Practice Questions of Input Probability Distributions.

12. The following data are generated randomly from a gamma distribution:

1.691	1.437	8.221	5.976
1.116	4.435	2.345	1.782
3.810	4.589	5.313	10.90
2.649	2.432	1.581	2.432
1.843	2.466	2.833	2.361

Compute the maximum-likelihood estimators $\widehat{\beta}$ and $\widehat{\theta}$.

12. Solution to Exercise 12:
$\ln \bar{X} - 1.255787$
$\sum_{i=1}^{20} \ln X_i = 21.35591$
$1/M = 5.319392$
$\theta = 0.3848516$
$\beta = 2.815$

16. Records pertaining to the monthly number of job-related injuries at an underground coal mine were being studied by a federal agency. The values for the past 100 months were as follows:

Injuries per Month	*Frequency of Occurrence*
0	35
1	40
2	13
3	6
4	4
5	1
6	1

(a) Apply the chi-square test to these data to test the hypothesis that the underlying distribution is Poisson. Use the level of significance $\alpha = 0.05$.
(b) Apply the chi-square test to these data to test the hypothesis that the distribution is Poisson with mean 1.0. Again let $\alpha = 0.05$.

16. Solution to Exercise 16:

(a) $\alpha = \bar{X} = 1.11$

x_i	O_i	p_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
0	35	.3296	32.96	.126
1	40	.3658	36.58	.320
2	13	.2030	20.30	2.625
3	6	.0751	7.51	
4	4	.0209	2.09	
5	1	.0046	.46	
≥ 6	1	.0010	.10	.333
Totals	100	1.0000	100	$3.404 = \chi_0^2$

$\chi^2_{.05,2} = 5.99$

Therefore, do not reject H_0. Notice that we have grouped cells $i = 3, 4, 5 \geq 6$ together into a single cell with $O_i = 12$ and $E_i = 10.16$.

(b) $\alpha = 1$

x_i	O_i	p_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
0	35	.3679	36.79	.087
1	40	.3679	36.79	.280
2	13	.1839	18.39	1.580
3	6	.0613	6.13	
4	4	.0153	1.53	
5	1	.0031	.31	
≥ 6	1	.0006	.06	1.963
Totals	100	1.0000	100	$3.910 = \chi_0^2$

$\chi^2_{.05,3} = 7.81$

Therefore, do not reject H_0. Notice that we have grouped cells $3, 4, 5 \geq 6$ into a single cell with $O_i = 12$ and $E_i = 8.03$.

17. The time required for 50 different employees to compute and record the number of hours worked during the week was measured, with the following results in minutes:

Employee	*Time (minutes)*	*Employee*	*Time (minutes)*
1	1.88	26	0.04
2	0.54	27	1.49
3	1.90	28	0.66
4	0.15	29	2.03
5	0.02	30	1.00
6	2.81	31	0.39
7	1.50	32	0.34
8	0.53	33	0.01
9	2.62	34	0.10
10	2.67	35	1.10
11	3.53	36	0.24
12	0.53	37	0.26
13	1.80	38	0.45
14	0.79	39	0.17
15	0.21	40	4.29
16	0.80	41	0.80
17	0.26	42	5.50
18	0.63	43	4.91
19	0.36	44	0.35
20	2.03	45	0.36
21	1.42	46	0.90
22	1.28	47	1.03
23	0.82	48	1.73
24	2.16	49	0.38
25	0.05	50	0.48

Use the chi-square test (as in Example 18) to test the hypothesis that these service times are exponentially distributed. Let the number of class intervals be $k = 6$. Use the level of significance $\alpha = 0.05$.

17. Solution to Exercise 17:

H_0 = Data are exponentially distributed

$\hat{\lambda} = \bar{X} = 1.206$

$S = 1.267$

i	O_i	$\frac{(O_i-E_i)^2}{E_i}$
1	8	.013
2	11	.853
3	9	.053
4	5	1.333
5	10	.333
6	7	.213
Totals	50	$2.798 = \chi_0^2$

$\chi^2_{.05,4} = 9.49$

Therefore, do not reject H_0

21. The time (in minutes) between requests for the hookup of electric service was accurately recorded at the Gotwatts Flash and Flicker Company, with the following results for the last 50 requests:

0.661	4.910	8.989	12.801	20.249
5.124	15.033	58.091	1.543	3.624
13.509	5.745	0.651	0.965	62.146
15.512	2.758	17.602	6.675	11.209
2.731	6.892	16.713	5.692	6.636
2.420	2.984	10.613	3.827	10.244
6.255	27.969	12.107	4.636	7.093
6.892	13.243	12.711	3.411	7.897
12.413	2.169	0.921	1.900	0.315
4.370	0.377	9.063	1.875	0.790

How are the times between requests for service distributed? Develop and test a suitable model.

21. H_0: Data are exponentially distributed

$\widehat{\lambda} = 1/\bar{X} = 1/9.459 = .106$

i	O_i	$\frac{(O_i - E_i)^2}{E_i}$
1	7	.8
2	3	.8
3	5	0.0
4	5	0.0
5	5	0.0
6	6	.2
7	5	0.0
8	7	.8
9	4	.2
10	3	.8
Totals	50	$3.6 = \chi_0^2$

$\chi^2_{.05,8} = 15.5$

Therefore, do not reject H_0

22. Daily demands for transmission overhaul kits for the D-3 dragline were maintained by Earth Moving Tractor Company, with the following results:

0	2	0	0	0
1	0	1	1	1
0	1	0	0	0
2	0	1	0	1
0	1	0	0	2
1	0	1	0	0
0	0	0	0	0
1	0	1	0	1
0	0	3	0	1
1	0	0	0	0

How are the daily demands distributed? Develop and test an appropriate model.

22. H_0: Data are Poisson distributed

$\alpha = \bar{X} = .48$

x_i	O_i	p_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
0	31	.6188	30.94	.0001
1	15	.2970	14.85	.0015
2	3	.0713	3.565	
≥ 3	1	.0129	.645	.0140
Totals	50	1.0000	50.00	$.0120 = \chi_0^2$

$\chi^2_{.05,1} = 3.84$

Therefore, do not reject H_0. Notice that we grouped cells $i = 2, 3$ into a single cell with $O_i = 4$ and $E_i = 4.21$.

Note: In Section 9.4.1 it was stated that there is no general agreement regarding the minimum size of E_i and that values of 3, 4 and 5 have been widely used. We prefer $E_i > 5$. If we follow our suggestion in this case, the degrees of freedom will equal zero, which results in an undefined tabular value of χ^2. The concern is that a very small E_i will result in an undue contribution to χ_0^2. With $E_i = 4.21$ this is certainly not a cause for concern. Thus, combining cells as shown is appropriate.