(Use Trigonometric substitution)

1. Evaluate the following indefinite integrals.

(1)
$$\int \frac{1}{\sqrt{1-4x^2}} dx$$
 (2) $\int \frac{1}{x^2+25} dx$

(2)
$$\int \frac{1}{x^2 + 25} dx$$

$$(3) \quad \int \frac{x}{x^4 + 16} \, dx$$

$$(4) \quad \int \frac{1}{\sqrt{2-5x^2}} \, dx$$

(4)
$$\int \frac{1}{\sqrt{2-5x^2}} dx$$
 (5) $\int \frac{3}{x\sqrt{x^2-9}} dx$

$$(6) \quad \int \frac{x}{\sqrt{16 - 9x^4}} \, dx$$

(7)
$$\int \frac{1}{x\sqrt{16x^2-9}} dx$$
 (8) $\int \frac{e^x}{7+e^{2x}} dx$

$$(8) \quad \int \frac{e^x}{7 + e^{2x}} \, dx$$

$$(9) \quad \int \frac{\sin x}{\sqrt{2 - \cos^2 x}} \, dx$$

(10)
$$\int \frac{1}{\sqrt{x(1+x)}} dx$$
 (11) $\int \frac{1}{x^2\sqrt{4-x^2}} dx$ (12) $\int \frac{1}{x\sqrt{x^2+4}} dx$

(11)
$$\int \frac{1}{x^2 \sqrt{4 - x^2}} \, dx$$

$$(12) \quad \int \frac{1}{x\sqrt{x^2+4}} \, dx$$

(13)
$$\int \frac{\sqrt{9-x^2}}{x^2} dx$$

(13)
$$\int \frac{\sqrt{9-x^2}}{x^2} dx$$
 (14) $\int \frac{1}{x\sqrt{25-x^2}} dx$

$$(15) \quad \int \frac{1}{\sqrt{x^2 - a^2}} \, dx$$

(Use Partial fraction)

EXERCISES Find the indefinite integral.

1.
$$\int \frac{6x+5}{x+2} dx$$

2.
$$\int \frac{4x^2 - 12x - 25}{x - 5} dx$$

3.
$$\int \frac{5x^3 + 3x - 2}{x - 1} dx$$

4.
$$\int \frac{x^3 + 3x^2 - 4x - 6}{x^2 + 2x - 15} dx$$
 5.
$$\int \frac{4x^2 - 8x + 3}{x^2 - 3x - 4} dx$$

5.
$$\int \frac{4x^2 - 8x + 3}{x^2 - 3x - 4} dx$$

$$6. \int \frac{x^3 - 3x^2}{x^2 - 3x - 10} \, dx$$

EXERCISES Find the indefinite integral.

$$1. \int \frac{3-4x}{x^2+x} \, dx$$

2.
$$\int \frac{x}{x^2 + 7 + 10} dx$$

3.
$$\int \frac{6}{3x^2 - 14x + 8} \, dx$$

4.
$$\int \frac{3x^2 + 8x - 7}{(x+4)(x+3)(x+1)} dx$$

4.
$$\int \frac{3x^2 + 8x - 7}{(x+4)(x+3)(x+1)} dx$$
 5. $\int \frac{2-4x^2}{(x+2)(x-2)(x-5)} dx$ **6.** $\int \frac{3x}{(x+4)(x-1)(x-3)} dx$

6.
$$\int \frac{3x}{(x+4)(x-1)(x-3)} \, dx$$

7.
$$\int \frac{3-2x}{x^2+6x+9} dx$$

8.
$$\int \frac{3x-1}{x^3-2x^2} dx$$

9.
$$\int \frac{2x^2 + x + 4}{(x+1)(x-4)^2} dx$$

10.
$$\int \frac{5x^2 + 8x + 6}{(x+4)(x^2+2)} dx$$

11.
$$\int \frac{12x+18}{(x+3)(2x^2+8x+9)} dx$$
 12.
$$\int \frac{15-25x}{(x-4)(2x^2-6x+9)} dx$$

12.
$$\int \frac{15 - 25x}{(x - 4)(2x^2 - 6x + 9)} dx$$

ANSWERS

1.
$$\frac{1}{2}\arcsin 2x + C$$

2.
$$\frac{1}{5}\arctan(\frac{x}{5}) + C$$

3.
$$\frac{1}{8}\arctan(\frac{x^2}{4}) + C$$

4.
$$\frac{1}{\sqrt{5}}\arcsin(\frac{x\sqrt{5}}{\sqrt{2}}) + C$$

5.
$$\sec^{-1}(\frac{x}{3}) + C$$

6.
$$\frac{1}{6}\arcsin(\frac{3x^2}{4}) + C$$

7.
$$\frac{1}{3}\sec^{-1}(\frac{4x}{3}) + C$$

8.
$$\frac{1}{\sqrt{7}}\arctan(\frac{e^x}{\sqrt{7}}) + C$$

9.
$$-\arcsin(\frac{\cos x}{\sqrt{2}}) + C$$

10.
$$2 \arctan \sqrt{x} + C$$

11.
$$-\frac{\sqrt{4-x^2}}{4x} + C$$

12.
$$\frac{1}{2} \ln \left| \frac{\sqrt{x^2 + 4} - 2}{x} \right| + C$$

ANSWERS

1.
$$\int \frac{3-4x}{x(x+1)} dx = \int \left[\frac{3}{x} - \frac{7}{x+1} \right] dx = 3 \ln|x| - 7 \ln|x+1| + C$$

2.
$$\int \frac{x}{(x+5)(x+2)} dx = \int \left[\frac{5}{3} \cdot \frac{1}{x+5} - \frac{2}{3} \cdot \frac{1}{x+2} \right] dx = \frac{5}{3} \cdot \ln|x+5| - \frac{2}{3} \cdot \ln|x+2| + C$$

3.
$$\int \frac{6}{(3x-2)(x-4)} dx = \int \left[-\frac{9}{5} \cdot \frac{1}{3x-2} + \frac{3}{5} \cdot \frac{1}{x-4} \right] dx = -\frac{3}{5} \cdot \ln |3x-2| + \frac{3}{5} \cdot \ln |x-4| + C$$

4.
$$\int \left[\frac{3}{x+4} + \frac{2}{x+3} - \frac{2}{x+1} \right] dx = 3 \ln |x+4| + 2 \ln |x+3| - 2 \ln |x+1| + C$$

5.
$$\int \left[-\frac{1}{2} \cdot \frac{1}{x+2} + \frac{7}{6} \cdot \frac{1}{x-2} - \frac{14}{3} \cdot \frac{1}{x-5} \right] dx = -\frac{1}{2} \cdot \ln|x+2| + \frac{7}{6} \cdot \ln|x-2| - \frac{14}{3} \cdot \ln|x-5| + C$$

6.
$$\int \left[-\frac{12}{35} \cdot \frac{1}{x+4} - \frac{3}{10} \cdot \frac{1}{x-1} + \frac{9}{14} \cdot \frac{1}{x-3} \right] dx = -\frac{12}{35} \cdot \ln\left|x+4\right| - \frac{3}{10} \cdot \ln\left|x-1\right| + \frac{9}{14} \cdot \ln\left|x-3\right| + C$$

7.
$$\int \frac{3-2x}{(x+3)^2} dx = \int \left[\frac{-2}{x+3} - \frac{9}{(x+3)^2} \right] dx = -2 \ln |3x-2| + \frac{9}{x+3} + C$$

8.
$$\int \frac{3x-1}{x^2(x-2)} dx = \int \left[-\frac{5}{4} \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x^2} + \frac{5}{4} \cdot \frac{1}{x-2} \right] dx = -\frac{5}{4} \cdot \ln|x| - \frac{1}{2} \cdot \frac{1}{x} + \frac{5}{4} \cdot \ln|x-2| + C$$

9.
$$\int \left[\frac{1}{5} \cdot \frac{1}{x+1} + \frac{9}{5} \cdot \frac{1}{x-4} + \frac{8}{(x-4)^2} \right] dx = \frac{1}{5} \cdot \ln|x+1| + \frac{9}{5} \cdot \ln|x-4| - \frac{8}{x-4} + C$$

10.
$$\int \left[\frac{3}{x+4} + \frac{2x}{x^2+2} \right] dx = 3 \ln |x+4| + \ln |x^2+2| + C$$

11.
$$\int \left[\frac{-6}{x+3} + \frac{12x+24}{2x^2+8x+9} \right] dx = -6 \ln|x+3| + 3 \ln|2x^2+8x+9| + C$$

12.
$$\int \left[\frac{-5}{x-4} + \frac{10x-15}{2x^2-6x+9} \right] dx = -5 \ln |x-4| + \frac{5}{2} \ln |2x^2-6x+9| + C$$