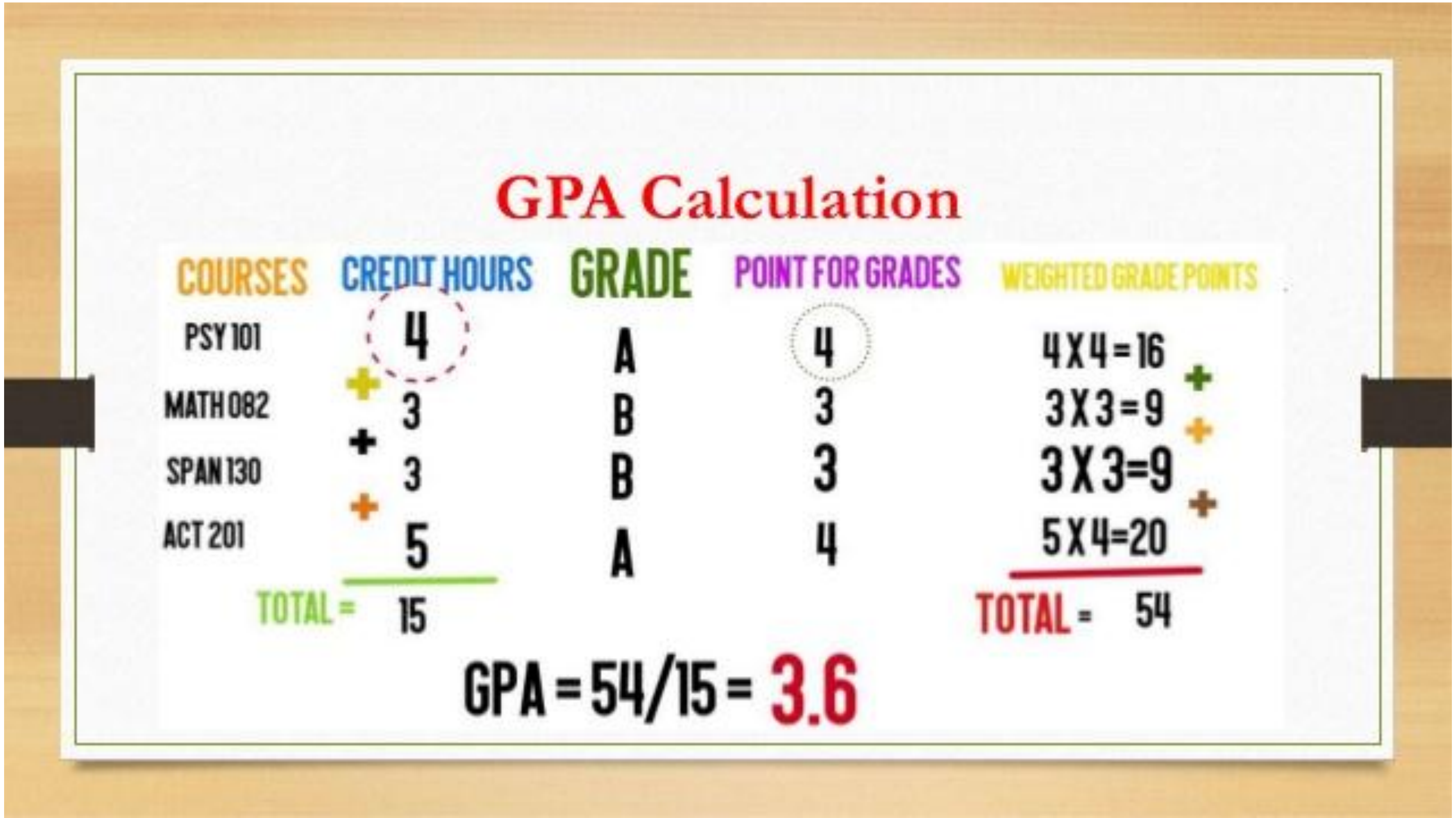
Grade point vs Marks:

BS / SE BBA/ EE	Equivalent %	Grade Points	Marks
A	86	4.00	86
A-	82	3.67	82
B+	78	3.33	78
B	74	3.00	74
B -	70	2.67*	70
C+	66	2.33*	66
C	62	2.00*	62
C-	58	1.67*	58
D+	54	1.33*	54
D	50	1.00*	50

Letter grades for BBA & BS programs

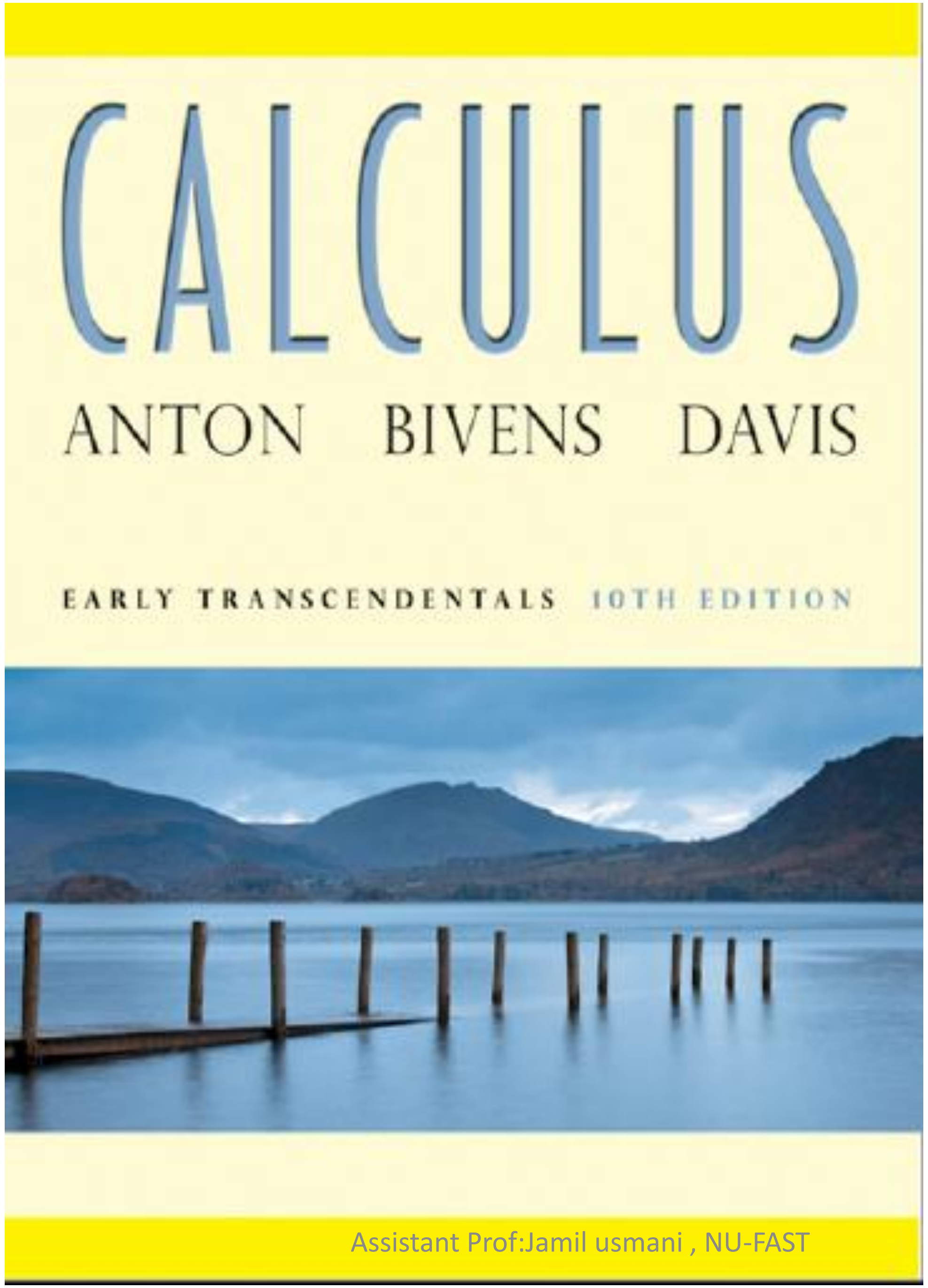
Grade	Points	Interpretation
A+	4.00	Excellent
A	4.00	Excellent
A-	3.67	Very Good
B+	3.33	Very Good
B	3.00	Good
B-	2.67	Average
C+	2.33	Below Average
C	2.00	Adequate
C-	1.67	Pass
D+	1.33	Pass
D	1.00	Pass

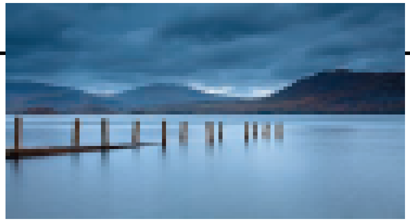
Grade Point Average : GPA (calculuation)



Course code: MT119
Course title: Calculus and Analytical Geometry
Credit hour :3+0

Book Title: *Calculus Early Transcendental 10th Edition*
Author(Howard Anton)


CALCULUS
ANTON BIVENS DAVIS
EARLY TRANSCENDENTALS 10TH EDITION



10th EDITION

CALCULUS

EARLY TRANSCENDENTALS

- **HOWARD ANTON** *Drexel University*
- **IRL BIVENS** *Davidson College*
- **STEPHEN DAVIS** *Davidson College*

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What is CALCULUS ?

- Calculus is a branch of mathematics that involves the study of rates of change.
- Leibniz and Isaac Newton, 17th-century mathematicians, both invented calculus independently. Newton invented it first, but Leibniz created the notations that mathematicians use today.
- There are two types of calculus:

  1-Differential calculus determines the rate of change of a quantity

  2-integral calculus finds the quantity where the rate of change is known.

Practical Applications

- Calculus has many practical applications in real life. Some of the concept that use calculus include motion, electricity, heat, light, harmonics, acoustics, and astronomy.
- Calculus is used in geography, computer vision (such as for autonomous driving of cars), photography, artificial intelligence, robotics, video games, and even movies.
- Calculus is also used to calculate the rates of radioactive decay in chemistry, and even to predict birth and death rates, as well as in the study of gravity and planetary motion, fluid flow, ship design, geometric curves, and bridge engineering.

Practical Applications

- In physics, for example, calculus is used to help define, explain, and calculate motion, electricity, heat, light, harmonics, acoustics, astronomy, and dynamics.
- Einstein's theory of relativity relies on calculus,
- A field of mathematics that also helps economists predict how much profit a company or industry can make and in shipbuilding.
- calculus has been used for many years to determine both the curve of the hull of the ship (using differential calculus), as well as the area under the hull (using integral calculus), and even in the general design of ships.
- In addition, calculus is used to check answers for different mathematical disciplines such as statistics, analytical geometry, and algebra.

Detail Outline:
Function,Limit and countinuity

- Introduction to Functions, vertical line test, Piecewise and Absolute value function, Domain and Range, Composition of function, Symmetry Test
- Basic Concepts of limit.
- Evaluation of limits.
- Continuity and point of discontinuity. Types of discontinuity

Differential Calculus:

- Secant line, Equation of Normal and tangent line, Slope ,Rate of change
- Concept and idea of differentiation, Geometrical and Physical meaning of derivatives,
- Rules and techniques of differentiation.
- Product and quotient rule
- Derivative of trigonometric function
- Chain rule

Applications of Derivative in Graphing:

- Implicit differentiation
- Indeterminate forms ,L' Hospital Rule
- Role's and Mean Value's Theorem.
- Concavity, Increasing and Decreasing interval
- Relative Extrema (Maxima and Minima), 1st derivative and 2nd derivative test
- Absolute Maxima and Minima

Integral Calculus:

- Concept and idea of Integration, Indefinite Integrals,Riemann sums
- Techniques of integration
- Basic Integration ,Integration by parts Trigonometric substitution
- Integration of Rational function by Partial fraction
- Improper integrals
- Applications of Integration, Definite Integrals ,
- Area bounded by the curves.
- Volume by Disk and washer method
- Applications of Integration : Arc length

Analytical Geometry:

- Parametric equations of lines in 3D
- Plane in 3-space ,
- Distance Problems involving planes,
- Intersecting planes

Grading Criteria:
Marks Distribution:

• 1. Class participation/Attendance	02
• 2. Quizzes	10
• 3. Assignments	08
• 4. First Mid Exam	15
• 5. Second Mid Exam	15
• 6. Final Exam	50
Total:-	**100**

Number systems:

$\mathbb{N}$ = the set of natural numbers

$\mathbb{Q}$ = the set of rational numbers

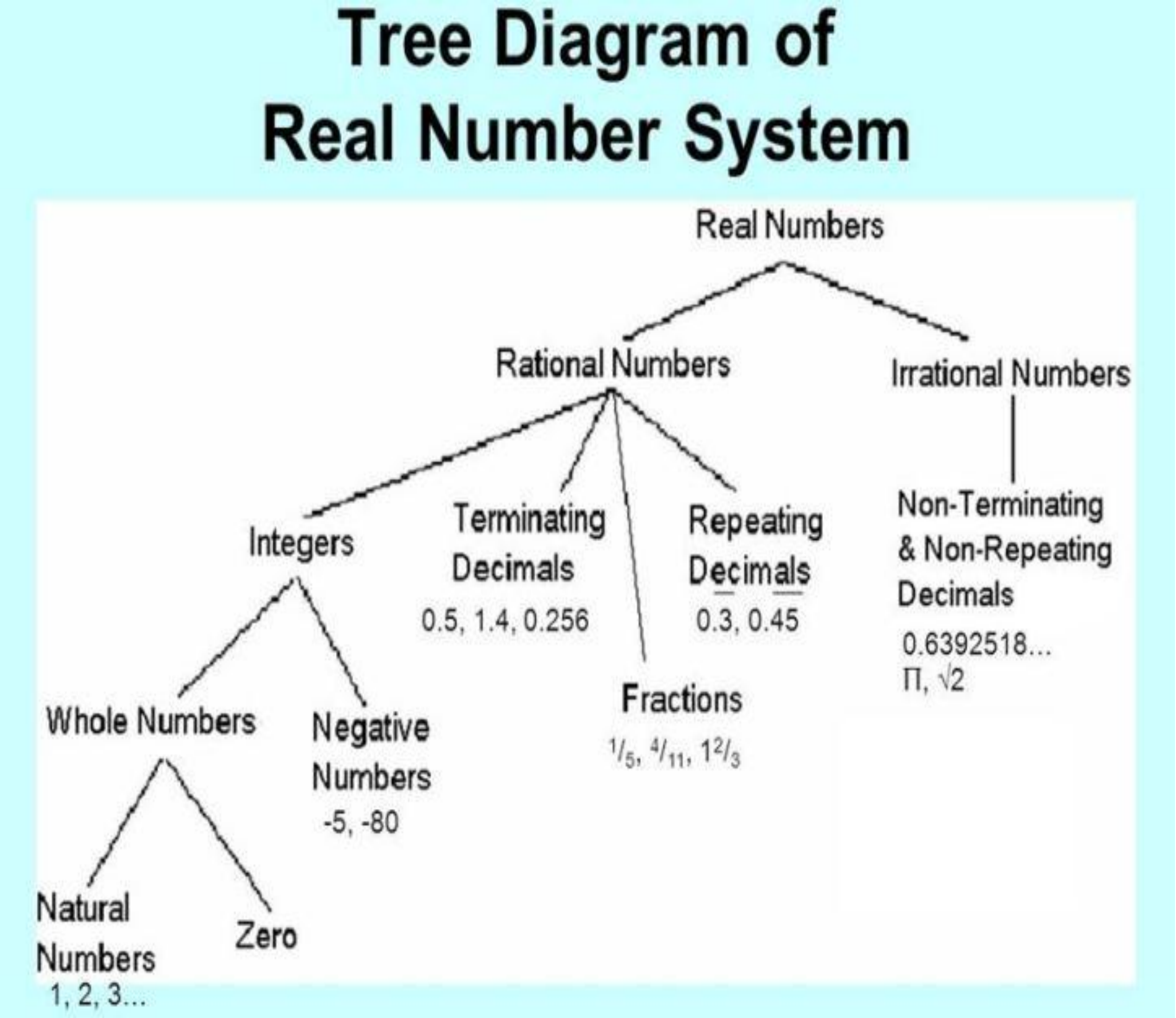
$\mathbb{R}$ = the set of real numbers

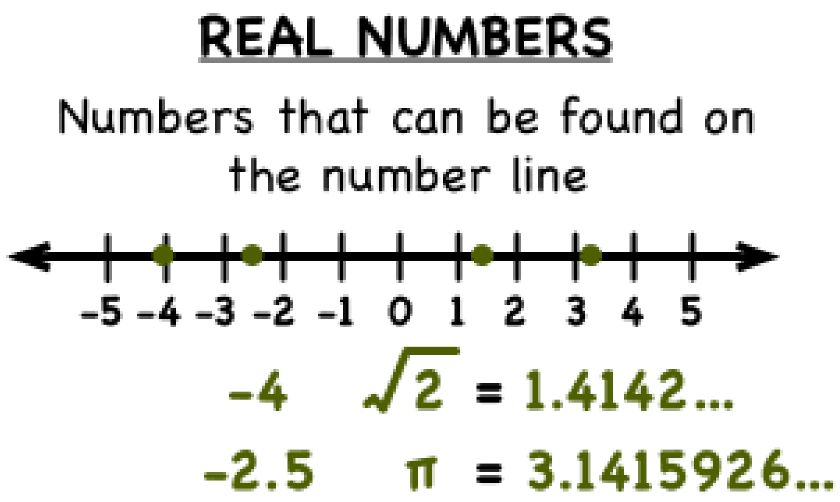
$\mathbb{P}$ = the set of prime numbers

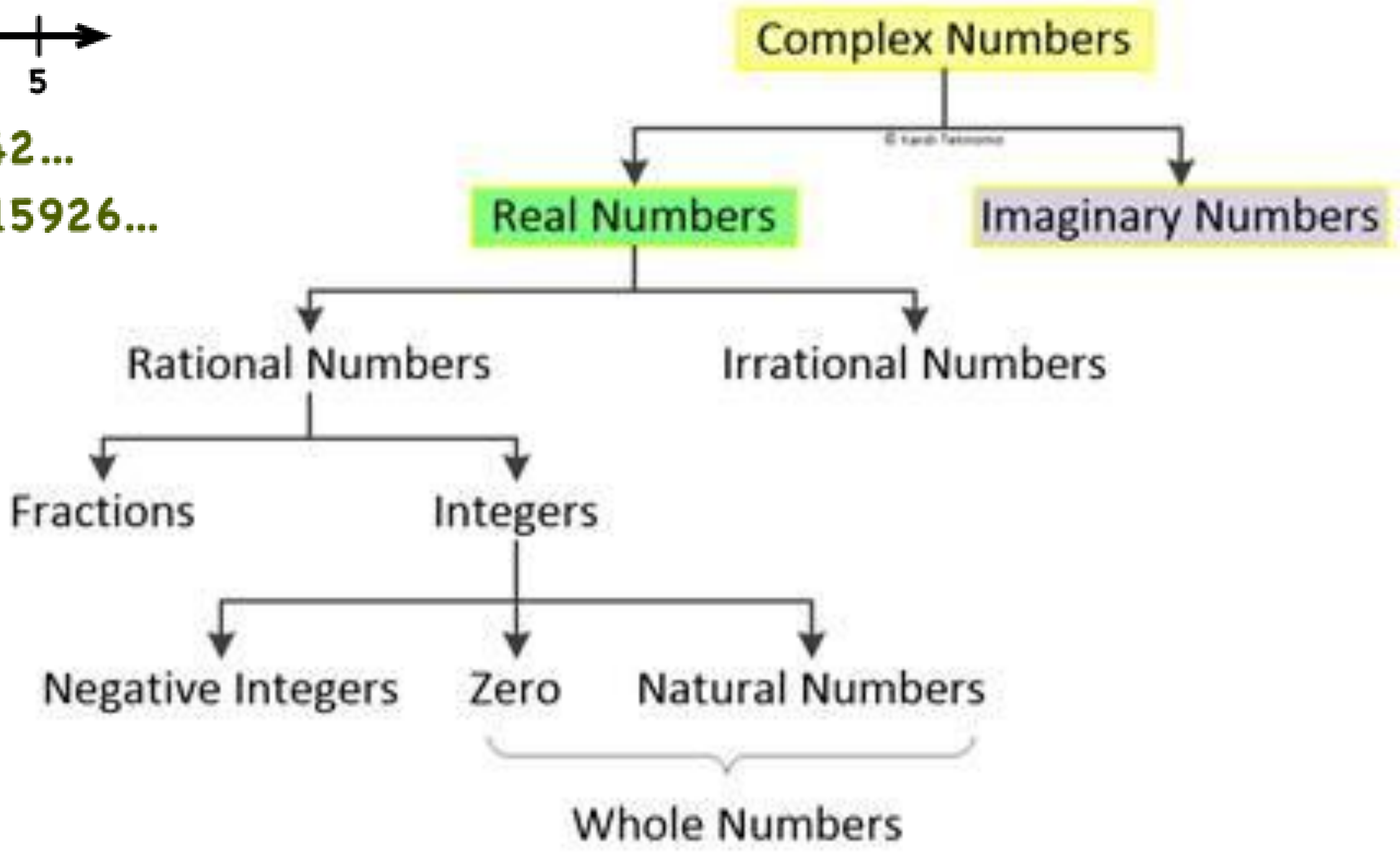
$\mathbb{Z}$ = the set of integers

$\mathbb{E}$ = the set of even integers

$\mathbb{O}$ = the set of odd integers




REAL NUMBERS
Numbers that can be found on the number line
-5 -4 -3 -2 -1 0 1 2 3 4 5
-4
√2 = 1.4142...
-2.5
π = 3.1415926...


Complex Numbers
Real Numbers
Imaginary Numbers
Rational Numbers
Irrational Numbers
Fractions
Integers
Negative Integers
Zero
Natural Numbers
Whole Numbers

Properties of Real Numbers

The real number system is an example of a mathematical structure called a ***field***.

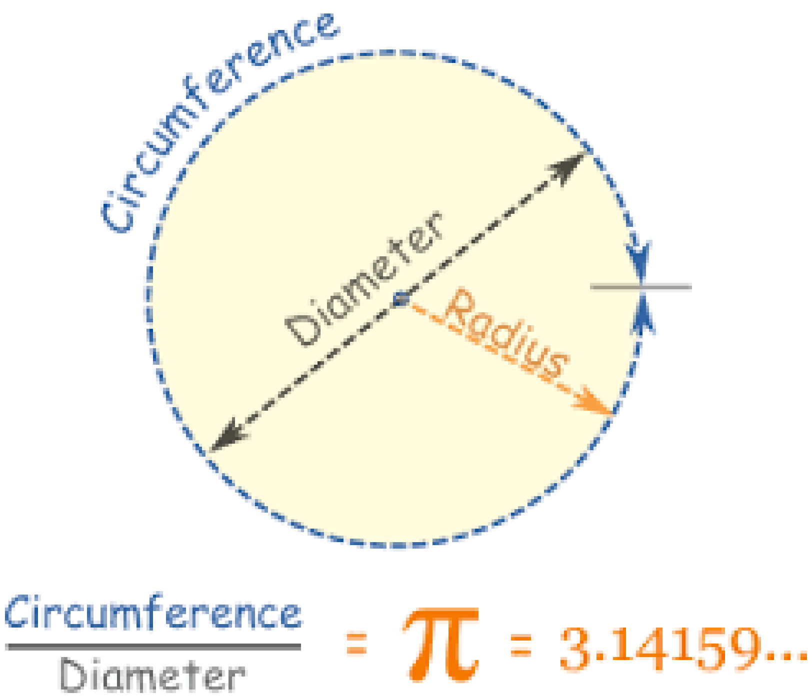
Some of the properties of a field are summarized in the table below:

Real Number Properties		
For any real numbers **a**, **b**, and **c**.		
Property	Addition	Multiplication
Commutative	$a+b=b+a$	$a \cdot b = b \cdot a$
Associative	$(a+b)+c = a+(b+c)$	$(a \cdot b)\cdot c = a \cdot (b \cdot c)$
Identity	$a+0 = a = 0+a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a+(-a) = 0 = (-a)+a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$
Distributive	$a(b+c) = ab+ac$ and $(b+c)a = ba+ca$	

LAWS OF SETS:

Name	Identities	
Idempotent laws	$A \cup A = A$	$A \cap A = A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double Complement law	$\overline{\overline{A}} = A$	
Complement laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = U$ $\overline{\varnothing} = U$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

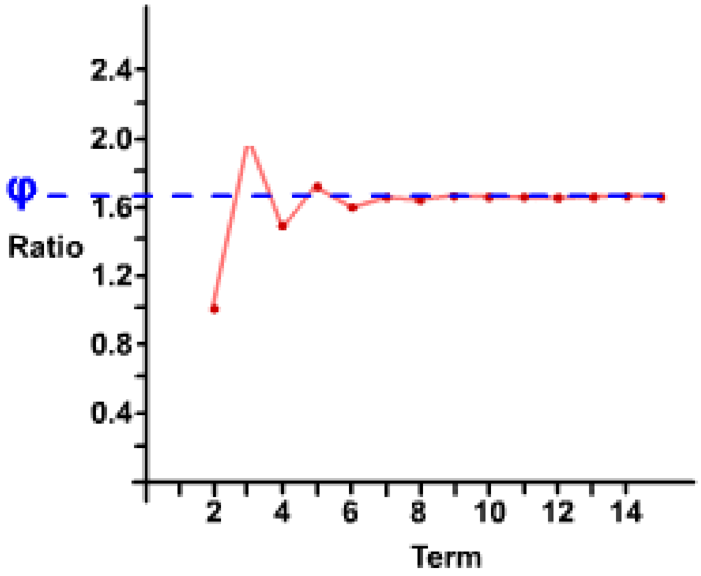
History of Pi and Golden Ratio



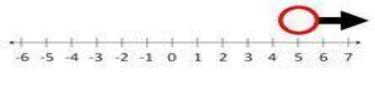
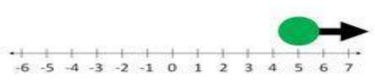
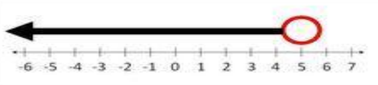
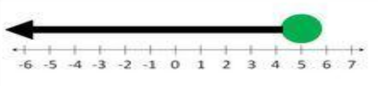
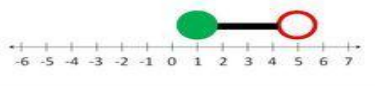
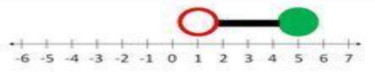
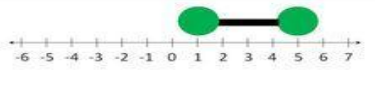
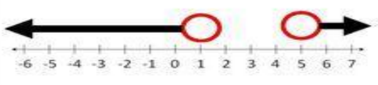
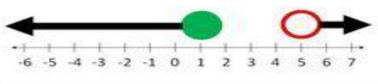
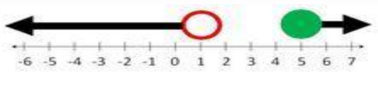
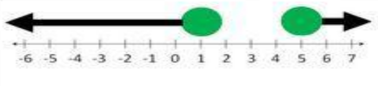
The Golden Ratio **φ** can be approximated by a process of successively dividing each term in the Fibonacci Sequence by the previous term.

With each successive division, the ration comes closer and closer to a value of 1.618033987...

2 ÷ 1 = 2.0000
3 ÷ 2 = 1.5000
5 ÷ 3 = 1.6666
8 ÷ 5 = 1.6000
13 ÷ 8 = 1.6250
21 ÷ 13 = 1.6154
34 ÷ 21 = 1.6190
55 ÷ 34 = 1.6176
89 ÷ 55 = 1.6182
etc...

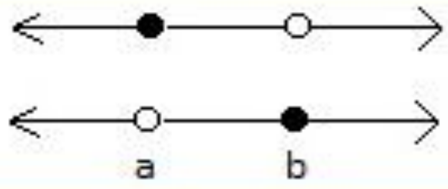
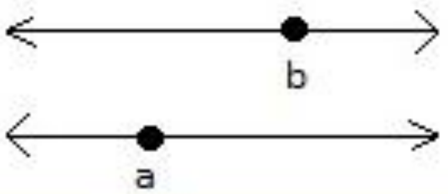
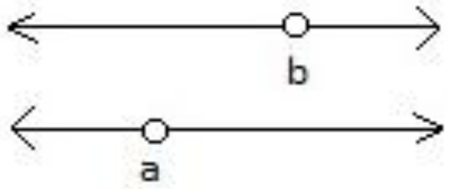


Inequality vs. Interval Notation

$x > 5$		$(5, \infty)$
$x \geq 5$		$[5, \infty)$
$x < 5$		$(-\infty, 5)$
$x \leq 5$		$(-\infty, 5]$
$1 < x < 5$		$(1, 5)$
$1 \leq x < 5$		$[1, 5)$
$1 < x \leq 5$		$(1, 5]$
$1 \leq x \leq 5$		$[1, 5]$
$x < 1$ or $x > 5$		$(-\infty, 1) \cup (5, \infty)$
$x \leq 1$ or $x > 5$		$(-\infty, 1] \cup (5, \infty)$
$x < 1$ or $x \geq 5$		$(-\infty, 1) \cup [5, \infty)$
$x \leq 1$ or $x \geq 5$		$(-\infty, 1] \cup [5, \infty)$

INTERVALS :set of real numbers called intervals

Intervals

Name of interval	Notation	Inequality description	Number line representation
Finite and closed	[a, b]	$a \leq x \leq b$	
Finite and open	(a, b)	$a < x < b$	
Finite and half-open	[a, b) (a, b]	$a \leq x < b$ $a < x \leq b$	
Infinite and closed	(- ∞ , b] [a, + ∞)	$-\infty < x \leq b$ $a \leq x < +\infty$	
Infinite and open	(- ∞ , b) (a, + ∞)	$-\infty < x < b$ $a < x < +\infty$	
Infinite and open	(- ∞ , + ∞)	$-\infty < x < +\infty$	

UNIONS AND INTERSECTIONS OF INTERVALS

$$(0, 5) \cup (1, 7) = (0, 7)$$

$$(-\infty, 1) \cap [0, +\infty) = [0, 1)$$

$$(-\infty, 0) \cap (0, +\infty) = \varnothing$$

OR

$$\{x : 0 < x < 5\} \cup \{x : 1 < x < 7\} = \{x : 0 < x < 7\}$$

$$\{x : x < 1\} \cap \{x : x \geq 0\} = \{x : 0 \leq x < 1\}$$

$$\{x : x < 0\} \cap \{x : x > 0\} = \varnothing$$

Inequality Symbols	
$\neq$	not equal
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to

ALGEBRAIC PROPERTIES OF INEQUALITIES

E.1 THEOREM (*Properties of Inequalities*) *Let a, b, c, and d be real numbers.*

(a) *If $a < b$ and $b < c$, then $a < c$.*

(b) *If $a < b$, then $a + c < b + c$ and $a - c < b - c$.*

(c) *If $a < b$, then $ac < bc$ when c is positive and $ac > bc$ when c is negative.*

(d) *If $a < b$ and $c < d$, then $a + c < b + d$.*

(e) *If a and b are both positive or both negative and $a < b$, then $1/a > 1/b$.*

Inequalities:

Example 3 Solve $7 \leq 2 - 5x < 9$.

Example 4 Solve $x^2 - 3x > 10$.

Example 5 Solve $\dfrac{2x-5}{x-2} < 1$.

THE ABSOLUTE VALUE FUNCTION

Recall that the *absolute value* or *magnitude* of a real number x is defined by

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

0.1.4 PROPERTIES OF ABSOLUTE VALUE *If a and b are real numbers, then*

(a)	$\lvert -a \rvert = \lvert a \rvert$	A number and its negative have the same absolute value.
(b)	$\lvert ab \rvert = \lvert a \rvert\, \lvert b \rvert$	The absolute value of a product is the product of the absolute values.
(c)	$\lvert a/b \rvert = \lvert a \rvert / \lvert b \rvert,\ b \neq 0$	The absolute value of a ratio is the ratio of the absolute values.
(d)	$\lvert a+b \rvert \leq \lvert a \rvert + \lvert b \rvert$	The ***triangle inequality***

Rule for INEQUALITIES WITH ABSOLUTE VALUES

INEQUALITY ($k > 0$)	ALTERNATIVE FORMS OF THE INEQUALITY
$\lvert x - a\rvert < k$	$-k < x - a < k$ $a - k < x < a + k$
$\lvert x - a\rvert > k$	$x - a < -k$ or $x - a > k$ $x < a - k$ or $x > a + k$

Solve

(a) $|x - 3| < 4$ (b) $|x + 4| \geq 2$ (c) $\dfrac{1}{|2x - 3|} > 5$

INEQUALITIES WITH ABSOLUTE VALUES

Solve :

$|3x - 7| = -5$ No sol

$|x + 6| > 0$ $x \neq -6$

$|x - 4| \geq 0$ All Real

$|2x - 1| < 0$ No sol

$|x + 1| \leq 0$

$3|4x - 1| \leq 9$

$|3x - 7| = 2$

$|3x + 12| = 0$

$|x + 5| = |2x - 1|$

$|3x + 4| \leq -2$ No sol

$|x - 1| \geq 3$

$3|4x - 1| \leq 9$

Relation and Function:

- A relation is a set of ordered pairs (x, y) OR A subset of AxB

  Example: The set {(1,a), (1, b), (2,b), (3,c), (3, a), (4,a)} is a relation
- A relation is not a function.
- A function is a relation (so, it is the set of ordered pairs) that does not contain two pairs with the same first component.
- OR

> **0.1.1 DEFINITION** If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that *y is a function of x*.

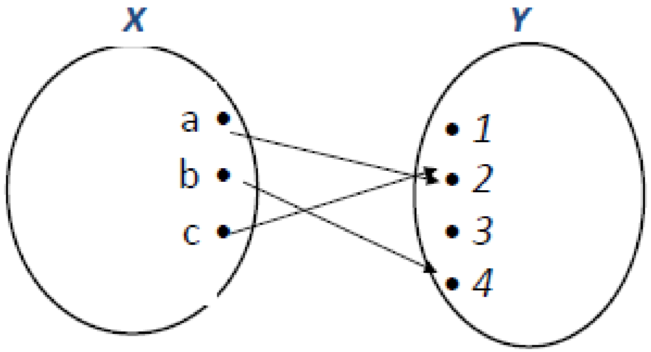
Function

Let X and Y be sets. A **function** f from X to Y is a rule that assigns every element x of X *to a unique y in* Y.
We write $\boldsymbol{f: X \to Y}$ and $\boldsymbol{f(x) = y}$

X = domain, Y = codomain
y = image of x under f,
x = preimage of y under f
range = subset of Y with preimages

Arrow Diagram of f:

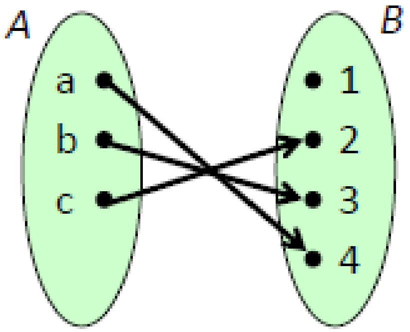
Domain X={a,b,c},
Co-domainY={1,2,3,4}
f={(a,2),(b,4),(c,2)},
preimage of 2 is{a,c}
Range={2,4}



A function f is **one-to-one** (or **injective**), if and only if $f(x) = f(y)$ implies $x = y$ for all x and y in the domain of f.

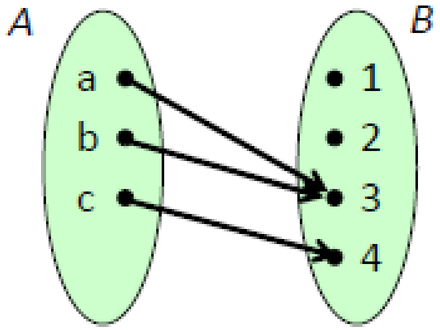
In words:

"All elements in the domain of f have different images"



one-to-one

all elements in A have a different image)



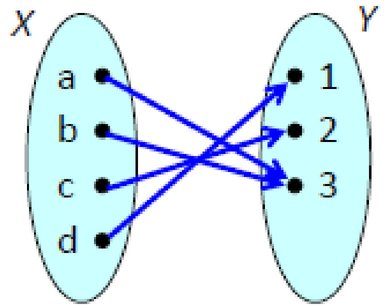
not one-to-one

(a and b have the same image)

Onto Functions

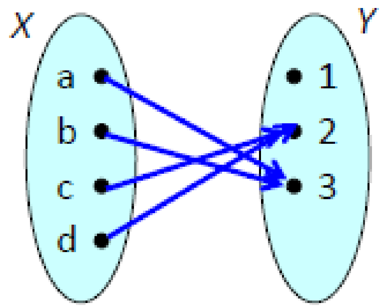
A function *f* from *X* to *Y* is **onto** (or **surjective**), if and only if for every element $y \in Y$ there is an element $x \in X$ with $f(x)=y$.

In words:
"Each element in the co-domain of *f* has a pre-image"



onto

(all elements in Y have a pre-image)

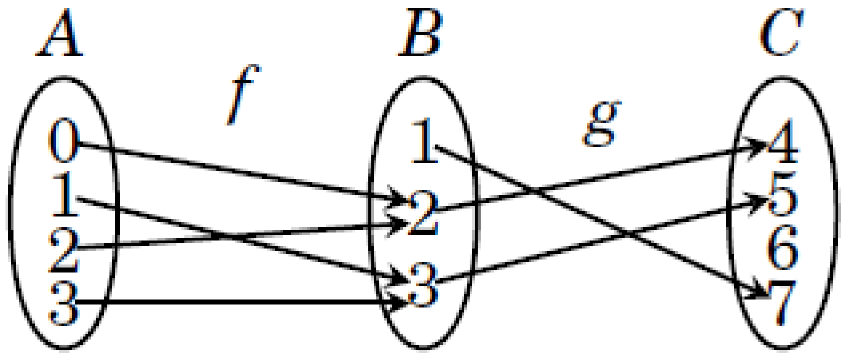


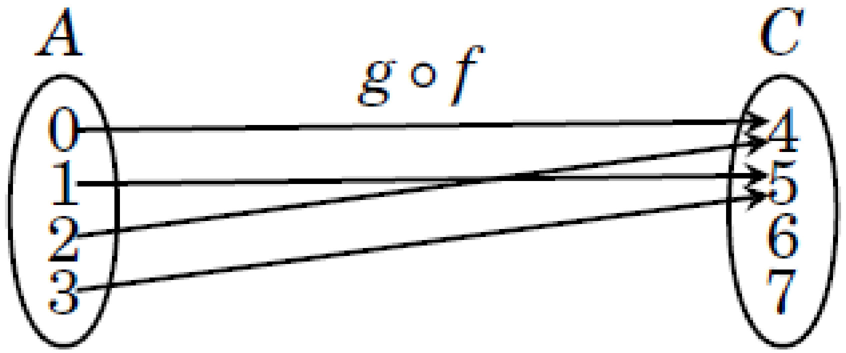
not onto

(1 has no pre-image)

Composition of two functions

Example:





Example:

Suppose $A=\{a,b,c\}$, $B=\{0,1\}$, $C=\{1,2,3\}$. Let $f:A\to B$ be the function $f=\{(a,0),(b,1),(c,0)\}$, and let $g:B\to C$ be the function $g=\{(0,3),(1,1)\}$. Then $g\circ f=\{(a,3),(b,1),(c,3)\}$.

Practice: [composite function]

1. Suppose $A = \{5,6,8\}$, $B = \{0,1\}$, $C = \{1,2,3\}$. Let $f : A \to B$ be the function $f = \{(5,1),(6,0),(8,1)\}$, and $g : B \to C$ be $g = \{(0,1),(1,1)\}$. Find $g \circ f$.
2. Suppose $A = \{1,2,3,4\}$, $B = \{0,1,2\}$, $C = \{1,2,3\}$. Let $f : A \to B$ be

$$f = \{(1,0),(2,1),(3,2),(4,0)\},$$

   and $g : B \to C$ be $g = \{(0,1),(1,1),(2,3)\}$. Find $g \circ f$.
3. Suppose $A = \{1,2,3\}$. Let $f : A \to A$ be the function $f = \{(1,2),(2,2),(3,1)\}$, and let $g : A \to A$ be the function $g = \{(1,3),(2,1),(3,2)\}$. Find $g \circ f$ and $f \circ g$.
4. Suppose $A = \{a,b,c\}$. Let $f : A \to A$ be the function $f = \{(a,c),(b,c),(c,c)\}$, and let $g : A \to A$ be the function $g = \{(a,a),(b,b),(c,a)\}$. Find $g \circ f$ and $f \circ g$.

0.2.2 DEFINITION Given functions f and g, the ***composition*** of f with g, denoted by $f \circ g$, is the function defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is defined to consist of all x in the domain of g for which $g(x)$ is in the domain of f.

Practice:

31–34 Find formulas for $f \circ g$ and $g \circ f$, and state the domains of the compositions.

31. $f(x) = x^2,\ g(x) = \sqrt{1-x}$

32. $f(x) = \sqrt{x-3},\ g(x) = \sqrt{x^2+3}$

33. $f(x) = \dfrac{1+x}{1-x},\ g(x) = \dfrac{x}{1-x}$

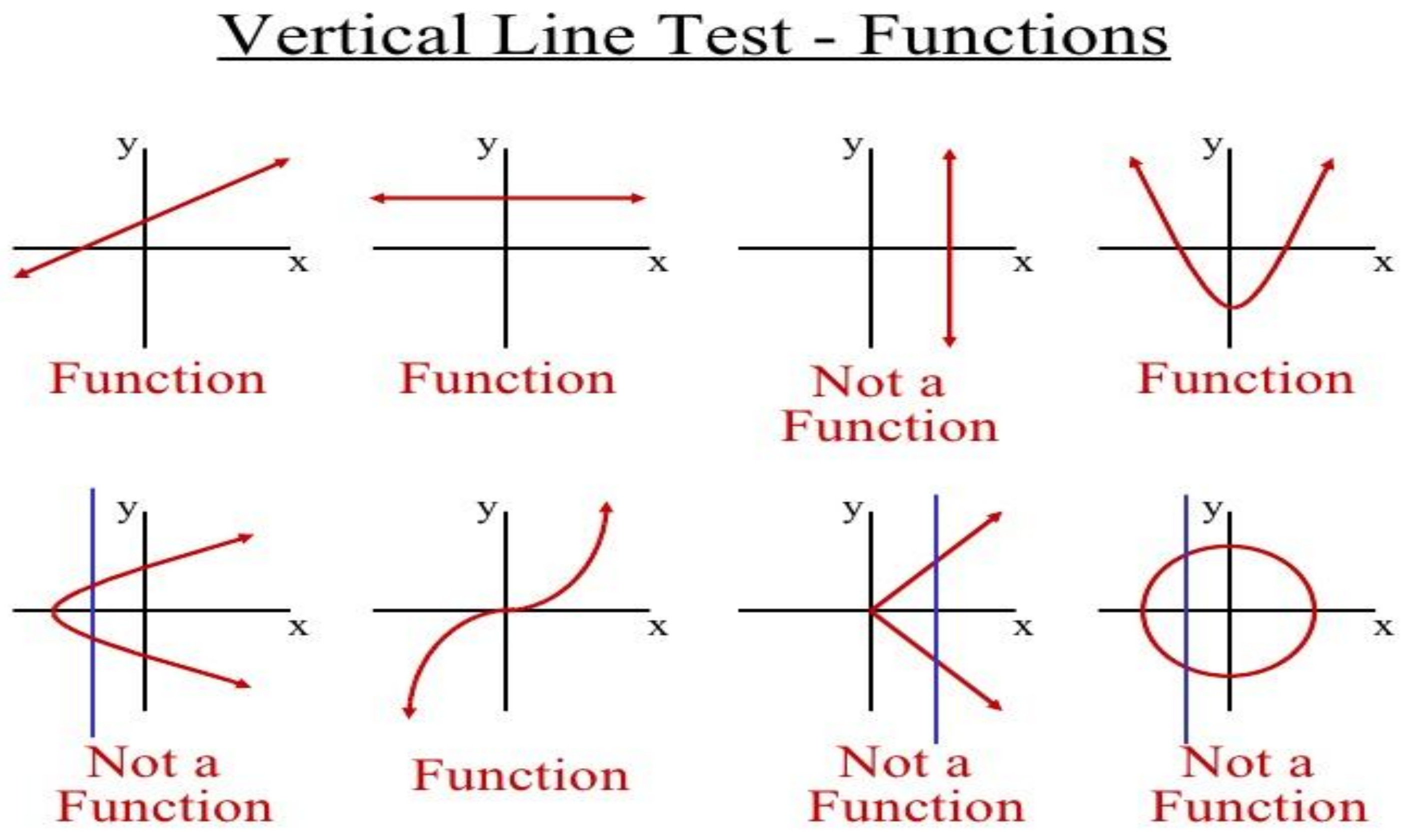
34. $f(x) = \dfrac{x}{1+x^2},\ g(x) = \dfrac{1}{x}$

35–36 Find a formula for $f \circ g \circ h$.

35. $f(x) = x^2+1,\ g(x) = \dfrac{1}{x},\ h(x) = x^3$

36. $f(x) = \dfrac{1}{1+x},\ g(x) = \sqrt[3]{x},\ h(x) = \dfrac{1}{x^3}$

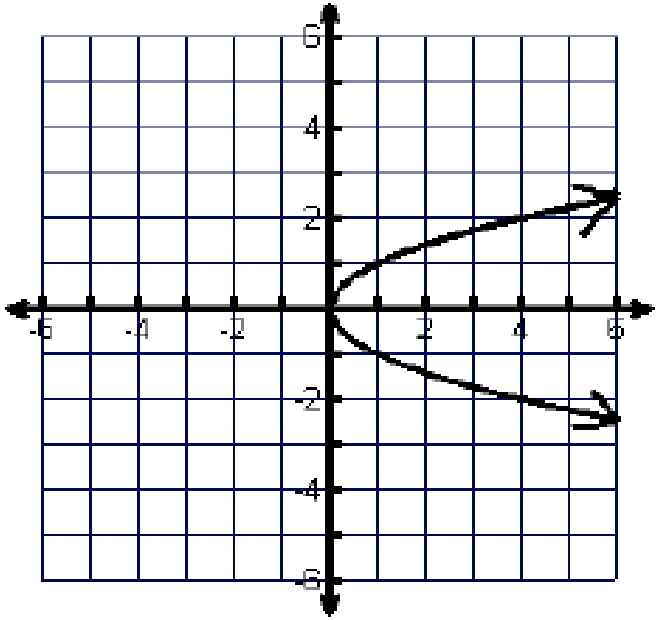
0.1.3 THE VERTICAL LINE TEST *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*

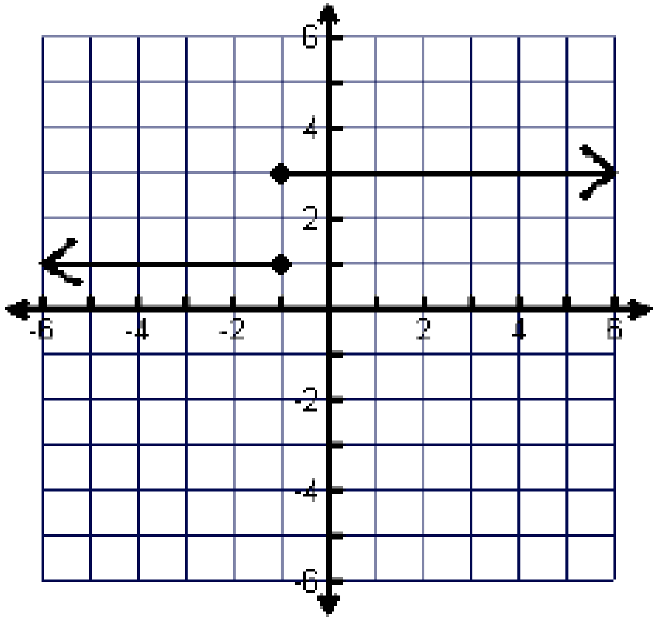


DOMAIN AND RANGE

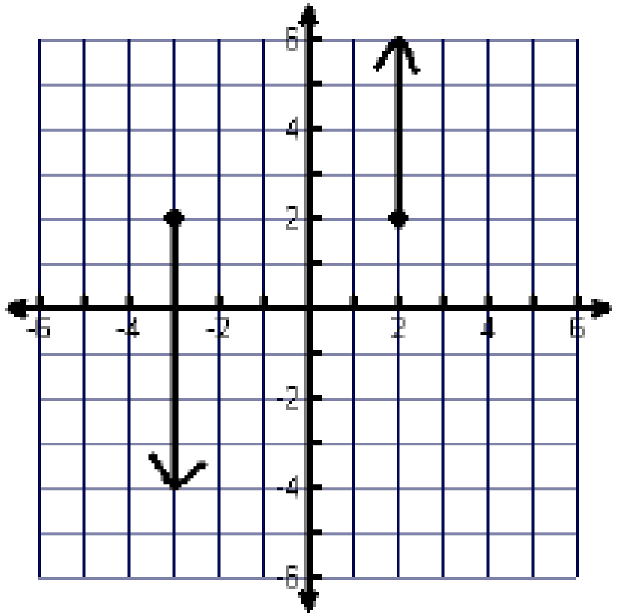
The set D of all possible input values is called the **domain** of the function. The set of all values of $f(x)$ as x varies throughout D is called the **range** of the function. The range may not include every element in the set Y.

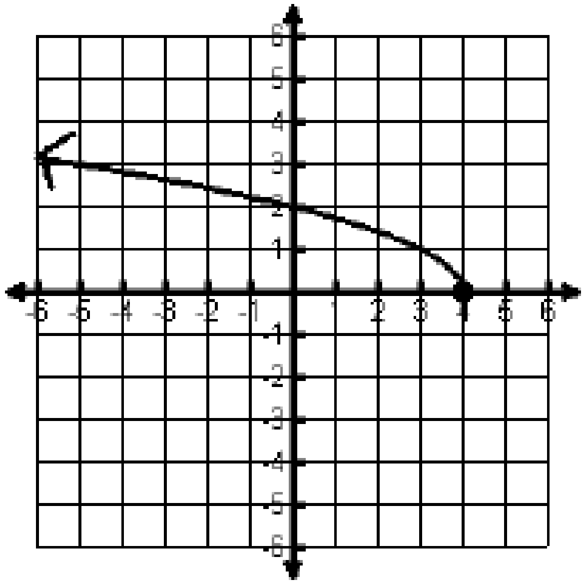
Function	**Domain (x)**	**Range (y)**
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

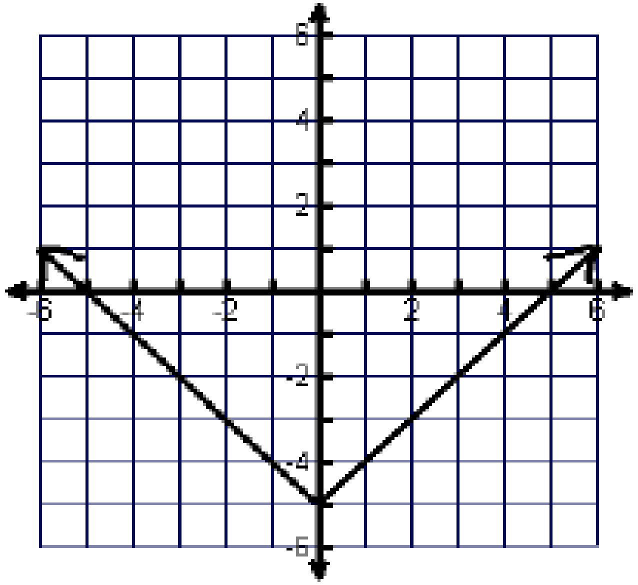


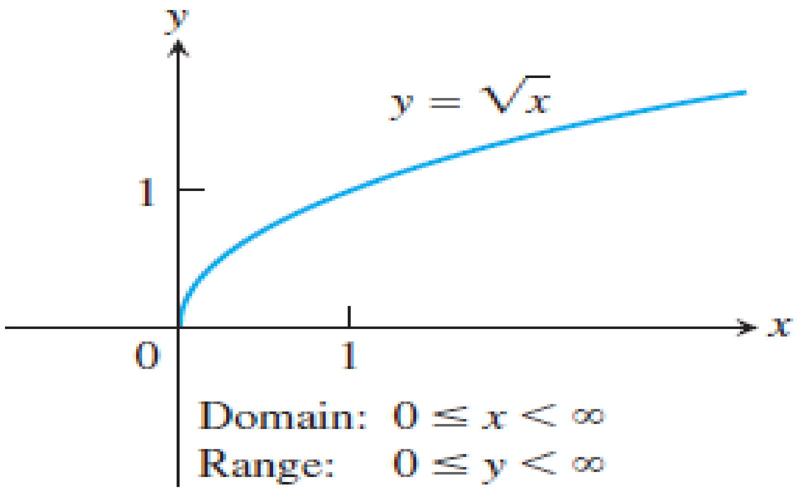


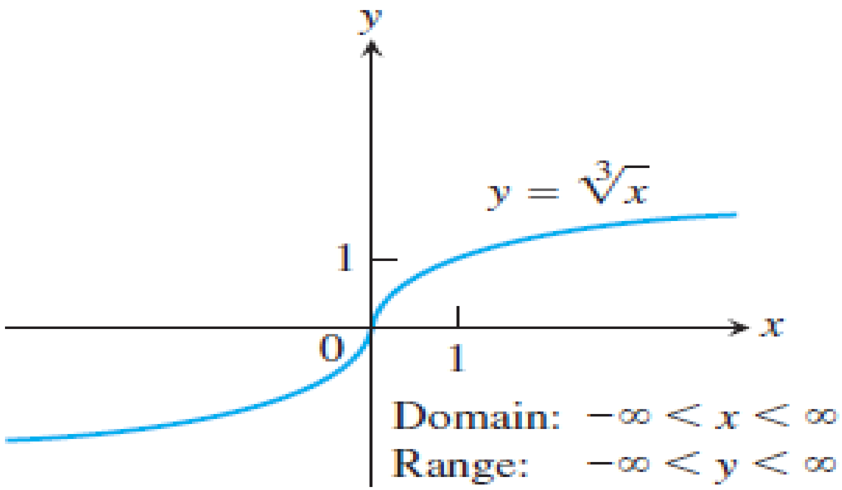
Write domain and Range of the given graph

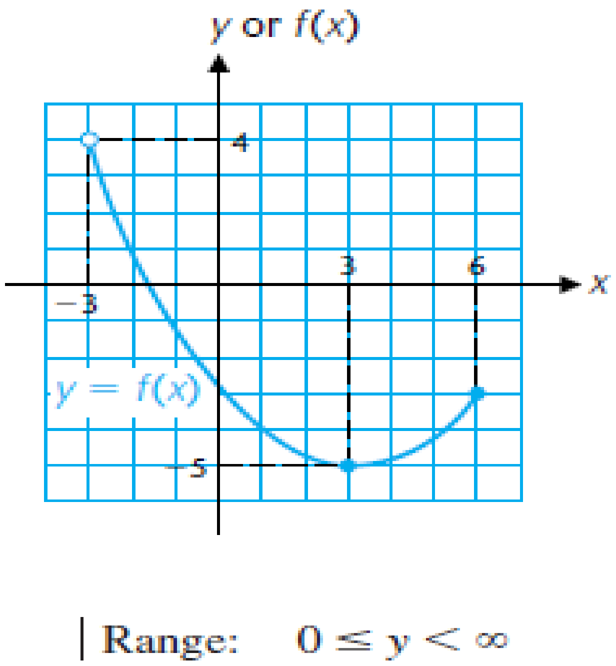


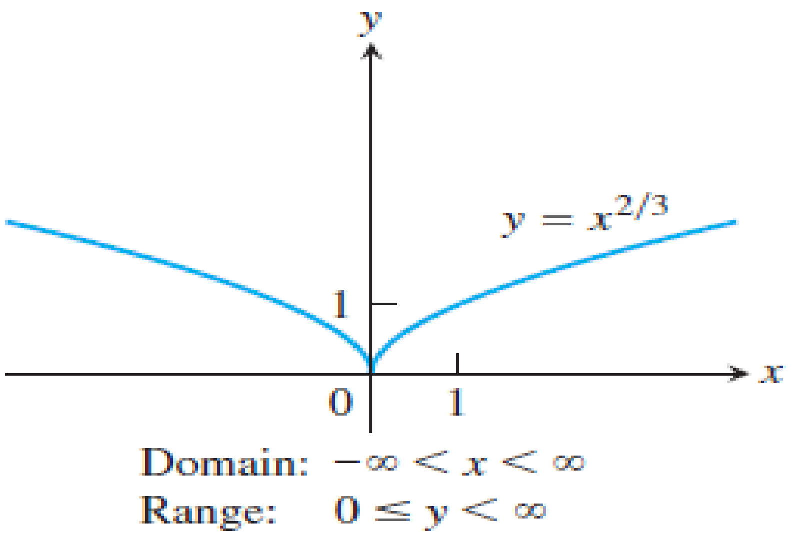





y
$y = \sqrt{x}$
1
0
1
x
Domain: $0 \le x < \infty$
Range: $0 \le y < \infty$


y
$y = \sqrt[3]{x}$
1
0
1
x
Domain: $-\infty < x < \infty$
Range: $-\infty < y < \infty$


y or f(x)
4
3
6
x
−3
$y = f(x)$
−5
Range: $0 \le y < \infty$


y
$y = x^{2/3}$
1
0
1
x
Domain: $-\infty < x < \infty$
Range: $0 \le y < \infty$

Domain and Range:

Example 6 Find the natural domain of

(a) $f(x) = x^3$ (b) $f(x) = 1/[(x-1)(x-3)]$

(c) $f(x) = \tan x$ (d) $f(x) = \sqrt{x^2 - 5x + 6}$

Example 8 Find the domain and range of

(a) $f(x) = 2 + \sqrt{x-1}$ (b) $f(x) = (x+1)/(x-1)$

Summary: Basic Trigonometric Functions

Function	Period	Domain	Range
$\sin x$	2π	$(-\infty, \infty)$	$[-1, 1]$
$\cos x$	2π	$(-\infty, \infty)$	$[-1, 1]$
$\tan x$	π	$x \neq \pi/2 + n\pi$	$(-\infty, \infty)$
$\cot x$	π	$x \neq n\pi$	$(-\infty, \infty)$
$\sec x$	2π	$x \neq \pi/2 + n\pi$	$(-\infty, -1] \cup [1, \infty)$
$\csc x$	2π	$x \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$

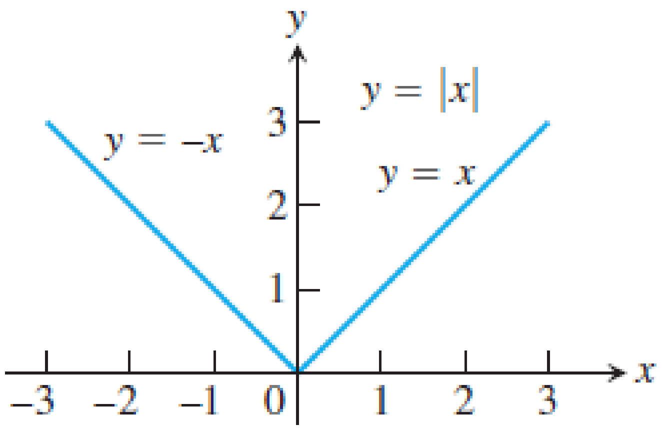
Inverse Trig.function:

Function	Domain	Range
$\sin^{-1}$ or $\arcsin$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}$ or $\arccos$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}$ or $\arctan$	$\mathbb{R}$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\csc^{-1}$ or arccsc	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$
$\sec^{-1}$ or arcsec	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$
$\cot^{-1}$ or arccot	$\mathbb{R}$	$[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$

Piecewise-Defined Functions

Sometimes a function is described by using different formulas on different parts of its do-
main. One example is the **absolute value function**

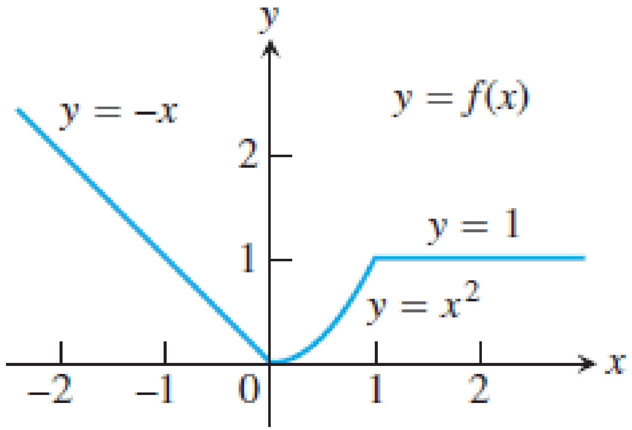
$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0, \end{cases}$$



Example:

The function

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

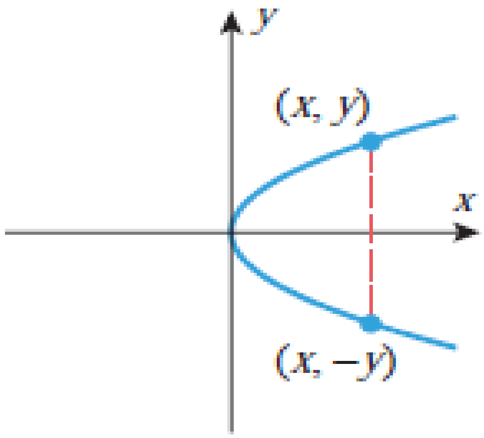


0.2.3 THEOREM (*Symmetry Tests*)

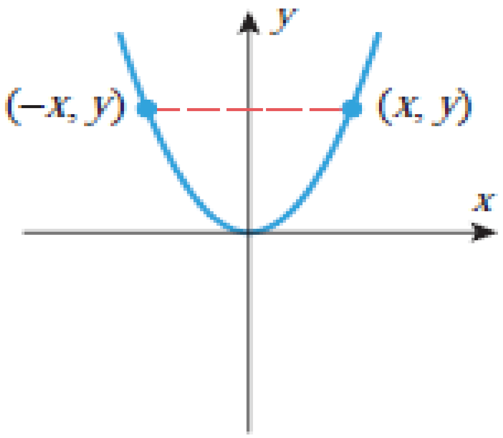
(a) *A plane curve is symmetric about the y-axis if and only if replacing x by $-x$ in its equation produces an equivalent equation.*

(b) *A plane curve is symmetric about the x-axis if and only if replacing y by $-y$ in its equation produces an equivalent equation.*

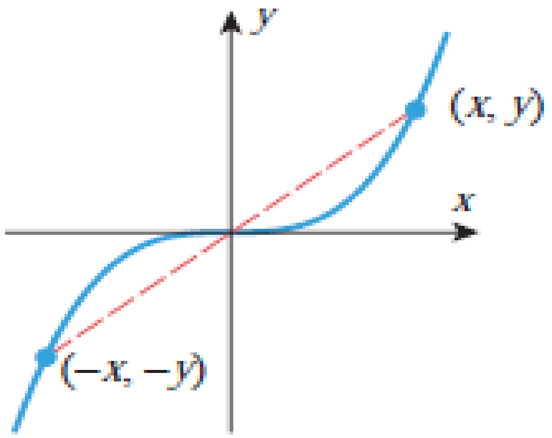
(c) *A plane curve is symmetric about the origin if and only if replacing both x by $-x$ and y by $-y$ in its equation produces an equivalent equation.*



Symmetric about the x-axis



Symmetric about the y-axis



Symmetric about the origin

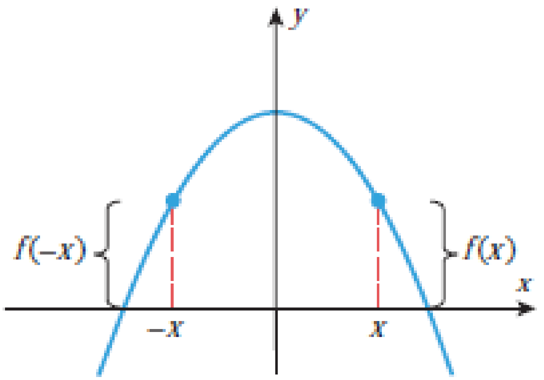
EVEN AND ODD FUNCTIONS

A function f is said to be an ***even function*** if

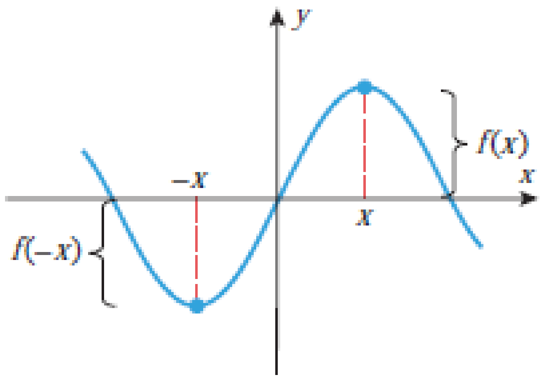
$$f(-x) = f(x)$$

and is said to be an ***odd function*** if

$$f(-x) = -f(x)$$



▲ **Figure 0.2.9** This is the graph of an even function since $f(-x) = f(x)$.



▲ **Figure 0.2.10** This is the graph of an odd function since $f(-x) = -f(x)$.

NOTE:

The graph of an even function is **symmetric about the y-axis.**

The graph of an odd function is **symmetric about the origin.**

Practice:

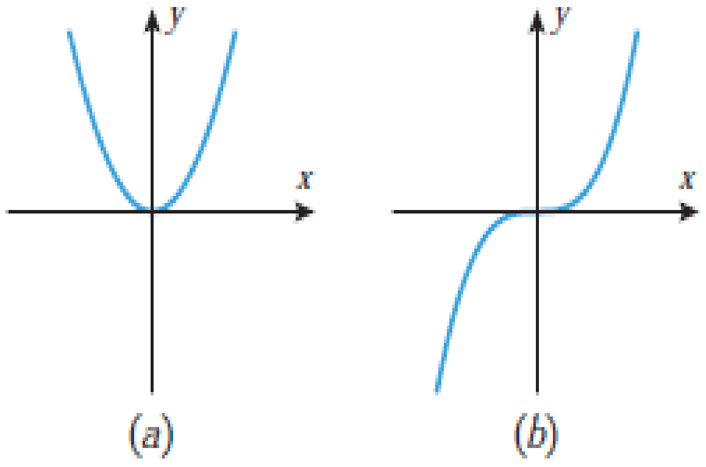
57. Classify the functions whose values are given in the accompanying table as even, odd, or neither.

x	-3	-2	-1	0	1	2	3
$f(x)$	5	3	2	3	1	-3	5
$g(x)$	4	1	-2	0	2	-1	-4
$h(x)$	2	-5	8	-2	8	-5	2

Practice:

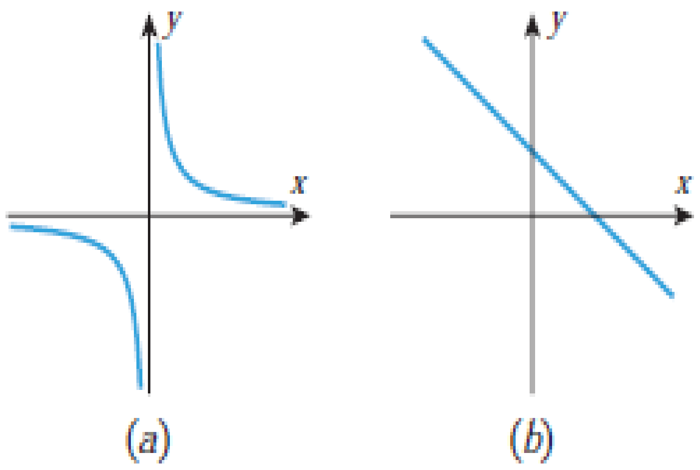
61–62 Classify the functions graphed in the accompanying figures as even, odd, or neither. ■

61.



▲ **Figure Ex-61**

62.



63. In each part, classify the function as even, odd, or neither.

(a) $f(x) = x^2$ (b) $f(x) = x^3$

(c) $f(x) = |x|$ (d) $f(x) = x + 1$

(e) $f(x) = \dfrac{x^5 - x}{1 + x^2}$ (f) $f(x) = 2$

66–67 Use Theorem 0.2.3 to determine whether the graph has symmetries about the x-axis, the y-axis, or the origin. ■

66. (a) $x = 5y^2 + 9$ (b) $x^2 - 2y^2 = 3$

(c) $xy = 5$

67. (a) $x^4 = 2y^3 + y$ (b) $y = \dfrac{x}{3 + x^2}$

(c) $y^2 = |x| - 5$