Electric Field and Applications

Course Title : Applied Physics

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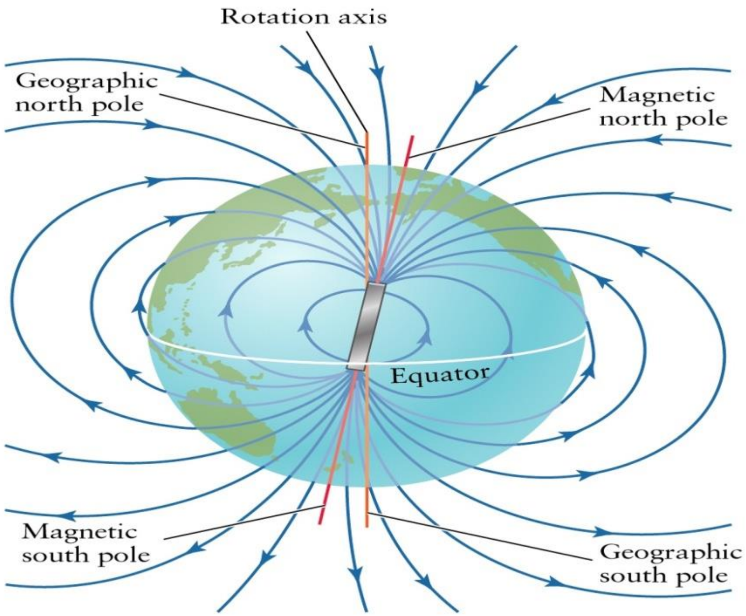
Type of forces

Contact forces

- Pushing a car up a hill or kicking a ball or pushing a desk across a room are some of the everyday **examples** where **contact forces** are at work

Field Forces

- **Examples** of **force fields** include magnetic **fields**, gravitational **fields**, and electrical **fields**.



Electric Field

- The concept of a field was developed by Michael Faraday (1791–1867) in the context of electric forces.
- An **electric field is said to exist** in the region of space around a charged object, the **source charge.**
- The presence of the electric field can be detected by placing a **test charge in the field and noting the** electric force on it.
- the electric field vector E at a point in space is defined as the electric force Fe acting on a positive test charge q_0 placed at that point divided by the test charge:

$$\vec{E} = \frac{\vec{F}_e}{q_o}$$

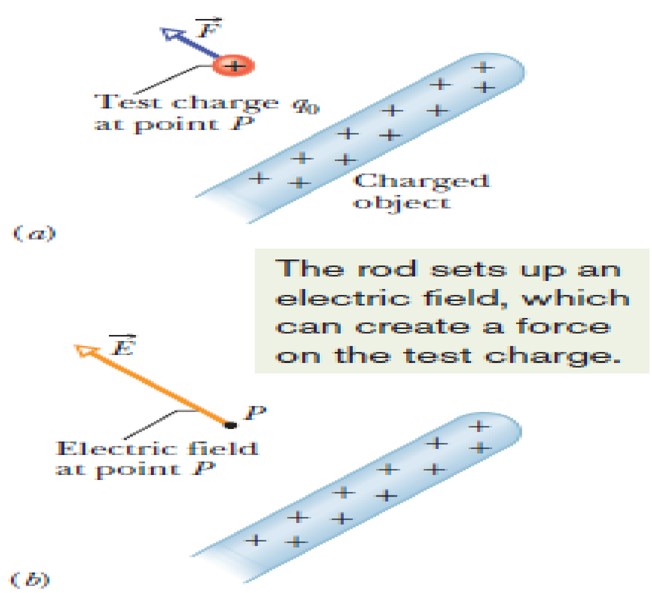
Electric Field

- It's a vector field
- We define the electric field at point *P due to the charged object as*

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{(electric field).}$$

Thus, the magnitude of the electric field $\vec{E}$ at point P is $E = F/q_0$, and the direction of $\vec{E}$ is that of the force $\vec{F}$ that acts on the *positive* test charge. As shown in Fig b

The SI unit for the electric field is the newton per coulomb (N/C).



Purpose of a Test Charge

- Note that E is the field produced by some charge or charge distribution separate from the test charge—it is not the field produced by the test charge itself.
- Also, note that the existence of an electric field is a property of its source—the presence of the test charge is **not necessary** for the field to exist.
- The test charge serves as a **detector of the electric field.**

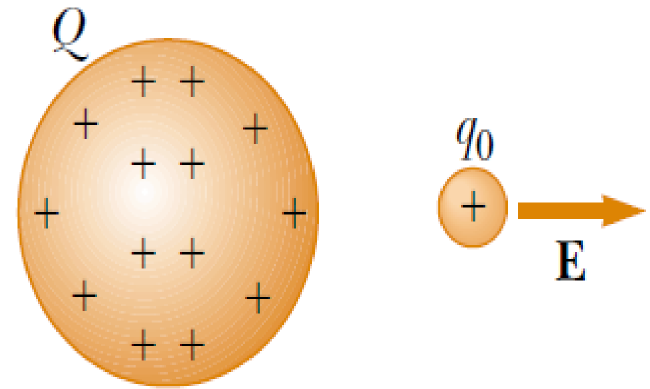


Figure A small positive test charge q_0 placed near an object carrying a much larger positive charge Q experiences an electric field **E** directed as shown.

Electric Field due to a Point Charge

- consider a point charge Q *as a* **source charge**.
- this charge creates an **electric field** at all points in space surrounding it.
- A **test charge** q is placed at point P, a **distance r** from the **source charge**
- We imagine using the test charge to determine the direction of the electric force and therefore that of the electric field
- However, the electric field does not depend on the existence of the test charge—it is established solely by the source charge.

Electric Field due to a Point Charge

- According to Coulomb's law, the force exerted by *q on the test* charge is:

$$\vec{F}_e = \frac{1}{4\pi\epsilon_o}\frac{Qq}{r^2}\hat{r}$$

- The electric field at *P, the position* of the test charge, is defined by,

$$\vec{E} = \frac{\vec{F}_e}{q}$$

- Substituting the value of Fe in Electric field equation we will get

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Qq}{r^2}\hat{r}\left(\frac{1}{q}\right)$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}}$$

Electric field created by source charge Q

To calculate E-Field by Group of Charges

- To calculate the electric field at a point P due to a group of point charges, we first calculate the electric field vectors at P individually using Equation, $\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}$ and then add them vectorially.

Or in other words….

- ***at any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.***
- Thus, the electric field at point *P due to a group* of source charges can be expressed as the vector sum,

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_o}\sum_{i}^{n}\frac{Q}{{r_i}^2}\hat{r_i}}$$

Field due to Two Charges (due to Point charge)

A charge $q_1 = 7.0\ \mu C$ is located at the origin, and a second charge $q_2 = -5.0\ \mu C$ is located on the x axis, 0.30 m from the origin (Fig.) Find the electric field at the point P, which has coordinates (0, 0.40) m.

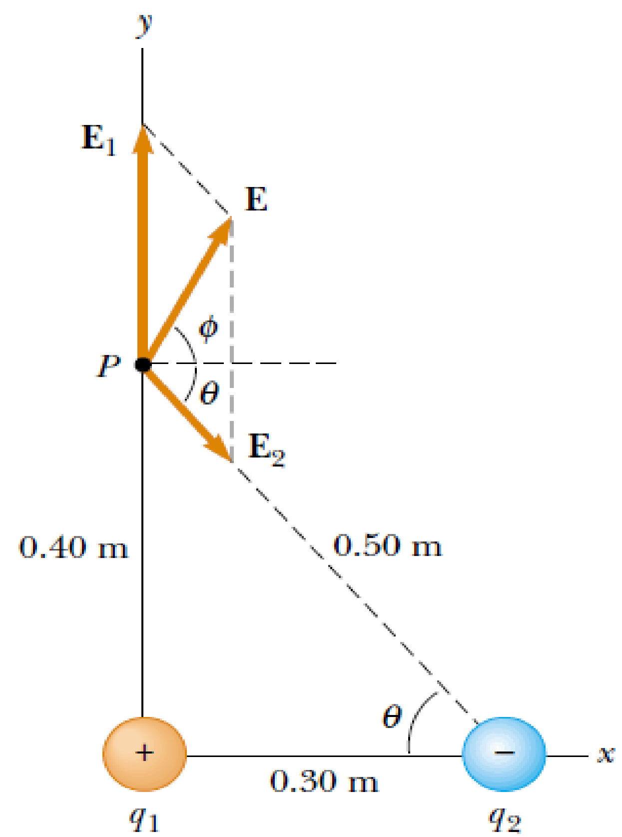
Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(7.0 \times 10^{-6}\ \mathrm{C})}{(0.40\ \mathrm{m})^2}$$

$$= 3.9 \times 10^5\ \mathrm{N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{(5.0 \times 10^{-6}\ \mathrm{C})}{(0.50\ \mathrm{m})^2}$$

$$= 1.8 \times 10^5\ \mathrm{N/C}$$



The total electric field $\mathbf{E}$ at P equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$, where $\mathbf{E}_1$ is the field due to the positive charge q_1 and $\mathbf{E}_2$ is the field due to the negative charge q_2.

The vector $\mathbf{E}_2$ has an x component given by

$$E_2 \cos\theta = \frac{3}{5}E_2$$

and a negative y component given by

$$-E_2 \sin\theta = -\frac{4}{5}E_2$$

Hence, we can express the vectors as

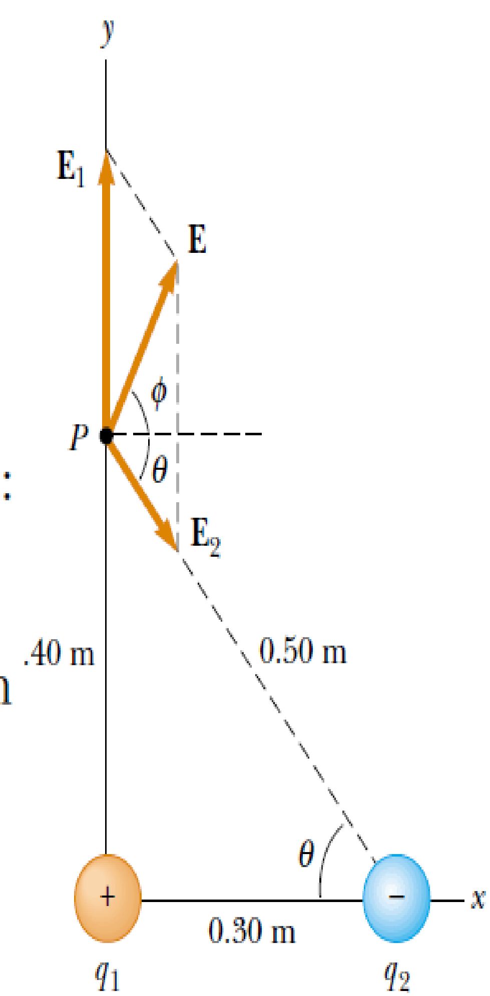
$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

$$E_1 = 3.9 \times 10^5 \text{ N/C}$$

$$E_2 = 1.8 \times 10^5 \text{ N/C}$$

The vector $\mathbf{E}_1$ has only a y component. coordinates (0, 0.40) m.



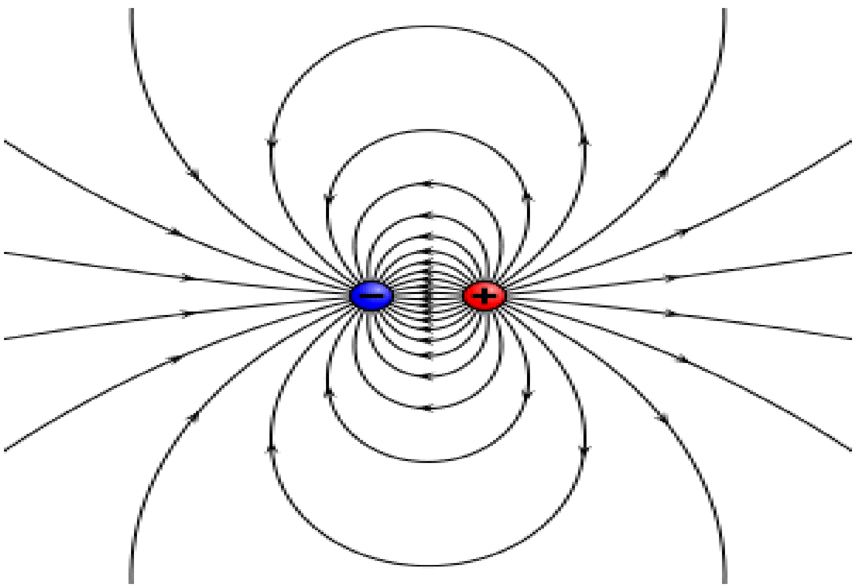
The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

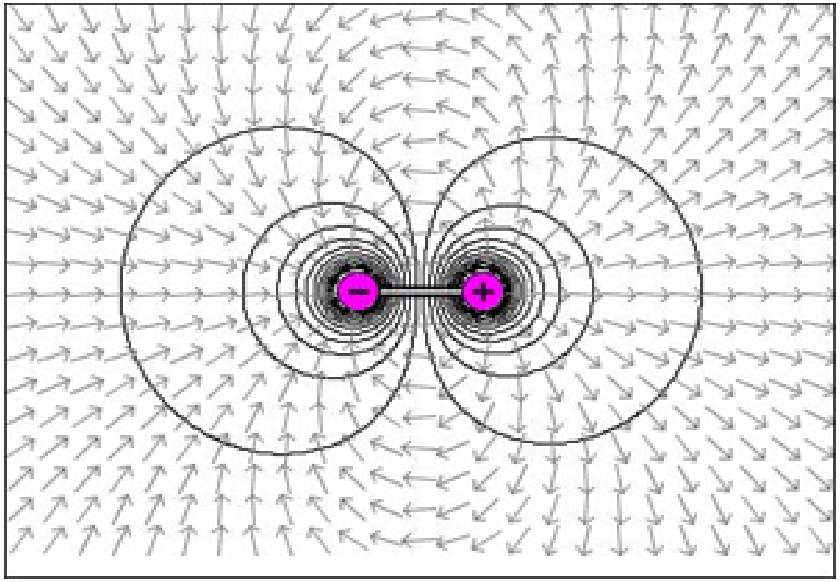
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

From this result, we find that $\mathbf{E}$ makes an angle ϕ of 66° with the positive x axis and has a magnitude of 2.7×10^5 N/C.

Electric Dipole & Dipole Moment

- **Electric Dipole**: Two **equal and opposite** point charges attached at a **fixed distance** is called Electric dipole.
- **Dipole Moment**: when the pair of these charges are placed in electric field they experience a turning effect this turning effect is known as **dipole moment**. $\vec{P} = 2ql$





Electric Field due to Dipole

An **electric dipole** is defined as a positive charge q and a negative charge $-q$ separated by a distance $2a$. For the dipole shown in Figure , find the electric field $\mathbf{E}$ at P due to the dipole, where P is a distance $y \gg a$ from the origin.

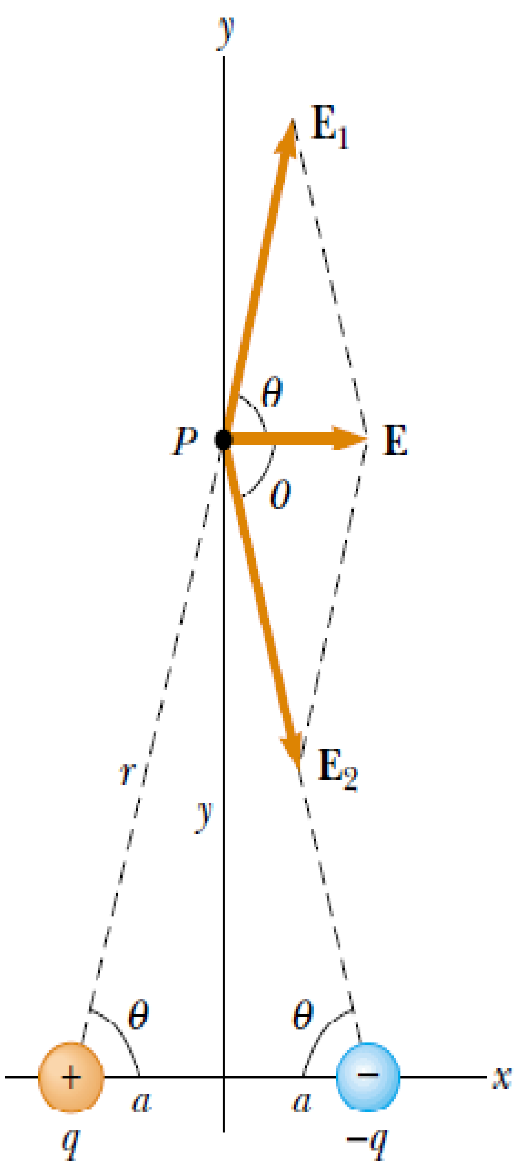


Figure The total electric field $\mathbf{E}$ at P due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $\mathbf{E}_1 + \mathbf{E}_2$. The field $\mathbf{E}_1$ is due to the positive charge q, and $\mathbf{E}_2$ is the field due to the negative charge $-q$.

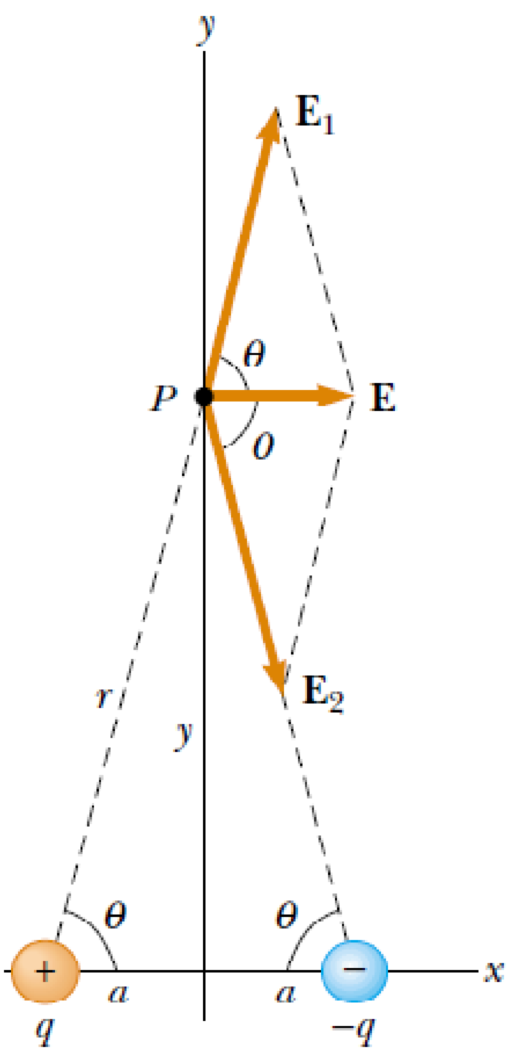
Solution At P, the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two charges are equal in magnitude because P is equidistant from the charges. The total field is $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$, where

$$E_1 = E_2 = k_e \frac{q}{r^2} = k_e \frac{q}{y^2 + a^2}$$

The y components of $\mathbf{E}_1$ and $\mathbf{E}_2$ cancel each other, and the x components are both in the positive x direction and have the same magnitude. Therefore, $\mathbf{E}$ is parallel to the x axis and has a magnitude equal to $2E_1 \cos\theta$. From Figure we see that $\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$. Therefore,

$$E = 2E_1 \cos\theta = 2k_e \frac{q}{(y^2 + a^2)} \frac{a}{(y^2 + a^2)^{1/2}}$$

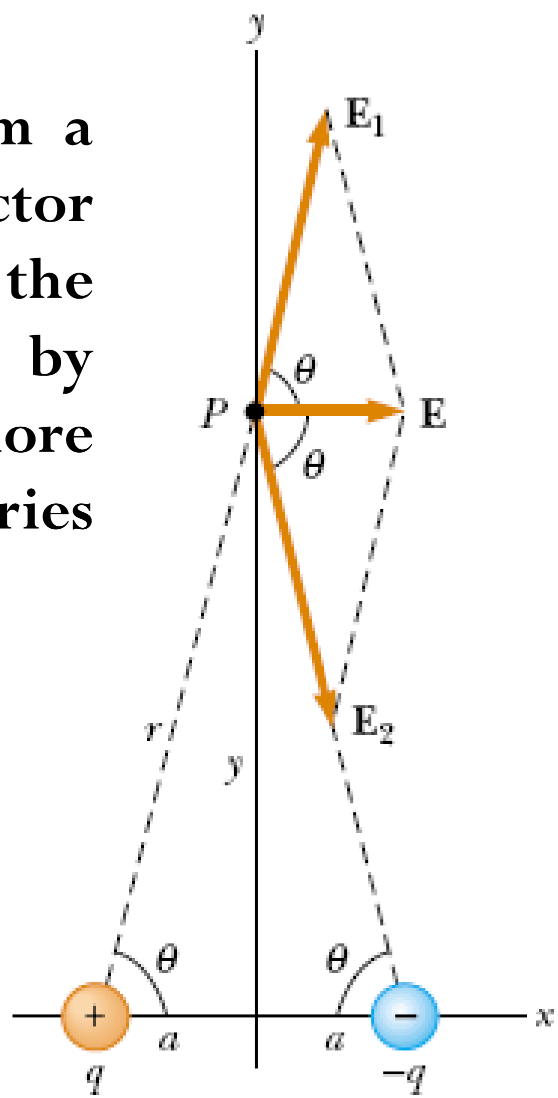
$$= k_e \frac{2qa}{(y^2 + a^2)^{3/2}}$$



Because $y \gg a$, we can neglect a^2 compared to y^2 and write

$$E \approx \boxed{k_e \frac{2qa}{y^3}}$$

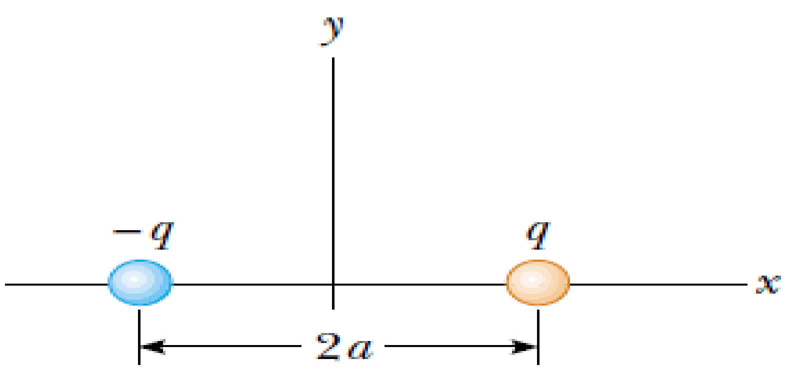
Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the line joining the two charges, the magnitude of the electric field created by the dipole varies as $1/r^3$, whereas the more slowly varying field of a point charge varies as $1/r^2$.



Home Work

Q 1 Two 2.00-μC point charges are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m. (a) Determine the electric field on the y axis at $y = 0.500$ m. (b) Calculate the electric force on a -3.00-μC charge placed on the y axis at $y = 0.500$ m.

Q 2 Consider the electric dipole shown in Figure . Show that the electric field at a *distant* point on the $+x$ axis is $E_x \approx 4k_e qa/x^3$.



Figure

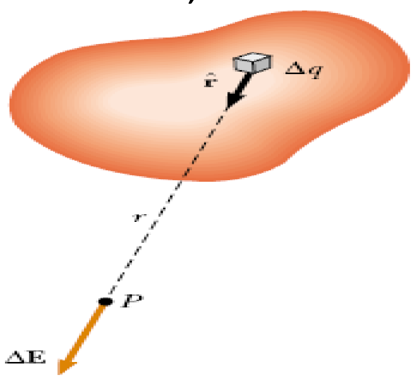
Electric Field

Electric Field of a Continuous Charge Distribution

Electric Field of a Continuous Charge Distribution

Electric field is said to exist in a region of space around a charged object, the source charge when another charged object, the test charge enters this electric field, an electric force acts on it.

To evaluate the electric field created by a continuous charge distribution, we use the following procedure:



- we divide the charge distribution into small elements, each of which contains a small charge Δq.

- we use Equation

$$\mathbf{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}$$

to calculate the electric field due to one of these elements at a point P.

- Finally, we evaluate the total electric field at P due to the charge distribution by summing the contributions of all the charge elements (that is, by applying the superposition principle).

The total electric field at P due to all elements in the charge distribution is approximately,

$$\mathbf{E} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i$$

where the index i refers to the ith element in the distribution.

Because the charge distribution is modeled as continuous, the total field at P in the limit $\Delta q_i \rightarrow 0$ is

$$\mathbf{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$

where the integration is over the entire charge distribution. This is a vector operation and must be treated appropriately.

- If a charge Q is uniformly distributed throughout a volume V, the **volume charge density** ρ is defined by

$$\rho \equiv \frac{Q}{V}$$

where ρ has units of coulombs per cubic meter (C/m^3).

- If a charge Q is uniformly distributed on a surface of area A, the **surface charge density** σ (lowercase Greek sigma) is defined by

$$\sigma \equiv \frac{Q}{A}$$

where σ has units of coulombs per square meter (C/m^2).

- If a charge Q is uniformly distributed along a line of length ℓ, the **linear charge density** λ is defined by

$$\lambda \equiv \frac{Q}{\ell}$$

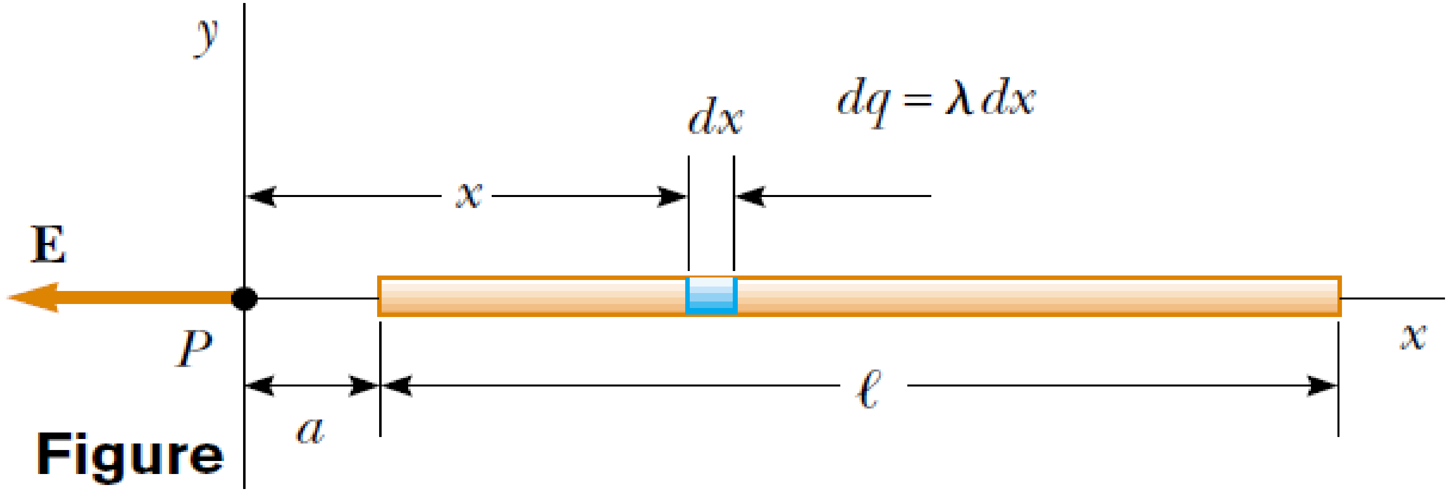
where λ has units of coulombs per meter (C/m).

- If the charge is nonuniformly distributed over a volume, surface, or line, the amounts of charge dq in a small volume, surface, or length element are

$$dq = \rho\, dV \qquad dq = \sigma\, dA \qquad dq = \lambda\, d\ell$$

Electric Field due to Charged Rod

A rod of length ℓ has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end (Fig.)



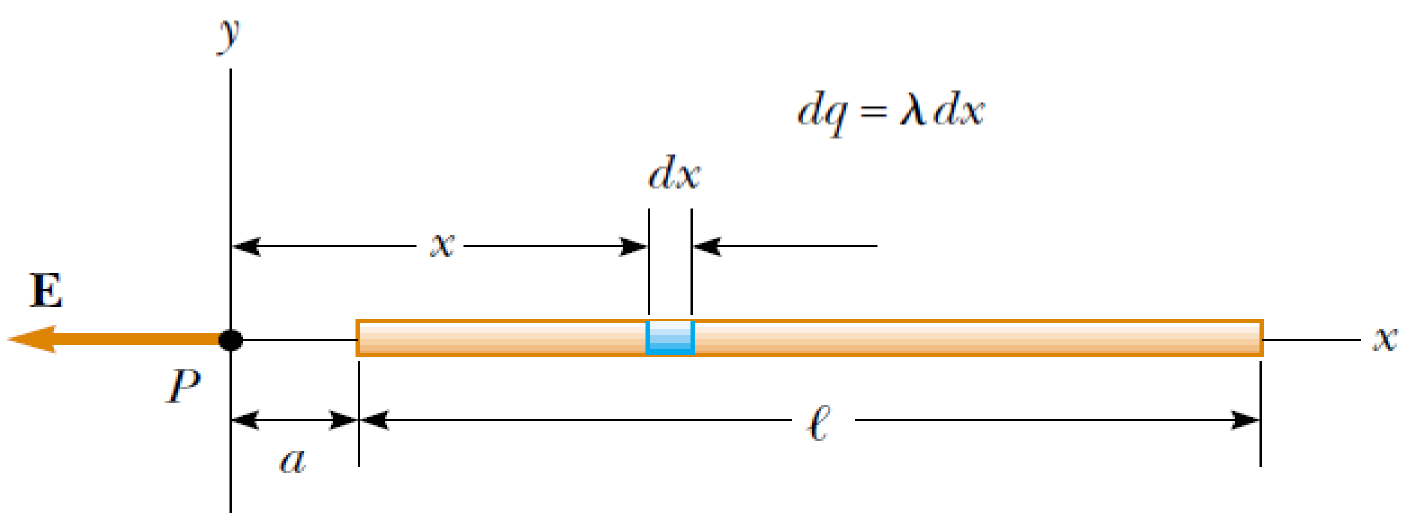
Figure

The electric field at P due to a uniformly charged rod lying along the x axis. The magnitude of the field at P due to the segment of charge dq is $k_e dq/x^2$. The total field at P is the vector sum over all segments of the rod.

Solution Let us assume that the rod is lying along the x axis, that dx is the length of one small segment, and that dq is the charge on that segment. Because the rod has a charge per unit length λ, the charge dq on the small segment is $dq = \lambda\, dx$.

The field $d\mathbf{E}$ at P due to this segment is in the negative x direction (because the source of the field carries a positive charge), and its magnitude is

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda\, dx}{x^2}$$

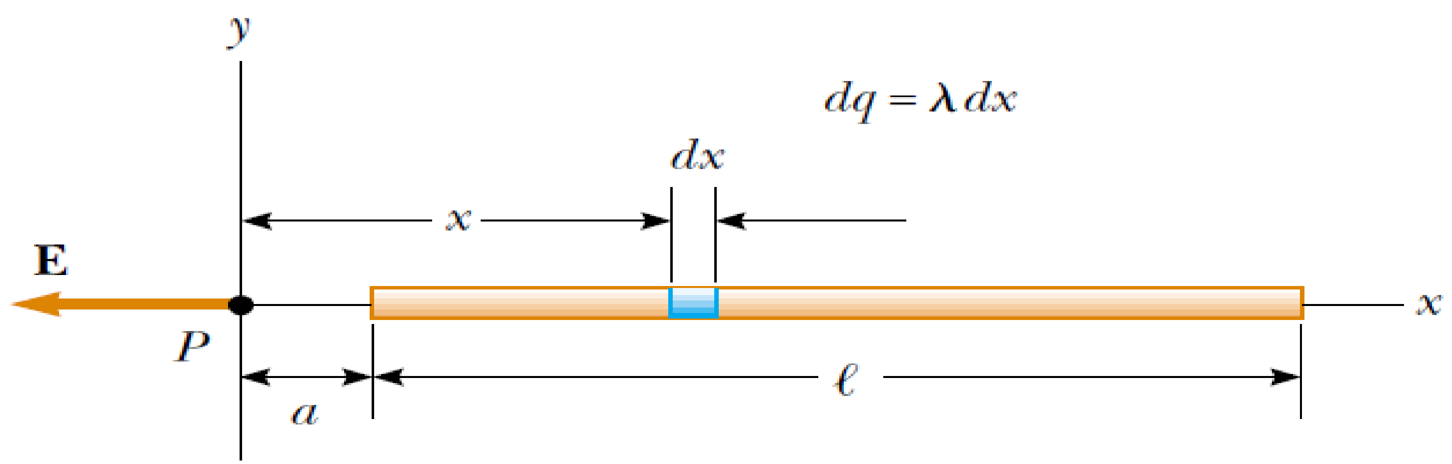


Because every other element also produces a field in the negative x direction, the problem of summing their contributions is particularly simple in this case. The total field at P due to all segments of the rod, which are at different distances from P, is given by Equation

$$= k_e \int \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

which in this case becomes[3]

$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$



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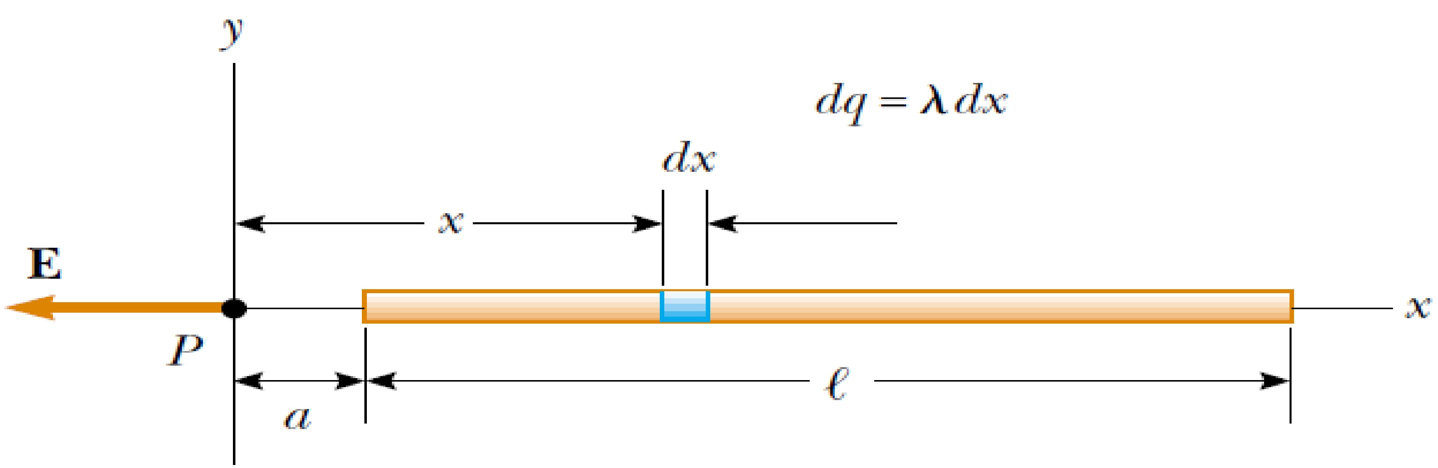
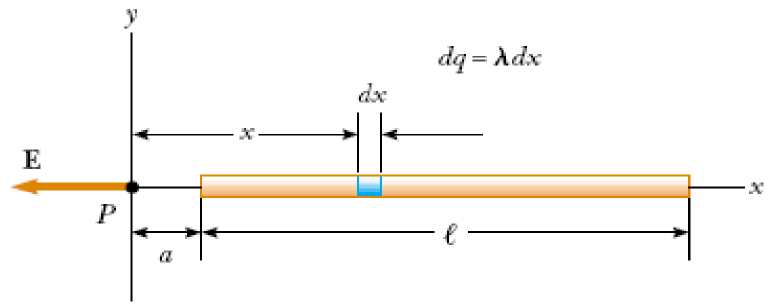
where the limits on the integral extend from one end of the rod ($x = a$) to the other ($x = \ell + a$). The constants k_e and λ can be removed from the integral to yield

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$= k_e \lambda \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$



where we have used the fact that the total charge $Q = \lambda \ell$.



What If? Suppose we move to a point *P* very far away from the rod. What is the nature of the electric field at such a point?

Answer If P is far from the rod ($a >> \ell$), then ℓ in the denominator of the final expression for E can be neglected, and $E \approx k_e Q/a^2$. This is just the form you would expect for a point charge. Therefore, at large values of a/ℓ, the charge distribution appears to be a point charge of magnitude Q—we are so far away from the rod that we cannot distinguish that it has a size. The use of the limiting technique ($a/\ell \rightarrow \infty$) often is a good method for checking a mathematical expression.

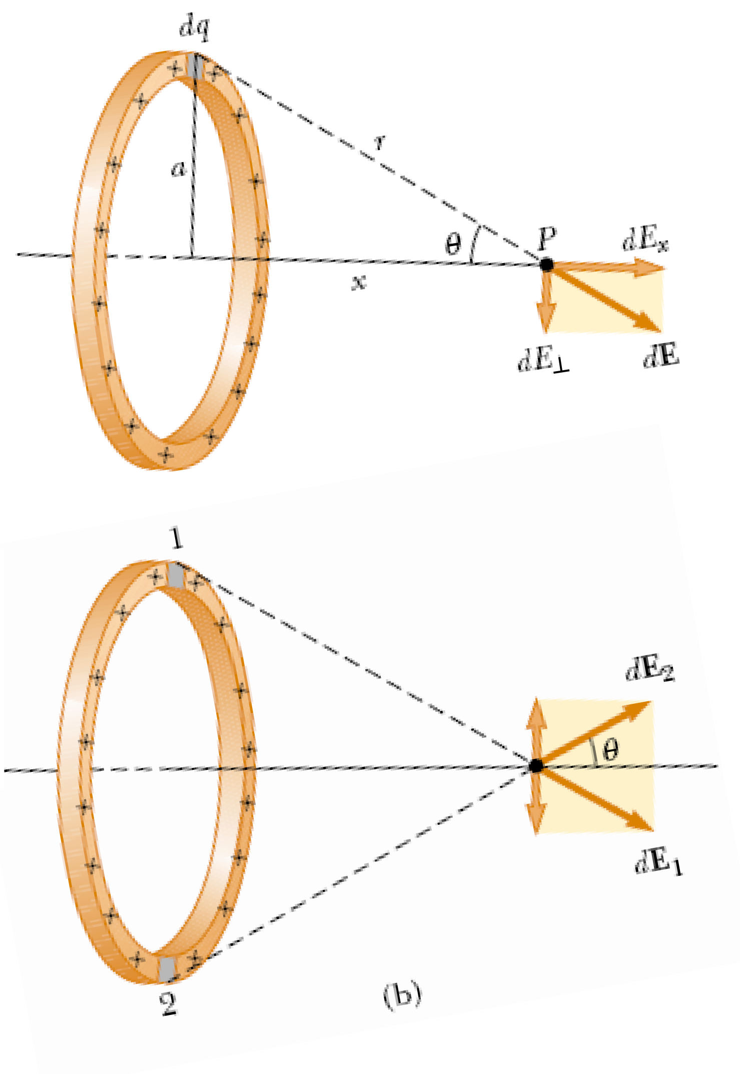
Electric Field of a Continuous Charge Distribution

Examples

- **Electric Field due to Uniform Ring of Charge**
- **Electric Field due to Uniformly Charged Disk**

Electric Field due to Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring (Fig. 23.18a).

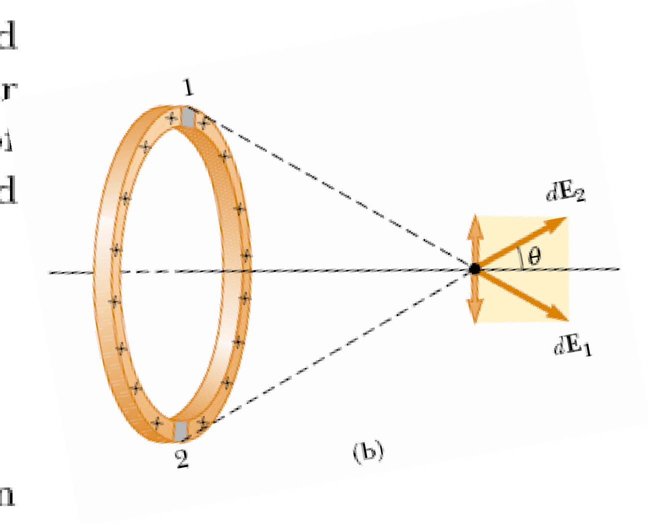


Solution The magnitude of the electric field at P due to the segment of charge dq is

$$dE = k_e \frac{dq}{r^2}$$

This field has an x component $dE_x = dE \cos\theta$ along the x axis and a component $dE_\perp$ perpendicular to the x axis. As we see in Figure 23.18b, however, the resultant field at P must lie along the x axis because the perpendicular components of all the various charge segments sum to zero.

That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because $r = (x^2 + a^2)^{1/2}$ and $\cos\theta = x/r$, we find that



$$dE_x = dE\cos\theta = \left(k_e\frac{dq}{r^2}\right)\frac{x}{r} = \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq$$

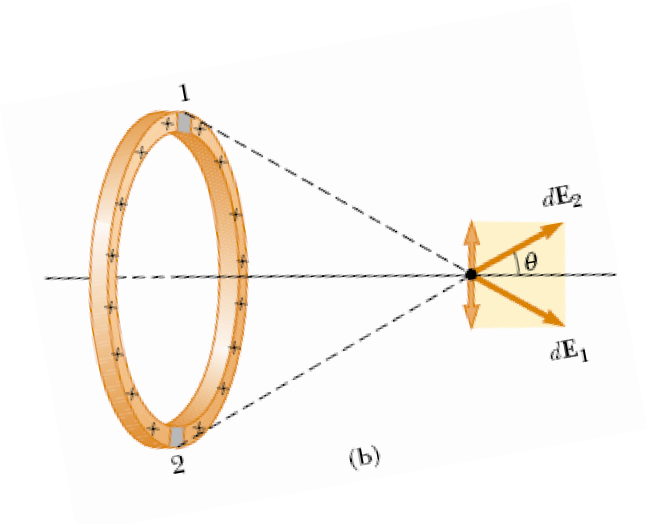
All segments of the ring make the same contribution to the field at P because they are all equidistant from this point. Thus, we can integrate to obtain the total field at P:

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\,dq = \frac{k_e x}{(x^2 + a^2)^{3/2}}\int dq$$

$$= \boxed{\frac{k_e x}{(x^2 + a^2)^{3/2}}\,Q}$$

This result shows that the field is zero at $x = 0$.

What If? Suppose a negative charge is placed at the center of the ring in Figure 23.18 and displaced slightly by a distance $x \ll a$ along the x axis. When released, what type of motion does it exhibit?



Answer In the expression for the field due to a ring of charge, we let $x \ll a$, which results in

$$E_x = \frac{k_e Q}{a^3} x$$

Thus, from Equation 23.8, the force on a charge $-q$ placed near the center of the ring is

$$F_x = -\frac{k_e q Q}{a^3} x$$