

Operations Research

K191048 (Amman Soomro)

2021

Section
SE - A

Instructor
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Assignment #2

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Sec: SE-A

Operational Research Assignment # 2

Assignment model Q# 1

Machines

	I	II	III	IV				
A	10	12	19	11	10			
jobs	B	5	10	7	18	5	11	1
C	12	14	13	11	11	P	P	A
D	8	15	11	9	8	P	0	8

I II III IV

	I	II	III	IV				
A	0	2	9	1				
B	0	5	21	3	11		1	
C	1	3	2	0		P	P	A
D	0	7	3	1		10	0	8

	I	II	III	IV				
A	0x	0	17	1	11			
B	0x	3	0	3				
C	1	1	0x	0				
D	0	5	1	1				

$$A \rightarrow II = 12$$

$$B \rightarrow III = 7$$

$$C \rightarrow IV = 11$$

$$D \rightarrow I = 8$$

Total cost: Rs. 38 Ans.

Assignment model Q# 3-4 : 09/09/2022

Gamer

Games

I II III IV

A 48 48 50 44 44

B 56 60 60 68 56

C 96 94 90 85 85

D 42 44 54 46 42

V VI VII VIII

II III IV V VI VII

A 4 4 6 8 0 11 5 8

B 0 4 4 12 12 21 8 0

C 11 9 5 0

D 0 2 12 4 11 1

0 2 4 0 0 0 0 0

I II III IV V VI VII

A 4 2 2 0 0 ★ 1 0

B 0 2 0 12 12 0 0 0

C 11 7 1 0 0 ★ 1 0

D 0 0 8 4 4 0 0 0

V VI VII VIII

I II III IV V VI VII

A 3 1 1 0 0 8 0 8

B 0 2 0 13 1 1 0 0

C 10 6 0 1 0 8 10 0

D 0 0 8 5 5 0 0 0

$$A \rightarrow IV = 44$$

$$B \rightarrow I = 56$$

$$C \rightarrow III = 90$$

$$D \rightarrow II = 44$$

$$\text{Total} = 234 \text{ Ans.}$$

Date: _____

Transportation Model Q# 1

	X	Y	Z	W	Supply
A	5	3	6	2	19
B	4	7	9	1	37
C	3	4	7	5	34
Demand.	16	18	31	25	90

$$\begin{aligned} \text{Min "W": } & 5x_{11} + 3x_{12} + 6x_{13} + 2x_{14} + \\ & 4x_{21} + 7x_{22} + 9x_{23} + x_{24} + \\ & 3x_{31} + 4x_{32} + 7x_{33} + 5x_{34}. \end{aligned}$$

$$\text{Supply: } \sum_i x_{ij} \leq a_{ij}$$

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 19$$

Supply

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 37$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 34$$

$$; x_{11} + x_{21} + x_{31} \leq 16$$

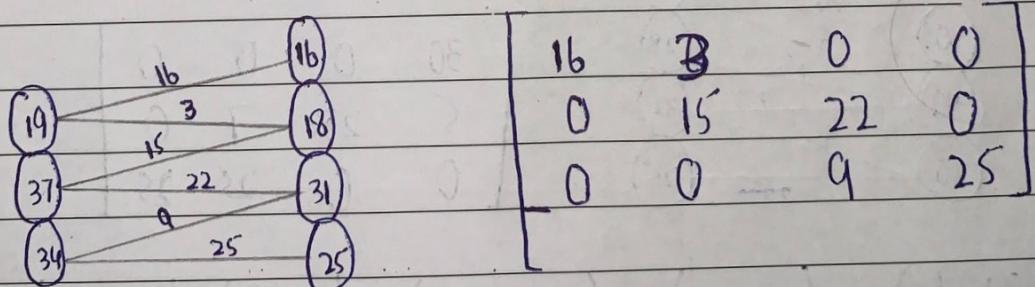
$$; x_{12} + x_{22} + x_{32} \leq 18$$

$$; x_{13} + x_{23} + x_{33} \leq 31$$

$$; x_{14} + x_{24} + x_{34} \leq 25$$

Demand.

$$\text{Non-Negative: } x_{ij} \geq 0 \quad i=1, 2, 3, 4; \quad j=1, 2, 3, 4.$$



$$\begin{aligned} W: & (16 \times 5) + (3 \times 3) + (7 \times 15) + (22 \times 9) + (9 \times 7) + (25 \times 5) \\ & = 80 + 9 + 105 + 198 + 63 + 125 \\ & = 580 \text{ Ans.} \end{aligned}$$

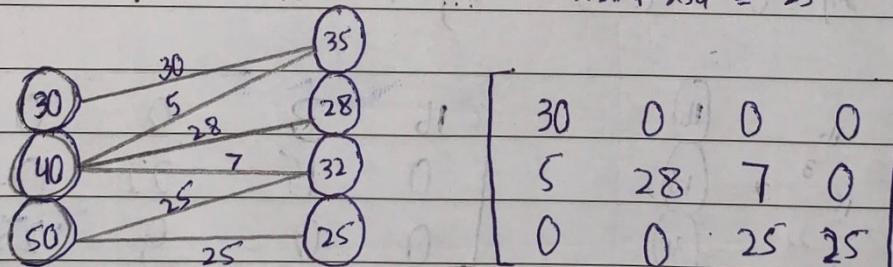
Transportation Model Q# 2

	Plague	L	N	V	X	
	x_1	x_2	x_3	x_4	Supply	
A	6	8	8	5	30	8
B	11	9	7	7	40	0
C	9	28	7	13	50	100
Demand	35	28	32	25	180	

$$\begin{aligned}
 \text{Min "W": } & 6x_{11} + 8x_{12} + 8x_{13} + 5x_{14} + \\
 & 11x_{21} + 9x_{22} + 9x_{23} + 7x_{24} + \\
 & 9x_{31} + 28x_{32} + 7x_{33} + 13x_{34} -
 \end{aligned}$$

subs:

$$\begin{aligned}
 x_{ij} &\leq a_{ij} & \sum_j x_{ij} &\leq b_{ij} \\
 x_{11} + x_{12} + x_{13} + x_{14} &\leq 30 & x_{11} + x_{21} + x_{31} &\leq 35 \\
 x_{21} + x_{22} + x_{23} + x_{24} &\leq 40 & x_{12} + x_{22} + x_{32} &\leq 28 \\
 x_{31} + x_{32} + x_{33} + x_{34} &\leq 50 & x_{13} + x_{23} + x_{33} &\leq 32 \\
 x_{14} + x_{24} + x_{34} &\leq 25
 \end{aligned}$$



$$W: (30 \times 6) + (5 \times 5) + (11 \times 28) + (7 \times 9) + (7 \times 25) + (13 \times 25)$$

$$W: 180 + 25 + 308 + 63 + 175 + 325$$

$$W: 1076 \text{ Ans}$$

Date: _____

North-West Corner Method Q. 1 min

	A	B	C	D	E	Supply	
I	2	3	11	10	3	7	4
II	1		4	2	7	4	8
III	3	9	4	8	12	3	9
Demand	3	3	4	5	6	0	brackets

$$\text{Min } "W" = 2x_{11} + 11x_{12} + 10x_{13} + 3x_{14} + 7x_{15} + \\ 1x_{21} + 4x_{22} + 7x_{23} + 2x_{24} + 1x_{25} + \\ 3x_{31} + 9x_{32} + 4x_{33} + 8x_{34} + 12x_{35}.$$

$$\text{Sul: } \sum_i^m x_{ij} \leq a_{ij} \rightarrow \text{Supply}$$

$$\sum_j x_{ij} \leq b_{ij} \rightarrow \text{Demand}.$$

$$W: (2 \times 3) + (11 \times 1) + (4 \times 2) + (7 \times 4) + (2 \times 2) + (8 \times 3) + (12 \times 6)$$

$$W: 6 + 11 + 8 + 28 + 4 + 24 + 72$$

$$W: 153 \text{ Ans.}$$

Date: _____

North-West Corner Method Q#2.

	1	2	3	Supply
A	1	2	6	7
B	0	4	2	12
C	3	1	5	11
Demand	10	10	10	

$$\begin{aligned}
 \text{Min } "W": & 1x_{11} + 2x_{12} + 6x_{13} + \\
 & 0x_{21} + 4x_{22} + 2x_{23} + \\
 & 3x_{31} + 1x_{32} + 5x_{33}
 \end{aligned}$$

$$\text{Sub: } \sum_i^n x_{ij} \leq a_{ij} \rightarrow \text{Supply.}$$

$$\sum_j^n x_{ij} \leq b_{ij} \rightarrow \text{Demand.}$$

$$W: ((7 \times 1) + (0 \times 3) + (9 \times 4) + (1 \times 1) + (5 \times 10))$$

$$W: 7 + 0 + 36 + 1 + 50$$

$$W: 94 \text{ Ans.}$$

Date _____

Project Evaluation Review Technique (PERT)

- Times to complete a project:
- Optimistic: t_0
- Most likely: t_m
- Pessimistic: t_p
- Average time:

$$\bar{T}_a = \frac{t_0 + 4t_m + t_p}{6}$$

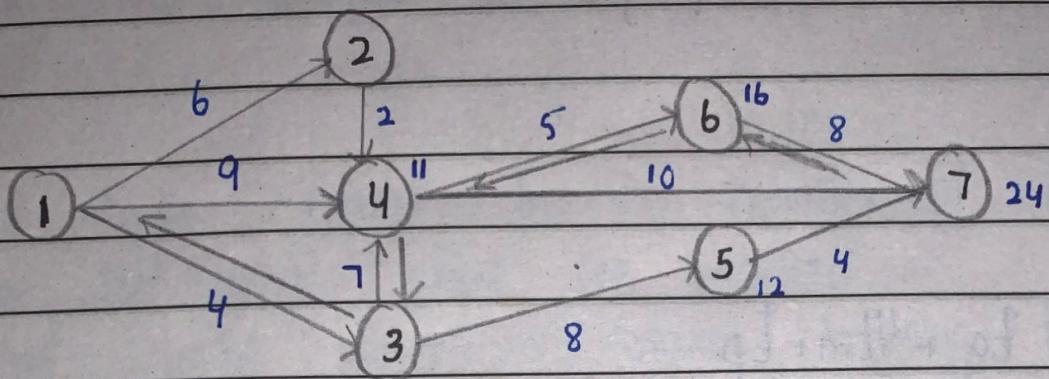
Variance

$$\sigma^2 = \frac{(t_p - t_0)^2}{6}$$

Question no #1

Activity	\bar{T}_a	σ^2
1-2	06	0.444
1-3	04	1.777
1-4	09	2.777
2-4	03	0.111
3-4	07	0.111
3-5	08	1.777
4-6	05	0.444
4-7	10	1.777
5-7	04	0.444
6-7	08	1.777

a) Draw a project network:



b) Critical Path:

$$\Rightarrow 1-3, 3-4, 4-6, 6-7$$

$$\Rightarrow 4 + 7 + 5 + 8$$

$\Rightarrow 24 \text{ days.}$

c) Probability of completion in 19 days.

$$Z = \frac{Ts - T_p}{\sqrt{\sum \sigma^2_{D_i} \text{ in C.P.}}}; Ts = 19; T_p = 24$$

$$\Rightarrow \sqrt{1.777 + 0.111 + 0.444 + 1.777}$$

$$\Rightarrow 2.027.$$

$$\Rightarrow Z = -5 / 2.027$$

$$\Rightarrow -2.470.$$

$$\Rightarrow 0.0068$$

$$\Rightarrow 0.0068 \times 100$$

$$\Rightarrow 0.68\% \text{ Ans.}$$

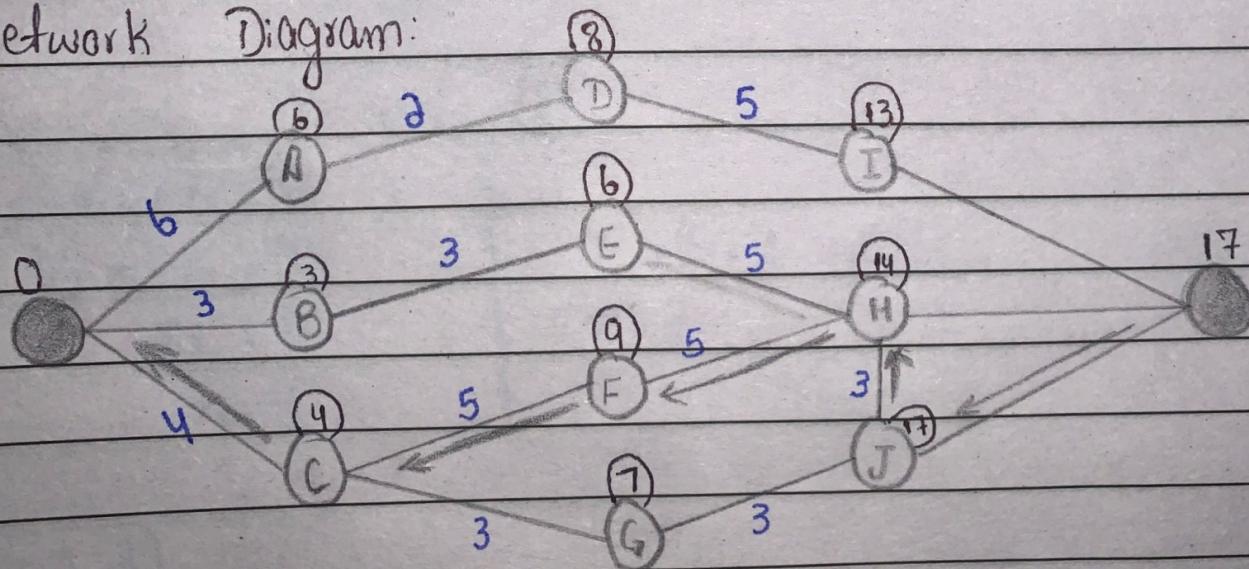


PERT

Question no: 2

Activity	Predecessors	T _a	θ^2
A	-	6	0.11
B	-	3	0.44
C	-	4	1.00
D	A	2	0.11
E	B	3	1.77
F	C	5	1.77
G	C	3	1.00
H	E, F	5	1.00
I	D	5	1.00
J	H, G	3	1.00

① Network Diagram:



• Critical Path:

$$\text{Start} - C ; C - F ; F - H ; H - J$$
$$4 + 5 + 5 + 3$$

\Rightarrow 17 days.

• Probability of completion in 22 days:

$$\Rightarrow \sqrt{1.00 + 1.77 + 1.00 + 1.00} \Rightarrow 2.18 \Rightarrow \sigma$$

$$\rightarrow P(22) = \frac{22-17}{2.18} \Rightarrow 2.29$$

$$Z = 0.988 \text{ Ans.}$$

Sensitivity Analysis

Q: $Z = 30x_1 + 20x_2$

$$2x_1 + x_2 \leq 8$$

$$x_1 + 3x_2 \leq 8$$

If change of co-efficient x_1 and x_2 of Z function is 35 and 25 respectively. Discuss sensitivity:

- Optimal range of co-efficient c_1/c_2 .
- Find optimal value of Z .
- The range of co-efficient c_1 and c_2 .

$$2x_1 + x_2 = 8$$

$$2x_1 \pm b x_2 = \pm 1b$$

$$-5x_2 - 8 \Rightarrow x_2 \Rightarrow 8/5$$

$$x_1 \Rightarrow 1b/5$$

$$Z \Rightarrow 30(1b/5) + 20(8/5)$$

$$Z = 188 \text{ Ans.}$$



$$a) Z = C_1 x_1 + C_2 x_2 \rightarrow$$

$$\cdot 1x_1 + 3x_2 \rightarrow ①$$

$$\cdot 2x_1 + 1x_2 \rightarrow ②$$

$$\therefore C_1/C_2$$

$$\Rightarrow 1/3 ; 2/1$$

$$\Rightarrow 1/3 \leq C_1/C_2 \leq 2$$

$$\Rightarrow 0.33 \leq C_1/C_2 \leq 2$$

$$\cancel{35/25}$$

$$1.6$$

$$1.6$$

\rightarrow This is within optimal range.

$$b) Z = 35(1b/5) + 25(8/5)$$

$$Z = 150.$$

$$c) 0.33 \leq C_1/C_2 \leq 2$$

$$C_2 \Rightarrow 0.33 \leq C_1/C_2 \leq 2$$

$$0.33 \leq C_1/20 \leq 2$$

$$0.6 \leq C_1 \leq 40.$$

$$C_1 \Rightarrow 0.33 \leq 30/C_2 \leq 2$$

$$90 \leq C_2 \leq 15$$

Duality

Q) Main $w = 8y_1 + 16y_2$

Dub

$$\begin{aligned} y_1 + 5y_2 &\geq 9 \\ 2y_1 + 3y_2 &\geq 10 \\ y_1, y_2 &\geq 0 \end{aligned}$$

- Simplex method

y_1	y_2	
1	5	9
2	3	10
8	16	

- Duality (Transpose)

y_1	y_2	
1	2	8
5	3	16
9	10	



$$\text{Max } Z = 9y_1 + 10y_2$$

Subject to

$$\begin{aligned} y_1 + 2y_2 &\leq 8 \\ 5y_1 + 2y_2 &\leq 16 \\ y_1, y_2 &\geq 0 \end{aligned} \quad \left\{ \begin{array}{l} \text{min to max} \end{array} \right\}$$

	y_1	y_2	S_1	S_2	Z
\rightarrow	1	2	1	0	0 8
	5	2	0	1	0 16
	-9	-10	0	0	1 0
	y_1	y_2	S_1	S_2	Z
	0	8	5	-1	0 24
	4	0	-1	1	0 8
	0	0	4	1	1 48

Basic

$$4y_1 = 8 \Rightarrow 2 ; 8y_2 = 24 \Rightarrow 3$$

$$Z = 9(2) + 10(3) \Rightarrow 48$$

Non-Basic

$$S_1 = 0 ; S_2 = 0$$

$$y_1 = 4 \quad y_2 = 1$$

$$W = 8(4) + 16(1) \Rightarrow 48 \text{ Ans.}$$

