Applications of moving charges in Magnetic Fields

Course Title : Applied Physics

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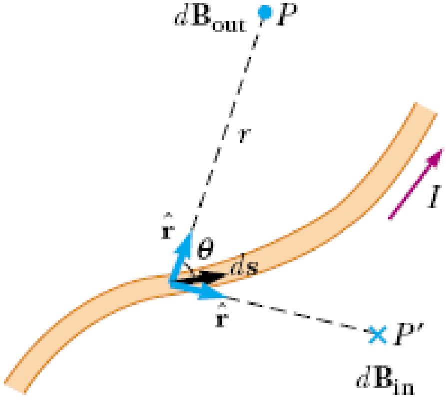
The Biot–Savart Law

- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ toward P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude ds of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

These observations are summarized in the mathematical expression known today as the **Biot–Savart law:**

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$



where μ_0 is a constant called the **permeability of free space:**

$$\mu_0 = 4\pi\times 10^{-7}\,\mathrm{T\cdot m/A}$$

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

The magnitude of the magnetic field varies as the inverse square of the distance from the source, as does the electric field due to a point charge. However, the directions of the two fields are quite different. The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\mathbf{s}$ and the unit vector $\hat{\mathbf{r}}$, as described by the cross product in Equation 30.1. Hence, if the conductor lies in the plane of the page, as shown in Figure 30.1, $d\mathbf{B}$ points out of the page at P and into the page at P'.

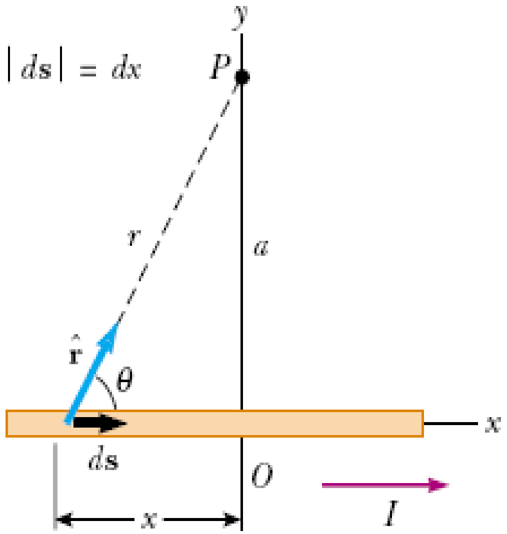
Another difference between electric and magnetic fields is related to the source of the field. An electric field is established by an isolated electric charge. The Biot–Savart law gives the magnetic field of an isolated current element at some point, but such an isolated current element cannot exist the way an isolated electric charge can. A current element *must* be part of an extended current distribution because we must have a complete circuit in order for charges to flow.

Magnetic Field Surrounding a Thin, Straight Conductor

Consider a thin, straight wire carrying a constant current I and placed along the x axis as shown in Figure 30.3. Determine the magnitude and direction of the magnetic field at point P due to this current.

Solution From the Biot–Savart law, we expect that the magnitude of the field is proportional to the current in the wire and decreases as the distance a from the wire to point P increases. We start by considering a length element $d\mathbf{s}$ located a distance r from P. The direction of the magnetic field at point P due to the current in this element is out of the page because $d\mathbf{s} \times \hat{\mathbf{r}}$ is out of the page. In fact, because *all* of the current elements $I\,d\mathbf{s}$ lie in the plane of the page, they all produce a magnetic field directed out of the page at point P. Thus, we have the direction of the magnetic field at point P, and we need only find the magnitude. Taking the origin at O and letting point P be along the positive y axis, with $\hat{\mathbf{k}}$ being a unit vector pointing out of the page, we see that

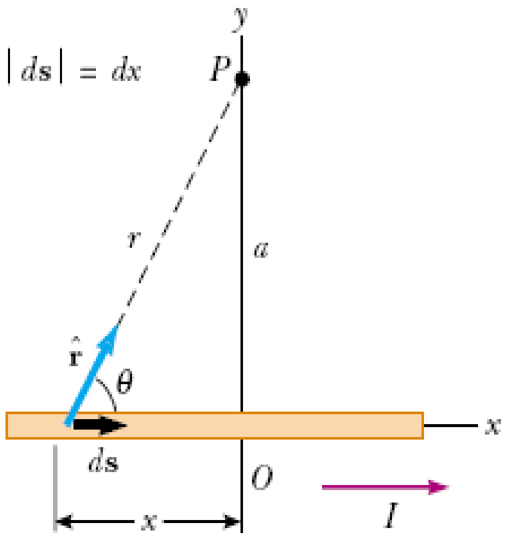
$$d\mathbf{s} \times \hat{\mathbf{r}} = |d\mathbf{s} \times \hat{\mathbf{r}}|\,\hat{\mathbf{k}} = (dx \sin\theta)\,\hat{\mathbf{k}}$$



(a)

where $|d\mathbf{s} \times \hat{\mathbf{r}}|$ represents the magnitude of $d\mathbf{s} \times \hat{\mathbf{r}}$. Because $\hat{\mathbf{r}}$ is a unit vector, the magnitude of the cross product is simply the magnitude of $d\mathbf{s}$, which is the length dx. Substitution into Equation 30.1 gives

$$d\mathbf{B} = (dB)\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2} \hat{\mathbf{k}}$$



(a)

Because all current elements produce a magnetic field in the $\hat{\mathbf{k}}$ direction, let us restrict our attention to the magnitude of the field due to one current element, which is

$$(1) \qquad dB = \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2}$$

To integrate this expression, we must relate the variables θ, x, and r. One approach is to express x and r in terms of θ. From the geometry in Figure 30.3a, we have

$$(2) \qquad r = \frac{a}{\sin\theta} = a \csc\theta$$

Because tan $\theta = a/(-x)$ from the right triangle in Figure 30.3a (the negative sign is necessary because $d\mathbf{s}$ is located at a negative value of x), we have

$$x = -a \cot \theta$$

Taking the derivative of this expression gives

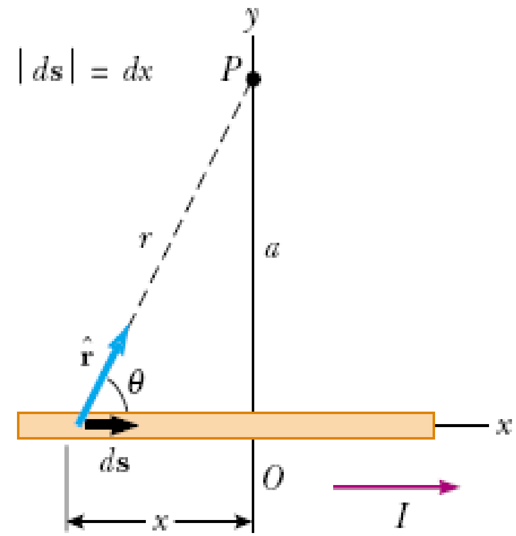
$$(3) \qquad dx = a \csc^2 \theta \, d\theta$$

Substitution of Equations (2) and (3) into Equation (1) gives

$$(4) \qquad dB = \frac{\mu_0 I}{4\pi} \frac{a \csc^2 \theta \sin \theta \, d\theta}{a^2 \csc^2 \theta} = \frac{\mu_0 I}{4\pi a} \sin \theta \, d\theta$$

an expression in which the only variable is θ. We now obtain the magnitude of the magnetic field at point P by integrating Equation (4) over all elements, where the subtending angles range from θ_1 to θ_2 as defined in Figure 30.3b:

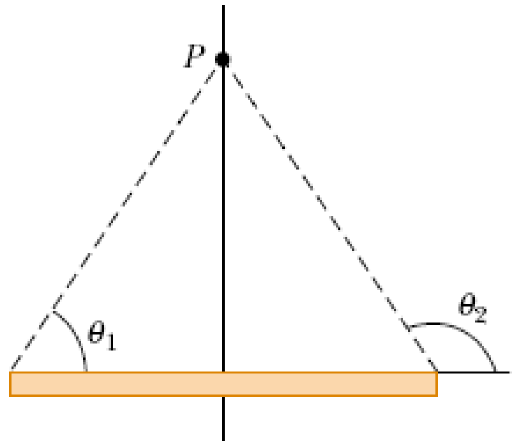
$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta = \boxed{\frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)} \qquad (30.4)$$



(a)

We can use this result to find the magnetic field of *any* straight current-carrying wire if we know the geometry and hence the angles θ_1 and θ_2. Consider the special case of an infinitely long, straight wire. If we let the wire in Figure 30.3b become infinitely long, we see that $\theta_1 = 0$ and $\theta_2 = \pi$ for length elements ranging between positions $x = -\infty$ and $x = +\infty$. Because $(\cos\theta_1 - \cos\theta_2) = (\cos 0 - \cos\pi) = 2$, Equation 30.4 becomes

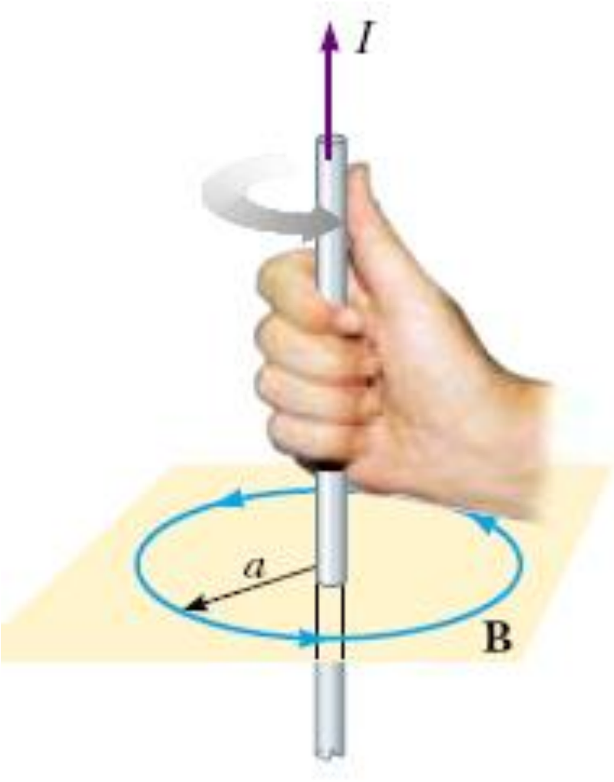
$$B = \boxed{\frac{\mu_0 I}{2\pi a}} \tag{30.5}$$



(b)

Equations 30.4 and 30.5 both show that the magnitude of the magnetic field is proportional to the current and decreases with increasing distance from the wire, as we expected. Notice that Equation 30.5 has the same mathematical form as the expression for the magnitude of the electric field due to a long charged wire (see Eq. 24.7). $\boxed{2k_e \frac{\lambda}{r}}$

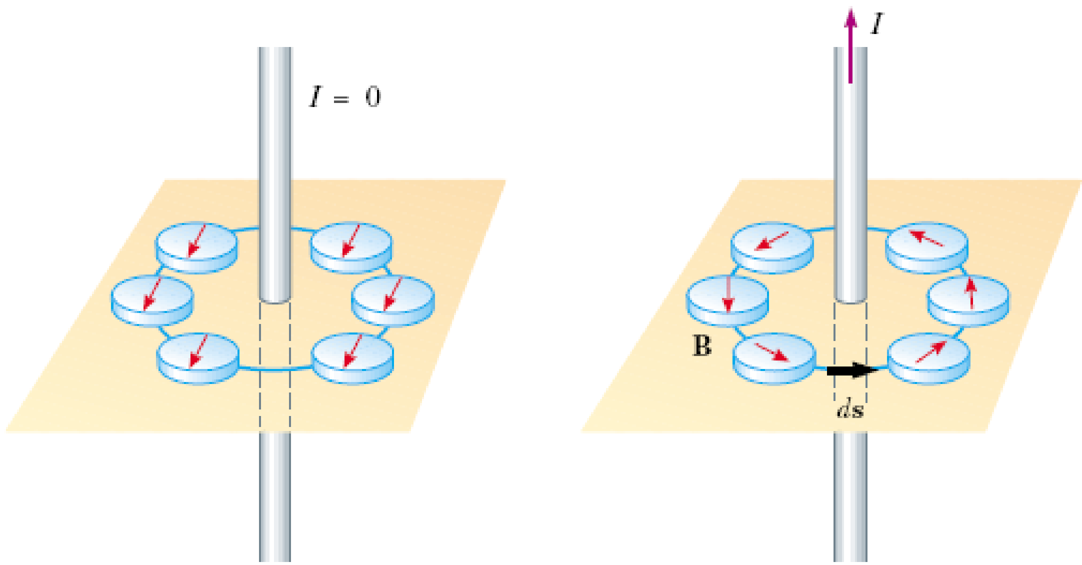
$$B = \boxed{\frac{\mu_0 I}{2\pi a}}$$



Because of the symmetry of the wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire.

magnetic field line shown has no beginning and no end. It forms a closed loop. This is a major difference between magnetic field lines and electric field lines, which begin on positive charges and end on negative charges.

Ampere's Law

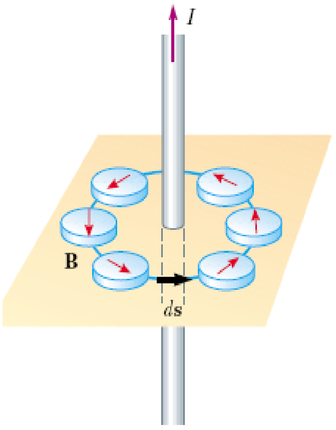


(a) (b)

Because the compass needles point in the direction of **B**, we conclude that the lines of **B** form circles around the wire, as discussed in the preceding section. By symmetry, the magnitude of **B** is the same everywhere on a circular path centered on the wire and lying in a plane perpendicular to the wire. By varying the current and distance a from the wire, we find that B is proportional to the current and inversely proportional to the distance from the wire, as Equation 30.5 describes.

$$B = \frac{\mu_0 I}{2\pi a}$$

Now let us evaluate the product $\mathbf{B} \cdot d\mathbf{s}$ for a small length element $d\mathbf{s}$ on the circular path defined by the compass needles, and sum the products for all elements over the closed circular path.[2] Along this path, the vectors $d\mathbf{s}$ and $\mathbf{B}$ are parallel at each point (see Fig. 30.9b), so $\mathbf{B} \cdot d\mathbf{s} = B\,ds$. Furthermore, the magnitude of $\mathbf{B}$ is constant on this circle and is given by Equation 30.5. Therefore, the sum of the products $B\,ds$ over the closed path, which is equivalent to the line integral of $\mathbf{B} \cdot d\mathbf{s}$, is



(b)

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

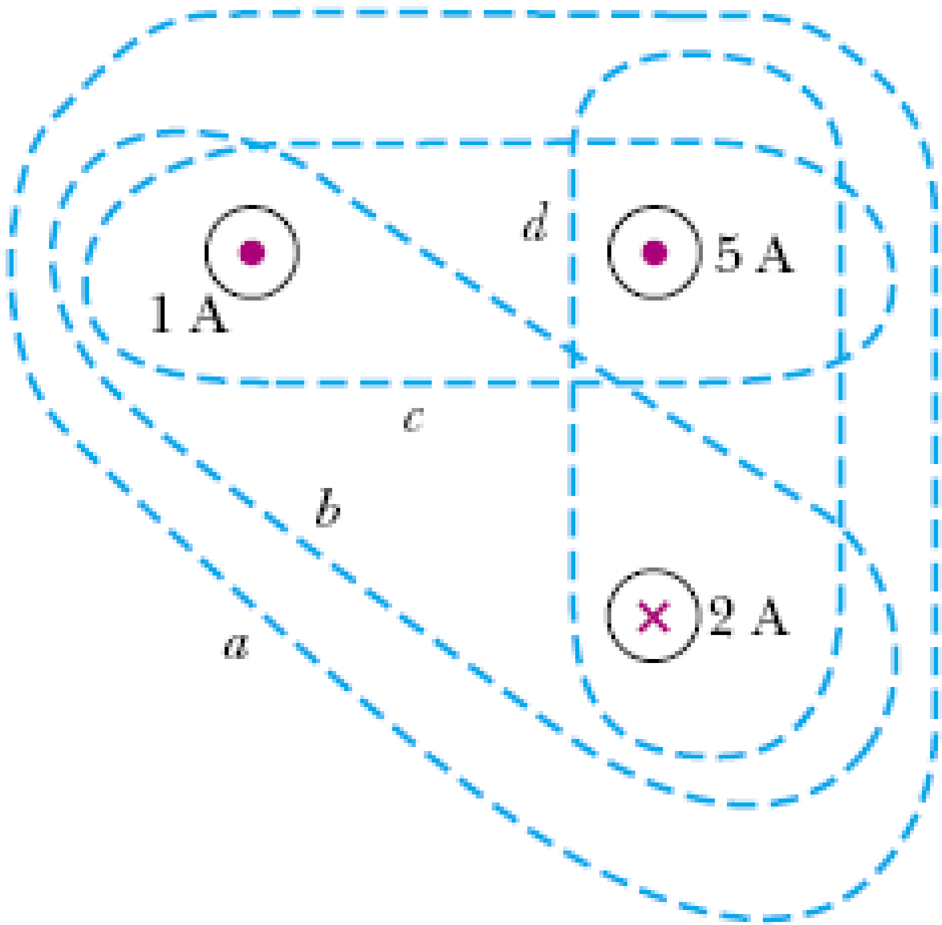
where $\oint ds = 2\pi r$ is the circumference of the circular path.

The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I$$

Ampère's law describes the creation of magnetic fields by all continuous current configurations, but at our mathematical level it is useful only for calculating the magnetic field of current configurations having a high degree of symmetry. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

Quick Quiz 30.4 Rank the magnitudes of $\oint \mathbf{B} \cdot d\mathbf{s}$ for the closed paths in Figure 30.10, from least to greatest.



Quick Quiz 30.5 Rank the magnitudes of $\oint \mathbf{B} \cdot d\mathbf{s}$ for the closed paths in Figure 30.11, from least to greatest.