Application of Derivative

We have already investigated some of the applications of derivatives, but now that we know the differentiation rules we are in a better position to pursue the applications of differentiation in greater depth. Here we learn how derivatives affect the shape of a graph of a function and, in particular, how they help us locate maximum and minimum values of functions. Many practical problems require us to minimize a cost or maximize an area or somehow find the best possible outcome of a situation. In particular, we will be able to investigate the optimal shape of a can and to explain the location of rainbows in the sky.

PopUp Quiz

Q-1

(a) Use implicit differentiation to find dy/dx for the Folium of Descartes $x^3 + y^3 = 3xy$.

(b) Find an equation for the tangent line to the Folium of Descartes at the point $\left(\frac{3}{2}, \frac{3}{2}\right)$.

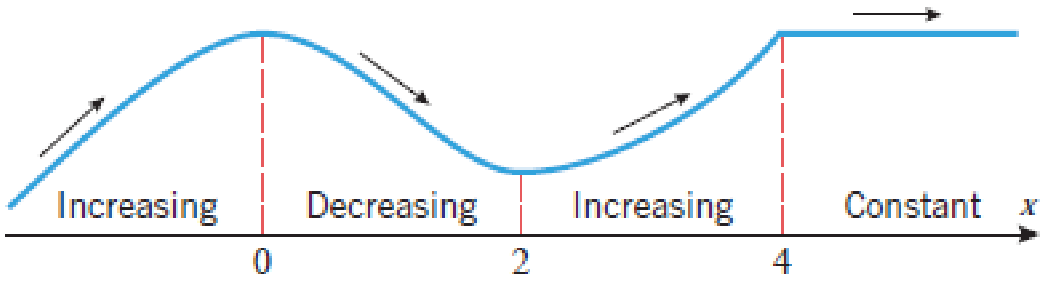
Q-2 Find dy/dx using logarithmic differentiation.

$$y = \frac{x^2 \sqrt[3]{7x - 14}}{(1 + x^2)^4} \qquad \text{OR} \qquad \tan(x - y) = \frac{y}{1 + x^2}$$

Q-3
Evaluate

$$\frac{d}{dx}\left[\ln\left(\frac{x^2 \sin x}{\sqrt{1 + x}}\right)\right] \qquad \text{OR} \qquad \frac{d}{dx}\left[\sqrt{x^3 + \csc x}\right]$$

4.1 ANALYSIS OF FUNCTIONS I: INCREASE, DECREASE, AND CONCAVITY



4.1.1 DEFINITION Let f be defined on an interval, and let x_1 and x_2 denote points in that interval.

(a) f is *increasing* on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.

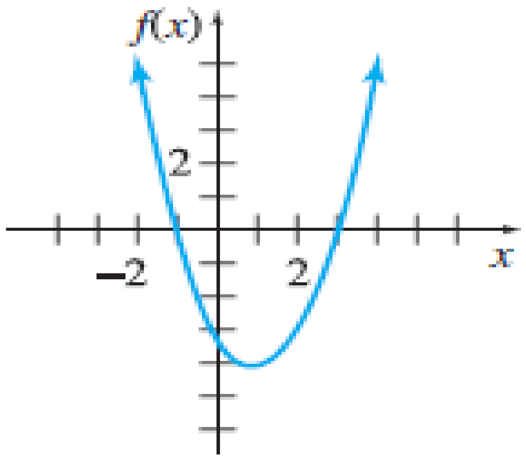
(b) f is *decreasing* on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.

(c) f is *constant* on the interval if $f(x_1) = f(x_2)$ for all points x_1 and x_2.

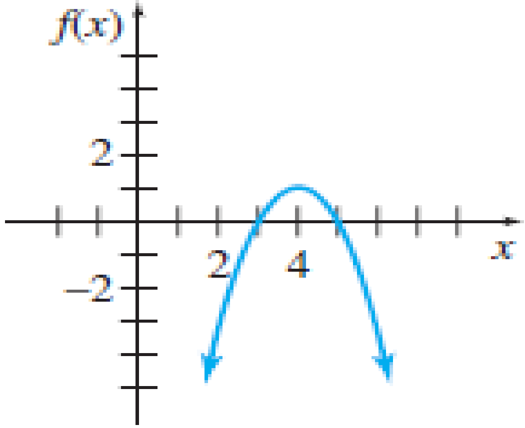
Find the open intervals where the functions graphed as follows are (a) increasing, or (b) decreasing.

1.

Increasing: (1, ∞)
Decreasing: (-∞, 1)



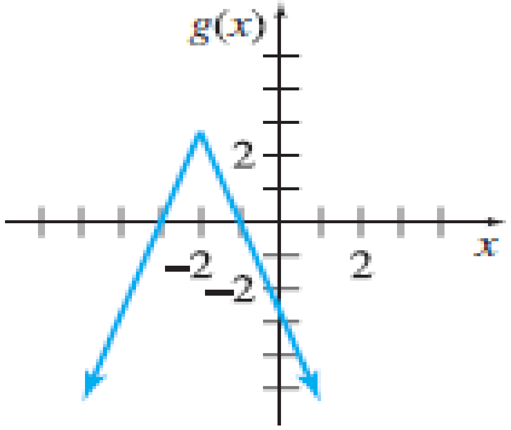
2.



Increasing: (-∞, 4)
Decreasing: (4, ∞)

3.

Increasing: (-∞, -2)
Decreasing: (-2, ∞)



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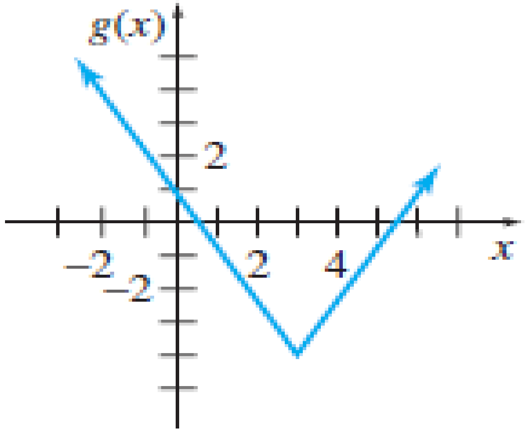
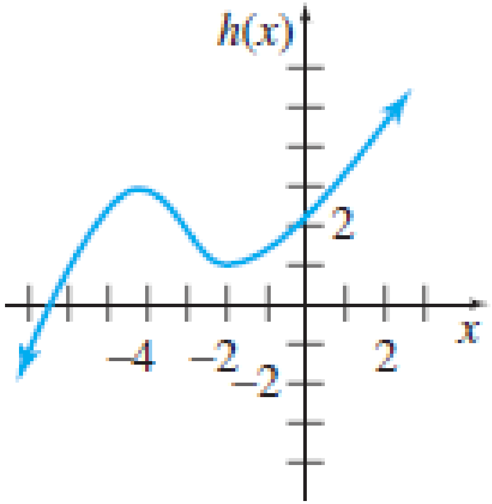


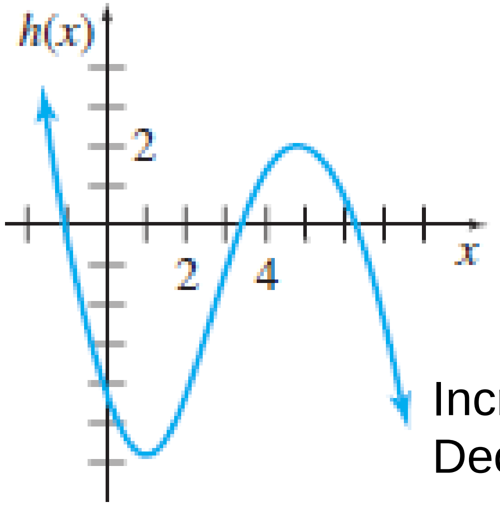
Increasing: (3, ∞)
Decreasing: (-∞, 3)

5.

Increasing: (-∞, -4)U(-2, ∞)
Decreasing: (-4, -2)



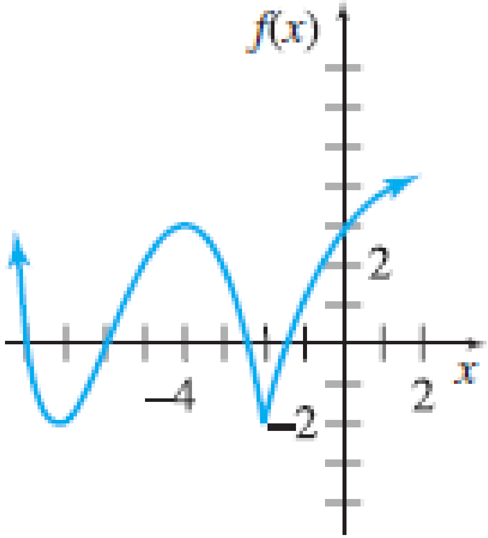
6.



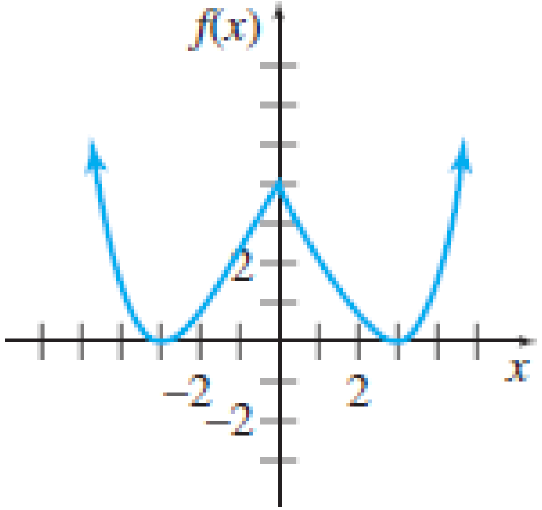
Increasing: (1, 5)
Decreasing: (-∞, 1)U(5, ∞)

7.

Increasing: (-7, -4)U(-2, ∞)
Decreasing: (-∞, -7)U(-4, -2)



8.



4.1.2 THEOREM *Let f be a function that is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b).*

(a) *If $f'(x) > 0$ for every value of x in (a, b), then f is increasing on $[a, b]$.*

(b) *If $f'(x) < 0$ for every value of x in (a, b), then f is decreasing on $[a, b]$.*

(c) *If $f'(x) = 0$ for every value of x in (a, b), then f is constant on $[a, b]$.*

► **Example 1** Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing.

► **Example 2** Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.

Example:

Determine the intervals where $f(x) = x^3 - 6x^2 + 1$ is increasing and where it is decreasing.

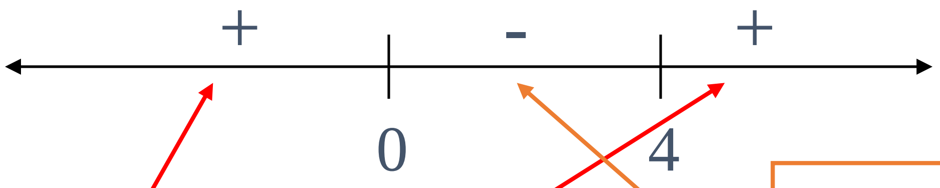
$$f'(x) = 3x^2 - 12x$$

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$3x = 0 \text{ or } x - 4 = 0$$

$$x = 0, 4$$



f is increasing on $(-\infty, 0) \cup (4, \infty)$

f is decreasing on $(0, 4)$

▸ Example 3

an opinion or conclusion formed on the basis of incomplete information

(a) Use the graph of $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ in Figure 4.1.6 to make a conjecture about the intervals on which f is increasing or decreasing.

(b) Use Theorem 4.1.2 to determine whether your conjecture is correct.

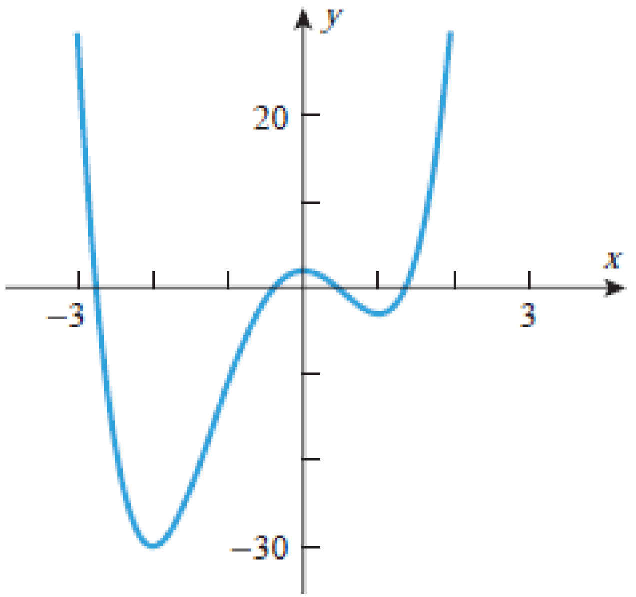


Table 4.1.1

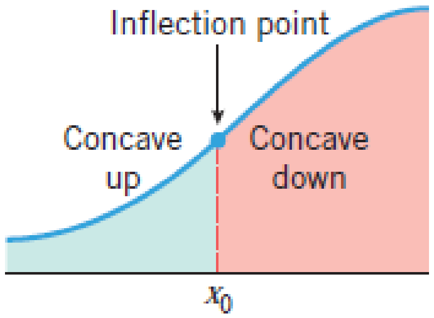
INTERVAL	$(12x)(x+2)(x-1)$	$f'(x)$	CONCLUSION
$x < -2$	(−) (−) (−)	−	f is decreasing on $(-\infty, -2]$
$-2 < x < 0$	(−) (+) (−)	+	f is increasing on $[-2, 0]$
$0 < x < 1$	(+) (+) (−)	−	f is decreasing on $[0, 1]$
$1 < x$	(+) (+) (+)	+	f is increasing on $[1, +\infty)$

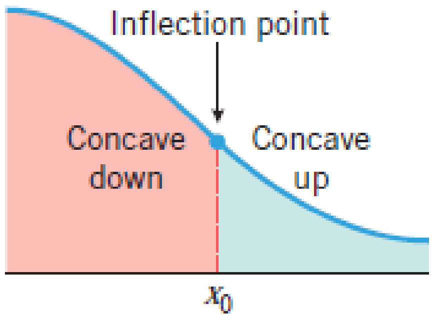
CONCAVITY

4.1.4 THEOREM *Let f be twice differentiable on an open interval.*

(a) *If $f''(x) > 0$ for every value of x in the open interval, then f is concave up on that interval.*

(b) *If $f''(x) < 0$ for every value of x in the open interval, then f is concave down on that interval.*





DEFINITION Critical Point

An interior point of the domain of a function f where f' is zero or undefined is a **critical point** of f.

$$f'(x) = 0 \text{ or } f'(x) \text{ does not exist.}$$

Example:

Find all Critical points $f(x)=\sqrt[3]{x^3-3x}$

$$f'(x)=\frac{x^2-1}{\sqrt[3]{\left(x^3-3x\right)^2}}$$

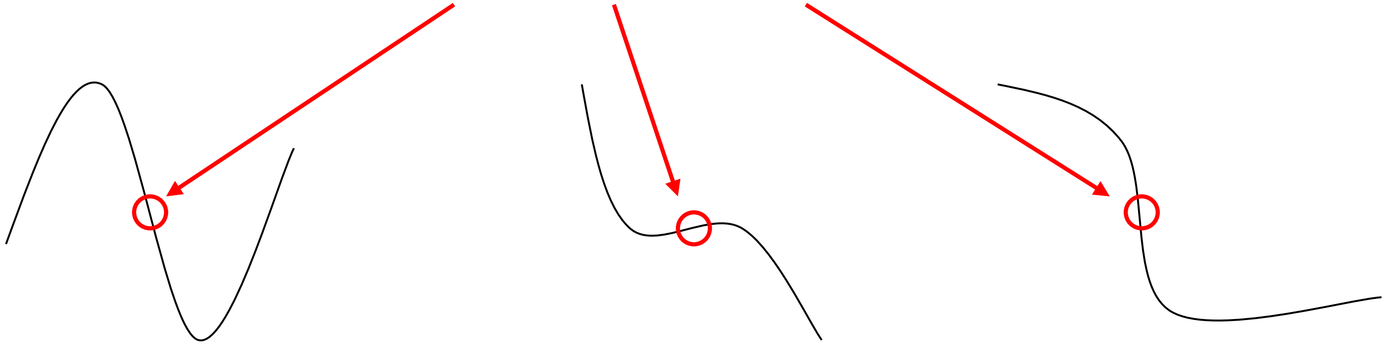
$\boxed{f'(x)=0}$ $\boxed{f'(x) \text{ undefined}}$

$\boxed{x^2-1=0}$ or $\boxed{x^3-3x=0}$

Critical points are : $x=0,\pm1,\pm\sqrt{3}$

4.1.5 Inflection Point

A point on the graph of f at which concavity changes is called an inflection point.



To find inflection points, find any point, c, in the domain where $f''(x) = 0$ or $f''(x)$ is undefined.

Then $(c, f(c))$ is an inflection point of f.

Example:

Determine where the function $f(x) = 9x^{5/3} + 5x^2$ is concave upward and concave downward and find any inflection points.

$$f'(x) = 15x^{2/3} + 10x$$

$$f''(x) = -10x^{-1/3} + 10 = -10(x^{-1/3} - 1)$$

f'' + +

-1

f concave up on $(-\infty, 1)$

f concave down on $(1, \infty)$

Inflection point *f* (-1) = ?

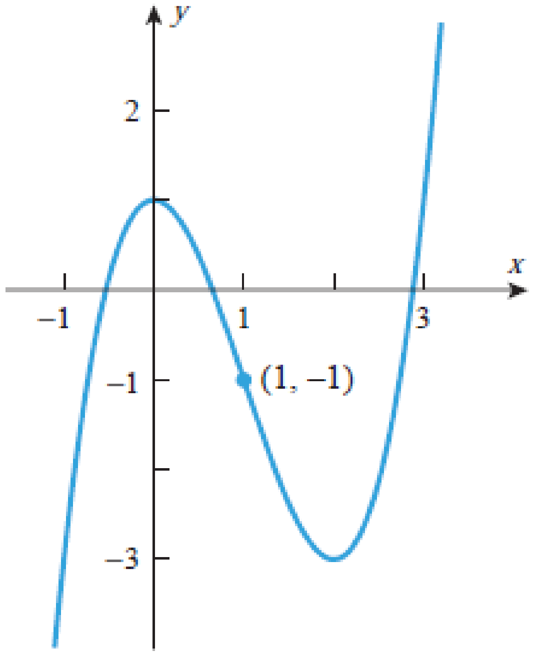
Example 5 Figure 4.1.10 shows the graph of the function $f(x) = x^3 - 3x^2 + 1$. Use the first and second derivatives of f to determine the intervals on which f is increasing, decreasing, concave up, and concave down. Locate all inflection points and confirm that your conclusions are consistent with the graph.

Using First derivative

INTERVAL	$(3x)(x-2)$	$f'(x)$	CONCLUSION
$x < 0$	$(-)(-)$	$+$	f is increasing on $(-\infty, 0]$
$0 < x < 2$	$(+)(-)$	$-$	f is decreasing on $[0, 2]$
$x > 2$	$(+)(+)$	$+$	f is increasing on $[2, +\infty)$

Using 2nd derivative

INTERVAL	$6(x-1)$	$f''(x)$	CONCLUSION
$x < 1$	$(-)$	$-$	f is concave down on $(-\infty, 1)$
$x > 1$	$(+)$	$+$	f is concave up on $(1, +\infty)$

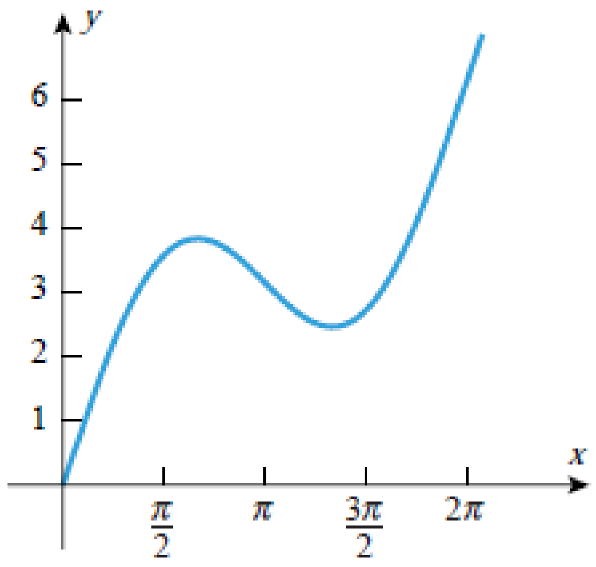


Example 6 Figure 4.1.11 suggests that the function $f(x) = xe^{-x}$ has an inflection point but its exact location is not evident from the graph in this figure. Use the first and second derivatives of f to determine the intervals on which f is increasing, decreasing, concave up, and concave down. Locate all inflection points.

▶ **Example 7** Figure 4.1.12 shows the graph of the function $f(x) = x + 2\sin x$ over the interval $[0, 2\pi]$. Use the first and second derivatives of f to determine where f is increasing, decreasing, concave up, and concave down. Locate all inflection points and confirm that your conclusions are consistent with the graph.

Using First derivative

INTERVAL	$f'(x) = 1 + 2\cos x$	CONCLUSION
$0 < x < 2\pi/3$	+	f is increasing on $[0, 2\pi/3]$
$2\pi/3 < x < 4\pi/3$	–	f is decreasing on $[2\pi/3, 4\pi/3]$
$4\pi/3 < x < 2\pi$	+	f is increasing on $[4\pi/3, 2\pi]$



Using 2nd derivative

INTERVAL	$f''(x) = -2\sin x$	CONCLUSION
$0 < x < \pi$	–	f is concave down on $(0, \pi)$
$\pi < x < 2\pi$	+	f is concave up on $(\pi, 2\pi)$

Find the Interval of a) Increasing b) Decreasing c) Concave up d) Concave Down e) Point of Inflection f) Critical point

Ex-1 $f(x) = \dfrac{x}{x^2 + 2}$

$$f'(x) = \frac{2 - x^2}{(x^2 + 2)^2} \quad f''(x) = \frac{2x(x^2 - 6)}{(x^2 + 2)^3}.$$

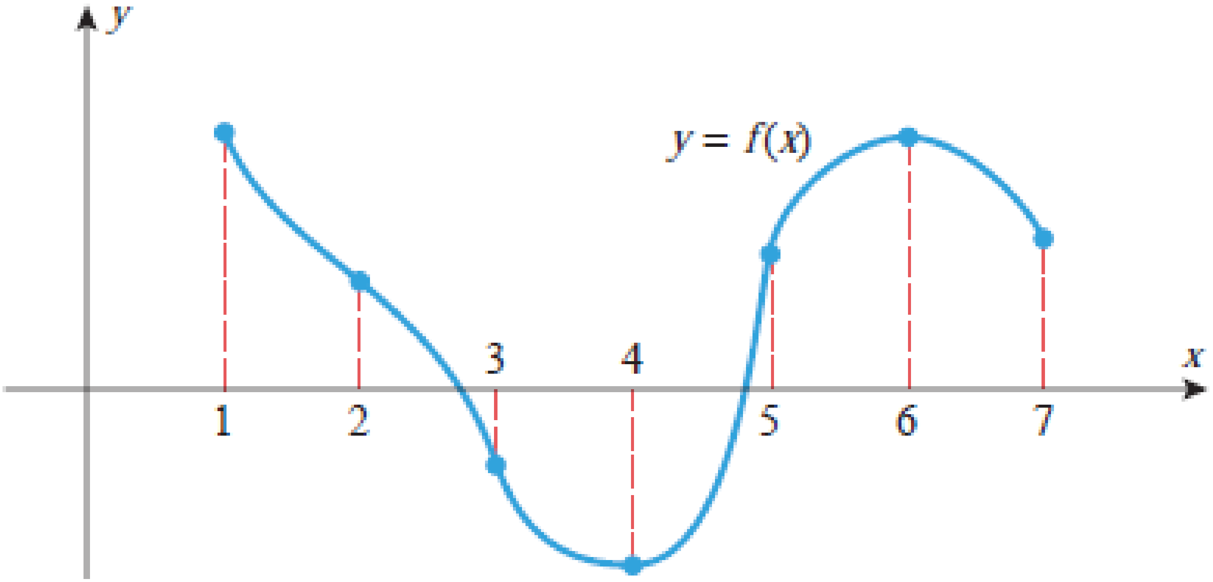
Ex-2 $f(x) = x^{4/3} - x^{1/3}$

$$f'(x) = \frac{4(x - 1/4)}{3x^{2/3}}, \quad f''(x) = \frac{4(x + 1/2)}{9x^{5/3}}.$$

Exercise 4.1

7. In each part, use the graph of $y = f(x)$ in the accompanying figure to find the requested information.

(a) Find the intervals on which f is increasing.
(b) Find the intervals on which f is decreasing.
(c) Find the open intervals on which f is concave up.
(d) Find the open intervals on which f is concave down.
(e) Find all values of x at which f has an inflection point.



(a) $[4, 6]$ (b) $[1, 4]$ and $[6, 7]$.

(c) $(1, 2)$ and $(3, 5)$. (d) $(2, 3)$ and $(5, 7)$.

9–10 A sign chart is presented for the first and second derivatives of a function f. Assuming that f is continuous everywhere, find: (a) the intervals on which f is increasing, (b) the intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d) the open intervals on which f is concave down, and (e) the x-coordinates of all inflection points. ■

9.

INTERVAL	SIGN OF $f'(x)$	SIGN OF $f''(x)$
$x < 1$	–	+
$1 < x < 2$	+	+
$2 < x < 3$	+	–
$3 < x < 4$	–	–
$4 < x$	–	+

(a) f is increasing on $[1, 3]$.

(b) f is decreasing on $(-\infty, 1], [3, +\infty)$.

(c) f is concave up on $(-\infty, 2), (4, +\infty)$.

(d) f is concave down on $(2, 4)$.

(e) Points of inflection at $x = 2, 4$.

Exercise : 4.1

15–32 Find: (a) the intervals on which f is increasing, (b) the intervals on which f is decreasing, (c) the open intervals on which f is concave up, (d) the open intervals on which f is concave down, and (e) the x-coordinates of all inflection points.

15. $f(x) = x^2 - 3x + 8$

16. $f(x) = 5 - 4x - x^2$

17. $f(x) = (2x + 1)^3$

18. $f(x) = 5 + 12x - x^3$

19. $f(x) = 3x^4 - 4x^3$

20. $f(x) = x^4 - 5x^3 + 9x^2$

21. $f(x) = \dfrac{x - 2}{(x^2 - x + 1)^2}$

22. $f(x) = \dfrac{x}{x^2 + 2}$

23. $f(x) = \sqrt[3]{x^2 + x + 1}$

24. $f(x) = x^{4/3} - x^{1/3}$

25. $f(x) = (x^{2/3} - 1)^2$

26. $f(x) = x^{2/3} - x$

27. $f(x) = e^{-x^2/2}$

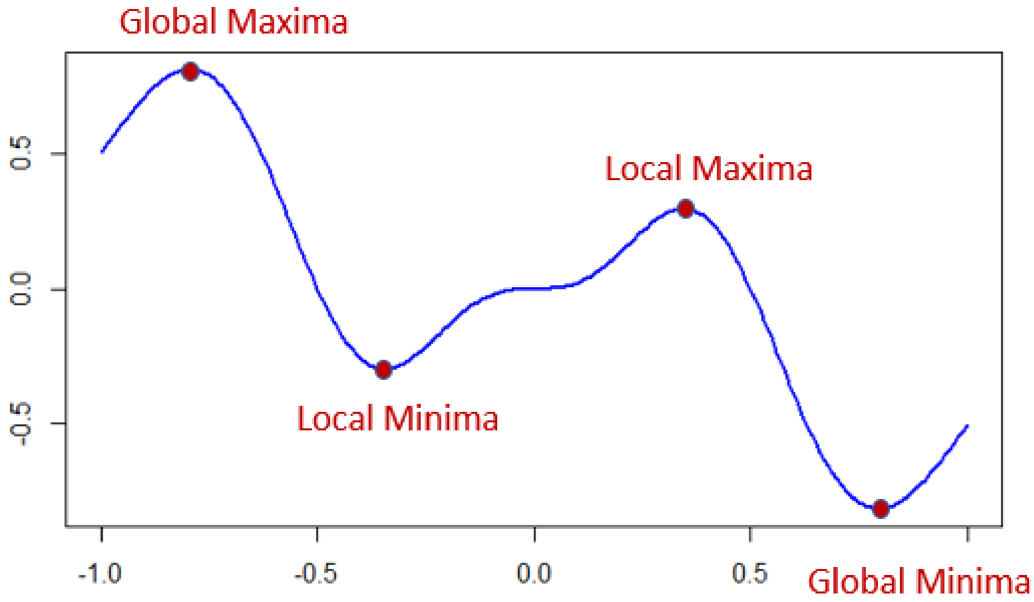
28. $f(x) = xe^{x^2}$

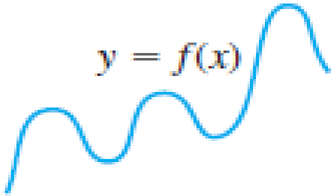
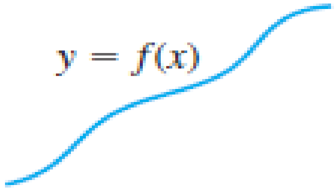
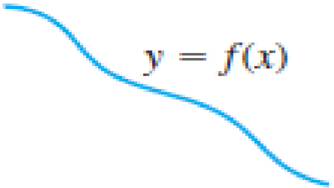
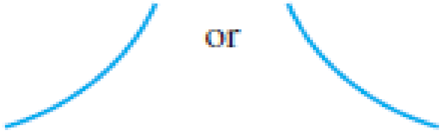
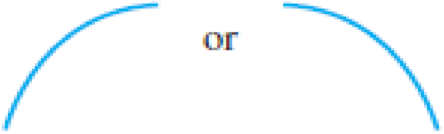
23. $f'(x) = \dfrac{2x+1}{3(x^2+x+1)^{2/3}}, \; f''(x) = -\dfrac{2(x+2)(x-1)}{9(x^2+x+1)^{5/3}}.$

27. $f'(x) = -xe^{-x^2/2}, \; f''(x) = (-1+x^2)e^{-x^2/2}.$

LOCAL / GLOBAL (EXTREMA):

- Global (or Absolute) Maximum and Minimum. The maximum or minimum over the entire function is called an "Absolute" or "Global" maximum or minimum.
- There is only one global maximum (and one global minimum) but there can be more than one local maximum or minimum.



 Differentiable $\Rightarrow$ smooth, connected; graph may rise and fall	 $y' > 0 \Rightarrow$ rises from left to right; may be wavy	 $y' < 0 \Rightarrow$ falls from left to right; may be wavy
 $y'' > 0 \Rightarrow$ concave up throughout; no waves; graph may rise or fall	 $y'' < 0 \Rightarrow$ concave down throughout; no waves; graph may rise or fall	 Inflection point
 y' changes sign $\Rightarrow$ graph has local maximum or local minimum	 $y' = 0$ and $y'' < 0$ at a point; graph has local maximum	 $y' = 0$ and $y'' > 0$ at a point; graph has local minimum

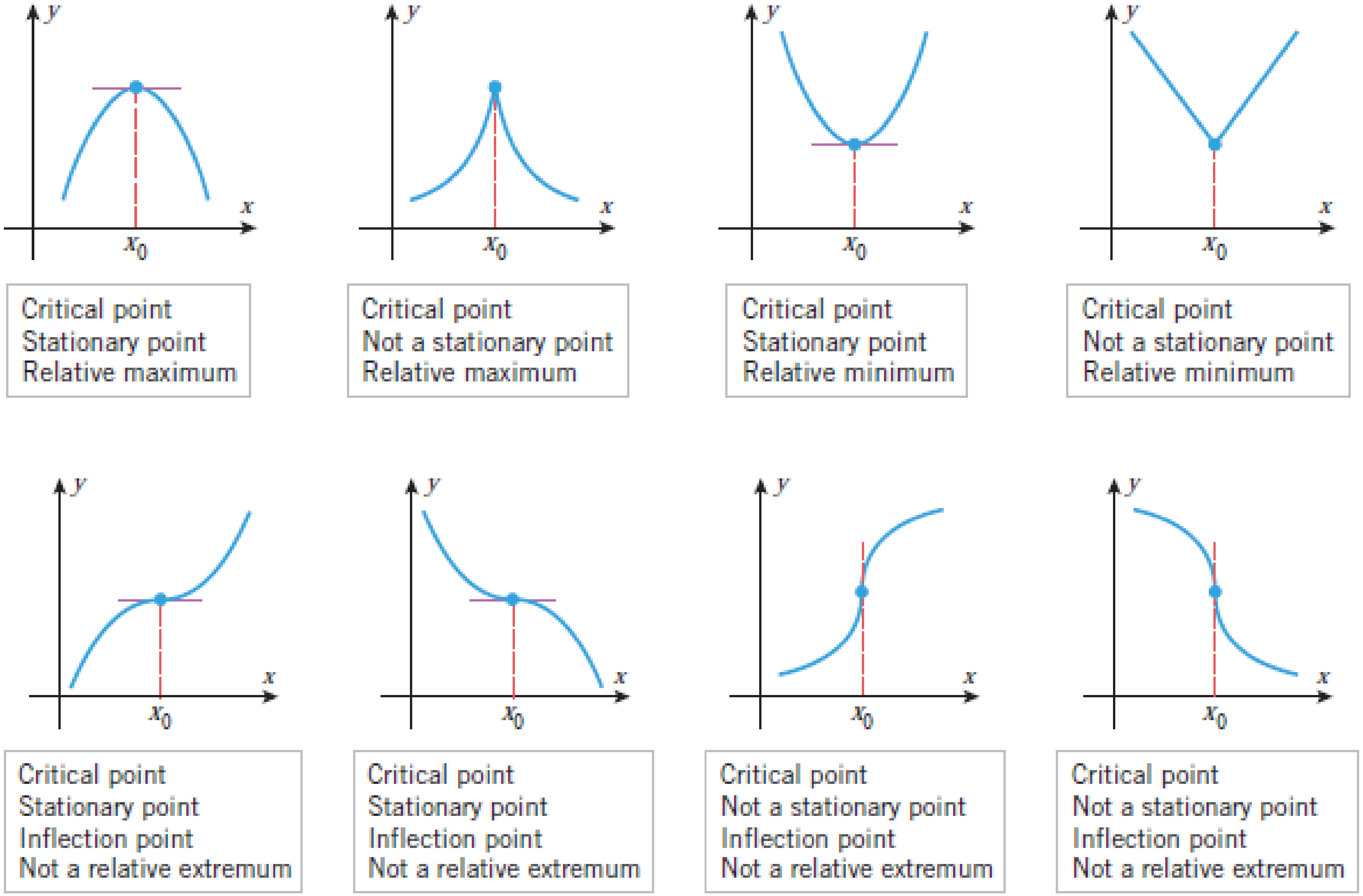
Relative Extremum:

4.2.2 THEOREM *Suppose that f is a function defined on an open interval containing the point x_0. If f has a relative extremum at $x = x_0$, then $x = x_0$ is a critical point of f; that is, either $f'(x_0) = 0$ or f is not differentiable at x_0.*

► **Example 2** Find all critical points of $f(x) = x^3 - 3x + 1$.

► **Example 3** Find all critical points of $f(x) = 3x^{5/3} - 15x^{2/3}$.

A function f has a relative extremum at those critical points where f' changes sign.


y
x
x0
Critical point
Stationary point
Relative maximum
Critical point
Not a stationary point
Relative maximum
Critical point
Stationary point
Relative minimum
Critical point
Not a stationary point
Relative minimum
Critical point
Stationary point
Inflection point
Not a relative extremum
Critical point
Stationary point
Inflection point
Not a relative extremum
Critical point
Not a stationary point
Inflection point
Not a relative extremum
Critical point
Not a stationary point
Inflection point
Not a relative extremum

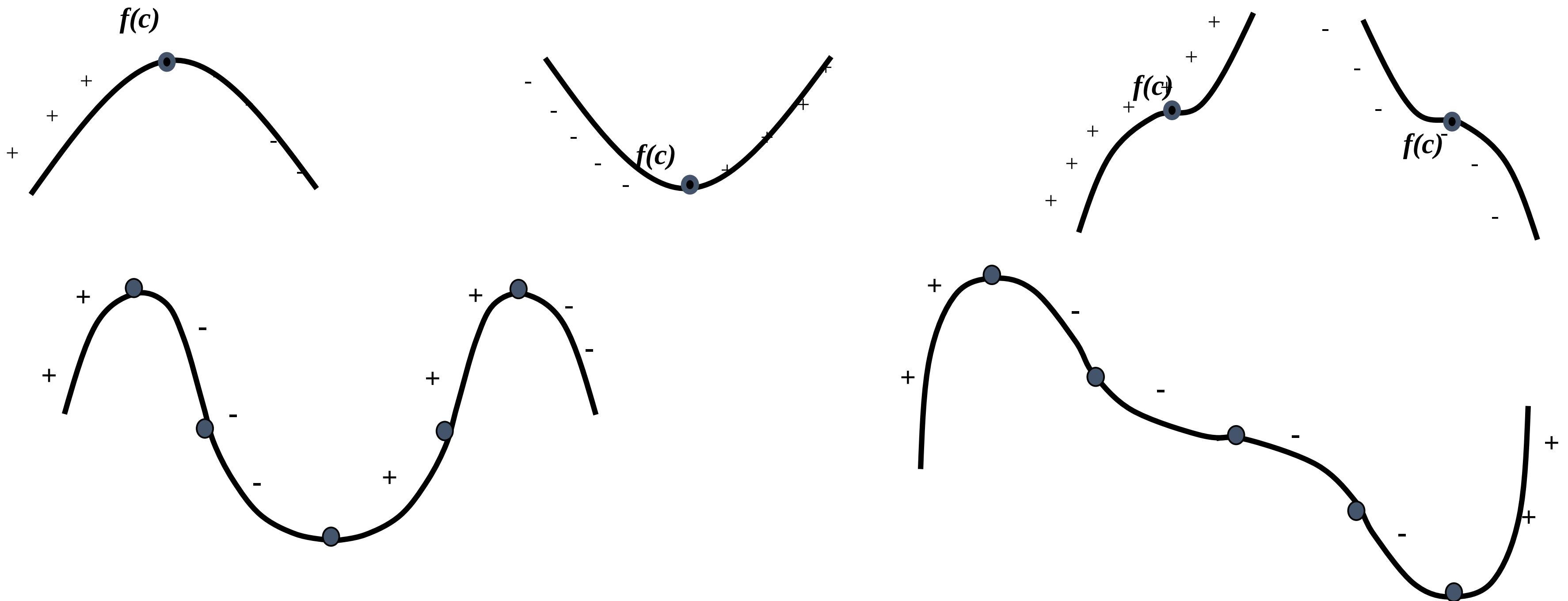
▲ **Figure 4.2.6**

4.2.3 Theorem:

First Derivative Test for Local Extrema

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across c from left to right,

1. if f' changes from negative to positive at c, then f has a local minimum at c;
2. if f' changes from positive to negative at c, then f has a local maximum at c;
3. if f' does not change sign at c (that is, f' is positive on both sides of c or negative on both sides), then f has no local extremum at c.



Example

Find all the relative extrema of $f(x) = x^3 - 6x^2 + 1.$
using the First derivative test.

$$f'(x) = 3x^2 - 12x$$

$$3x^2 - 12x = 0$$

$$3x(x-4) = 0$$

$$3x = 0 \text{ or } x - 4 = 0$$

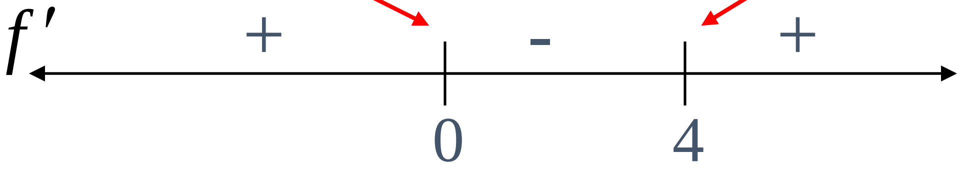
$$x = 0, 4$$

Relative max.

$f(0) = 1$

Relative min.

$f(4) = -31$



Example : g(x)= $x^3 - 27x + 3$

Example:

Find all the relative extrema of $f(x)=\sqrt[3]{x^3-3x}$

$$f'(x)=\frac{x^2-1}{\sqrt[3]{\left(x^3-3x\right)^2}}$$

$\boxed{f'(x)=0}$ $\qquad$ $\boxed{f'(x)\ \text{undefined}}$

$$\boxed{x^2-1=0} \quad \text{or} \quad \boxed{x^3-3x=0}$$

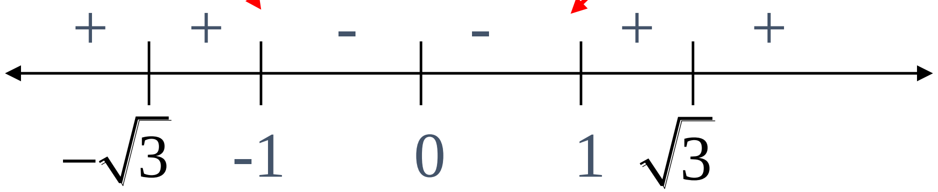
$$x=0,\pm 1,\pm\sqrt{3}$$

Relative max.

$f(-1)=\sqrt[3]{2}$

Relative min.

$f(1)=-\sqrt[3]{2}$



Practice with First derivative test: (Thomas)

Analyzing f Given f'

Answer the following questions about the functions whose derivatives are given in Exercises 1–8:

a. What are the critical points of f?

b. On what intervals is f increasing or decreasing?

c. At what points, if any, does f assume local maximum and minimum values?

1. $f'(x) = x(x - 1)$
2. $f'(x) = (x - 1)(x + 2)$
3. $f'(x) = (x - 1)^2(x + 2)$
4. $f'(x) = (x - 1)^2(x + 2)^2$
5. $f'(x) = (x - 1)(x + 2)(x - 3)$
6. $f'(x) = (x - 7)(x + 1)(x + 5)$
7. $f'(x) = x^{-1/3}(x + 2)$
8. $f'(x) = x^{-1/2}(x - 3)$

2^{nd} derivative test:

4.2.4 THEOREM (*Second Derivative Test*) *Suppose that f is twice differentiable at the point x_0.*

(*a*) *If $f'(x_0) = 0$ and $f''(x_0) > 0$, then f has a relative minimum at x_0.*

(*b*) *If $f'(x_0) = 0$ and $f''(x_0) < 0$, then f has a relative maximum at x_0.*

(*c*) *If $f'(x_0) = 0$ and $f''(x_0) = 0$, then the test is inconclusive; that is, f may have a relative maximum, a relative minimum, or neither at x_0.*

Example:

Find the relative Extrema of $f(x) = x^4 - 4x^3 + 4x^2 - 5$ using the second derivative test.

$$f'(x) = 4x^3 - 12x^2 + 8x$$
$$= 4x(x-2)(x-1)$$

Critical points are : $x = 0, 1, 2$

$$f''(x) = 12x^2 - 24x + 8$$

$$f''(0) = 8 > 0$$
$$f''(1) = -4 < 0$$
$$f''(2) = 8 > 0$$

▶ Example 5 Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

STATIONARY POINT	$30x(2x^2 - 1)$	$f''(x)$	SECOND DERIVATIVE TEST
$x = -1$	-30	$-$	f has a relative maximum
$x = 0$	0	0	Inconclusive
$x = 1$	30	$+$	f has a relative minimum

The test is inconclusive at $x = 0$, so we will try the first derivative test at that point.

INTERVAL	$15x^2(x+1)(x-1)$	$f'(x)$
$-1 < x < 0$	$(+)(+)(-)$	$-$
$0 < x < 1$	$(+)(+)(-)$	$-$

Exercise 4.2

7–14 Locate the critical points and identify which critical points are stationary points.

7. $f(x) = 4x^4 - 16x^2 + 17$
8. $f(x) = 3x^4 + 12x$
9. $f(x) = \dfrac{x+1}{x^2+3}$
10. $f(x) = \dfrac{x^2}{x^3+8}$
11. $f(x) = \sqrt[3]{x^2 - 25}$
12. $f(x) = x^2(x-1)^{2/3}$

25–32 Use the given derivative to find all critical points of f, and at each critical point determine whether a relative maximum, relative minimum, or neither occurs. Assume in each case that f is continuous everywhere.

25. $f'(x) = x^2(x^3 - 5)$
26. $f'(x) = 4x^3 - 9x$
27. $f'(x) = \dfrac{2-3x}{\sqrt[3]{x+2}}$
28. $f'(x) = \dfrac{x^2-7}{\sqrt[3]{x^2+4}}$
29. $f'(x) = xe^{1-x^2}$
30. $f'(x) = x^4(e^x - 3)$

Exercise 4.2

37–50 Use any method to find the relative extrema of the function f. ■

37. $f(x) = x^4 - 4x^3 + 4x^2$

38. $f(x) = x(x-4)^3$

39. $f(x) = x^3(x+1)^2$

40. $f(x) = x^2(x+1)^3$

41. $f(x) = 2x + 3x^{2/3}$

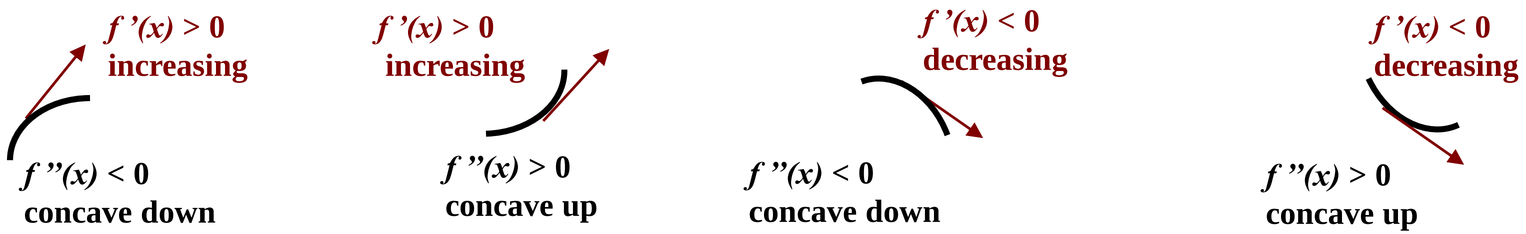
42. $f(x) = 2x + 3x^{1/3}$

43. $f(x) = \dfrac{x+3}{x-2}$

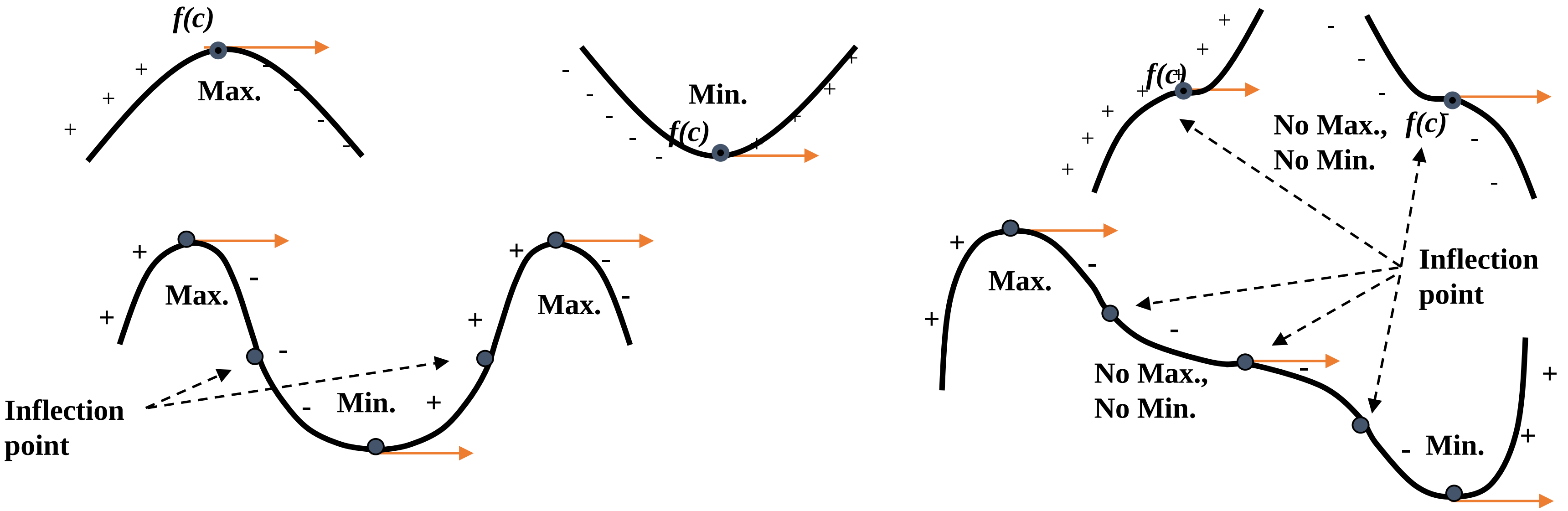
44. $f(x) = \dfrac{x^2}{x^4+16}$

SUMMARY

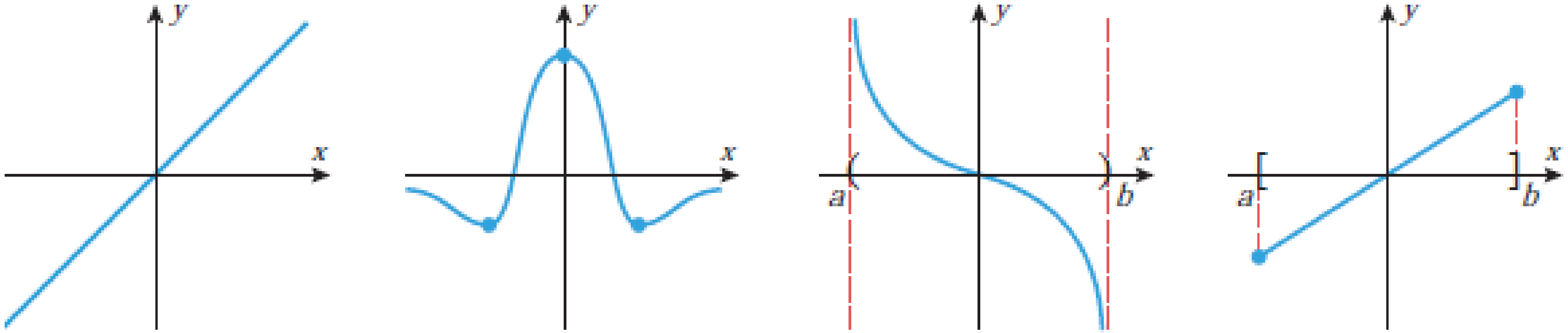
- $f'(x)$ indicates if the function is: **Increasing** when $f'(x) > 0$; **Decreasing** when $f'(x) < 0$
- $f''(x)$ indicates if the function is: **Concave up** when $f''(x) > 0$; **Concave down** when $f''(x) < 0$



- **Critical Point** c is where $f'(c) = 0$ *(tangent line is horizontal* → *)* ;
- **Inflection Point**: where $f''(x) = 0$ or where the function changes concavity, **No Min and no Max**
- If the sign of $f'(c)$ changes from + to - , then there is a **local Maximum**
- If the sign of $f'(c)$ changes from - to + , then there is a **local Minimum**
- If $f'(c) = 0$ but there is no sign change for $f'(c)$, then there is no local extreme, it is an **Inflection Point** (*concavity changes*)



4.4 ABSOLUTE MAXIMA AND MINIMA



f has no absolute extrema on $(-\infty, +\infty)$.

f has an absolute maximum and minimum on $(-\infty, +\infty)$.

f has no absolute extrema on (a, b).

f has an absolute maximum and minimum on $[a, b]$.

4.4.2 THEOREM (*Extreme-Value Theorem*) *If a function f is continuous on a finite closed interval $[a, b]$, then f has both an absolute maximum and an absolute minimum on $[a, b]$.*

Extrema on Interval:

***A Procedure for Finding the Absolute Extrema of a Continuous Function f on a Finite Closed Interval* [a, b]**

Step 1. Find the critical points of f in (a, b).

Step 2. Evaluate f at all the critical points and at the endpoints a and b.

Step 3. The largest of the values in Step 2 is the absolute maximum value of f on $[a, b]$ and the smallest value is the absolute minimum.

▶ **Example 1** Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[1, 5]$, and determine where these values occur.

▶ **Example 2** Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$, and determine where these values occur.

Exercise : 4.4

7–16 Find the absolute maximum and minimum values of f on the given closed interval, and state where those values occur.

7. $f(x) = 4x^2 - 12x + 10;\ [1, 2]$

8. $f(x) = 8x - x^2;\ [0, 6]$

9. $f(x) = (x - 2)^3;\ [1, 4]$

10. $f(x) = 2x^3 + 3x^2 - 12x;\ [-3, 2]$

11. $f(x) = \dfrac{3x}{\sqrt{4x^2 + 1}};\ [-1, 1]$

12. $f(x) = (x^2 + x)^{2/3};\ [-2, 3]$

13. $f(x) = x - 2\sin x;\ [-\pi/4, \pi/2]$

14. $f(x) = \sin x - \cos x;\ [0, \pi]$

Class Activity

- **Example:** Analyze the function $f(x) = 3x^5 - 20x^3$

Increasing :	$\boldsymbol{x < -2}$ **and** $\boldsymbol{x > 2}$
Decreasing :	$\boldsymbol{-2 < x < 2}$
Local Max. points and Max values:	Max. at $x = -2$, **Max (-2 , 64)**
Local Min. points and Min values:	Min. at $x = 2$, **Max (2 , -64)**
Inflection points at:	**(-1.414 , 39.6) , (0 , 0) , (-1.414 , -39.6)**
Concave Up :	$\boldsymbol{-1.414 < x < 0}$ and $\boldsymbol{x > 1.414}$
Concave Down :	$\boldsymbol{x < -1.414}$ and $\boldsymbol{0 < x < 1.414}$

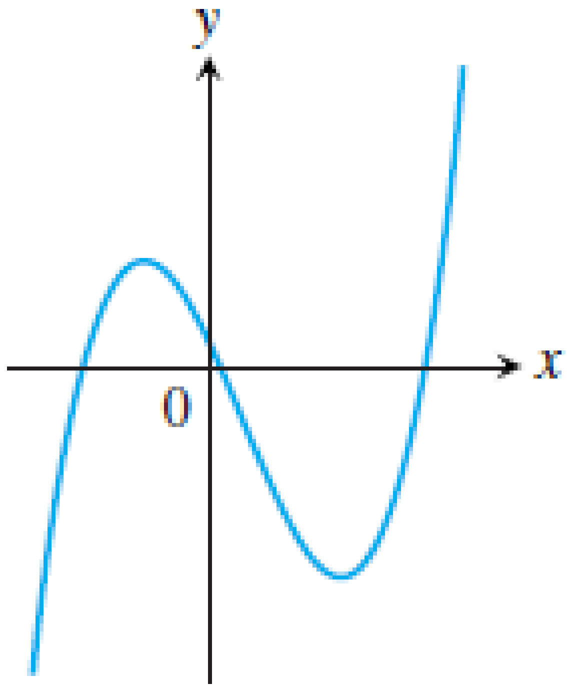
$$Example:\ Given\ \ that\ f'(x) = 4x^3 - 12x^2 \quad find\ Extrema$$

Quiz-Class Activity

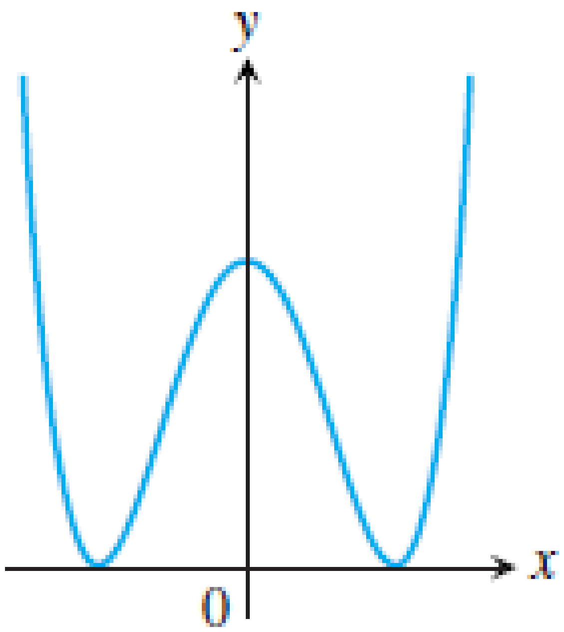
Analyzing Graphed Functions

Identify the inflection points and local maxima and minima of the functions graphed in Exercises 1–8. Identify the intervals on which the functions are concave up and concave down.

1. $y = \dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x + \dfrac{1}{3}$



2. $y = \dfrac{x^4}{4} - 2x^2 + 4$



Solution format:

Increasing in the intervals:	
Decreasing in the intervals:	
Local Max. points and Max values:	
Local Min. points and Min values:	
Inflection points at:	
Concave Up in the intervals:	
Concave Down in the intervals:	

Soln:Final results in tabular form:

EX-1

Increasing	$(-\infty,-1)U(2,\infty)$
Decreasing	(-1,2)
Concave up	$(\frac{1}{2},\infty)$
Concave down	$(-\infty,\frac{1}{2})$
Local Maxima	$(-1,\frac{3}{2})$
Local Minima	(2,-3)
Inflection	$(\frac{1}{2},\frac{-3}{4})$

Ex-2

Increasing	$(-2,0)U(2,\infty)$
Decreasing	(-∞,-2)U(0,2)
Concave up	$(-\infty,\frac{-2}{\sqrt{3}})U(\frac{2}{\sqrt{3}},\infty)$
Concave down	$(\frac{-2}{\sqrt{3}},\frac{2}{\sqrt{3}})$
Local Maxima	(0,4)
Local Minima	$(\pm 2,0)$
Inflection	$(\pm\frac{2}{\sqrt{3}},\frac{16}{9})$

Q-5(Mid1) Draw the graph

- $f(x) = 5 + 12x - x^3$

  Locate Intercepts, critical point

  Inflection point, Extrema

  then verify Inc/Dec

  and up/down interval