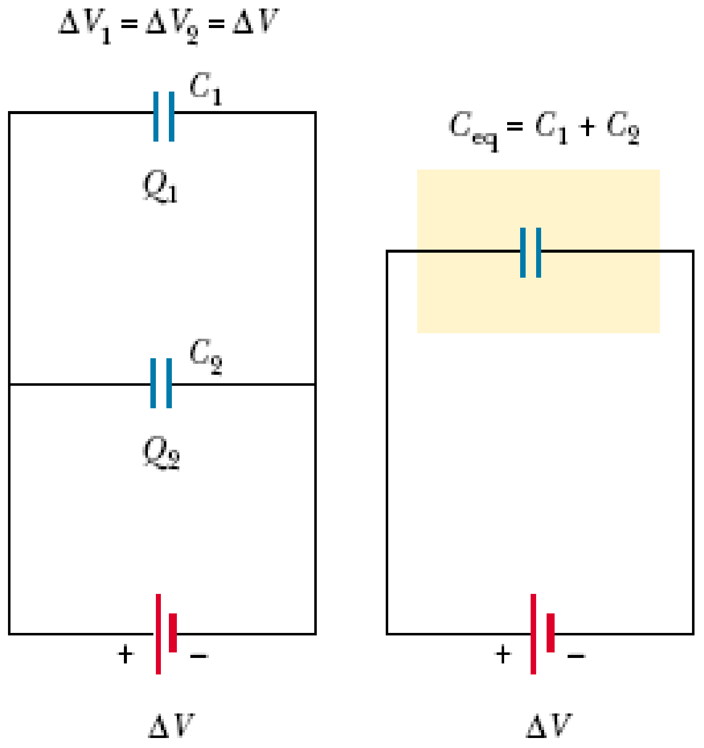
The *total charge* Q stored by the two capacitors is

$$Q = Q_1 + Q_2$$

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V \qquad Q = C_{eq} \Delta V$$

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \begin{pmatrix}\text{parallel}\\ \text{combination}\end{pmatrix}$$



$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

COMBINATIONS OF CAPACITORS

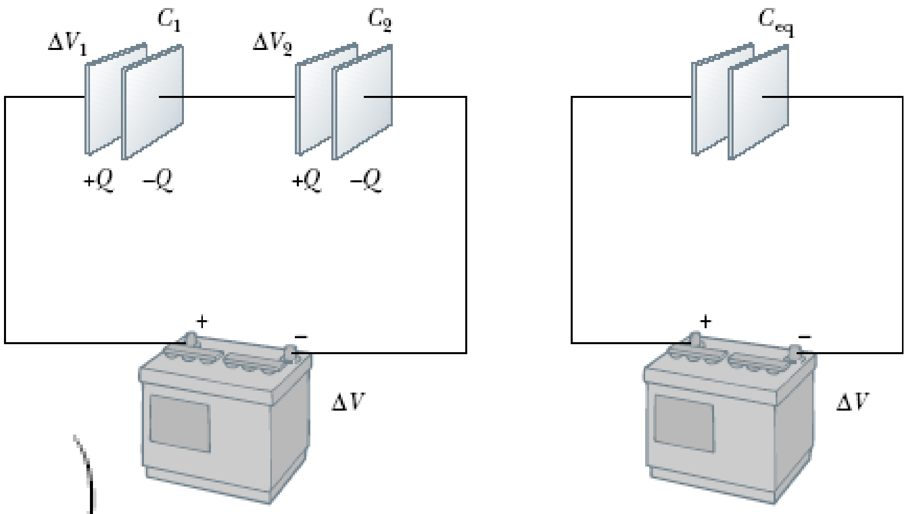
Series Combination

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2} \qquad \Delta V = \frac{Q}{C_{eq}}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right)$$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \left(\begin{array}{l}\text{series}\\ \text{combination}\end{array}\right)$$

This demonstrates that **the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.**

ENERGY STORED IN A CHARGED CAPACITOR

Suppose that q is the charge on the capacitor at some instant during the charging process. At the same instant, the potential difference across the capacitor is ΔV = q/C. The work necessary to transfer an increment of charge dq from the plate carrying charge -q to the plate carrying charge q (which is at the higher electric potential) is

$$dW = \Delta V\, dq = \frac{q}{C}\, dq$$

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C} = \tfrac{1}{2} Q\,\Delta V = \tfrac{1}{2} C(\Delta V)^2$$

Energy stored in a parallel-plate capacitor

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

energy per unit volume $u_E = U/V = U/Ad$,

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

Energy density in an electric field

energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

CAPACITORS WITH DIELECTRICS

A dielectric is a non conducting material, such as rubber, glass, or waxed paper.

When a dielectric is inserted between the plates of a capacitor, the capacitance increases.

If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor κ, which is called the dielectric constant.

$$\Delta V = \frac{\Delta V_0}{\kappa} \qquad C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa \frac{\epsilon_0 A}{d}$$

$$\boxed{C = \kappa C_0}$$

advantages:

- Increase in capacitance
- Increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

Types of Capacitors

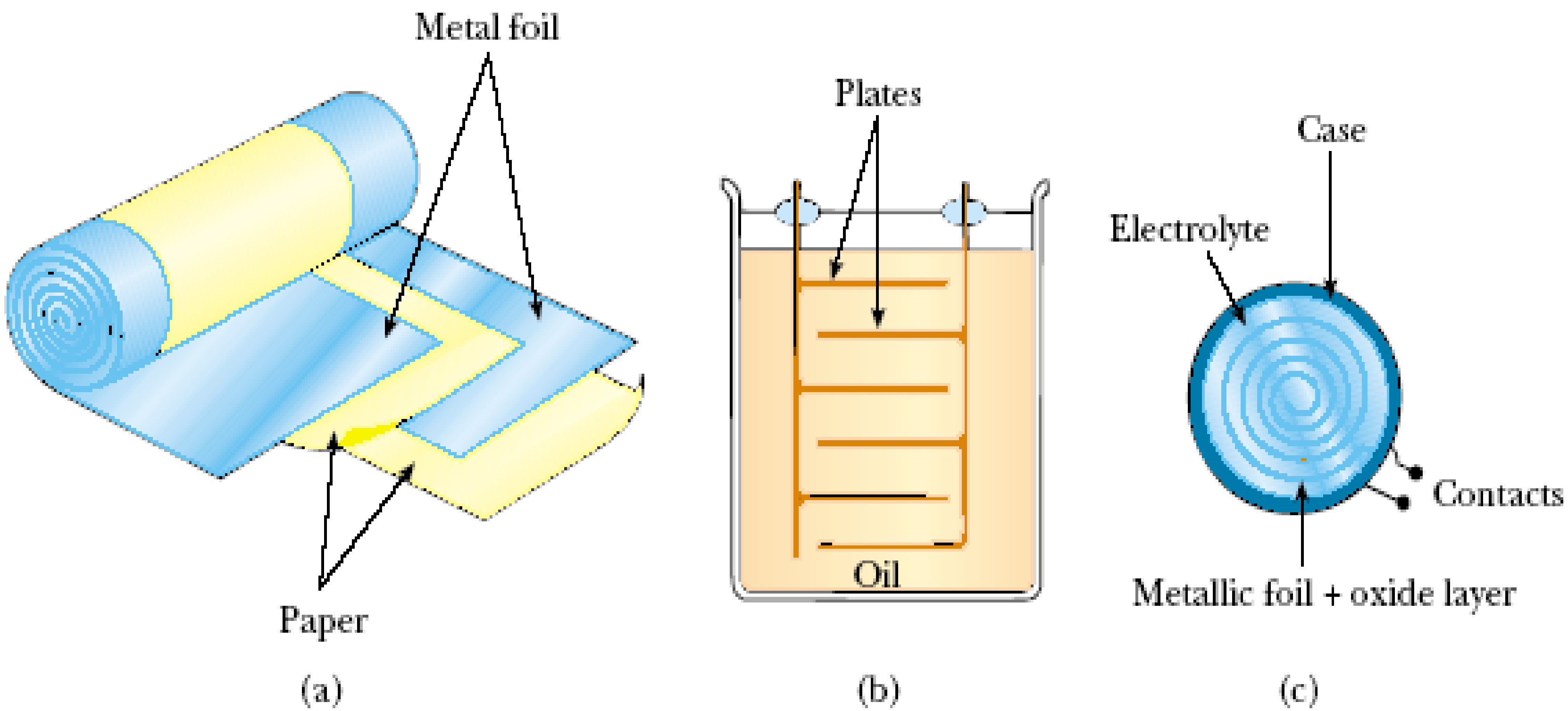


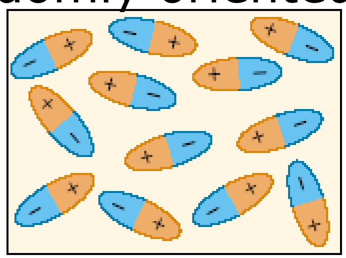
Figure 26.15 Three commercial capacitor designs. (a) A tubular capacitor, whose plates are separated by paper and then rolled into a cylinder. (b) A high-voltage capacitor consisting of many parallel plates separated by insulating oil. (c) An electrolytic capacitor.

AN ATOMIC DESCRIPTION OF DIELECTRICS

the field in the presence of a dielectric is

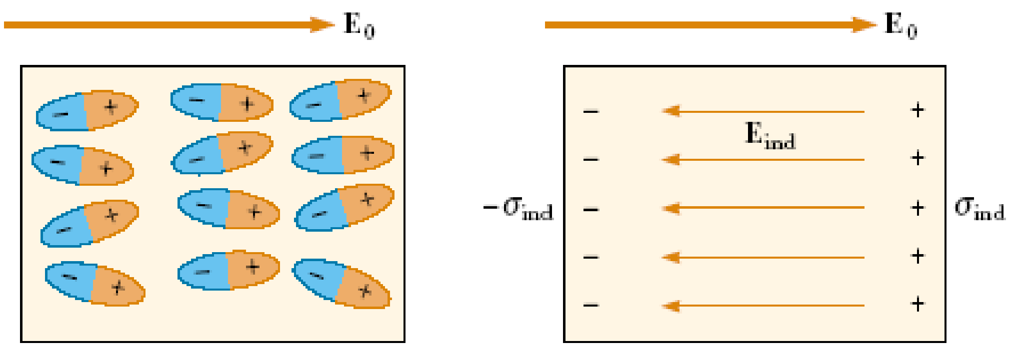
$$\mathbf{E} = \frac{\mathbf{E}_0}{\kappa}$$

(a)Polar molecules are randomly oriented in the absence of an external electric field.



(a)

(b) When an external field is applied, the molecules partially align with the field.



$$E = E_0 - E_{\rm ind}$$