Applications of Ampere's Law

Course Title : Applied Physics

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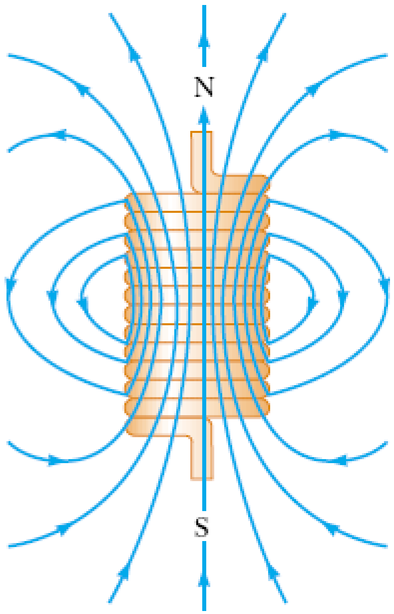
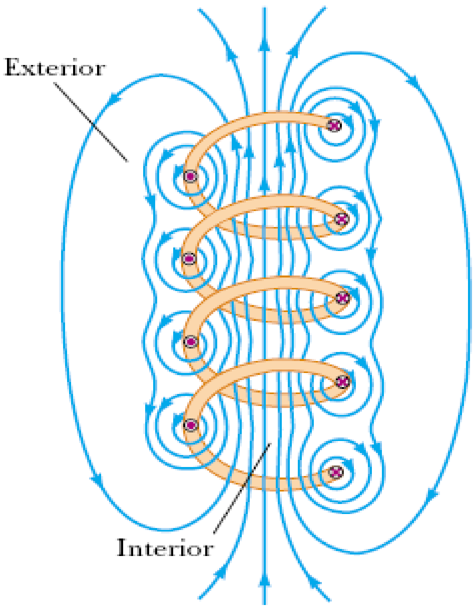
The Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire, which we shall call the interior of the solenoid, when the solenoid carries a current.
When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

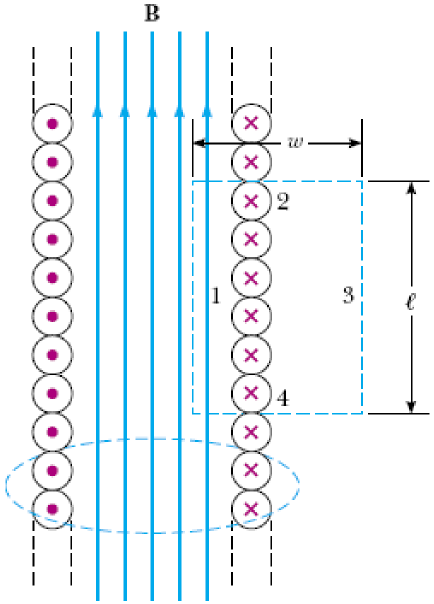
Note that the field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is strong and almost uniform.

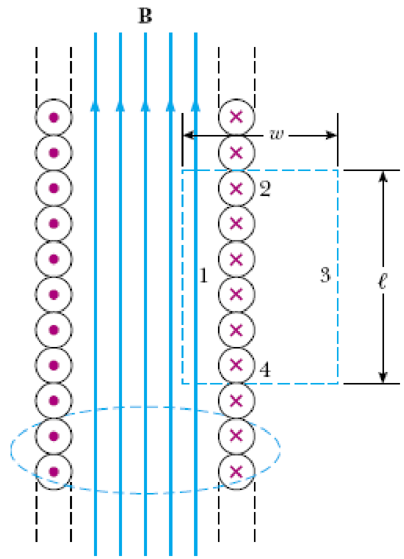




We can use Ampère's law to obtain a quantitative expression for the interior magnetic field in an ideal solenoid. Because the solenoid is ideal, **B** in the interior space is uniform and parallel to the axis, and the magnetic field lines in the exterior space form circles around the solenoid. The planes of these circles are perpendicular to the page. Consider the rectangular path of length ℓ and width w shown in Figure 30.19. We can apply Ampère's law to this path by evaluating the integral of $\mathbf{B}\cdot d\mathbf{s}$ over each side of the rectangle. The contribution along side 3 is zero because the magnetic field lines are perpendicular to the path in this region. The contributions from sides 2 and 4 are both zero, again because **B** is perpendicular to $d\mathbf{s}$ along these paths, both inside and outside the solenoid. Side 1 gives a contribution to the integral because along this path **B** is uniform and parallel to $d\mathbf{s}$. The integral over the closed rectangular path is therefore

$$\oint \mathbf{B}\cdot d\mathbf{s} = \int_{\text{path 1}} \mathbf{B}\cdot d\mathbf{s} = B\int_{\text{path 1}} ds = B\ell$$



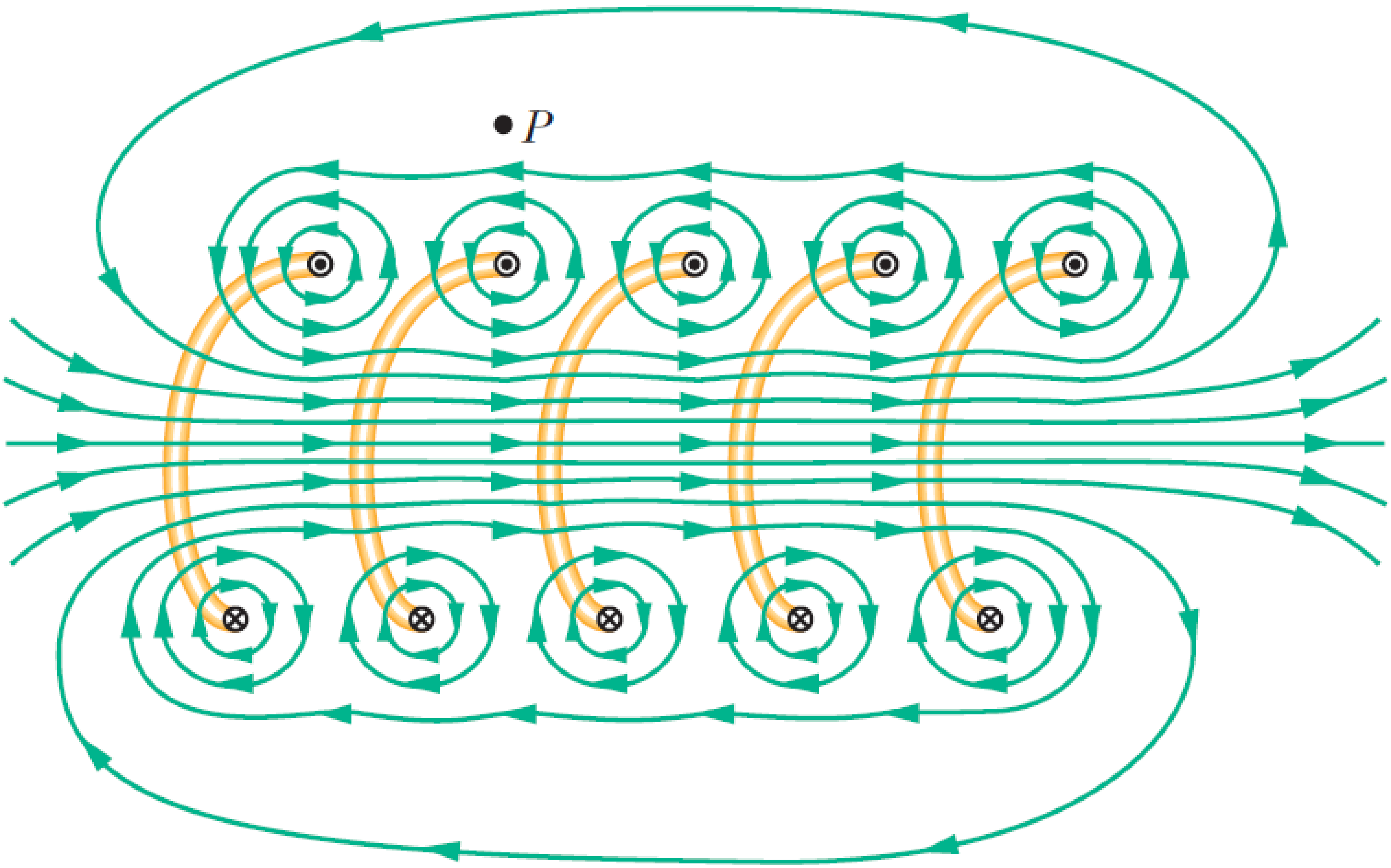


The right side of Ampère's law involves the total current I through the area bounded by the path of integration. In this case, the total current through the rectangular path equals the current through each turn multiplied by the number of turns. If N is the number of turns in the length ℓ, the total current through the rectangle is NI. Therefore, Ampère's law applied to this path gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B\ell = \mu_0 NI$$

$$\boxed{B = \mu_0 \frac{N}{\ell} I = \mu_0 n I} \qquad (30.17)$$

where $n = N/\ell$ is the number of turns per unit length.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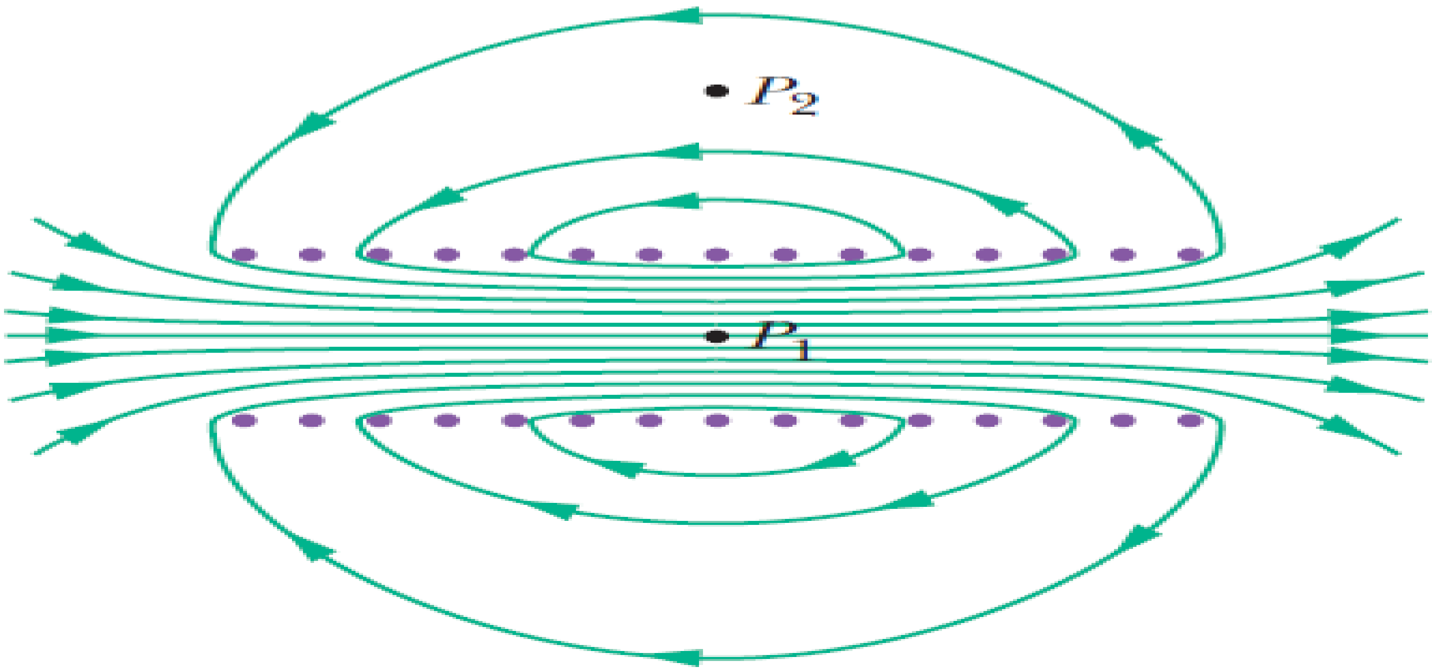


Figure 29-19 Magnetic field lines for a real solenoid of finite length. The field is strong and uniform at interior points such as P_1 but relatively weak at external points such as P_2.

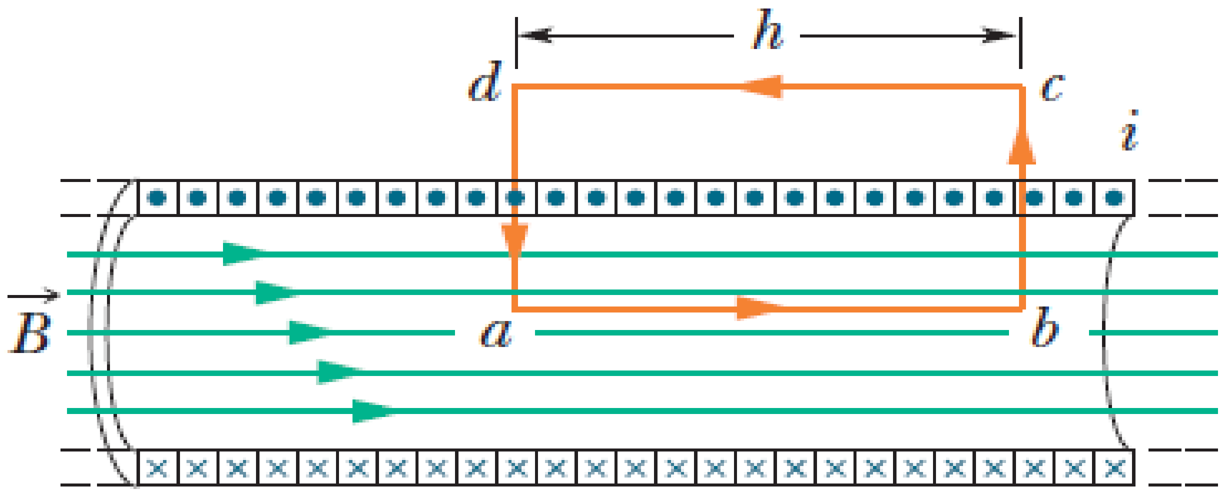


Figure 29-20 Application of Ampere's law to a section of a long ideal solenoid carrying a current *i*. The Amperian loop is the rectangle *abcda*.

Magnetic Field inside a Toroid

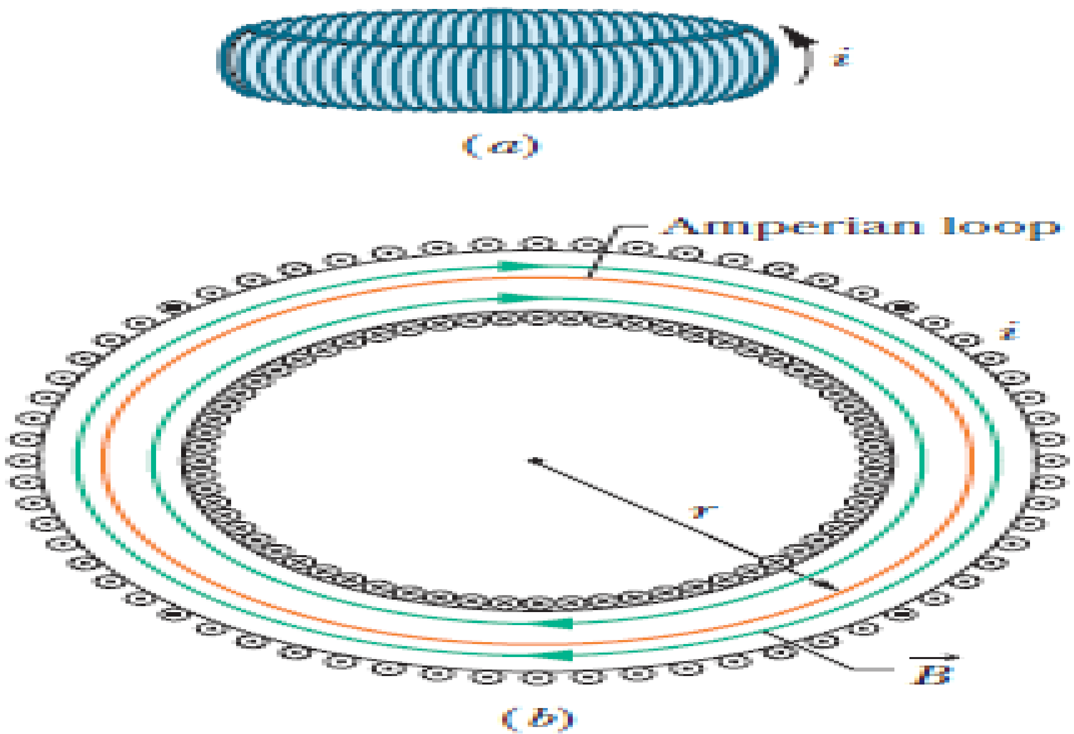


Figure 29-21 (*a*) A toroid carrying a current *i*. (*b*) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.

From the symmetry, we see that the lines of $\vec{B}$ form concentric circles inside the toroid, directed as shown in Fig. 29-21*b*. Let us choose a concentric circle of radius r as an Amperian loop and traverse it in the clockwise direction. Ampere's law (Eq. 29-14) yields

$$(B)(2\pi r) = \mu_0 i N,$$

where i is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and N is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r} \quad \text{(toroid).} \tag{29-24}$$

In contrast to the situation for a solenoid, B is not constant over the cross section of a toroid.

It is easy to show, with Ampere's law, that $B = 0$ for points outside an ideal toroid (as if the toroid were made from an ideal solenoid). The direction of the

Sample Problem 29.04 The field inside a solenoid (a long coil of current)

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

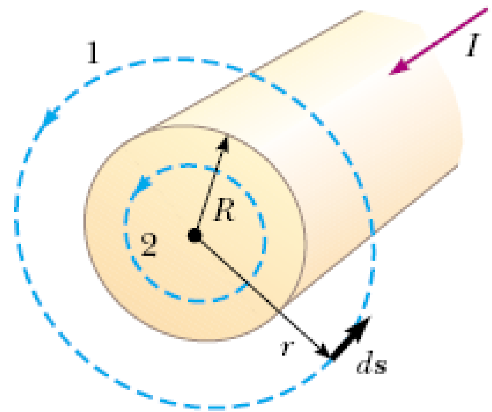
$$B = \mu_0 in = (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(5.57\ \mathrm{A})\frac{5 \times 850\ \text{turns}}{1.23\ \mathrm{m}}$$

$$= 2.42 \times 10^{-2}\ \mathrm{T} = 24.2\ \mathrm{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.

The Magnetic Field Created by a Long Current-Carrying Wire

A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. 30.12). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.

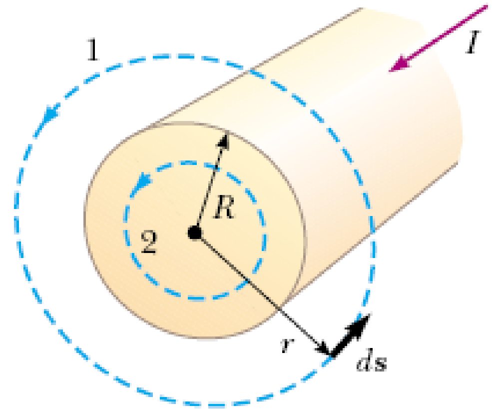


Solution Figure 30.12 helps us to conceptualize the wire and the current. Because the wire has a high degree of symmetry, we categorize this as an Ampère's law problem. For the $r \geq R$ case, we should arrive at the same result we obtained in Example 30.1, in which we applied the Biot–Savart law to the same situation. To analyze the problem, let us choose for our path of integration circle 1 in Figure 30.12. From symmetry, $\mathbf{B}$ must be constant in magnitude and parallel to $d\mathbf{s}$ at every point on this circle. Because the total current passing through the plane of the circle is I, Ampère's law gives

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \boxed{\frac{\mu_0 I}{2\pi r}} \qquad (\text{for } r \geq R)$$

Now consider the interior of the wire, where $r < R$. Here the current I' passing through the plane of circle 2 is less than the total current I. Because the current is uniform over the cross section of the wire, the fraction of the current enclosed by circle 2 must equal the ratio of the area πr^2 enclosed by circle 2 to the cross-sectional area πR^2 of the wire:[3]



$$\frac{I'}{I} = \frac{\pi r^2}{\pi R^2}$$

$$I' = \frac{r^2}{R^2} I$$

Following the same procedure as for circle 1, we apply Ampère's law to circle 2:

$$\oint \mathbf{B} \cdot d\mathbf{s} = B(2\pi r) = \mu_0 I' = \mu_0 \left(\frac{r^2}{R^2} I \right)$$

$$B = \boxed{\left(\frac{\mu_0 I}{2\pi R^2} \right) r} \qquad (\text{for } r < R)$$

To finalize this problem, note that this result is similar in form to the expression for the electric field inside a uniformly charged sphere (see Example 24.5). The magnitude of the magnetic field versus r for this configuration is plotted in Figure 30.13. Note that inside the wire, $B \to 0$ as $r \to 0$. Furthermore, we see that Equations 30.14 and 30.15 give the same value of the magnetic field at $r = R$, demonstrating that the magnetic field is continuous at the surface of the wire.