Q.1: 1) True: Boston is capital of Massachusetts.

2) False: Miami is not the capital of Florida.

3) True: $2+3=5$.

4) False: $5+7 \neq 10$

5) Not a proposition: $x+y=11$

6) Not a proposition.

Q.3)

a) False

b) True

c) True

d) True

e) True.

Q.9)

a) True

b) True

c) False

d) False

e) False

Q.4: p: You have the flu.
q: You miss the final examination.
r: You pass the course.

a) $p \rightarrow q$: If you have flu, then you'll miss the fin examination.

b) $\neg q \leftrightarrow r$: You won't miss the final examination if only if you pass the course.

c) $q \rightarrow \neg r$: If you will miss the final examination, then you will fail the course.

d) $p \vee q \vee r$: You have the flu, or you miss the final examination, or you pass the course.

e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$: If you have the flu, then you will not pass the course or if you miss the final examination then you'll fail the course.

f) $(p \wedge q) \vee (\neg q \wedge r)$: You have the flu and you miss the fin examination, or you will not miss the final examination and pass the course.

Q.5 p: You get an A on the final exam.
q: You do every exercise in this book.
r: You get an A in this class.

a) $r \wedge \neg q$

b) $p \wedge q \wedge r$

c) $p \rightarrow r$

d) $p \wedge \neg q \wedge r$

e) $(p \wedge q) \rightarrow r$

f) $r \leftrightarrow (q \vee p)$

Q.6) if p. then q $(p \rightarrow q)$:

a) If I remember to send you the address then you will send me an e-mail message.

b) If you are born in US then you will be a citizen of this country.

c) If you keep your textbook then it will be a useful reference in your future courses.

d) If goalie plays well then Red wings will win the Stanley Cup.

e) If you have the best credentials then you get the job.

f) If there is a storm then the beach erodes.

g) If the password is valid then you log on to the server.

h) If you begin your climb too late then you will not reach the summit.

8)

a) Jan is not rich or not happy.

b) Carlos will not bicycle and not run tomorrow.

c) The fan is fast and it is very cold.

d) Akram is fit or Saleem is not injured.

Q. 11:

a) $(p \to \gamma) \wedge (q \to \gamma)$ and $(p \vee q) \to \gamma$

p	q	γ	$p \to \gamma$	$q \to \gamma$	$(p \to \gamma) \wedge (q \to \gamma)$
True	True	True	True	True	True
True	True	False	False	False	False
True	False	True	True	True	True
True	False	False	False	True	False
False	True	True	True	True	True
False	True	False	True	False	False
False	False	True	True	True	True
False	False	False	True	True	True

p	q	γ	$p \vee q$	$(p \vee q) \to \gamma$
True	True	True	True	True
True	True	False	True	False
True	False	True	True	True
True	False	False	True	False
False	True	True	True	True
False	True	False	True	False
False	False	True	False	True
False	False	False	False	True

Proved!

b) $(P \to q) \vee (P \to r)$ and $P \to (q \vee r)$

P	q	r	$P \to q$	$P \to r$	$(P \to q) \vee (P \to r)$
True	True	True	True	True	True
True	True	False	True	False	True
True	False	True	False	True	True
True	False	False	False	False	False
False	True	True	True	True	True
False	True	False	True	True	True
False	False	True	True	True	True
False	False	False	True	True	True.

P	q	r	$q \vee r$	$P \to (q \vee r)$
True	True	True	True	True.
True	True	False	True	True
True	False	True	True	True
True	False	False	False	False
False	True	True	True	True
False	True	False	True	True
False	False	True	True	True
False	False	False	False	True

Proved!

c) $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$

p	q	r	s	$(p \to q)$	$(r \to s)$	$(p \to q) \to (r \to$
True	True	True	True	True	True	True
True	True	True	False	True	False	False
True	True	False	True	True	True	True
True	True	False	False	True	True	True
True	False	True	True	False	True	True
True	False	True	False	False	False	True
True	False	False	True	False	True	True
True	False	False	False	False	True	True
False	True	True	True	True	True	True
False	True	True	False	True	False	False
False	True	False	True	True	True	True
False	True	False	False	True	True	True
False	False	True	True	True	True	True
False	False	True	False	True	False	False
False	False	False	True	True	True	True
False	False	False	False	True	True	True

p	q	r	s	$p \to r$	$q \to s$	$(p \to r) \to (q \to$
True	True	True	True	True	True	True
True	True	True	False	True	False	False
True	True	False	True	False	True	True
True	True	False	False	False	False	True
True	False	True	True	True	True	True
True	False	True	False	True	False	False
True	False	False	True	False	True	True
True	False	False	False	False	True	True
False	True	True	True	True	True	True
False	True	True	False	True	False	False
False	True	False	True	True	True	True
False	True	False	False	True	False	False
False	False	True	True	True	True	True
False	False	True	False	True	False	False
False	False	False	True	True	True	True
False	False	False	False	True	True	True

Q. 7: P: It is sunny tomorrow.
q: I will go for a walk in the woods.

a) 5 different ways:

q whenever p
I will go for a walk in the woods whenever it is sunny tomorrow.

P iff q
It is sunny tomorrow if and only if I will go for a walk in the woods.

q if p
I will go for a walk in the woods if it is sunny tomorrow.

P implies q
It is sunny tomorrow implies that I will go for a walk in the woods.

If p, then q.
If it is sunny tomorrow, then I will go for a walk in the woods.

b). Converse: I will go for a walk in the woods if it is sunny tomorrow.

Inverse: If it is not sunny tomorrow, then I will not go for a walk in the woods.

Contrapositive: If I will not go for a walk in the woods then it is not sunny tomorrow.

C). Inverse of inverse: If it is sunny tomorrow then I will go for a walk in the woods

- Inverse of converse: I will not go for a walk in the woods if it is not sunny tomorrow
- Inverse of contrapositive: If I will go for a walk in the woods then it is sunny tomorrow.

Q. 9:

① Exclusive.

② Inclusive.

③ Inclusive

④ Inclusive.

Q.12) $P(m,n)$ "m divides n"

a) $P(4,5)$: False - 4 does not divides 5.

b $P(9,4)$: True - 9 divides 4.

c) $\forall m \forall n\ P(m,n)$: False - Not possible.

d) $\exists m \forall n\ P(m,n)$: True - Number 1

e) $\exists n \forall m\ P(m,n)$: False - Not Possible

f) $\forall n P(1,n)$: True - Every number is divisible by 1.

Q.13) Determine truth values:

a) $\exists x (x^2 = 9)$: True $= \sqrt{9}$

b) $\exists x (x^2 = -1)$: False = Square of a number is never -ve.

c) $\forall x (x^2 + 9 \geq 1)$: True = The equation is true even at 0.

d) $\exists x (x^2 = x)$: True = $(1, 0,)$

Q.14) $F(x,y)$ "x can fool y"

a) Everybody can fool Bob: $\forall x F(x, Bob)$

b) Alice can fool everybody: $\forall y F(Alice, y)$

c) Everybody can fool somebody: $\forall x \exists y F(x,y)$

d) There is nooone who can fool everybody: $\neg \exists x \forall y F(x,y)$

e) Everybody can be fooled by somebody: $\forall y \exists x F(x,y)$.

Q.15) $P(x)$: x can speak Russian.
$Q(x)$: x knows the computer language C++.

a) $\exists x (P(x) \wedge Q(x))$

b) $\exists x (P(x) \wedge \neg Q(x))$

c) $\forall x (P(x) \vee Q(x))$

d) $\neg \exists x (P(x) \vee Q(x))$.

Q.16) $Q(x,y)$ "x has sent an email message to y"

a) $\exists x \exists y Q(x,y)$: There is a student in your class who has sent a message to some student in your class.

b) $\exists x \forall y Q(x,y)$: There is a student in your class who has sent a message to every student in your class.

c) $\forall x \exists y Q(x,y)$: Every student in your class has sent a message to atleast one student in your class.

d) $\exists y \forall x Q(x,y)$: There is a student in your class who has been sent a message by every student in your class.

e) $\forall y \exists x Q(x,y)$: Every student in your class has been sent a message from atleast one student in your class.

f) $\forall x \forall y Q(x,y)$: Every student in your class has sent a message to every student in your class.

Q.17 $P(x,y)$ "Student x has taken class y"

x: All students in your class.

y: All CS courses at your school.

a) $\exists x \exists y P(x,y)$: There is a student on your class that has taken a computer science course.

b) $\exists x \forall y P(x,y)$: There is a student in your class that has taken every computer science courses.

c) $\forall x \exists y P(x,y)$: Every student in your class has taken a CS course.

d) $\exists y \forall x P(x,y)$: There is a CS course that every student in your class has taken.

e) $\forall y \exists x P(x,y)$: Every CS course has been taken by some student in your class.

f) $\forall x \forall y P(x,y)$: All students in your class have taken all computer science courses.

Q.18) Rules of Inference:

a) p: Alice is a mathematics major.

q: Alice is a computer science major.

$$\frac{p}{\therefore p \vee q}$$

- Addition.

b) p: Jerry is a mathematics major.
q: Jerry is a CS major.

$$\frac{p \wedge q}{\therefore p}$$

- Simplification.

c) p: It is raining.
q: Pool is closed.

$$\frac{p \rightarrow q \quad p}{\therefore q}$$

- Modus Ponen.

d) p: It snows.
q: University is closed.

$$\frac{p \rightarrow q \quad \neg q}{\therefore \neg p}$$

- Modus Tollens.

e) p: I go swimming
q: I stay in the sun too long.
r: I will get sunburn.

$$\frac{p \rightarrow q \quad q \rightarrow r}{p \rightarrow r}$$

- Hypothetical syllogism.

Q.30: Let;

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A = \{1, 2, 4, 5\}$

$B = \{2, 3, 5, 6\}$

$C = \{4, 5, 6, 7\}$

a) $(A \cap B) \cap \bar{C}$

- $A \cap B = \{1, 2, 4, 5\} \cap \{2, 3, 5, 6\} \Rightarrow \{2, 5\}$
- $\bar{C} = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5, 6, 7\} \Rightarrow \{1, 2, 3, 8\}$
- $(A \cap B) \cap \bar{C} = \{2, 5\} \cap \{1, 2, 3, 8\} \Rightarrow \{2\}$ Ans.

b) $\bar{A} \cup (B \cup C)$

- $\bar{A} = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{1, 2, 4, 5\} \Rightarrow \{3, 6, 7, 8\}$
- $B \cup C = \{2, 3, 5, 6\} \cup \{4, 5, 6, 7\} \Rightarrow \{2, 3, 4, 5, 6, 7\}$
- $\bar{A} \cup (B \cup C) = \{3, 6, 7, 8\} \cup \{2, 3, 4, 5, 6, 7\} \Rightarrow \{2, 3, 4, 5, 6, 7, 8\}$ Ans.

c) $(A - B) \cap C$

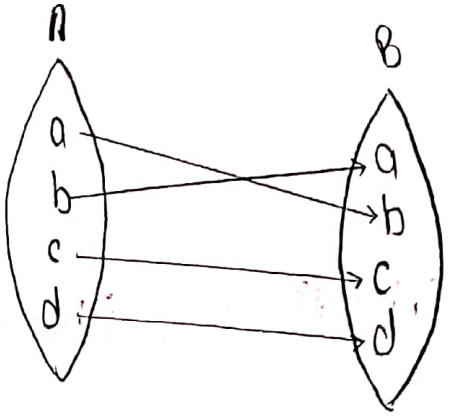
- $A - B = \{1, 2, 4, 5\} - \{2, 3, 5, 6\} \Rightarrow \{1, 4\}$
- $(A - B) \cap C = \{1, 4\} \cap \{4, 5, 6, 7\} \Rightarrow \{4\}$ Ans.

d) $(A \cap \bar{B}) \cup \bar{C}$

- $\bar{B} = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 3, 5, 6\} \Rightarrow \{1, 4, 7, 8\}$
- $\bar{C} = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{4, 5, 6, 7\} \Rightarrow \{1, 2, 3, 8\}$
- $A \cap \bar{B} = \{1, 2, 4, 5\} \cap \{1, 4, 7, 8\} \Rightarrow \{1, 4\}$
- $(A \cap \bar{B}) \cup \bar{C} = \{1, 4\} \cup \{1, 2, 3, 8\} \Rightarrow \{1, 2, 3, 4, 8\}$ Ans.

Q.93: Let $A = \{a, b, c, d\}$; $B = \{a, b, c, d\}$

a) $f(a) = b$; $f(b) = a$; $f(c) = c$; $f(d) = d$.

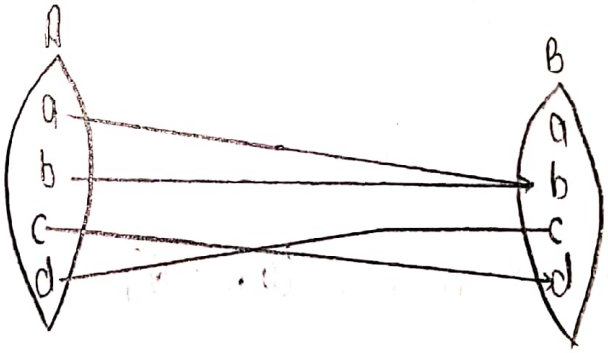


i) Domain = $\{a, b, c, d\}$ Co-domain = $\{a, b, c, d\}$ Range = $\{a, b, c, d\}$

ii) Bijective.

iii) $f^{-1}(b) = a$ $f^{-1}(a) = b$ $f^{-1}(c) = c$ $f^{-1}(d) = d$.

b) $f(a) = b$; $f(b) = b$; $f(c) = d$; $f(d) = c$

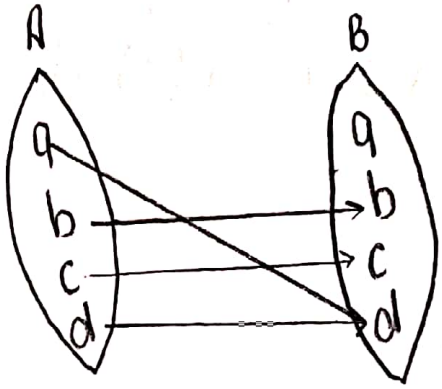


i) Domain = $\{a, b, c, d\}$; Co-domain = $\{a, b, c, d\}$ Range = $\{b, c, d\}$.

ii) neither

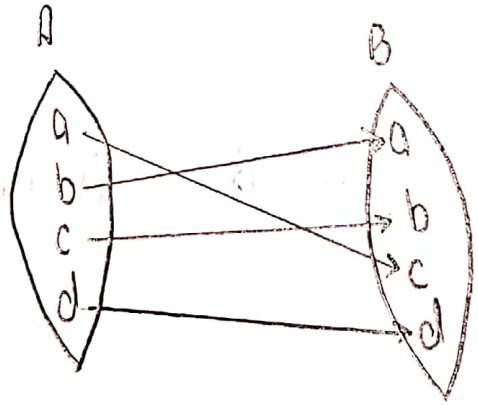
iii) no inverse

c) $f(a) = d$; $f(b) = b$; $f(c) = c$; $f(d) = d$.



i) Domain = {a, b, c, d}
ii) Neither — Codomain = {a, b, c, d} — Range = {b, c, d}
iii) No inverse.

d) $f(a) = c$; $f(b) = a$; $f(c) = b$; $f(d) = d$



i) Domain = {a, b, c, d} — Co-Domain = {a, b, c, d} — Range = {a, b, c, d}
ii) Bi-jective.
iii) $f(c) = a$ $f^{-1}(a) = b$ $f^{-1}(b) = c$ $f^{-1}(d) = d$.

Q.94: Let $f(x) = \left|\frac{x^2}{3}\right|$, Find $f(S)$ if:

i) $S = \{-2, -1, 0, 1, 2, 3\}$.

Sol: $f(-2) = \left|\frac{(-2)^2}{3}\right| = \left|\frac{4}{3}\right| = \underline{1}$

$f(-1) = \left|\frac{(-1)^2}{3}\right| = \left|\frac{1}{3}\right| = \underline{0}$

$f(0) = \left|\frac{0^2}{3}\right| = \left|\frac{0}{3}\right| = \underline{0}$

$f(1) = \left|\frac{1^2}{3}\right| = \left|\frac{1}{3}\right| = \underline{0}$,

$f(2) = \left|\frac{2^2}{3}\right| = \left|\frac{4}{3}\right| = \underline{1}$

$f(3) = \left|\frac{3^2}{3}\right| = \left|\frac{9}{3}\right| = \underline{3}$

• $f(S) = \{0, 1, 3\}$ Ans.

ii) $S = \{0, 1, 2, 3, 4, 5\}$.

Sol: $f(0) = \left|\frac{0^2}{3}\right| = |0/3| = \underline{0}$

$f(1) = |1^2/3| = |1/3| = \underline{0}$

$f(2) = |2^2/3| = |4/3| = \underline{1}$

$f(3) = |3^2/3| = |9/3| = \underline{3}$

$f(4) = |4^2/3| = |16/3| = \underline{5}$

$f(5) = |5^2/3| = |25/3| = \underline{8}$

• $f(S) = \{0, 1, 3, 5, 8\}$. Ans.

iii) $S = \{1, 5, 7, 11\}$.

Sol: $f(1) = |1^2/3| = |1/3| = \underline{0}$

$f(5) = |5^2/3| = |25/3| = \underline{8}$

$f(7) = |7^2/3| = |49/3| = \underline{16}$

$f(11) = |11^2/3| = |121/3| = \underline{40}$

• $f(S) = \{0, 8, 16, 40\}$. Ans.

iv) $S = \{2, 6, 10, 14\}$.

Sol: $f(2) = |2^2/3| = |4/3| = \underline{1}$

$f(6) = |6^2/3| = |36/3| = \underline{12}$

$f(10) = |10^2/3| = \underline{33}$

$f(14) = |14^2/3| = \underline{65}$

• $f(S) = \{1, 12, 33, 65\}$ Ans.

b) Floor and ceiling function:

i) $\lceil 3/4 \rceil = 1$

ii) $\lfloor 7/8 \rfloor = 0$

iii) $\lceil -3/4 \rceil = 0$

iv) $\lfloor -\frac{7}{8} \rfloor = -1$

v) $\lceil 3 \rceil = 3$

vi) $\lfloor -1 \rfloor = -1$

vii) $\left\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \right\rfloor = 2$

viii) $\left\lfloor \frac{1}{2} \cdot \lfloor \frac{5}{2} \rfloor \right\rfloor = 1$

c) → $\lfloor -x \rfloor = -\lceil x \rceil$

- $x = 2$
- $\lfloor -2 \rfloor = -\lceil 2 \rceil$
- $-2 = -2$

Proved.

→ $\lceil -x \rceil = -\lfloor x \rfloor$

- $x = 5$
- $\lceil -5 \rceil = -\lfloor 5 \rfloor$
- $-5 = -5$

Proved.

Q.10

d) $(P \wedge Q) \rightarrow (P \rightarrow Q) \equiv T$

- $(P \wedge Q) \rightarrow (\neg P \vee Q)$
- $\neg (P \wedge Q) \vee (\neg P \vee Q)$
- $(\neg P \vee \neg Q) \vee (\neg P \vee Q)$
- $(\neg P \vee \neg P) \vee (\neg Q \vee Q)$
- $\neg P \vee T$
- $T \equiv T$ Proved.

e) ~~$(P \wedge Q)$~~

$\neg (P \vee \neg (P \wedge Q)) \equiv F$

- $\neg (P \vee (\neg P \vee \neg Q))$
- $\neg P \wedge (P \wedge Q)$
- $(\neg P \wedge P) \wedge Q$
- $F \wedge Q$
- $Q \vee F$
- F Proved

c) $\neg P \leftrightarrow Q \equiv P \leftrightarrow \neg Q$

- $(\neg P \rightarrow Q) \wedge (Q \rightarrow \neg P)$
- $(P \vee Q) \wedge (\neg Q \vee \neg P)$
- $(Q \vee P) \wedge (\neg P \vee \neg Q)$
- $(\neg Q \rightarrow P) \wedge (P \rightarrow \neg Q)$
- $P \leftrightarrow \neg Q$ proved.

b) $\neg (P \leftrightarrow Q) \equiv (P \leftrightarrow \neg Q)$

- $\neg ((P \rightarrow Q) \wedge (Q \rightarrow P)) \equiv (P \rightarrow \neg Q) \wedge (\neg Q \rightarrow P)$
- $\neg ((\neg P \vee Q) \wedge (\neg Q \vee P)) \equiv (\neg P \vee \neg Q) \wedge (Q \vee P)$
- $(P \wedge Q) \vee \neg (\neg Q \vee P) \equiv (P \vee Q) \wedge (\neg Q \vee \neg P)$
- $(P \wedge Q) \vee (Q \wedge \neg P)$
- $[(P \wedge \neg Q) \vee Q] \wedge [(P \wedge \neg Q) \vee \neg P]$
- $[(P \vee Q) \wedge (\neg Q \vee Q)] \wedge [(P \vee \neg P) \wedge (\neg Q \vee \neg P)]$
- $(P \wedge T) \wedge (T \wedge (\neg Q \vee \neg P))$
- $(P \wedge T) \wedge (\neg Q \vee \neg P)$
- $(P \wedge Q) \wedge (\neg Q \vee \neg P) \equiv (P \vee Q) \wedge (\neg Q \vee \neg P)$

Proved.

a) $(P \wedge (\neg(\neg P \vee q))) \vee (P \wedge q) \equiv P$

$(P \wedge (P \wedge \neg q)) \vee (P \wedge q)$

$[(P \wedge P) \wedge \neg q] \vee (P \wedge q)$

$(P \wedge \neg q) \vee (P \wedge q)$

$P \wedge (\neg q \vee q)$

$P \wedge T$

$P \equiv P$ Proved.

Q.19:

a): t = Today is ~~Thursday~~ Tuesday

m = I have a test in Maths

e = I have a test in Economics

s - my Economy Sir is sick.

$t \rightarrow (m \vee e)$

$s \rightarrow \neg e$

$t \wedge s$

$\therefore m$ (disjunctive syllogism).

b)

b) L = Ali is a lawyer.

a = He is ambitious.

γ = He is an early riser.

$\neg C$ = He does not like chocolates.

$L \rightarrow a$

$\gamma \rightarrow \neg C$

$a \rightarrow \gamma$

$\therefore L \rightarrow \neg C$

$L \rightarrow a \ : a \rightarrow \gamma = L \rightarrow \gamma$

$L \rightarrow \gamma$

$\gamma \rightarrow \neg C$

$\therefore L \rightarrow \neg C$

$L \rightarrow \gamma \ : \gamma \rightarrow \neg C$

99)

a) Total = 100

Worm = 30 [A]

Bruises = 15 [B]

Both = 10

A+B = 35

$(A+B) - (A \cap B) \Rightarrow 35 - 10 \Rightarrow 25$

$\Rightarrow 100 - [30 + 15 - 10]$

$\Rightarrow 100 - 95 = 75$ Ans.

c) Mixed Berry = 78 [M]

Irish Cream = 39 [I]

Tiramisu = 57 [T]

$M \cap T = 16$

$T \cap I = 21$

$M \cap I = 13$

$M \cap T \cap I = 5$

$\Rightarrow 78 + 39 + 57 - 16 - 21 - 13 + 5 = 199$. Ans.

b) Total = 1000

CS = 350 [A]

SE = 450 [B]

Both (A∩B) = 100

$1000 - (350 + 450 - 100) = 700$ like either

300 like neither

d) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$\{(x,y) \mid x \in A \text{ and } y \in B \cap C\}$

$\{(x,y) \mid x \in A \text{ and } y \in B \text{ and } y \in C\}$

$\{(x,y) \mid (x,y) \in A \times B \text{ and } (x,y) \in A \times C\}$

$(A \times B) \cap (A \times C)$ Ans.

Q.95: a) $f(a) = g(a) + 3$; $g(a) = 3a + 9$

$f(g(a)) = f(3a+9)$

$= g(3a+9) + 3$

$= 6a + 4 + 3$

$= 6a + 7$. Ans.

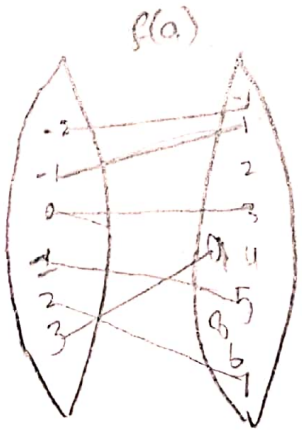
$g(f(a)) = g(2a+3)$

$= 3(2a+3) + 9$

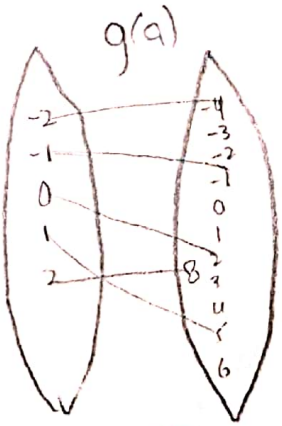
$= 6a + 9 + 9$

$= 6a + 11$ Ans.

b)



neither



neither

c) Not invertible for g.