Waves & Wave Motion

Course Title : Applied Physics

Instructor: Ms. Sonia Nasir

Contents

- Waves & Type of Waves
- Properties of Wave

  (Frequency, Amplitude, Phase, Wavelength)
- Mathematical Representation of a Wave

Visualize this image and tell what happens?

Visualize this image too and tell what happens?


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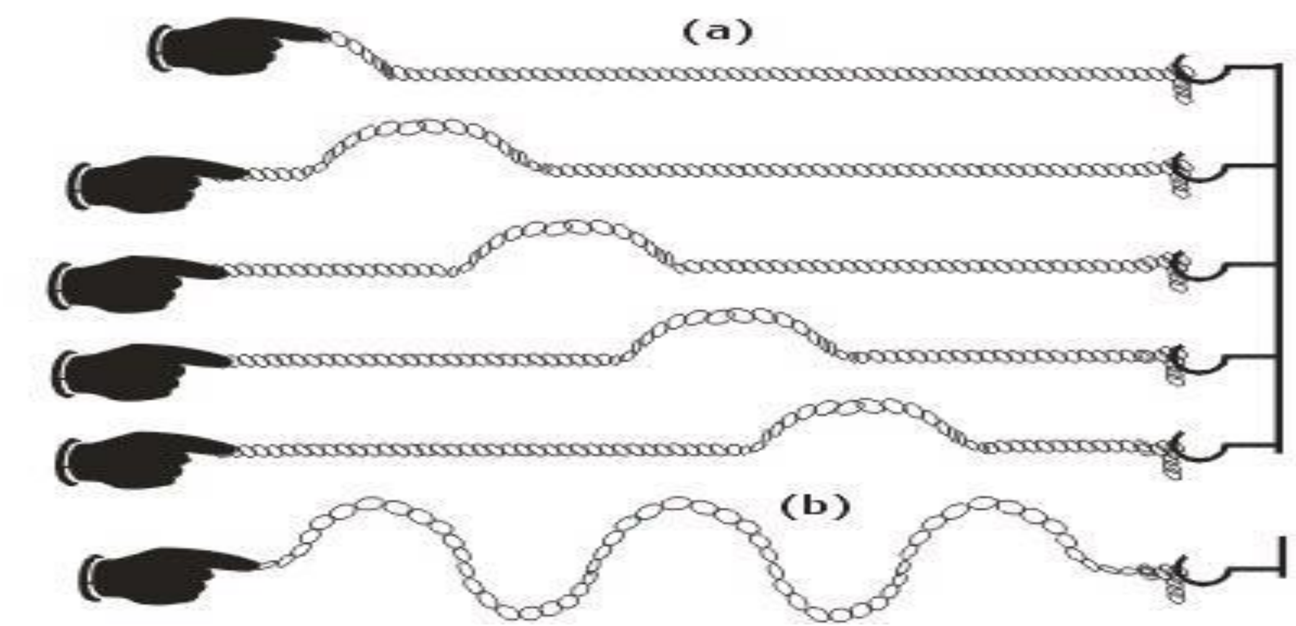
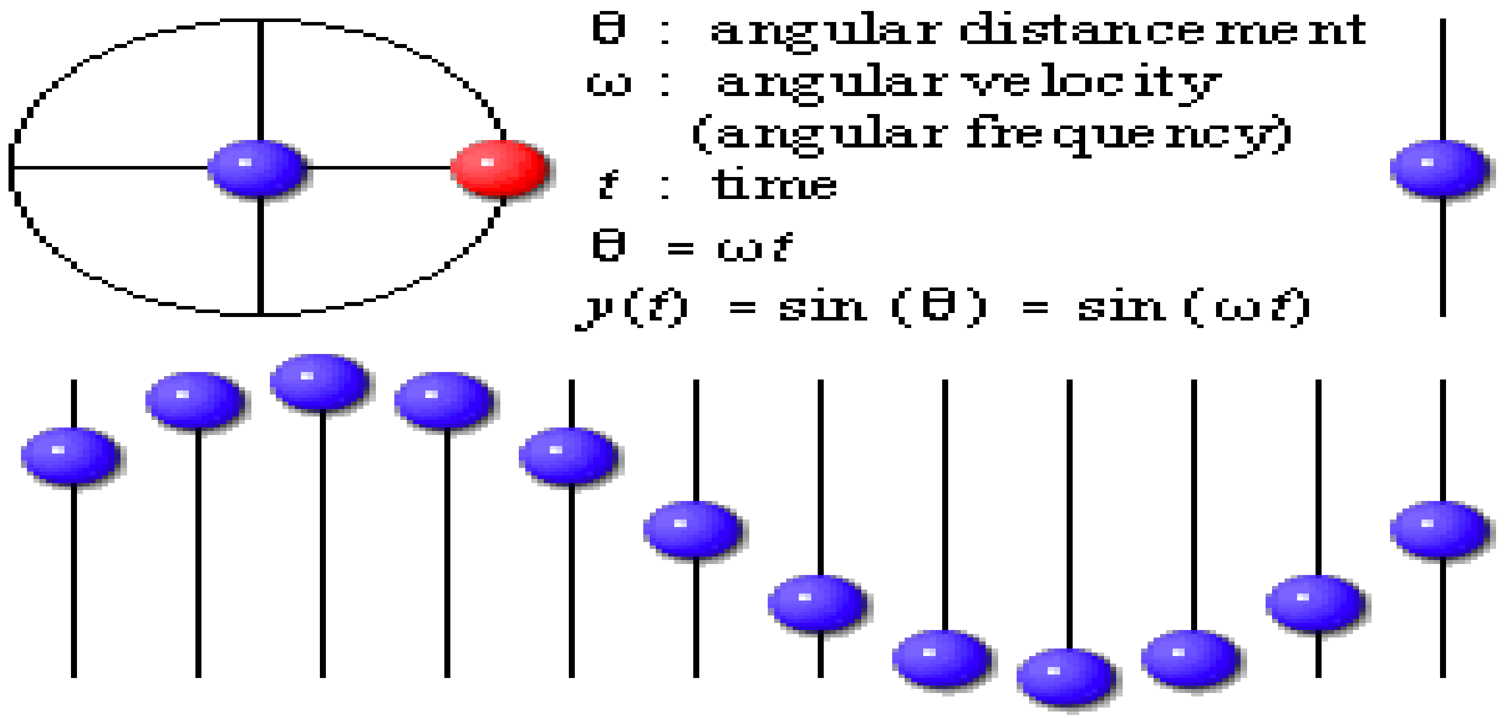
Q…What is a Wave?
Ans…Wave is… pattern of motion of particles of the medium which transport disturbance without transporting the medium.

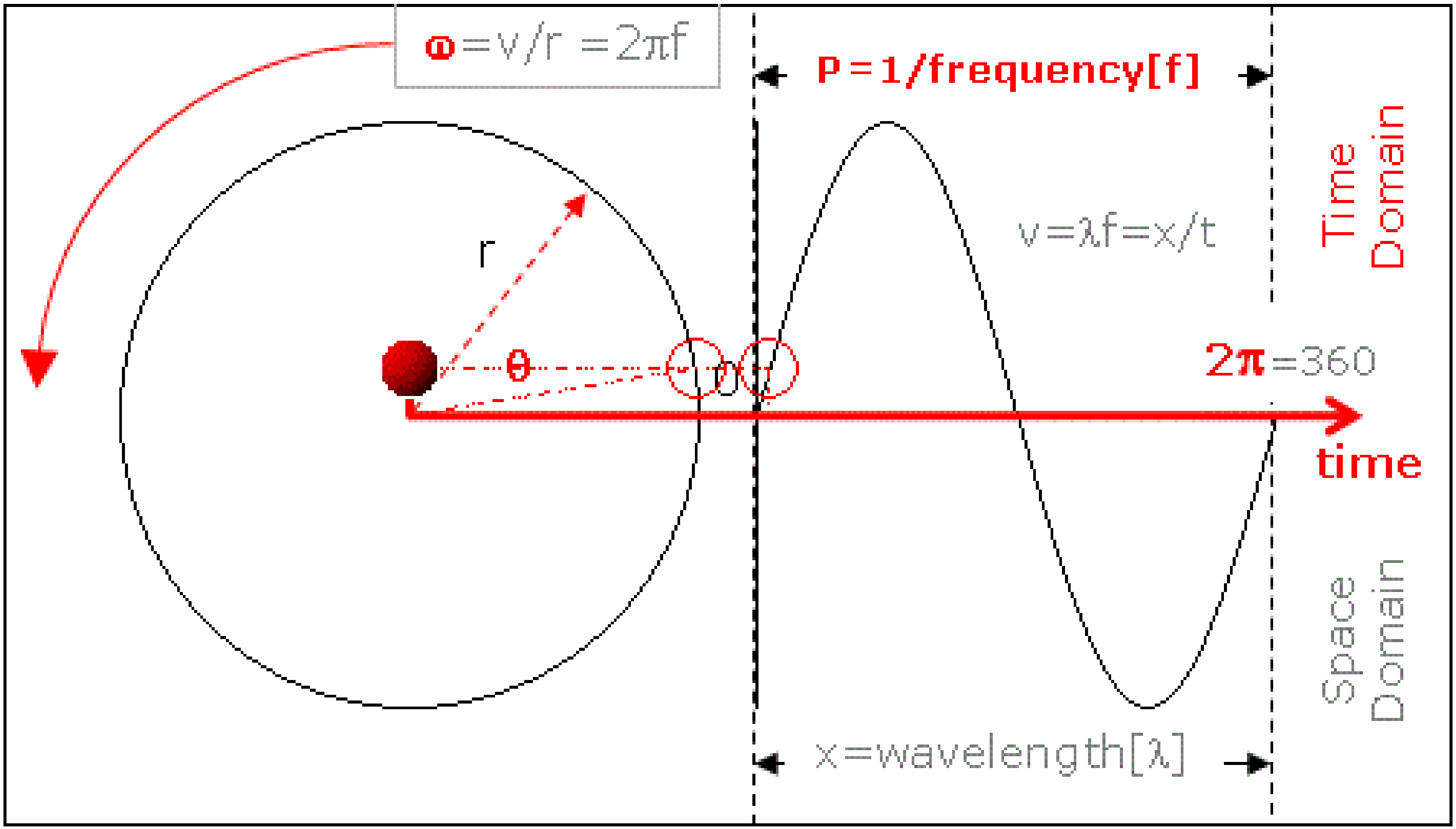


Motion of Particles in the Medium



It is a sinusoidal graph because particles oscillate according to the Sine function

Formation of Sinusoidal Wave



Types of Waves

1. MECHANICAL WAVES
2. ELECTROMAGNETIC WAVES(EMW)
3. MATTER WAVES

Mechanical Waves

- These waves are most familiar because we encounter them almost constantly; common examples include water waves, sound waves, and seismic waves.
- All these waves have **two central features**:

1. They are governed by Newton's laws,
2. and they can exist only within a material medium,

Examples: water, air, and rock.

EMW

- The disturbance of Electric and Magnetic Field is transported.
- These waves require no material medium.
- Examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar waves.
- All electromagnetic waves travel through a vacuum at

speed $c = 299\,792\,458$ m/s.

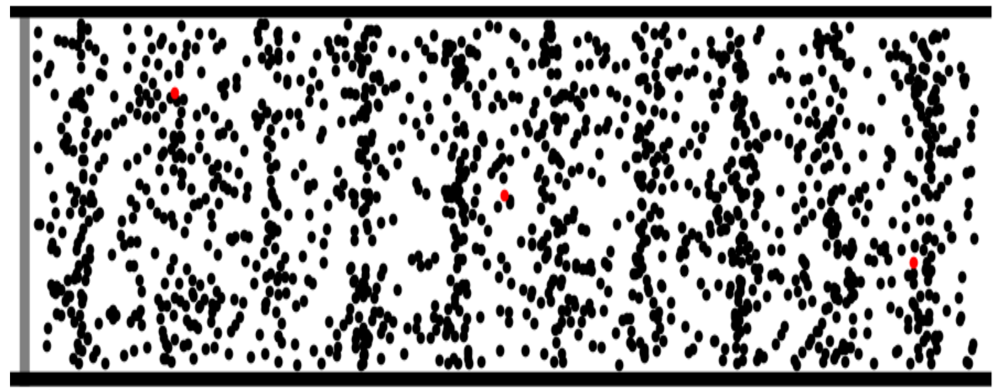
Matter Waves

- Although these waves are commonly used in modern technology, they are probably very unfamiliar to you.
- These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. Because we commonly think of these particles as constituting matter, such waves are called matter waves.(study in modern physics)

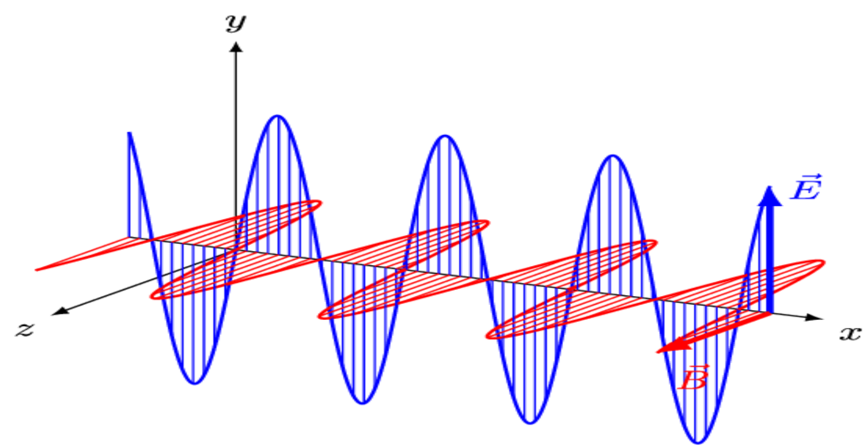
Classification of Waves

<u>On the basis of Medium</u>

Mechanical waves: require medium to travel



Electromagnetic waves: do not require medium to travel.
Travel in free space

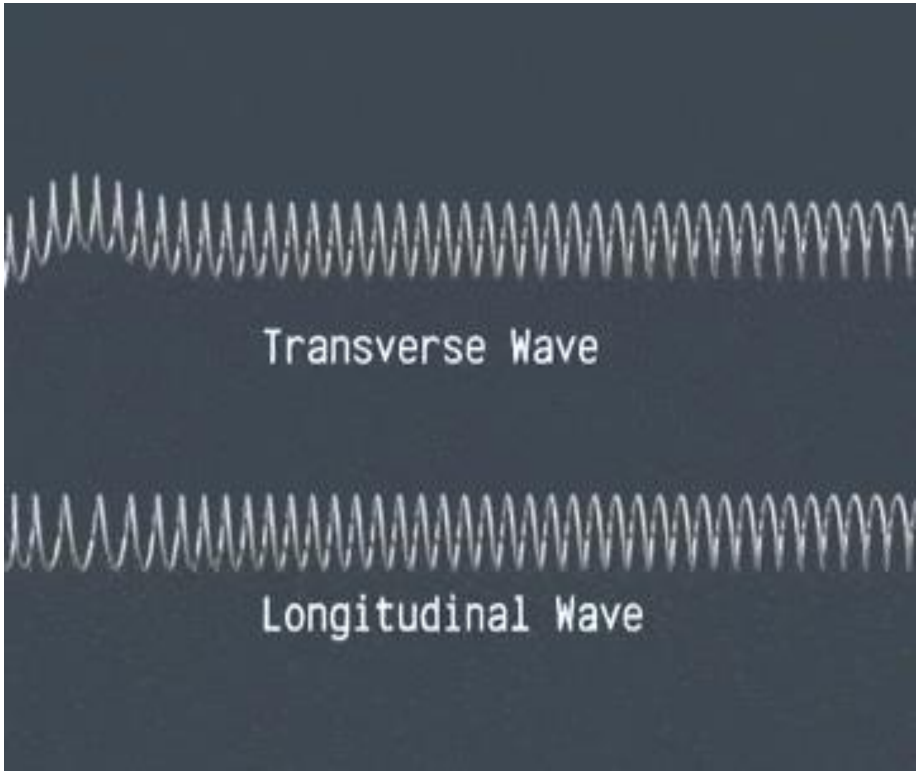


Classification of Waves

On the basis of vibration of particles

(in material/mechanical wave)

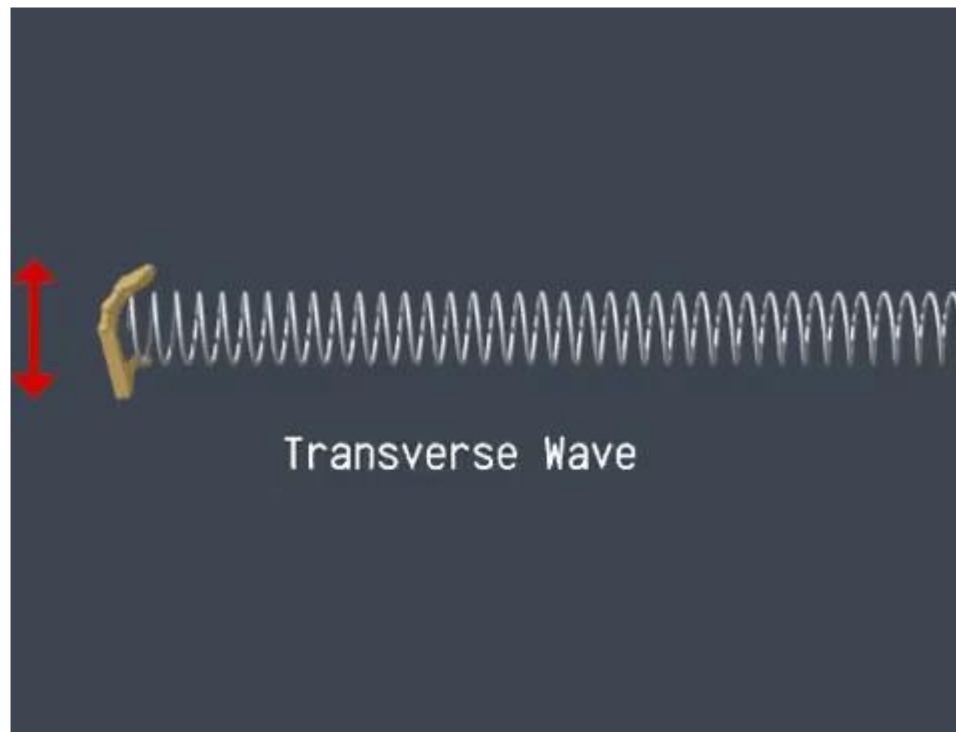
- Transverse Waves
- Longitudinal Waves



Transverse Waves

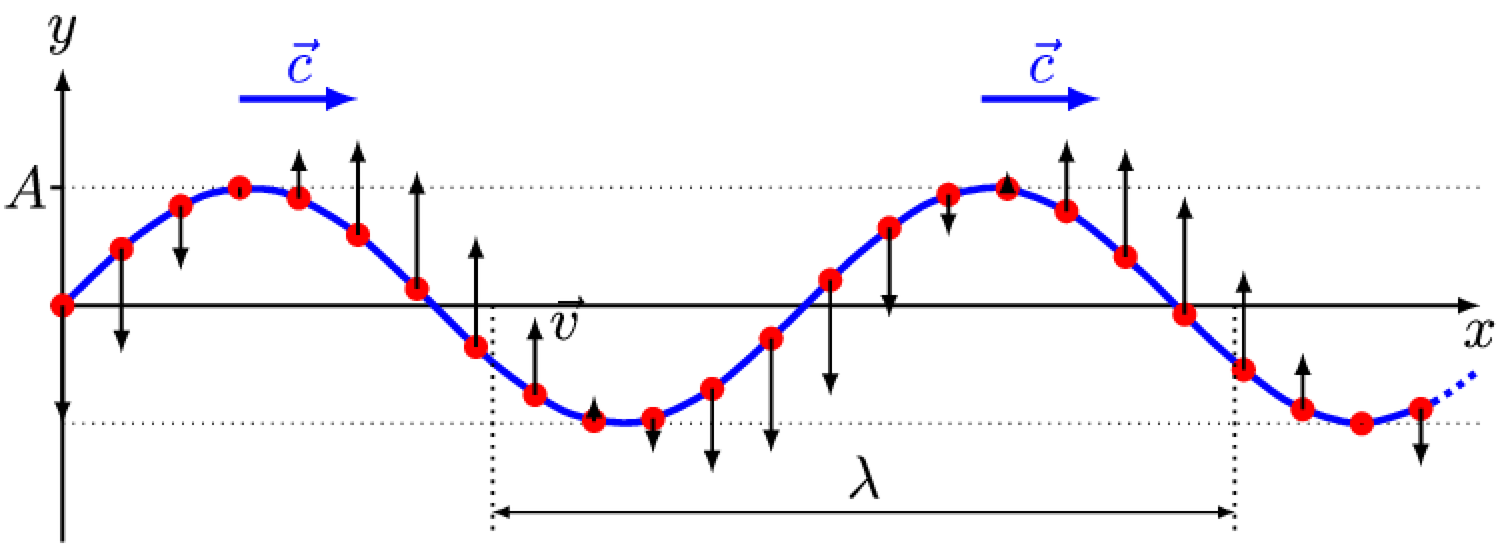
- The waves in which particles of the medium vibrates perpendicular to the direction of propagation of wave are said to be transverse waves.
- It travels in the form of crest

and trough.



Transverse Waves

Transverse waves vibrate at right angles to the direction of travel of the wave.



Light and radio waves are transverse waves.

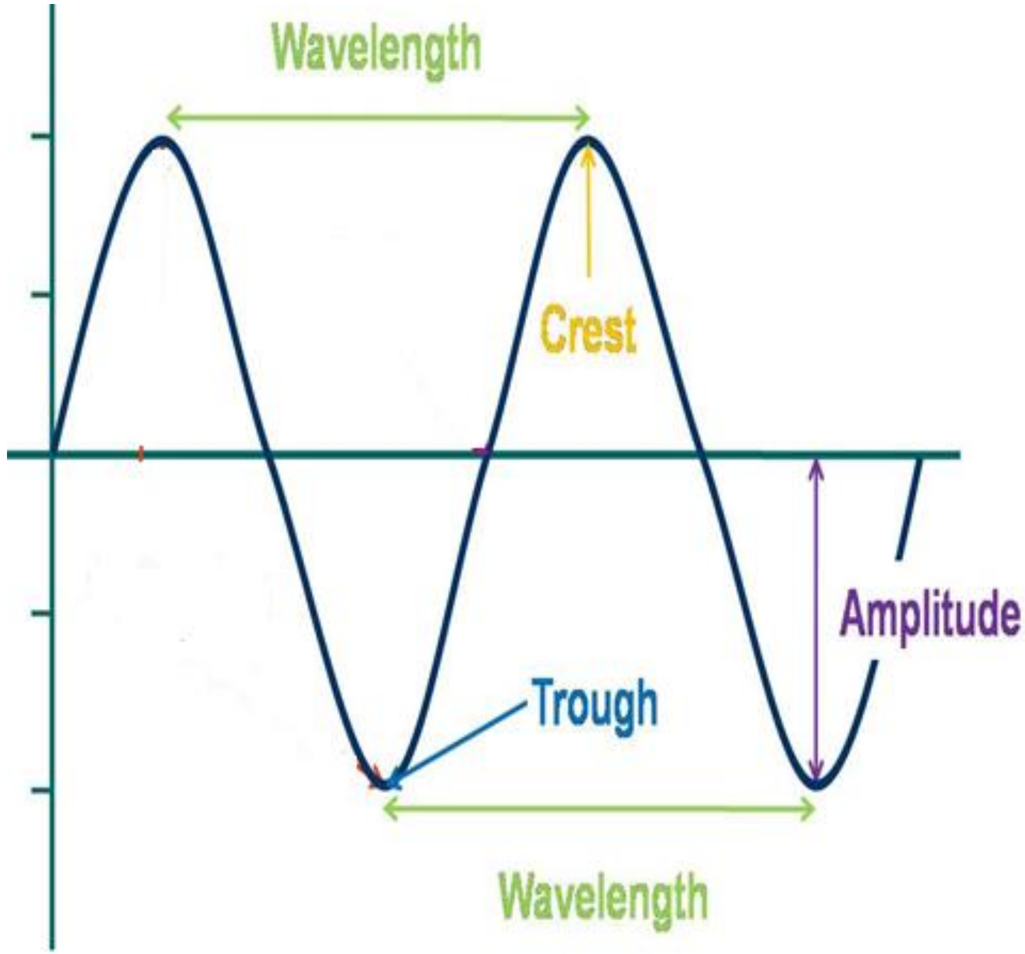
Transverse Waves(TW) …Cont'd

Limitation for the propagation of TW:

- It transmits **only** in the medium having **rigidity** , means in solid and on the surface of liquids **except gases.**
- There is **no effect** on Temperature and Pressure of the Medium while the transverse waves is being processed on it(medium).

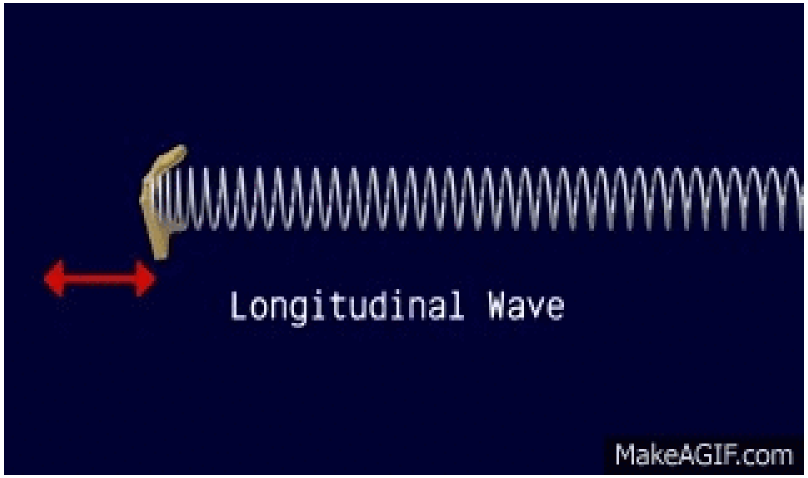
Parameters of TW

- Amplitude ($\sim$A)
- Frequency($\sim \nu_{(neu)}$)
- Time Period($\sim$ T)
- Wavelength($\sim\lambda$)
- Crest & Trough



Longitudinal Waves

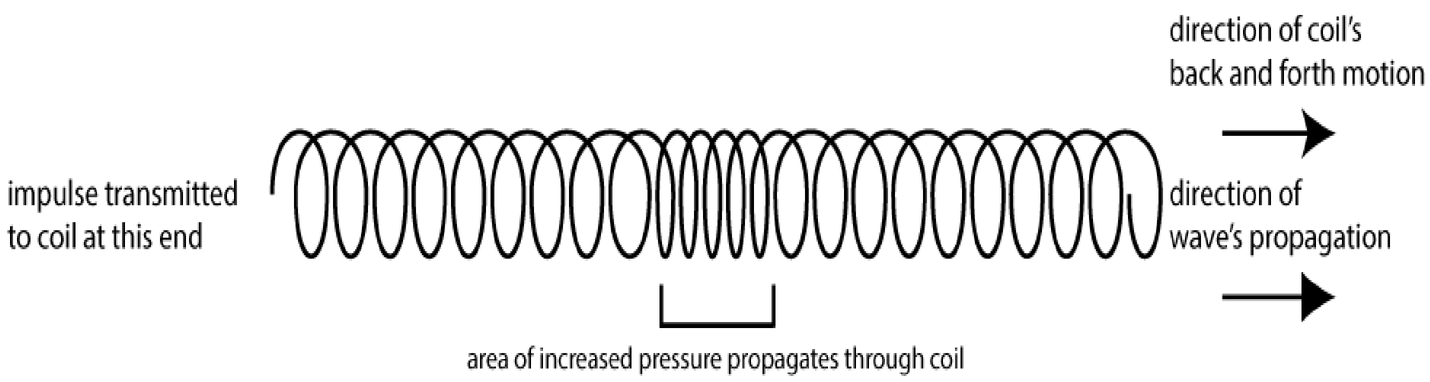
- Waves in which particles of the medium vibrates in the same direction of propagation of the wave are said to be longitudinal waves.
- It travels in the form of compression and rarefaction.



Longitudinal Waves (LW) …Cont’d

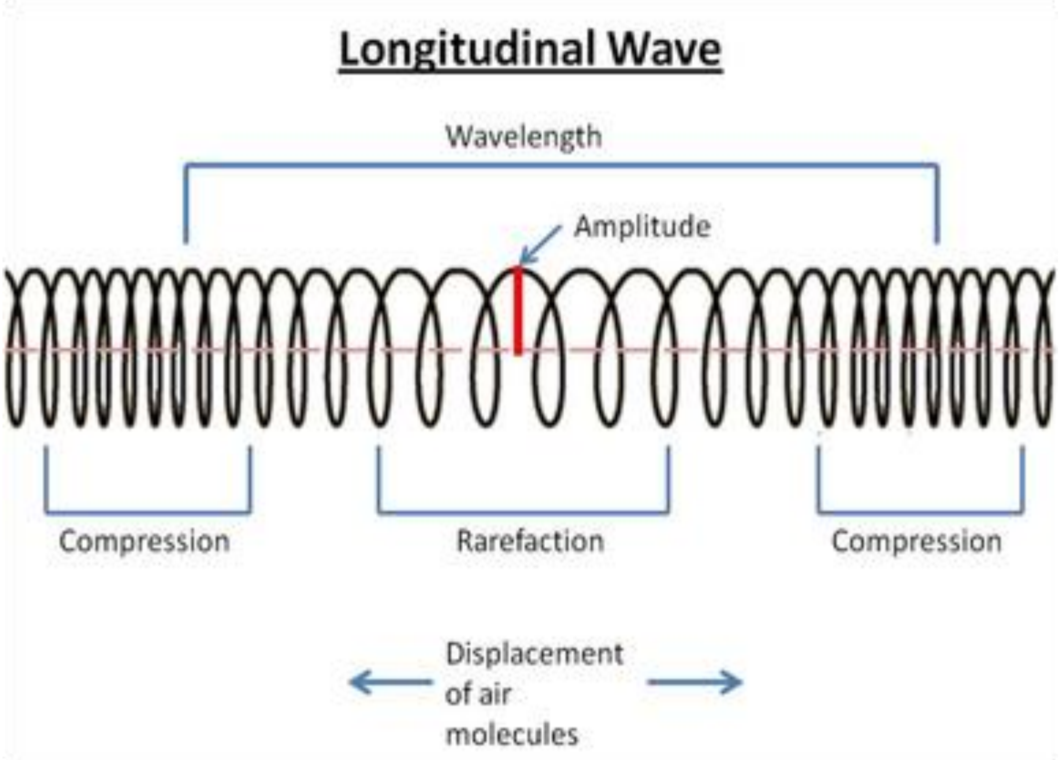
Limitation for the propagation of LW:

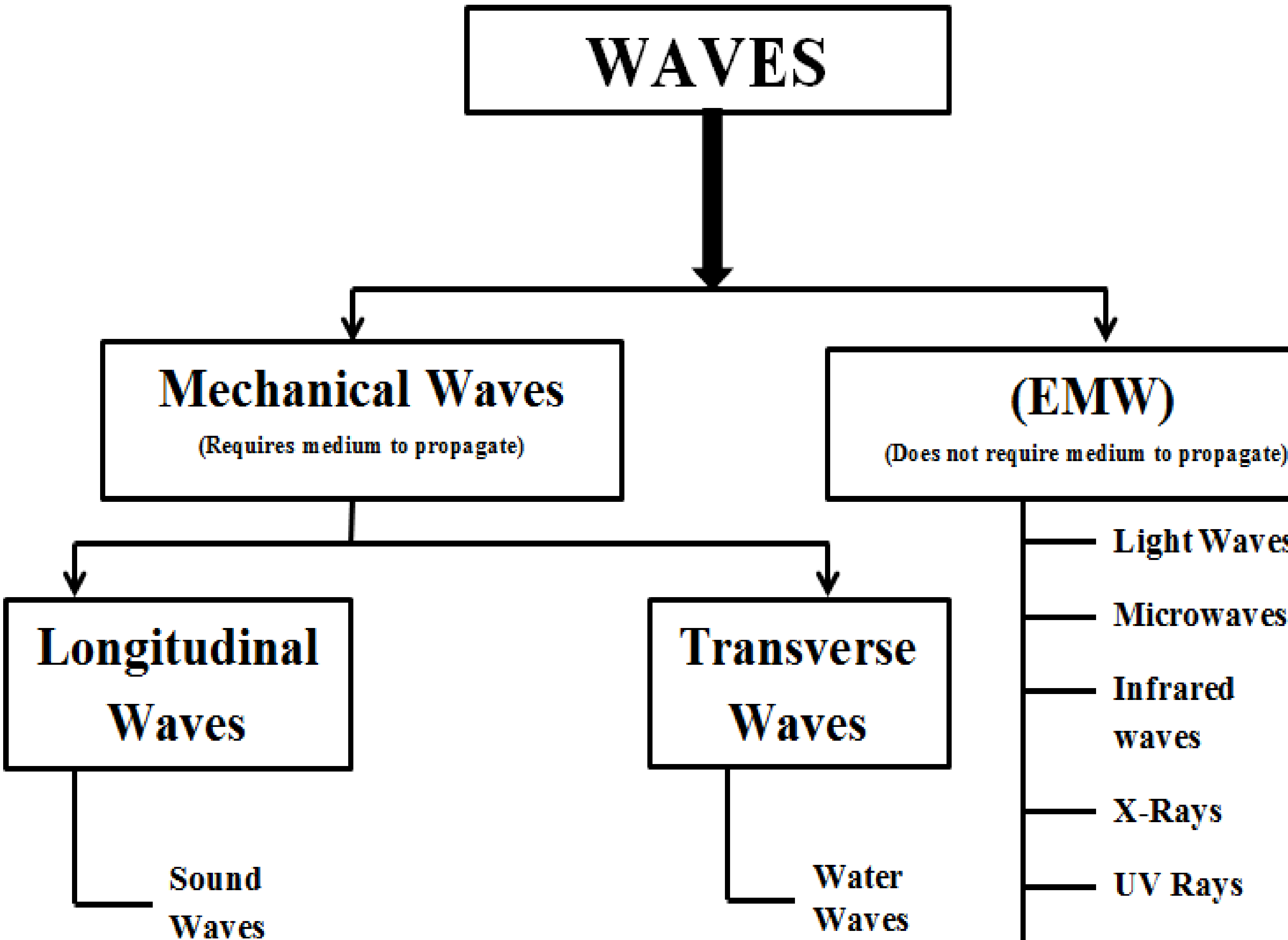
- It travels in all types of medium i.e, solid, liquid and gases.
- Temperature and Pressure of the Medium rises when the longitudinal waves is being processed on it.



Parameters of LW

- Amplitude (~A)
- Frequency(~ $\nu_{(neu)}$)
- Time Period(~ T)
- Wavelength(~λ)
- Compression & Rarefaction




WAVES
Mechanical Waves
(Requires medium to propagate)
(EMW)
(Does not require medium to propagate)
Longitudinal Waves
Transverse Waves
Sound Waves
Water Waves
Light Waves
Microwaves
Infrared waves
X-Rays
UV Rays
Radio waves

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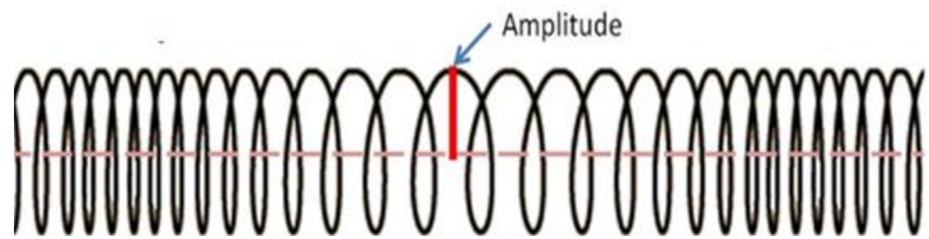
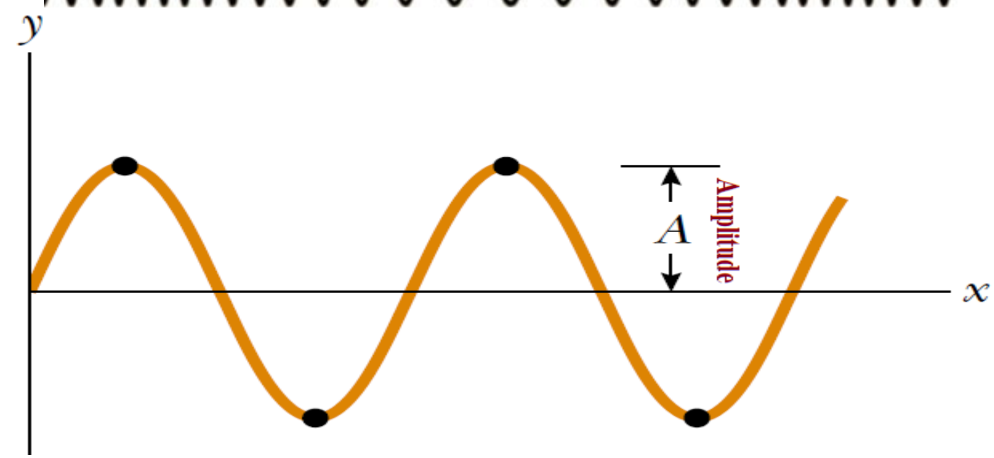
Amplitude (A)

- The maximum displacement from equilibrium of an element of the medium is called the amplitude *A of the wave.*
- Larger amplitude more energy *(A* αE^2)

In case of Longitudinal Wave →



In case of Transverse Wave →

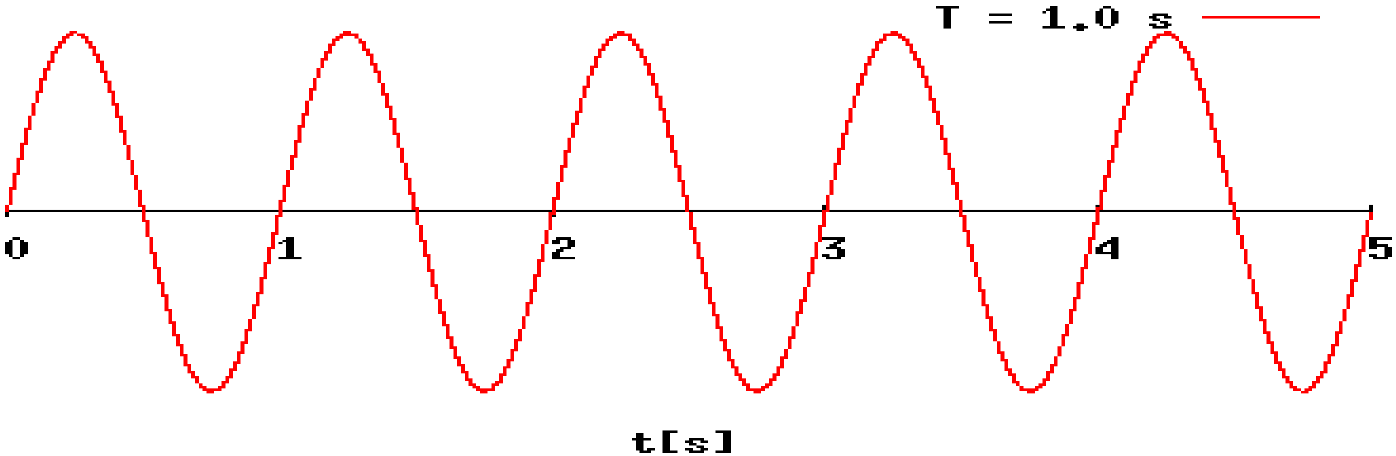


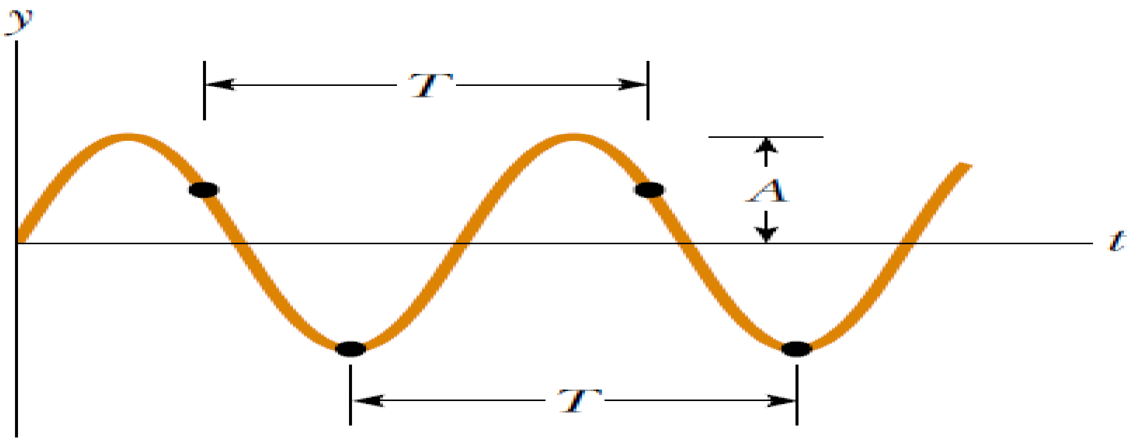
Time Period (T) & Frequency $\nu_{(neu)}$

- **The period** is the time interval required for two identical points (such as the crests) of adjacent waves to pass by a point.
- The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium.
- The **inverse** of the **period**, is called the **frequency** f.
- In general, the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval.

$$f = 1/T \; (\textit{hertz})$$

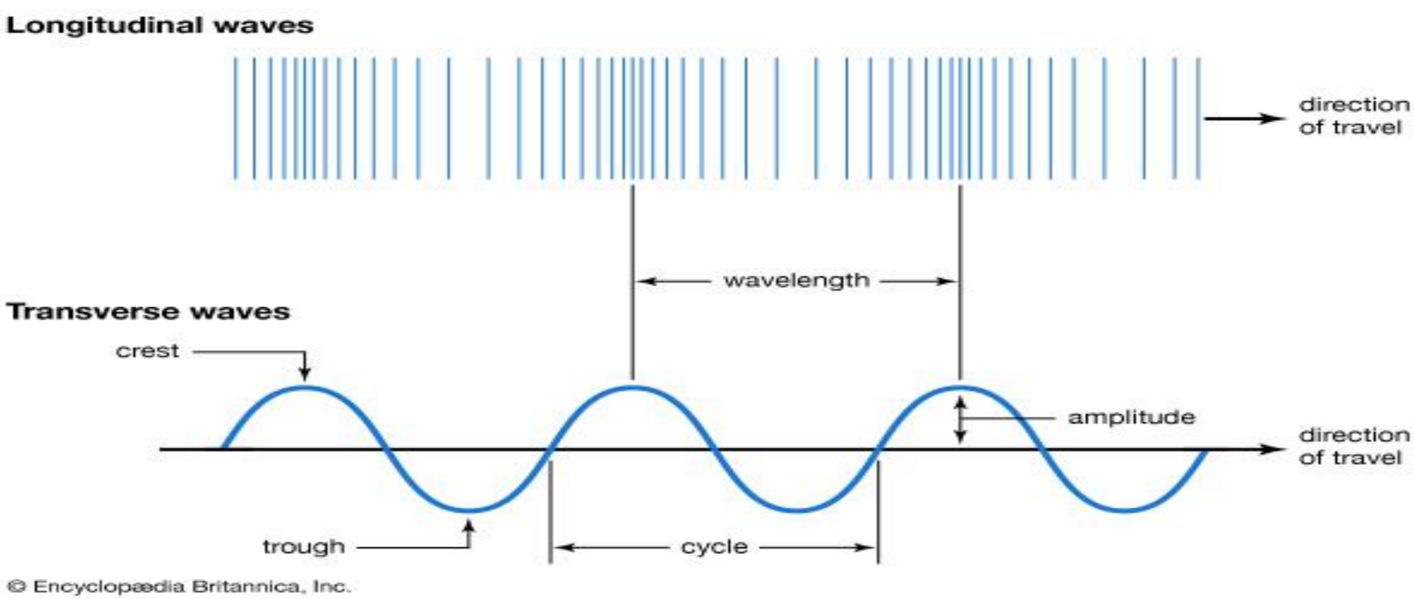
Visual Concept of Time Period & Frequency





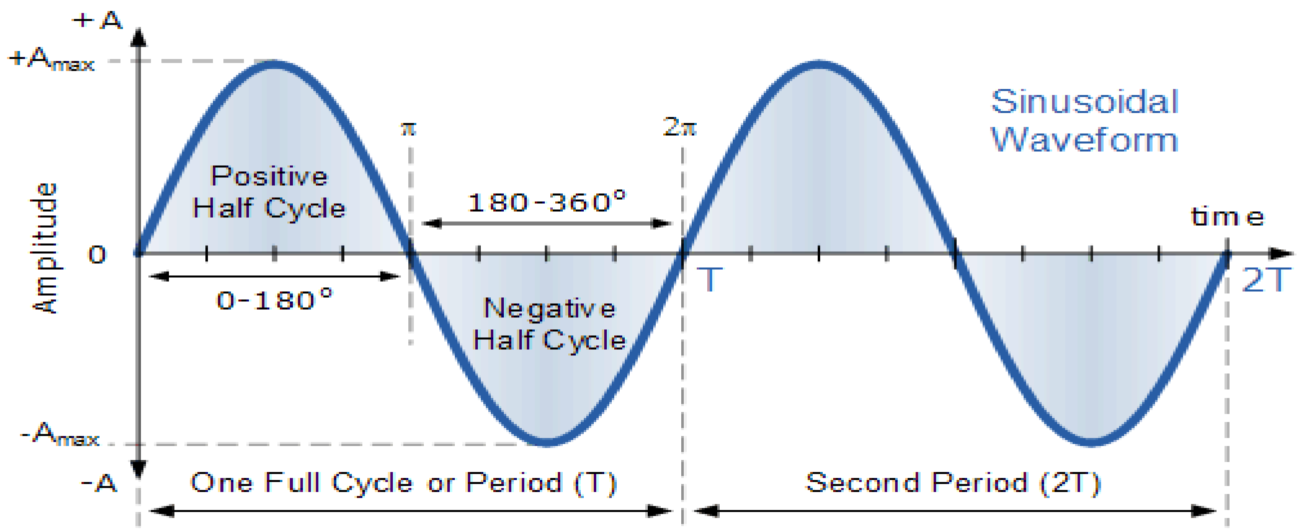
Wave length (λ)

- (In terms of TW) **The wavelength** is the minimum distance between any two identical points (such as the crests) on adjacent waves.
- (In terms of LW) **The Wavelength** is the minimum distance between any two identical points (such as the compressions) on adjacent waves.



Phase of Waveform

- **Phase** is the position of a point in time (an instant) on a waveform cycle. A complete cycle is defined as the interval required for the waveform to return to its arbitrary initial value.
- The graph below shows how one cycle constitutes 360° of phase.



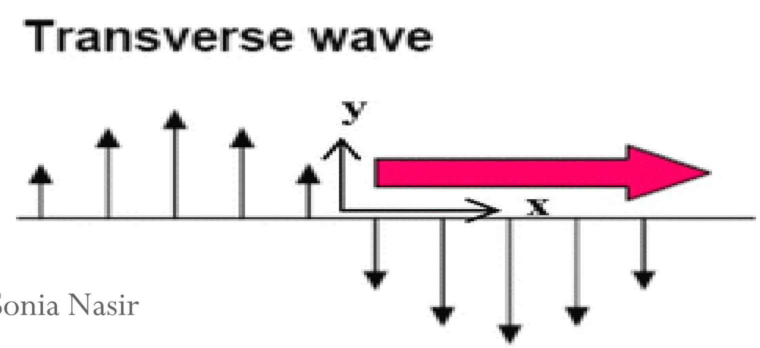
Comparison of LW & TW

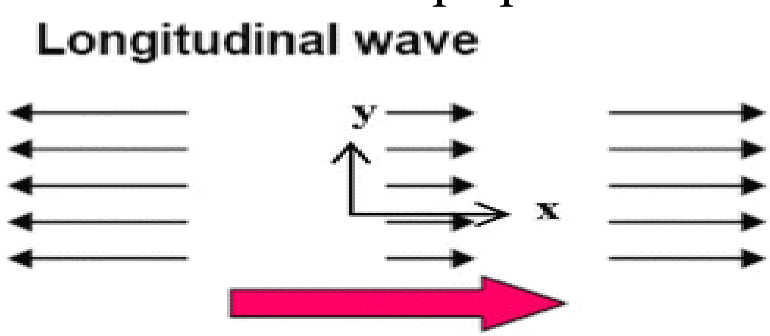
Transverse Waves

- Travels perpendicular to direction of propagation.
- Travels in the form of crests and troughs.
- It does not effect medium's temperature and pressure.
- Travels through only in solid and surface of liquids except gases.

Longitudinal Waves

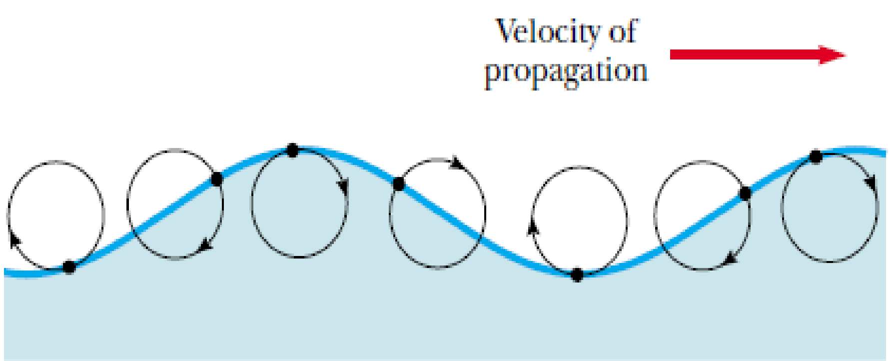
- Travels parallel to direction of propagation.
- Travels in the form of compression and rarefactions.
- Temperature and pressure rises in its propagation.
- Travels through all type of medium except plasma.

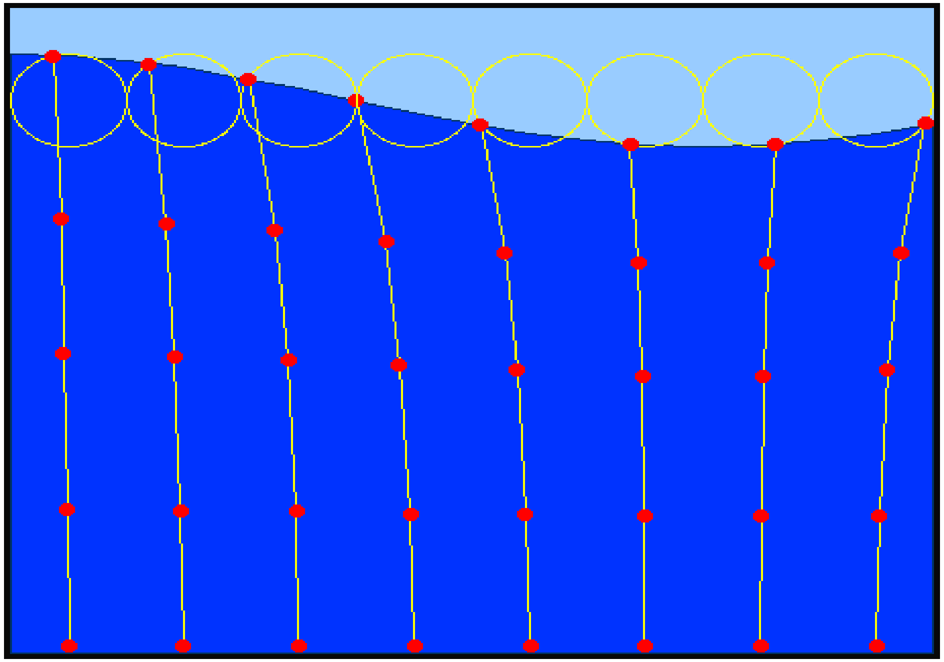




Combination of Transverse and Longitudinal Waves

- Some waves in nature exhibit a combination of transverse and longitudinal displacements.
- Surface water waves are a good example.
- Note that in this disturbance has both transverse and longitudinal waves.





Question :

Think of some other examples in nature in which they exhibits both the features (TW & LW) at a time.

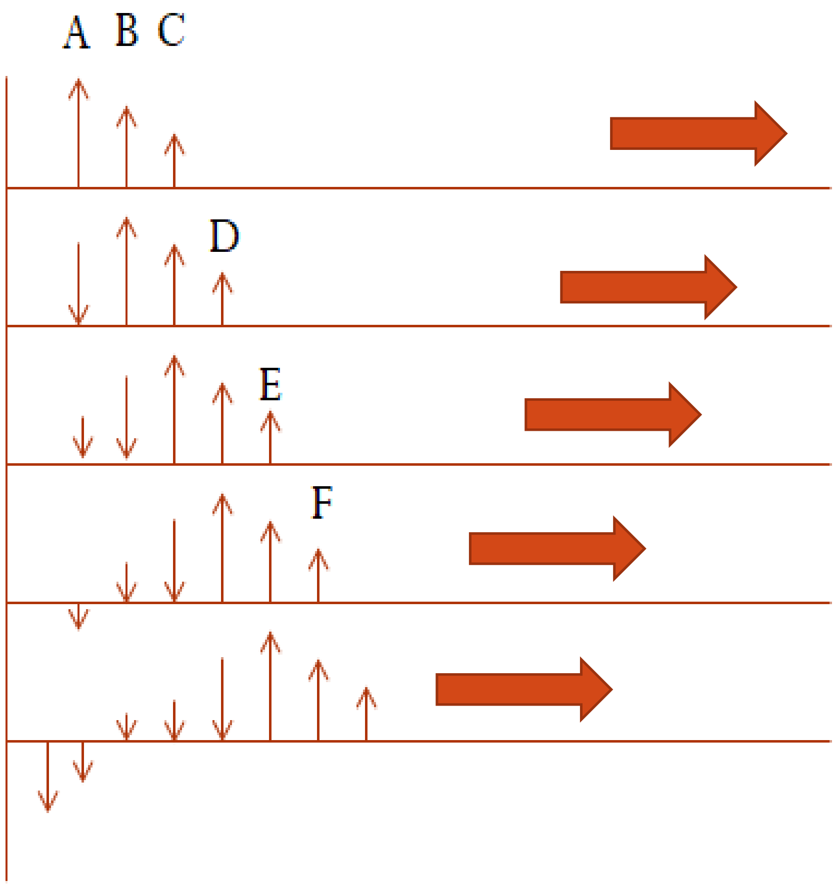
Formation of Wave

- In order to have a wave, our medium should have two properties

1. Elasticity
2. Inertia(measure of mass)

Or

we can say that …..for oscillation our particle should have inertia and elasticity.



Brainstorming

- Can you tell what is the direction of wave?

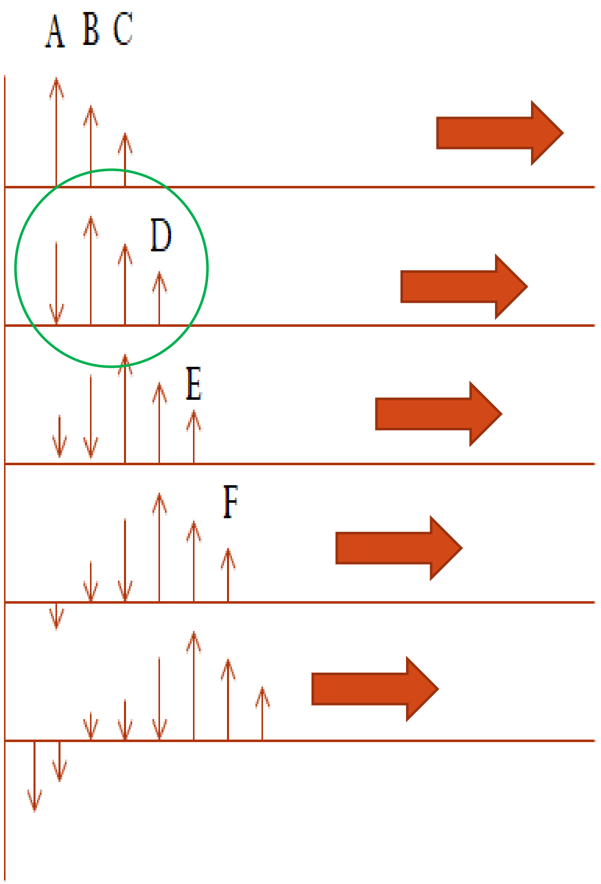
  Ans: Left to right

  (Watch the movement of crest)
- Which particle has the maximum phase? B,C or A

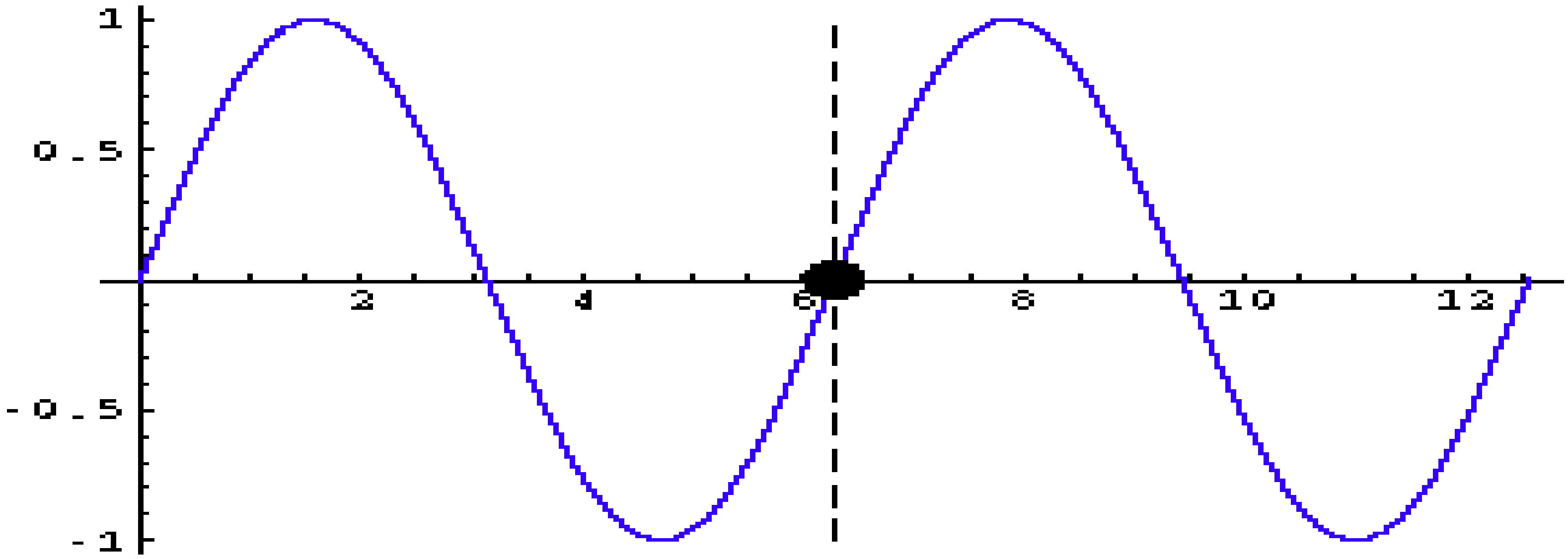
  Ans: **A** has the maximum phase.

- To which size of phase is wave travelling?

  Ans: Wave moves towards minimum phase forming particle i.e, C.

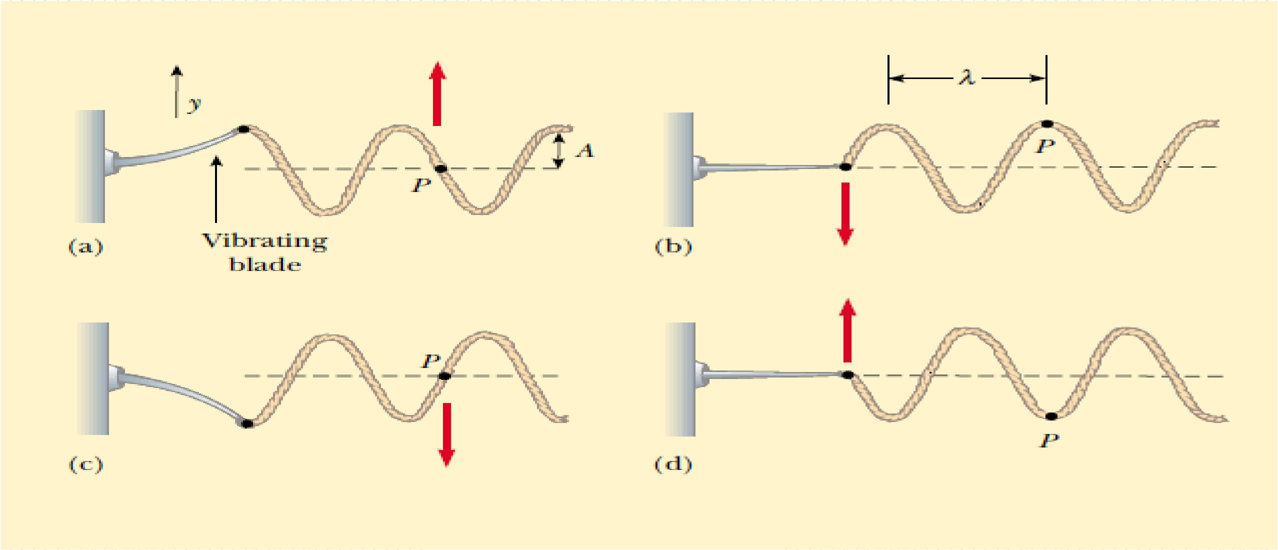


Effect of travelling wave on Particle



Representation of a Wave

Mathematical Approach



$$y = A \sin(kx - \omega t)$$

Wave Velocity

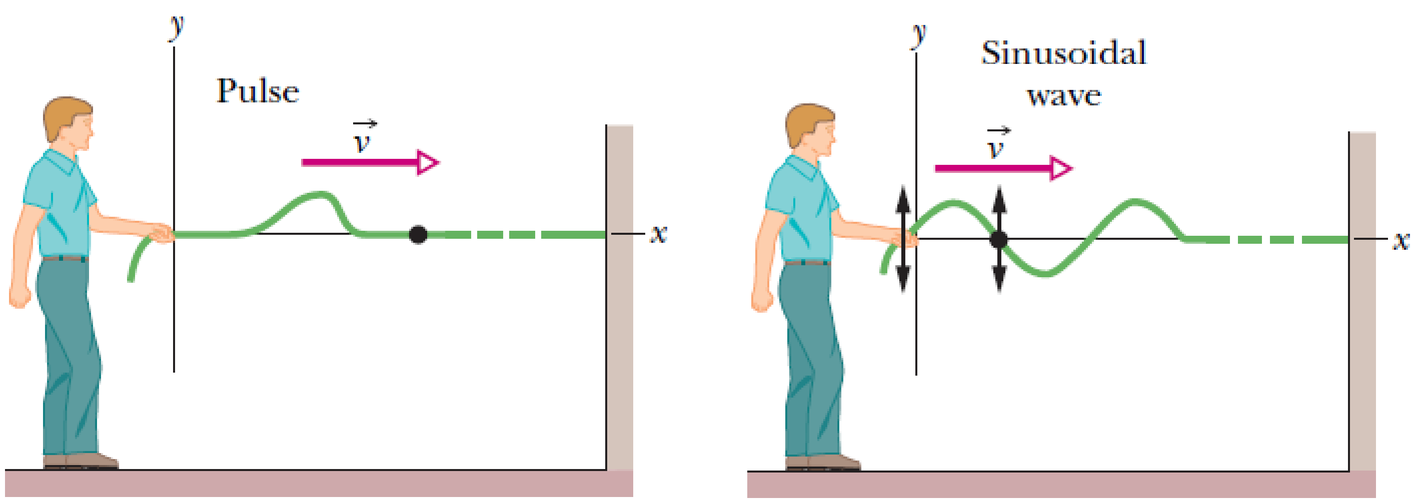
- Below is the fundamental wave velocity equation where υ is the wave velocity , λ is the wavelength and ν is the frequency of wave.

$$\upsilon = \lambda \nu$$

- Frequency is the number of consecutive risings of wave /cycles per second measured in hertz.
- This is a fundamental equation obeyed by all waves.

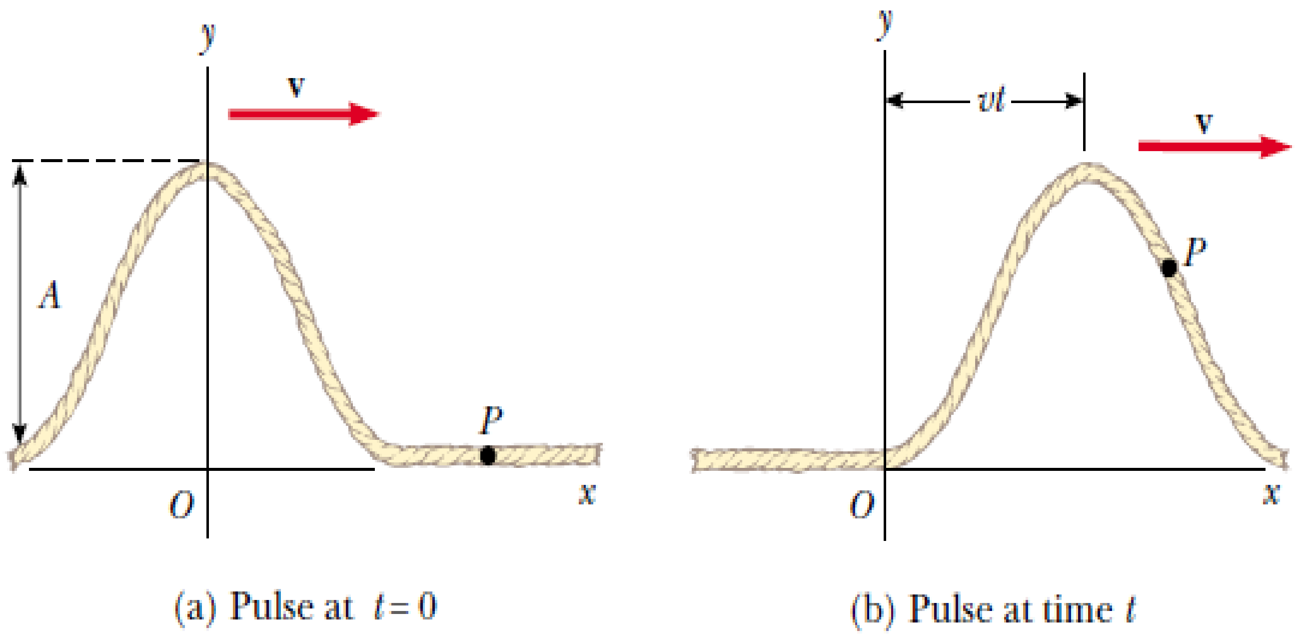
One Dimensional Pulse and a Wave Function

- We need a function that gives the shape of the wave



(a) (b)

One Dimensional Pulse and a Wave Function



A one-dimensional pulse traveling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical position of an element of the medium any point P is given by $y = f(x - vt)$.

Wave Function

- To completely describe a wave on a string (and the motion of any element along its length), we need a function that gives the shape of the wave.
- y is the perpendicular displacement , x is the horizontal distance covered by wave in time t.

Wave Function

Consequently, an element of the string at x at this time has the same y position as an element located at $x - vt$ had at time $t = 0$:

$$y(x, t) = y(x - vt, 0)$$

In general, then, we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = f(x - vt) \qquad \textbf{Pulse traveling to the right}$$

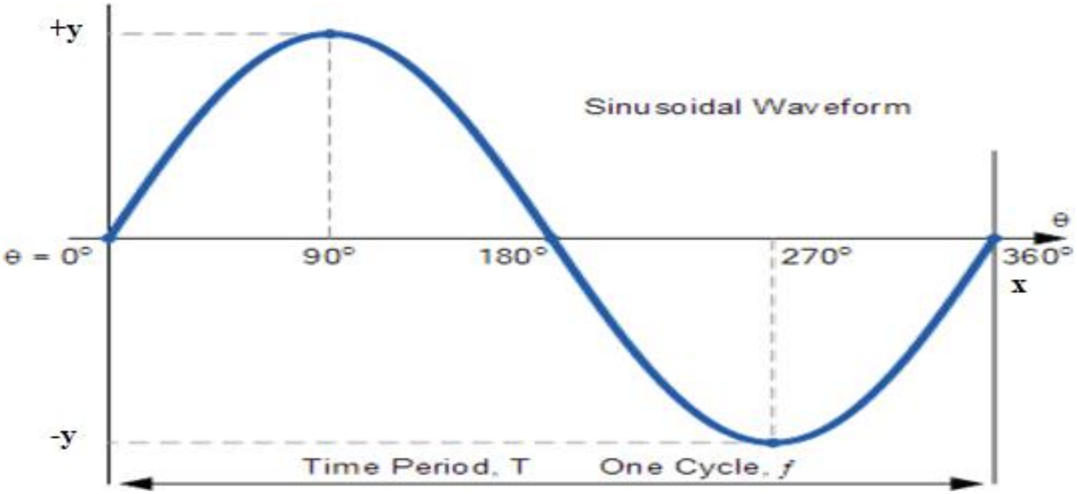
Similarly, if the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x, t) = f(x + vt) \qquad \textbf{Pulse traveling to the left}$$

The function y, sometimes called the **wave function,** depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t."

Wave Form

It is important to understand the meaning of y. Consider an element of the string at point P, identified by a particular value of its x coordinate. As the pulse passes through P, the y coordinate of this element increases, reaches a maximum, and then decreases to zero. **The wave function $y(x, t)$ represents the y coordinate—the transverse position—of any element located at position x at any time t.** Furthermore, if t is fixed (as, for example, in the case of taking a snapshot of the pulse), then the wave function $y(x)$, sometimes called the **waveform,** defines a curve representing the actual geometric shape of the pulse at that time.



Example Problem 1

- Pulse moving to the Right:

A pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is measured in seconds. Plot the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

Solution First, note that this function is of the form $y = f(x - vt)$. By inspection, we see that the wave speed is $v = 3.0$ cm/s. Furthermore, the maximum value of y is given by $A = 2.0$ cm. (We find the maximum value of the function representing y by letting $x - 3.0t = 0$.) The wave function expressions are

$$y(x, 0) = \frac{2}{x^2 + 1} \qquad \text{at } t = 0$$

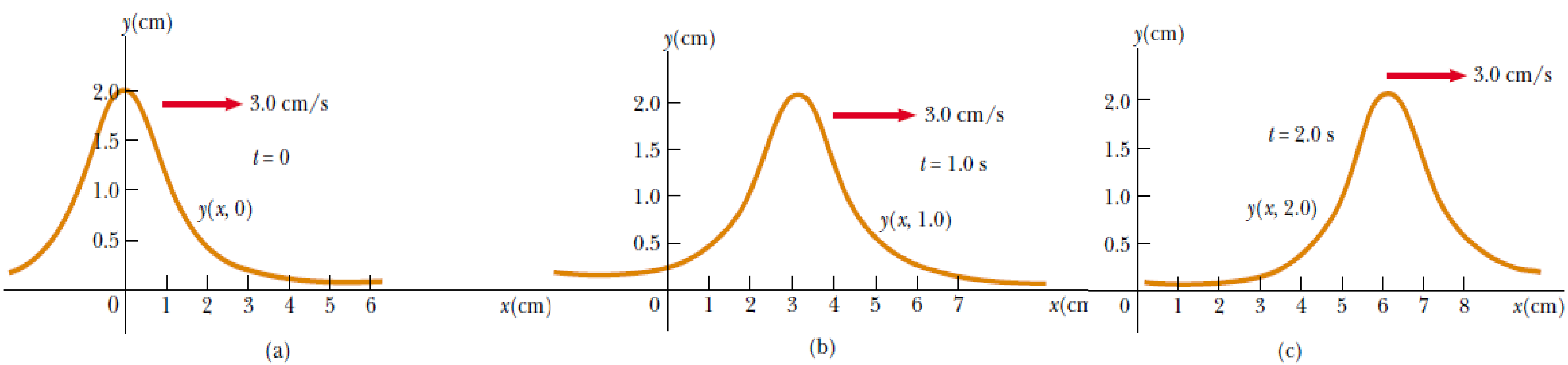
$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \qquad \text{at } t = 1.0 \text{ s}$$

$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1} \qquad \text{at } t = 2.0 \text{ s}$$

We now use these expressions to plot the wave function versus x at these times. For example, let us evaluate $y(x, 0)$ at $x = 0.50$ cm:

$$y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise, at $x = 1.0$ cm, $y(1.0, 0) = 1.0$ cm, and at $\mathbf{x} = 2.0$ cm, $y(2.0, 0) = 0.40$ cm. Continuing this procedure for other values of x yields the wave function shown in Figure **a**. In a similar manner, we obtain the graphs of $y(x, 1.0)$ and $y(x, 2.0)$, shown in Figure b and c respectively. These snapshots show that the pulse moves to the right without changing its shape and that it has a constant speed of 3.0 cm/s.



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What If? (A) What if the wave function were

$$y(x, t) = \frac{2}{(x + 3.0t)^2 + 1}$$

How would this change the situation?

Answer (A) The new feature in this expression is the plus sign in the denominator rather than the minus sign. This results in a pulse with the same shape as that in Figure **a,b,c** but moving to the left as time progresses.

(B) What if the wave function were

$$y(x, t) = \frac{4}{(x - 3.0t)^2 + 1}$$

How would this change the situation?

(B) The new feature here is the numerator of 4 rather than 2. This results in a pulse moving to the right, but with twice the height of that in Figure **a,b,c**

Sinusoidal Wave

Consider the sinusoidal wave in Figure below, which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Figure The next value of x for which y is zero is $x = \lambda/2$. Thus,

$$y(x, 0) = A \sin ax,$$

$$y\left(\frac{\lambda}{2}, 0\right) = A \sin a\left(\frac{\lambda}{2}\right) = 0$$

For this to be true, we must have

$$a(\lambda/2) = \pi,$$

$$\text{or } a = 2\pi/\lambda.$$

therefore

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$ at t=0

$$y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$ at time "t" .....equation 1

we know $$v = \frac{\lambda}{T}$$

Substituting this expression for v into Equation 1

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

We can express the wave function in a convenient form by defining two other quantities, the **angular wave number** k (usually called simply the **wave number**) and the **angular frequency** ω:

$$k \equiv \frac{2\pi}{\lambda}$$ Angular wave number

$$\omega \equiv \frac{2\pi}{T}$$ Angular frequency

$$k \equiv \frac{2\pi}{\lambda}$$ $$\omega \equiv \frac{2\pi}{T}$$

$$y = A \sin(kx - \omega t)$$ Wave function for a sinusoidal wave ...equation 2

we can express the wave speed v originally

in the alternative forms $$v = \frac{\omega}{k}$$

$$v = \lambda f$$

The wave function given by Equation 2 assumes that the vertical position y of an element of the medium is zero at $x = 0$ and $t = 0$. This need not be the case. If it is not, we generally express the wave function in the form

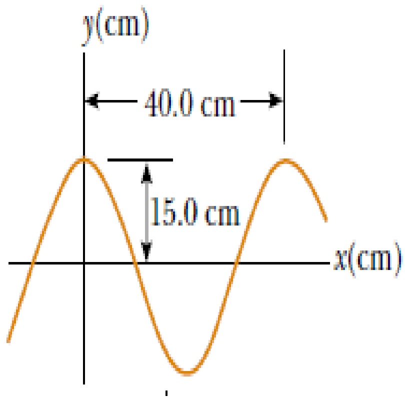
$$y = A \sin(kx - \omega t + \phi)$$ ...Equation 3 General expression for a sinusoidal wave

where ϕ is the **phase constant,**

This constant can be determined from the initial conditions.

Example Problem 2

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure below



A sinusoidal wave of wavelength $\lambda = 40.0$ cm and amplitude $A = 15.0$ cm. The wave function can be written in the form $y = A\cos(kx - \omega t)$.

(A) Find the wave number k, period T, angular frequency ω, and speed v of the wave.

(B) Determine the phase constant ϕ, and write a general expression for the wave function.

Solution A Using Equations below we find the following:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = \boxed{0.157 \text{ rad/cm}}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = \boxed{0.125 \text{ s}}$$

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = \boxed{50.3 \text{ rad/s}}$$

$$v = \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = \boxed{320 \text{ cm/s}}$$

Solution B Because $A = 15.0$ cm and because $y = 15.0$ cm at $x = 0$ and $t = 0$, substitution into

Equation $y = A\sin(kx - \omega t + \phi)$

$$15.0 = (15.0)\sin\phi \quad \text{or} \quad \sin\phi = 1$$

We may take the principal value $\phi = \pi/2$ rad (or 90°). Hence, the wave function is of the form

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right) = A\cos(kx - \omega t)$$

By inspection, we can see that the wave function must have this form, noting that the cosine function has the same shape as the sine function displaced by 90°. Substituting the values for A, k, and ω into this expression, we obtain

$$y = \boxed{(15.0 \text{ cm})\cos(0.157x - 50.3t)}$$

Example Problem 3

The string shown in Figure 16.10 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Determine the angular frequency ω and wave number k for this wave, and write an expression for the wave function.

Solution Using Equations below we find that

$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00\text{ Hz}) = \boxed{31.4\text{ rad/s}}$$

$$k = \frac{\omega}{v} = \frac{31.4\text{ rad/s}}{20.0\text{ m/s}} = \boxed{1.57\text{ rad/m}}$$

Because $A = 12.0\text{ cm} = 0.120\text{ m}$, we have

$$y = A\sin(kx - \omega t)$$

$$= \boxed{(0.120\text{ m})\sin(1.57x - 31.4t)}$$

Sinusoidal Wave on String

We can use this expression to describe the motion of any element of the string. An element at point P (or any other element of the string) moves only vertically, and so its x coordinate remains constant. Therefore, the **transverse speed** v_y (not to be confused with the wave speed v) and the **transverse acceleration** a_y of elements of the string are

$$v_y = \frac{dy}{dt}\bigg]_{x\,=\,\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t)$$

$$a_y = \frac{dv_y}{dt}\bigg]_{x\,=\,\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t)$$

In these expressions, we must use partial derivatives because y depends on both x and t. In the operation $\partial y/\partial t$, for example, we take a derivative with respect to t while holding x constant. The maximum values of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y,\,\max} = \omega A$$

$$a_{y,\,\max} = \omega^2 A$$

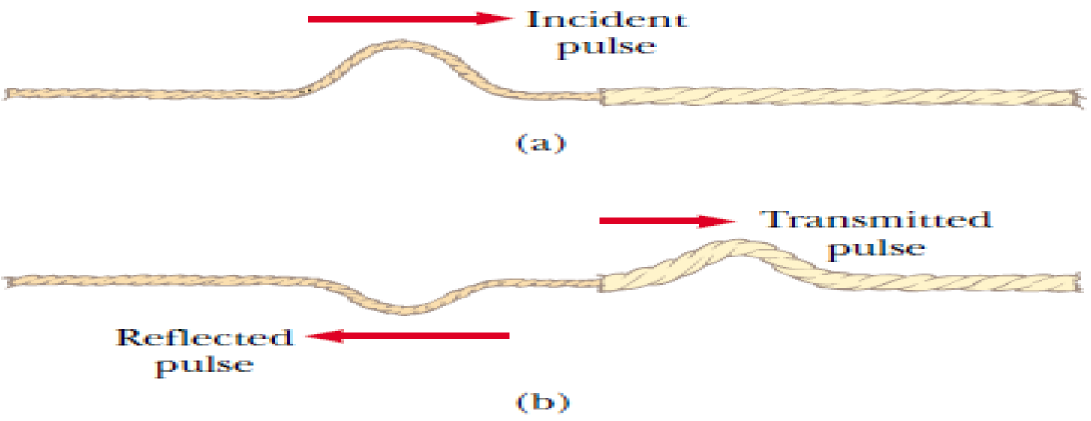
The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches its maximum value (ωA) when $y = 0$, whereas the magnitude of the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = \pm A$.

Reflection and Transmission of Wave

The speed of a wave traveling on a taut string of mass per unit length μ and tension T is

$$v = \sqrt{\frac{T}{\mu}} \qquad \mu = \frac{m}{\ell}$$

A wave is totally or partially reflected when it reaches the end of the medium in which it propagates or when it reaches a boundary where its speed changes discontinuously. If a wave traveling on a string meets a fixed end, the wave is reflected and inverted. If the wave reaches a free end, it is reflected but not inverted.



Power Transmitted by Sine Wave & Linear Wave Equation

The **power** transmitted by a sinusoidal wave on a stretched string is

$$\mathcal{P} = \tfrac{1}{2}\mu\omega^2 A^2 v$$

Wave functions are solutions to a differential equation called the **linear wave equation:**

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Example Problem 4

A taut string for which $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution The wave speed on the string is,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80.0\ \text{N}}{5.00 \times 10^{-2}\ \text{kg/m}}} = 40.0\ \text{m/s}$$

Because $f = 60.0$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0\ \text{Hz}) = 377\ \text{s}^{-1}$$

Using these values in Equation 16.21 for the power, with $A = 6.00 \times 10^{-2}$ m, we obtain

$$\begin{aligned}\mathscr{P} &= \tfrac{1}{2}\mu\omega^2 A^2 v \\ &= \tfrac{1}{2}(5.00 \times 10^{-2}\ \text{kg/m})(377\ \text{s}^{-1})^2 \\ &\quad \times (6.00 \times 10^{-2}\ \text{m})^2(40.0\ \text{m/s}) \\ &= \boxed{512\ \text{W}}\end{aligned}$$