
بسم الله الرحمن الرحيم

Physics For Engineers EE (117)

Lecture

Date: 11th Oct, 2019

Oscillations

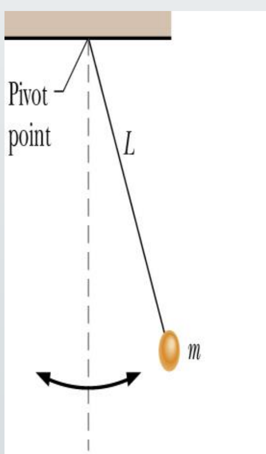
Oscillatory motion

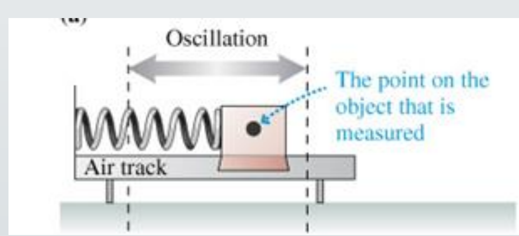
Motion which is periodic in time, that is, motion that repeats itself in time.

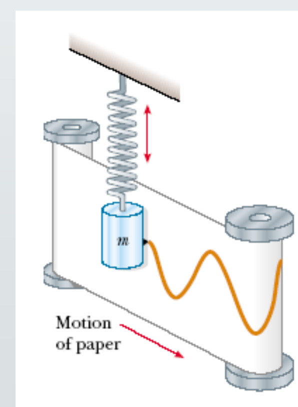
Examples:

- Earthquake oscillations move buildings etc

  Sometimes the oscillations are so severe, that the system exhibiting oscillations break apart.







Simple Harmonic Motion

- Simple harmonic motion (SHM) refers to a certain kind of oscillatory, or wave-like motion that describes the behavior of many physical phenomena:
  - a pendulum
  - a bob attached to a spring
  - low amplitude waves in air (sound), water, the ground
  - the electromagnetic field of laser light
  - vibration of a plucked guitar string
  - the electric current of most AC power supplies

Simple Harmonic Motion

When the block is displaced a small distance x from equilibrium, the spring exerts on the block a force that is proportional to the displacement and given by Hooke's law

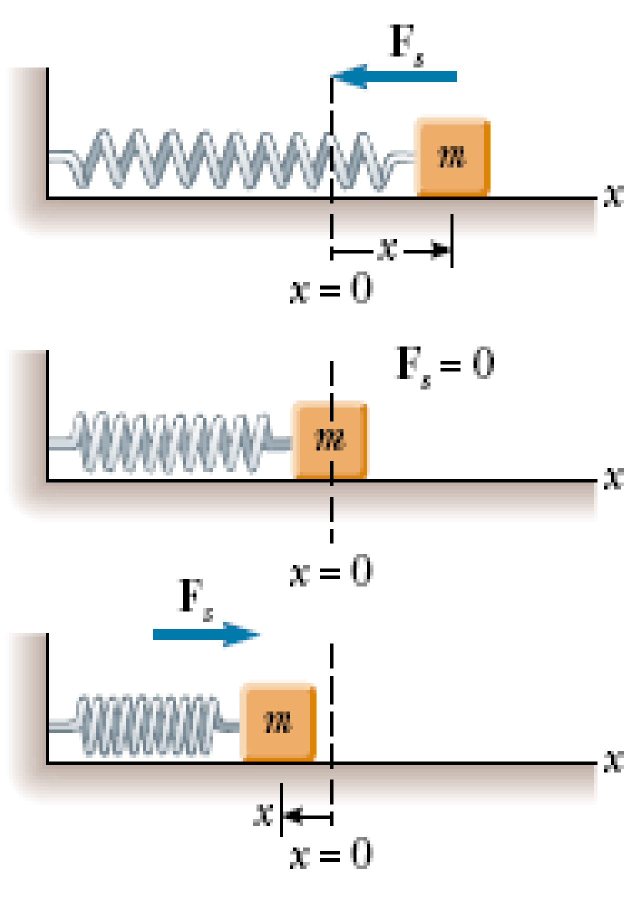
$$F_s = -kx$$

Applying Newton's second law to the motion of the block,

$$F_s = -kx = ma$$

$$a = -\frac{k}{m}x$$

An object moves with simple harmonic motion whenever its acceleration is proportional to its displacement from some equilibrium position and is oppositely directed.

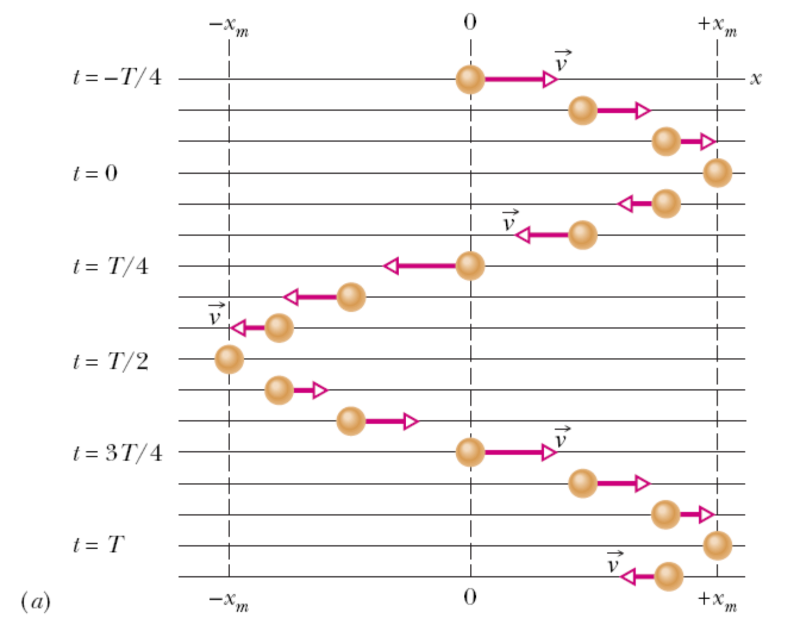


Simple Harmonic Motion

In the figure snapshots of a simple oscillatory system is shown. A particle repeatedly moves back and forth about the point x=0.

The time taken for one complete oscillation is the period, T. In the time of one T, the system travels from $x=+x_m$, to $-x_m$, and then back to its original position x_m.

The velocity vector arrows are scaled to indicate the magnitude of the speed of the system at different times. At $x=\pm x_m$, the velocity is zero.



Simple Harmonic Motion

Frequency of oscillation is the number of oscillations that are completed in each second.

The symbol for frequency is f, and the SI unit is the hertz (abbreviated as Hz).

It follows that

$$T = \frac{1}{f}$$

Simple Harmonic Motion

Any motion that repeats itself is periodic or harmonic.

If the motion is a sinusoidal function of time, it is called simple harmonic motion (SHM).

Mathematically SHM can be expressed as:

$$x(t) = x_m \cos(\omega t + \phi)$$

Here,

- x_m is the amplitude (maximum displacement of the system)
- t is the time
- ω is the angular frequency, and
- ϕ is the phase constant or phase angle

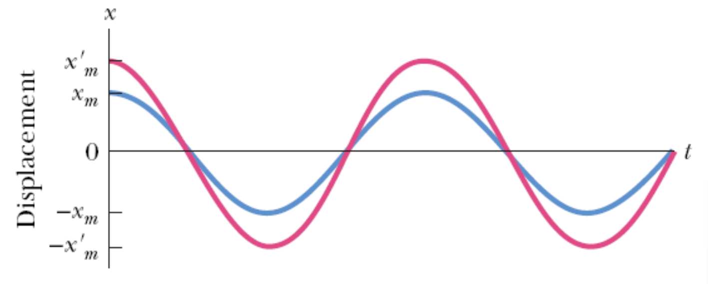
Simple Harmonic Motion

Figure a plots the displacement of two SHM systems that are different in amplitudes, but have the same period.

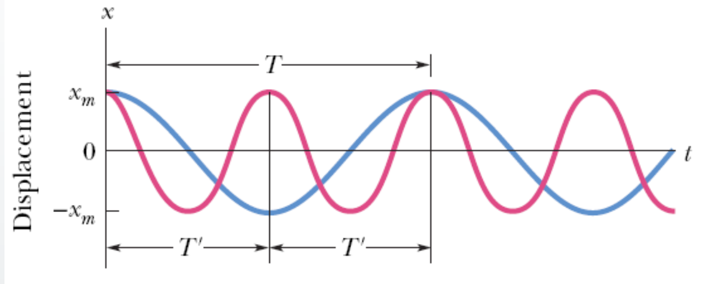
Figure b plots the displacement of two SHM systems which are different in periods but have the same amplitude.

The value of the phase constant term, ϕ, depends on the value of the displacement and the velocity of the system at time t = 0.

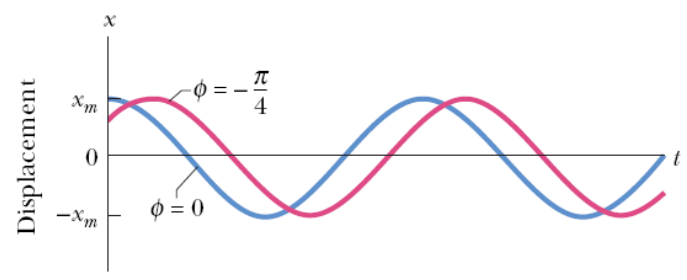
Figure c plots the displacement of two SHM systems having the same period and amplitude, but different phase constants.



(a)



(b)



(c)

For an oscillatory motion with period T,

$$x(t) = x(t+T)$$

The cosine function also repeats itself when the argument increases by 2π. Therefore,

$$\omega(t+T) = \omega t + 2\pi$$

$$\rightarrow \omega T = 2\pi$$

$$\rightarrow \omega = \frac{2\pi}{T} = 2\pi f$$

Here, ω is the angular frequency, and measures the angle per unit time. Its SI unit is radians/second. To be consistent, ϕ then must be in radians.

Simple Harmonic Motion

The velocity of SHM:

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$$\rightarrow v(t) = -\omega x_m \sin(\omega t + \phi$$

The maximum value (amplitude) of velocity is ωx_m. The phase shift of the velocity is $\pi/2$, making the cosine to a sine function.

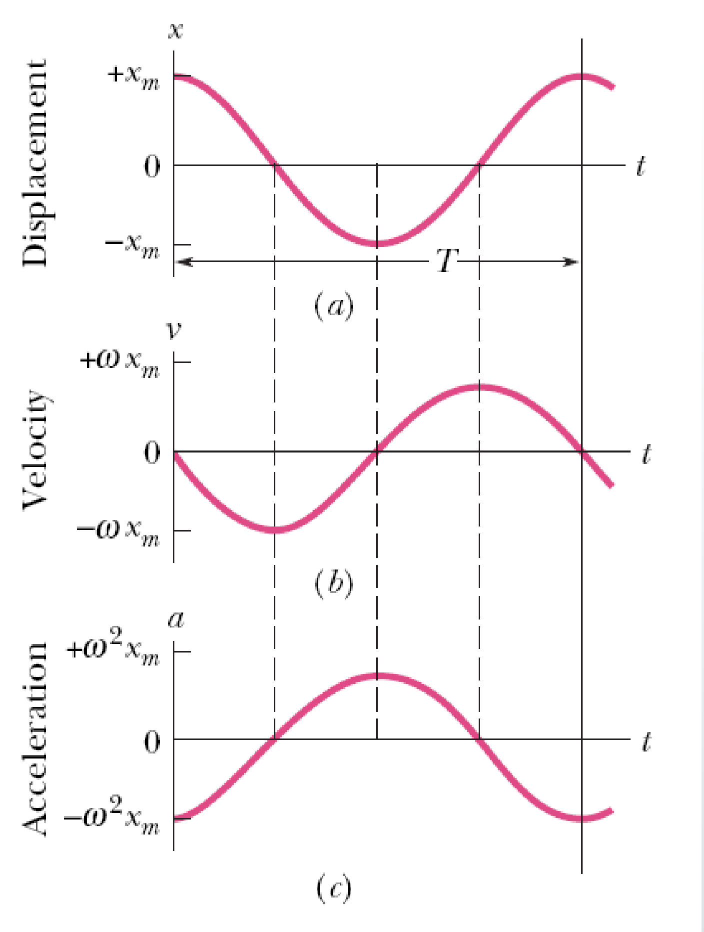
The acceleration of SHM is:

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

$$\rightarrow a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$\rightarrow a(t) = -\omega^2 x(t)$$

The acceleration amplitude is $\omega^2 x_m$.



In SHM a(t) is proportional to the displacement but opposite in sign.

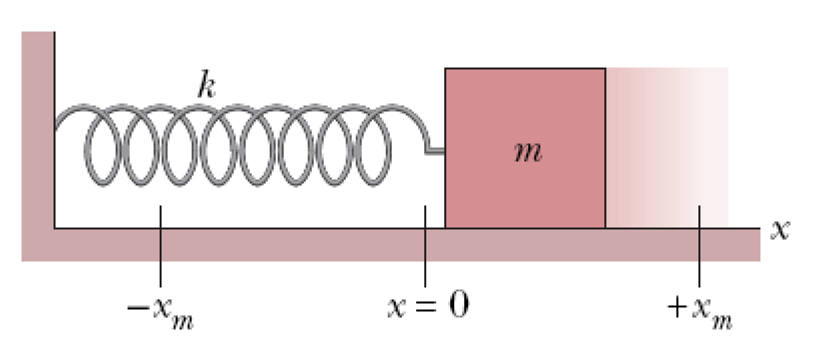
Force Law for SHM

From Newton's 2nd law:

$$F = ma = -(m\omega^2)x = -kx$$

SHM is the motion executed by a system subject to a force that is proportional to the displacement of the system but opposite in sign.

Force Law for SHM

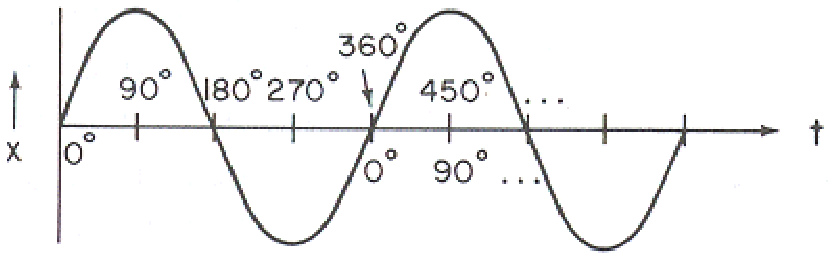


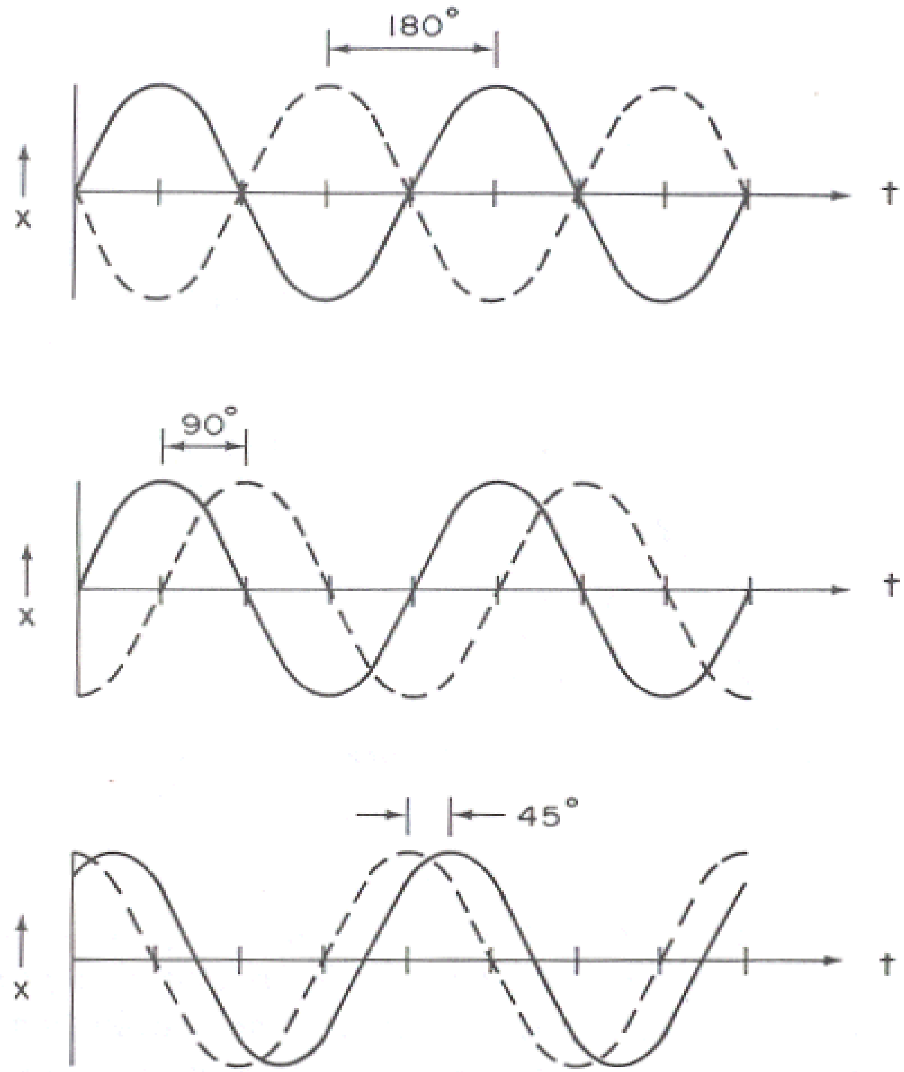
The block-spring system shown on the right forms a linear SHM oscillator.

The spring constant of the spring, k, is related to the angular frequency, ω, of the oscillator:

$$\omega = \sqrt{\frac{k}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

Phase (Time) Phase Diffference





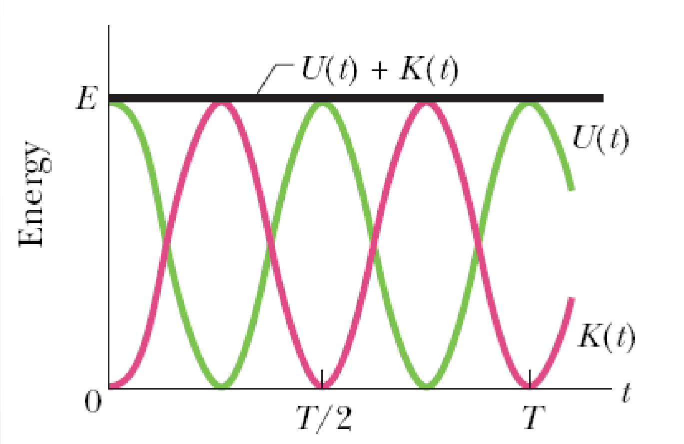
Properties of simple harmonic motion

- The acceleration of the particle is proportional to the displacement but is in the opposite direction. This is the *necessary and sufficient condition for simple harmonic motion*, as opposed to all other kinds of vibration.
- The displacement from the equilibrium position, velocity, and acceleration all vary sinusoidally with time but are not in phase
- The frequency and the period of the motion are independent of the amplitude.

Energy in SHM

The potential energy of a linear oscillator is associated entirely with the spring.

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m{}^2\cos^2(\omega t + \phi)$$

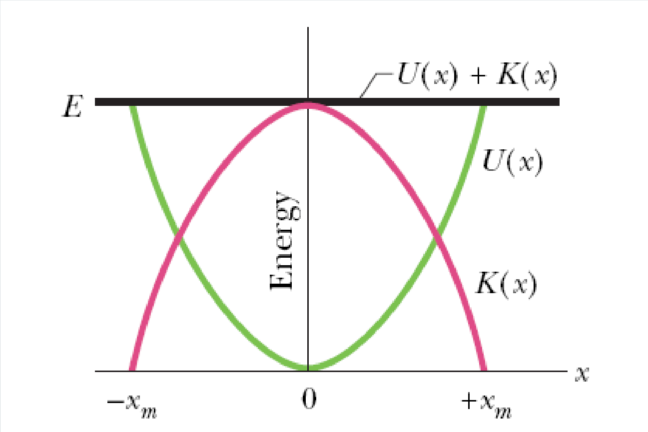


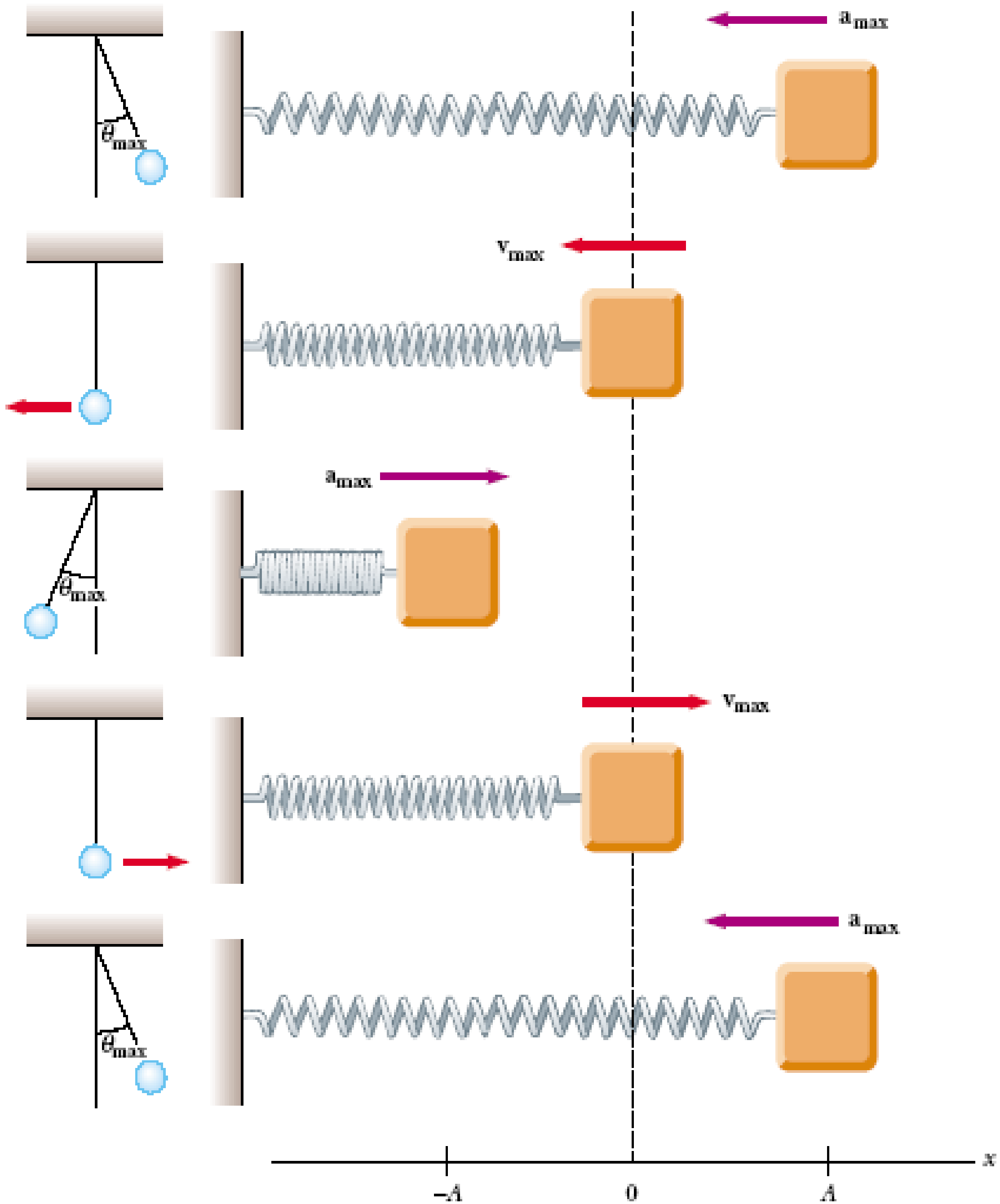
The kinetic energy of the system is associated entirely with the speed of the block.

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 x_m{}^2\sin^2(\omega t + \phi) = \frac{1}{2}kx_m{}^2\sin^2(\omega t + \phi)$$

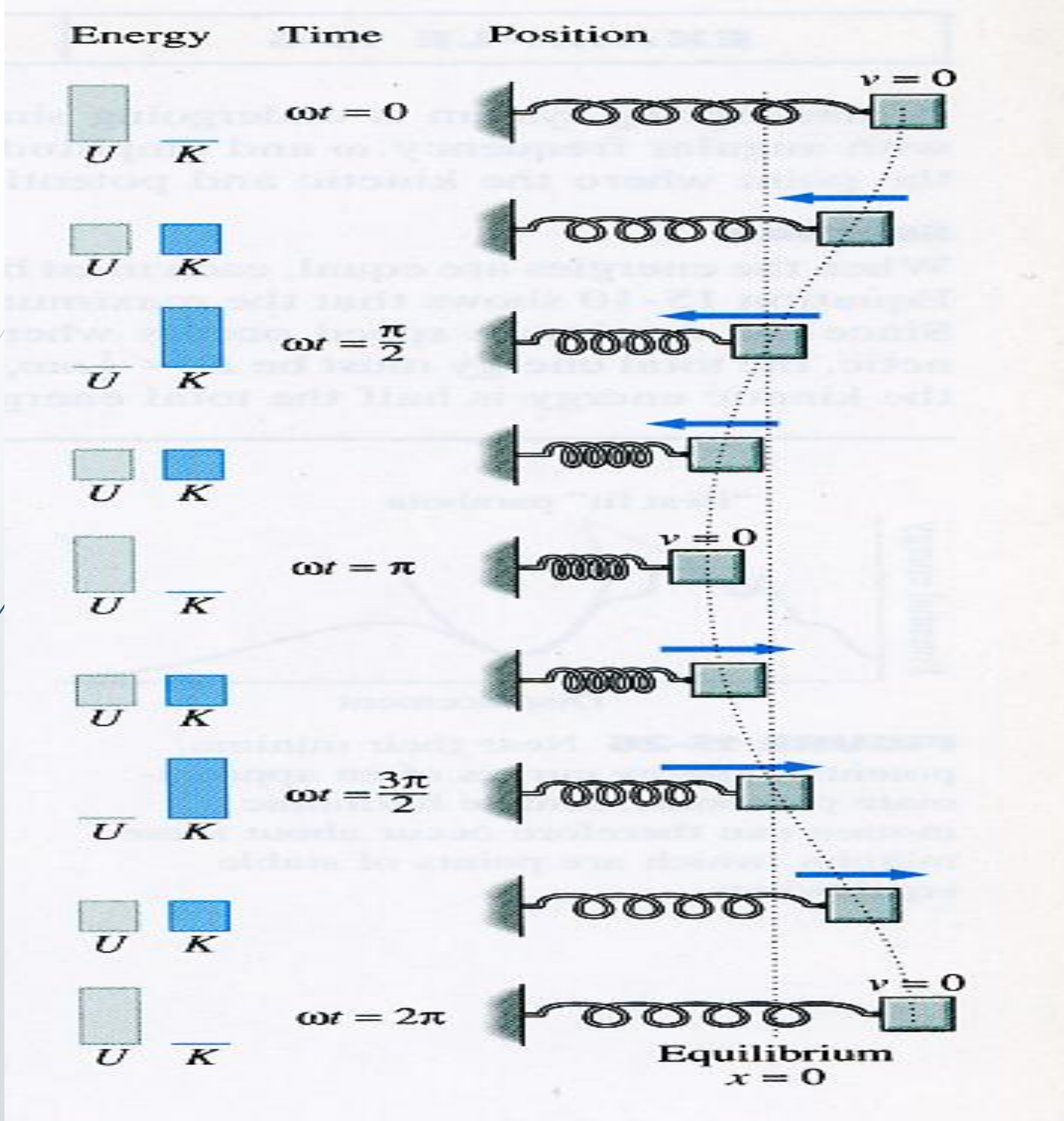
The total mechanical energy of the system:

$$E = U + K = \frac{1}{2}kx_m{}^2$$





t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
$T/4$	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
$T/2$	$-A$	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
$3T/4$	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$

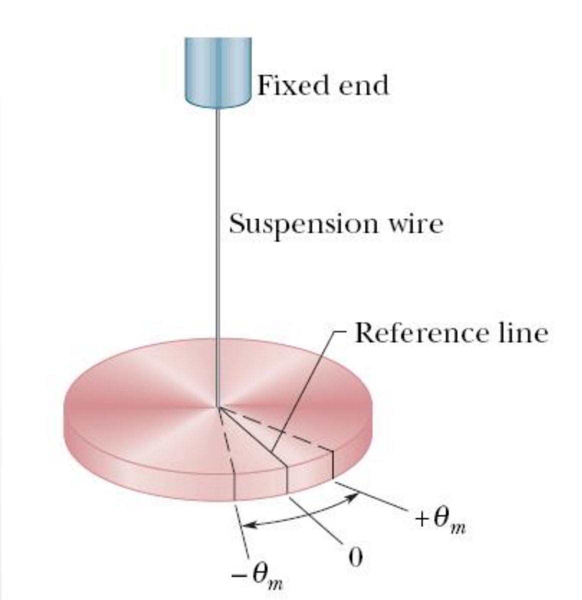


Energy
Time
Position
v = 0
U
K
ωt = 0
ωt = π/2
ωt = π
ωt = 3π/2
ωt = 2π
Equilibrium
x = 0

Simple Harmonic Motion

An angular SHM:

The figure shows a **torsion pendulum,** which involves the twisting of a suspension wire as the disk oscillates in a horizontal plane. The torque associated with an angular displacement of θ is given by:



$$t = - kq = I\frac{d^2 q}{dt^2}$$

κ*(kappa)* is the torsion constant, which depends on the length, diameter, and material of the suspension wire, and *I* is the moment of inertia (rotational inertia) of the disk.

The period, *T*, is then

$$w = \sqrt{\frac{k}{I}} \qquad T = 2p\sqrt{\frac{I}{k}}$$