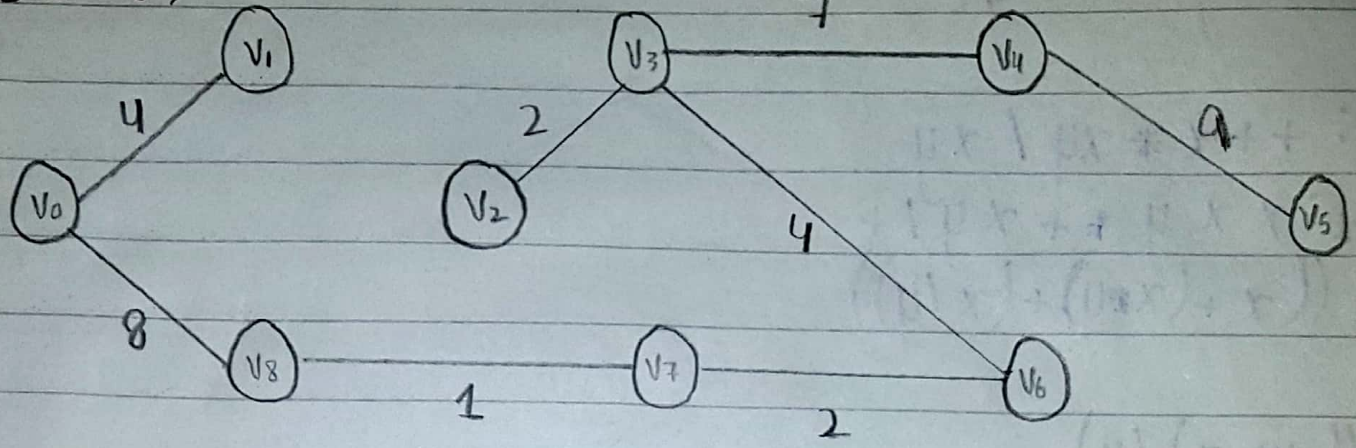
Q.1

a) 3
b) 0
c) 5
d) u, v
e) d
f) k, l
g) m → s, t → x, y
h) b, c, d, e, f, h, i, k, m, n, o, t
i) v, n, h, d, a.
j) j, l, q, r, s, x, y, g, u, z, w, p

Q.2: b)

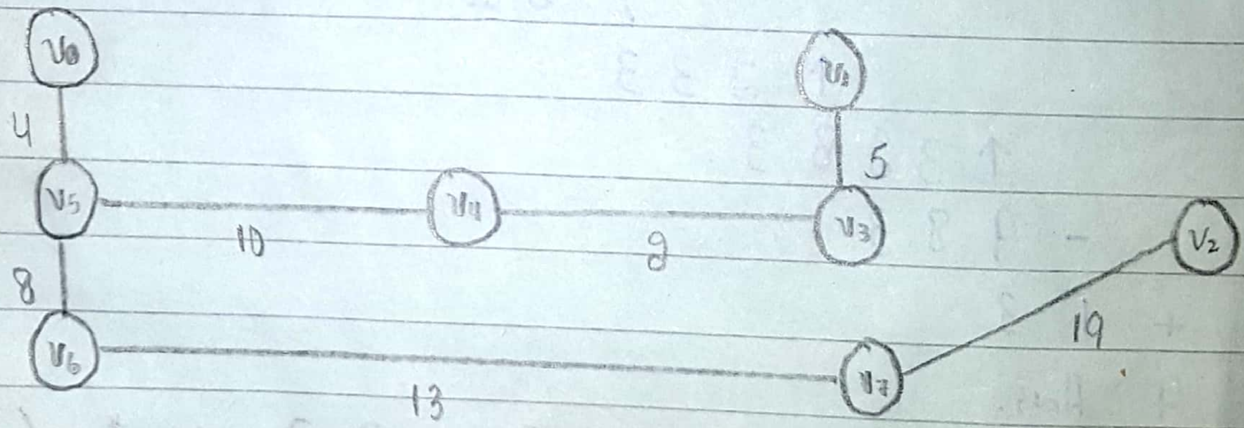


Prim's: $(V_8, V_7), (V_7, V_6), (V_6, V_3), (V_3, V_2), (V_3, V_4),$
$(V_8, V_0), (V_0, V_1), (V_4, V_5) \Rightarrow 37$

Kruskal's: $(V_8, V_7), (V_7, V_6), (V_2, V_3), (V_3, V_6), (V_0, V_1),$
$(V_3, V_4), (V_0, V_8), (V_4, V_5). \Rightarrow 37$

$\Rightarrow \quad 4+8+1+2+4+2+7+9 \Rightarrow 37.$

a)

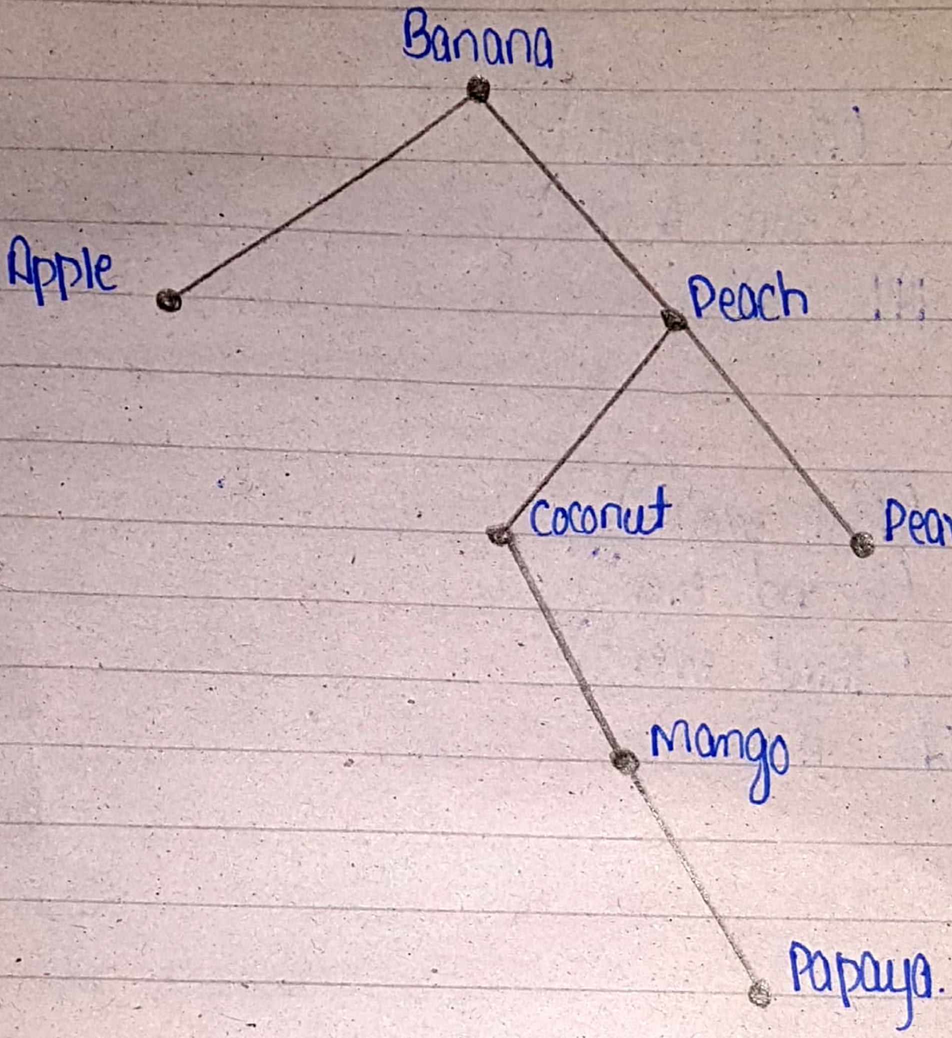


Prim's: $(V_4, V_3), (V_3, V_1), (V_4, V_5), (V_5, V_0), (V_5, V_6),$
$(V_6, V_7), (V_7, V_2) \Rightarrow 61$

Kruskal's: $(V_4, V_3), (V_5, V_0), (V_3, V_1), (V_5, V_6), (V_5, V_4),$
$(V_6, V_7), (V_7, V_2) \Rightarrow 61$

$\Rightarrow 4+8+10+9+13+5+19 \Rightarrow 61$

Q.3 a)



b)(i)

→ A tree with n vertices has $n-1$ edges.

$= 10{,}000 - 1 \Rightarrow 9999$ edges.

ii) A full binary has 2 edges for each vertex.

$1000 \cdot 2 \Rightarrow 2000$ edges.

Q.4 (i)

(a) PreOrder: a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q.

InOrder: k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

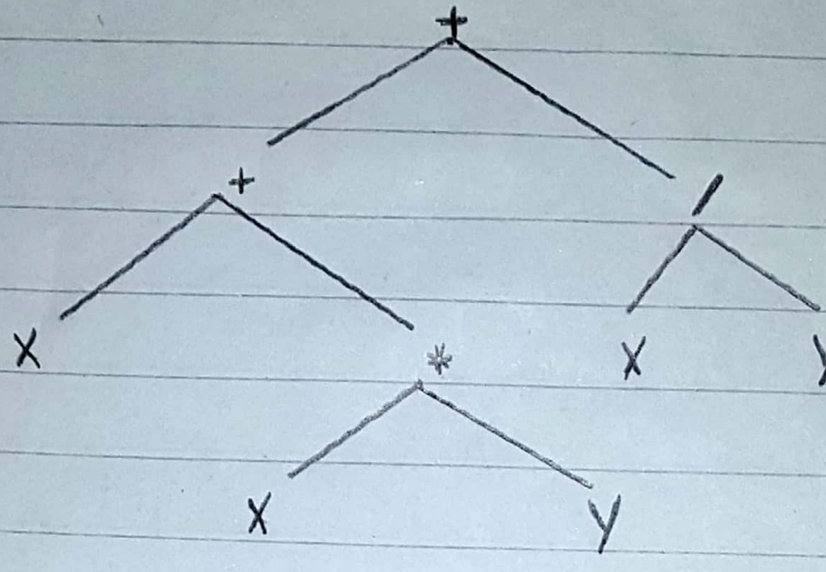
PostOrder: k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a.

b) PreOrder: a, b, d, e, i, j, m, n, o, c, f, g, h, k, l, p

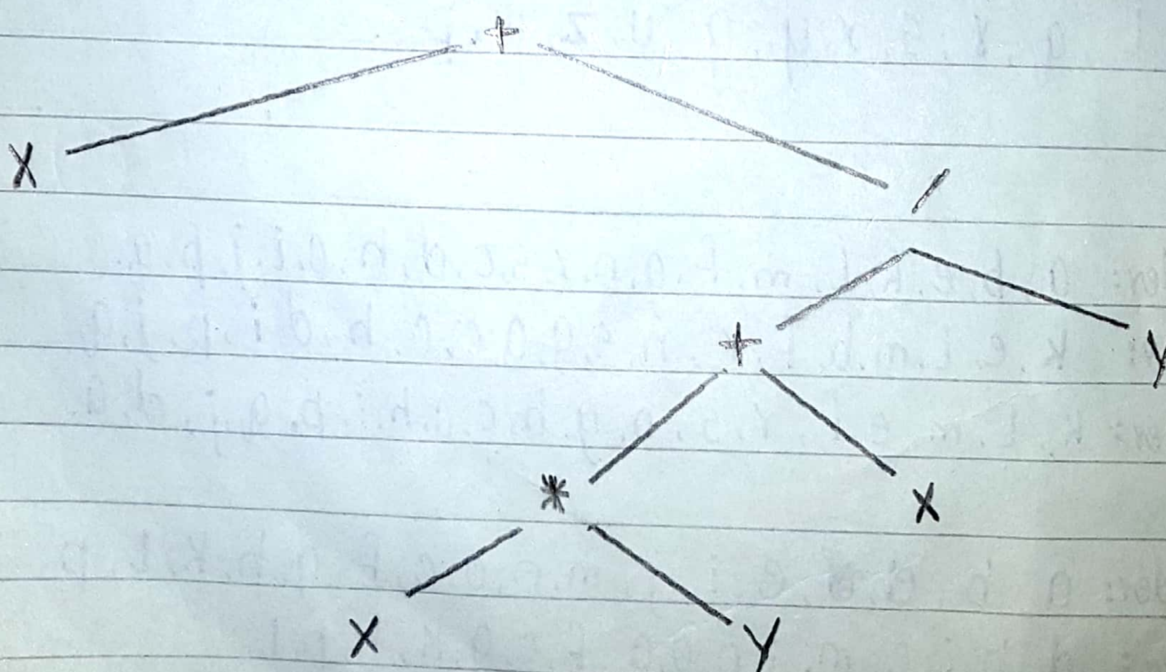
InOrder: d, b, i, e, m, j, n, o, a, f, c, g, k, h, p, l.

PostOrder: d, i, m, n, o, j, e, b, f, g, k, p, l, h, c, a.

b) $(x+xy)+(x/y)$



• $x+((xy+x)/y)$



Q.5: $(x+xy)+(x/y)$

Prefix: $++x*xy/xy$
Post Fix: $xxy*+xy/+$
Infix: $((x+(x*y))+(x/y))$

- $x+((xy+x)/y)$

Prefix: $+x/+*xyxy$
Post Fix: $xxy*x+y/+$
Infix: $(x+(((x*y)+x)/y))$

b) + − ↑ 3 2 ↑ 2 3 / 6 − 4 2

/ 6 2

↑ 2 3 3

↑ 3 2 8 3

− 9 8 3

+ 1 3

⇒ 4 Ans.

- 4 8 + 6 5 − * 3 2 − 2 2 + * /

12 6 5 −

12 1 *

12 3 2 −

12 1 2 2 +

12 1 4 *

12 4 /

⇒ 3 Ans.

Q.6: a)

Floors = 27. (First event)
Offices = 37. (Second event)
$27 \cdot 37 \Rightarrow 999.$ Ans.

Q.6(b):

Color = 12 (First event)
Gender = 2 (Second event)
Size = 3. (Third event)
$12 \cdot 2 \cdot 3 \Rightarrow 72.$ Ans.

Q.7(a):

First = 26
Second = 26
Third = 26
$26 \cdot 26 \cdot 26 \Rightarrow 17576$ Ans.

Q.7(b):

First = 26
Second = 25
Third = 24
$26 \cdot 25 \cdot 24 \Rightarrow 15.600$ Ans.

Q.8(b):

- Number of Alphabets $\Rightarrow 26$
- Total 4 lettered words $\Rightarrow 26 \cdot 26 \cdot 26 \cdot 26 \rightarrow 456,976.$
- Letters without "x" $\Rightarrow 25 \cdot 25 \cdot 25 \cdot 25 \Rightarrow 390,625.$
- Strings that are not strings of length 4 without "x":

$\Rightarrow 456,976 - 390,625$

$\Rightarrow 66,351$ strings.

Q.8(a)

- Possible Hexa-Digits (16): 0 - 9 , A - F

- String of length 10: $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdots 16 = 16^{10}$
- String of length 26: $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdots 16 = 16^{26}$
- String of length 58: $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdots 16 = 16^{58}$

$\Rightarrow 16^{94}$ Ans.

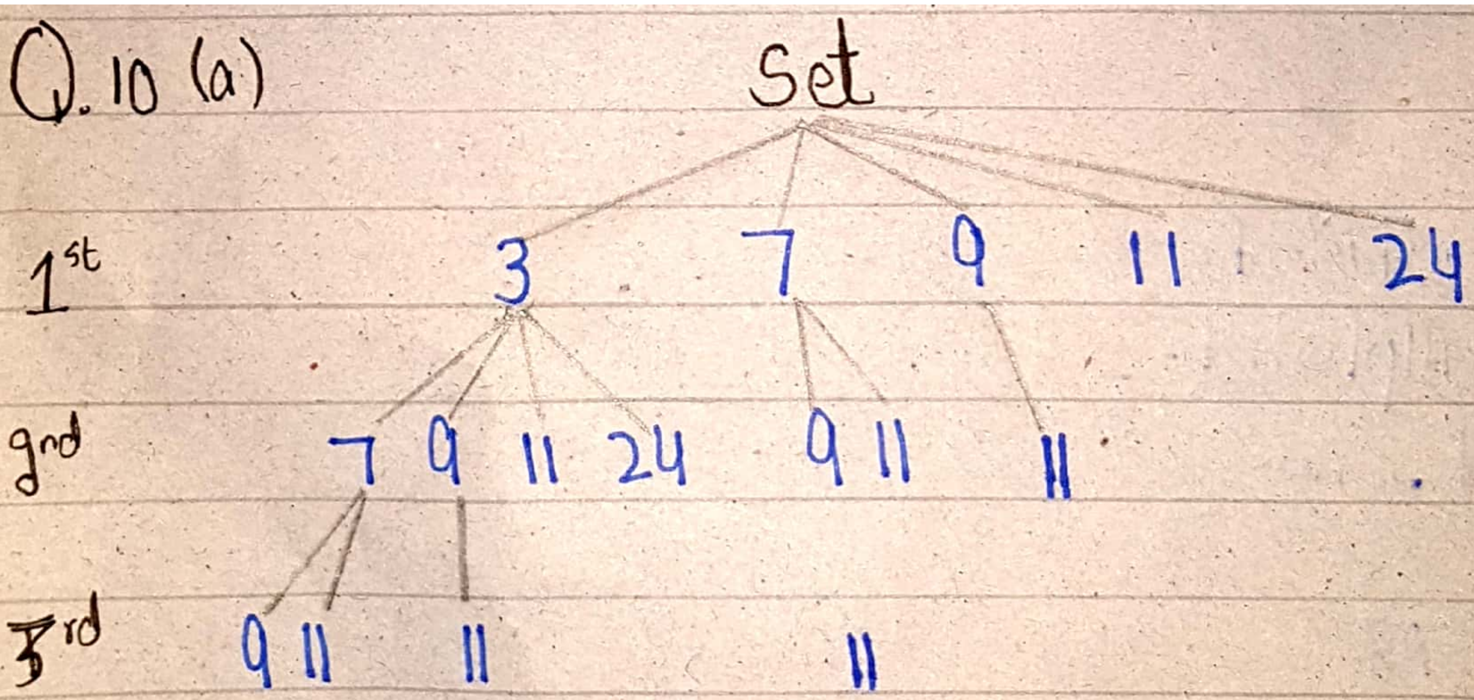
Q.9 (a)

⇒ First element = two ways.
⇒ Second element = two ways.
→ Third element = two ways.
⋮
⇒ n^{th} element = two ways.
$= 2 \cdot 2 \cdot 2 \cdots 2 = 2^n$ Answer.

9(b):

Product Rule:
$\Rightarrow 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
$\Rightarrow 120$ ways Ans.

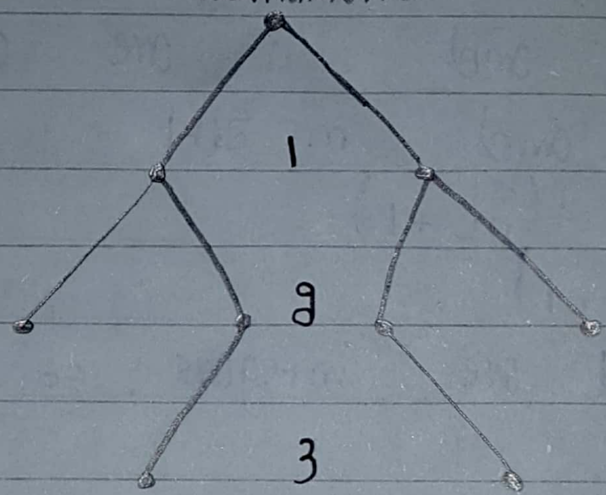
Q. 10 (a)



• 17 subsets.

Q.10 (b)

Tournament



won by A — Won by B

won by B — Won by A

Won by A — Won by B

Total number of matches to be played: 3

Q.11 (a)

$\Rightarrow {}^{8}C_3$

$\Rightarrow \dfrac{8 \cdot 7 \cdot \cancel{6}}{3 \cdot \cancel{2}}$

$= 56$ ways.

Q.11 (b)

$\Rightarrow {}^{12}C_6$

$\Rightarrow \dfrac{\cancel{12} \cdot 11 \cdot \cancel{10} \cdot \cancel{9}^{3} \cdot \cancel{8}^{4} \cdot 7}{\cancel{6}^{2} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$

$\Rightarrow 11 \cdot 3 \cdot 4 \cdot 7$

$\Rightarrow 924$ ways

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Q.11 (c)

$\Rightarrow$ 9.8.7.6.5

$\Rightarrow$ $\Rightarrow$ 15120. ways.

Q.19 (a):

$\Rightarrow \; ^{20}P_5 \Rightarrow 20 \times 19 \times 18 \times 17 \times 16$

$\Rightarrow \; 1,860,480.$

b)

$\Rightarrow \; ^{16}P_4 \Rightarrow 16 \cdot 15 \cdot 14 \cdot 13$

$\Rightarrow \; 43,680.$

c)

$\rightarrow \; ^{15}P_2 \Rightarrow 15 \cdot 14.$

$\rightarrow \; 210.$

Q.13(b):

Sol: $15 \times 48 \times 24 \times 34 \times 28 \times 28$

$\Rightarrow$ 460,615,680 Ans.

Q.13(a):

Sol: ${}^{5}C_{1} \cdot {}^{3}C_{2} \cdot {}^{4}C_{1} \cdot {}^{6}C_{3}$

$\Rightarrow 5 \cdot 3 \cdot 4 \cdot 20$

$\Rightarrow$ 1200 Ans.

Q. 14(a):

- String of length 10 that begin with three 0's:

- First, second, third digit = 1 way.
- Fourth, fifth, ..., tenth = 2 ways.
- $= 1 \cdot 1 \cdot 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7 \Rightarrow 128$
- String of length 10 that end with two 0's:
- First, second, ..., Eight = 2 ways.
- Ninth, Tenth = 1 way.
- $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 2^8 \Rightarrow 256$

- $2^7 + 2^8 - 2^5 \Rightarrow 352$ Ans.

Q.14(b):

= 1^{st} digit = 1 way

= 2^{nd}, 3^{rd}, 4^{th}, 5^{th} = 2 ways.

= $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^4 \Rightarrow 16.$

= 1^{st}, 2^{nd}, 3^{rd} = 2 ways.

= 4^{th}, 5^{th} = 1 way.

= $2 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 2^3 \Rightarrow 8$

$\Rightarrow (16+8) - 2^2$

$\Rightarrow (16+8) - 4$

$\Rightarrow 20$ Ans.

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Q. 15

a) $M = 30 \rightarrow$ Students

$N = 26 \rightarrow$ Alphabets

$\Rightarrow \left\lceil \frac{30}{26} \right\rceil = 2$. Ans.

b) $M = 8{,}008{,}278$.

$N = 1{,}000{,}000$

$\left\lceil \frac{8{,}008{,}278}{1{,}000{,}000} \right\rceil \Rightarrow 9$.

c) $M = 677 \rightarrow$ Classes.

$N = 38 \rightarrow$ Periods

$\left\lceil \frac{677}{38} \right\rceil \Rightarrow 18$.

Q. 16 (a): Co-efficient of x^5 in $(1+x)^{11}$

$\Rightarrow \binom{11}{6} x^5 \Rightarrow \frac{11!}{6!\,(11-6)!} \Rightarrow \frac{11!}{6!\,5!}$

$\Rightarrow \frac{11 \times \cancel{10}^{1} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times \cancel{6!}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}} \Rightarrow 462$

$\Rightarrow$ Co-efficient of $x^5 = 462.$ Ans.

Q. 16 (b): Co-efficient of $a^7 b^{17}$ in $(2a-b)^{24}$

$\Rightarrow \binom{24}{17} a^7 b^{17} \Rightarrow \frac{24!}{17!\,(24-17)!} \Rightarrow \frac{24!}{17!\,7!}$

$\Rightarrow \frac{\cancel{24}^{12\,2} \times 23 \times 22 \times \cancel{21} \times \cancel{20} \times 19 \times 18 \times \cancel{17!}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{17!}} \Rightarrow 346{,}104$

$\Rightarrow$ Co-efficient of $a^7 b^{17} = 346{,}104$ Ans.

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Q.17 (a):

If $a|b$ and $b|c$.

Then there are integers m and n such that

$b = am$; $c = bn$.

Then

$c = bn$

$= (am)n = a(mn)$

So, $a|c$.

17(b): Consider the above statement.

$b = am$; $c = an$

Then $b+c$:

$am + an = a(m+n)$

So, $a | (b+c)$

18(a):

For $n = 7$ (Prime)

$2^n - 1 \Rightarrow 2^7 - 1$

$= 128 - 1$

$= 127$ (Prime)

- This statement is correct.

Q.18(b): Assumption.

suppose P/a and $P/(a+1)$ 'r' and 's'.

Then there exists:

$a = pr$ AND $a+1 = ps$.

subtract:

$$a+1-a = ps - pr$$

$$1 = p(s-r)$$

since $(s-r)$ is also constant: $P/1$

Divisor of 1 are 1 and -1 and $P>1$, because P is prime from statement.

This is a contradiction and makes our assumption false.

. Initial statement is true.

Q.19(a): $\sqrt{(a+b)} = \sqrt{a} + \sqrt{b}$

• Sq on b/s

$\Rightarrow a+b = a + 2\sqrt{ab} + b$

$\Rightarrow a-a+b-b = 2\sqrt{ab}$

$\Rightarrow 0 = 2\sqrt{ab}$ • Sq on b/s

$\Rightarrow ab = 0$

• Either $a=0$ or $b=0$.

• Now $a=16$ and $b=0$

$= \sqrt{16+0} = \sqrt{16} + \sqrt{0}$

$= \sqrt{16} = \sqrt{16}$

$= 4 = 4$ Ans Proved.

Q. 19 (b): By Contra positive:

Suppose: $x = -y$

ⓐ $-y <= -1$ Then $|-y| <= 1$

$y <\neq 1$

ⓑ suppose $x = y$

$y >= -1$

$|y| <= 1$

$y <\neq 1$

Our assumption is wrong.

Proved!

20 (a):

$x = \sqrt{5}$ irrational

$y = \frac{\sqrt{5}}{5}$ irrational.

$$x \cdot y = \sqrt{5} \cdot \frac{\sqrt{5}}{5}$$

$$x \cdot y = \frac{5}{5} \Rightarrow 1$$

→ 1 is not irrational and so $x \cdot y$ is not irrational

→ statement Disproved.

20 (b): rational + irrational = rational

a/b x $\frac{m}{n}$

- $\frac{a}{b} + x = \frac{m}{n}$
- $x = \frac{m}{n} - \frac{a}{b}$
- $x = \frac{bm - an}{nb}$ > rational.

→ According to this "x" must be rational.

- rational + irrational = irrational proved!

Q.91(a):

- Suppose $n = 19$ → Prime.

Now $n+9 \Rightarrow 19+9 = \boxed{91}$

91 is not a prime number.

This a counter to this, and thus disproving this proposition.

Q.91(b)

- Suppose there are finite prime numbers.

$\{p_1, p_2, p_3, \ldots, p_n.\}$

Now a new number P.

$P = P_1 \times P_2 \times P_3 \times \cdots \times P_n + 1$

Now accordingly P is larger than any of the prime numbers, and since $\{p_1, p_2, \ldots, p_n\}$ consist of all the prime numbers P can't be prime.

Thus P must be divisible by atleast one of the many primes $(1 < n < n)$

- If we divide P by P_n we get a remainder 1. This is a contradiction.
- Thus there are infinitely many primes.

Q. 22(a):

- Suppose m and n are odd integers
- Then $n = 2k+1$ and $m = 2l+1$
- $m+n = (2k+1) + (2l+1)$
  $= 2(k+l+1)$
- Since k and l are integers, so is $k+l+1$.
- ∴ $p = k+l+1$.
- Hence $m+n = 2p$
- This shows that $m+n$ is even.

Q. 22(b): Proof by contradiction:

Suppose there exist integers m and n such that $m+n$ is even and either m is even and n is odd or m is odd and n is even, then sum $n+m$ is odd.
This ~~cond~~ contradicts the supposition that $m+n$ is even. Hence supposition is false and original statement is true.

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Q.23 (a):

- Suppose there are integers a and b for which

$\rightarrow \quad 6 - 7\sqrt{2} = \dfrac{a}{b}$

$\rightarrow \quad -7\sqrt{2} = \dfrac{a}{b} - 6$

$\rightarrow \quad \sqrt{2} = \dfrac{6b - a}{7b}$ > Rational

- So $\sqrt{2}$ is also rational
- But $\sqrt{2}$ is irrational and this contradicts our assumption.

- Hence $6 - 7\sqrt{2}$ is irrational.

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Q.23(b) Prove by contradiction that $\sqrt{2} + \sqrt{3}$ is irrational.

→ Suppose $\sqrt{2} + \sqrt{3}$ is rational.

- $\sqrt{2} + \sqrt{3} = \dfrac{a}{b}$ a and b are integars.
- Squaring on both sides.
- $(\sqrt{2} + \sqrt{3})^2 = \dfrac{a^2}{b^2}$
- $5 + 2\sqrt{6} = \dfrac{a^2}{b^2}$
- $2\sqrt{6} = \dfrac{a^2}{b^2} - 5$
- $2\sqrt{6} = \dfrac{a^2 - 5b^2}{b^2}$
- $\sqrt{6} = \dfrac{a^2 - 5b^2}{2b^2}$ > rational
- So, $\sqrt{6}$ is also rational.
- But $\sqrt{6}$ is irrational, so this contrdicts our assumption.

- Hence $\sqrt{2} + \sqrt{3}$ is irrational.

Q. 34 (a):

i) (1,5), (2,4), (3,3), (4,2), (5,1)

ii) (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

iii) (1,4), (2,4), (3,4)

b) Possible outcomes = 9

Total outcomes = 99

$\Rightarrow \frac{9}{99} \Rightarrow \frac{1}{11}$ Ans.

Q. 95(a):

$\Rightarrow$ $n = 1$

$n^2 = (n(n+1)(2n+1))/6$

$1 = (1(1+1)(2n+1))/6$

$1 = 6/6$

$1 = 1$ proved.

Rough work: $n = k+1$

$(k+1)^2 = ((k+1)(k+1+1)(2(k+1)+1))/6$

$k^2+2k+1 = (k+1)(2k^2+3k+4k+6)/6$

$k^2+2k+1 = 2k^3+9k^2+13k+6/6.$

• Now $n = k$

• $1^2+2^2+3^2+\dots+k^2 = k(k+1)(2k+1)/6$

$n = k^2+2k+1$ add.

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$= 1^2 + 2^2 + \dots + k^2 + k^2 + 2k + 1 = \dfrac{2k^3 + 3k^2 + k + k^2 + 2k + 1}{6}$

$=$ " " " " " $= \dfrac{2k^3 + 3k^2 + k + 6k^2 + 12k + 6}{6}$

$= 1^2 + 2^2 + 3^2 +$ " " " $+ \dots = \dfrac{2k^3 + 9k^2 + 13k + 6}{6}$

= proved.

ii) $n = 0$

$= 2^0 = 2^{0+1} - 1$

$= 1 = 1$ proved.

Rough Work

$2^{k+1} = 2^{k+1+1} - 1$

$2^{k+1} = 2^{k+2} - 1$

. Add 2^{k+1} b/s. and $n = k$

$1 + 2 + 2^2 + \dots + 2^k + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1}$

$= 2^{k+1} + 2^{k+1} - 1$

$= 2^{k+1}(1+1) - 1$

$= \cancel{2^{k+1}}\ 2^{k+1}(2^1) - 1$

$= 2^{k+1+1} - 1$

$= 2^{k+2} - 1$ proved.