

# Physics Undergraduate Research (PHYS497)

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## 1 Introduction

This paper shows the idea of realization of braiding of Majorana fermions but as series of measurement instead. And then using Jordan-Wigner transformation to write Majorana operators in terms of spin (fermionic) system. Hence, we can use methods provided by `qiskit` package to simulate such an idea and system.

## 2 As Series of Measurement

As a demonstration for the idea, we'll start with a system of 4 Majorana fermions, corresponding to two fermions. The configuration of Majorana fermions is shown in fig. The true braiding operator between  $\gamma_0$  and  $\gamma_3$  is given by

$$U = e^{\frac{\pi}{4} \gamma_0 \gamma_3} \quad (1)$$

Then, to realize this braiding operator as just series of measurement we do this in four steps:

1.  $(1 + i\gamma_1\gamma_2)$
2.  $(1 + i\gamma_1\gamma_0)$
3.  $(1 + i\gamma_3\gamma_1)$
4.  $(1 + i\gamma_1\gamma_2)$

## 3 Jordan-Wigner Transformation

We shall redefine the our  $\gamma$  s in term of fermionic spin operators, giving us a way to model this system in much more familiar systems, such as qubits in quantum computing information. So we'll have:

- $\gamma_0 = Z_0$
- $\gamma_1 = X_0 Z_1$
- $\gamma_2 = X_0 Y_1$
- $\gamma_3 = Y_0$

Note: tensor product is understood, if there is one gate, tensor product with identity of that subsystem is implicit. Then 4-step series of measurement on the system becomes

1.  $(1 + iX_0Z_1X_0Y_1) = (1 + X_1)$
2.  $(1 + iX_0Z_1Z_0) = (1 + Y_0Z_1)$
3.  $(1 + iY_0X_0Z_1) = (1 + Z_0Z_1)$
4.  $(1 + X_1)$

Also for true braiding operator we get

$$e^{i\frac{\pi}{4}X_0} = \frac{1}{\sqrt{2}}(1 + iX_0) \quad \text{or} \quad e^{-i\frac{\pi}{4}X_0} = \frac{1}{\sqrt{2}}(1 - iX_0) \quad (2)$$

## 4 Applying all Measurements

Let's understand the possible outcomes from the general case of the measurement operator, that is,

$$(1 + S_3X_1)(1 + S_2Z_0Z_1)(1 + S_1Y_0Z_1)(1 + S_0X_1) \quad (3)$$

Expanding the middle two factors as

$$(1 + S_3X_1)(1 + S_2Z_0Z_1 + S_1Y_0Z_1 + S_2S_1Z_0Z_1Y_0Z_1)(1 + S_0X_1) \quad (4)$$

Utilizing the Pauli gates anticommutation relations, we move the LHS factor to RHS, as for first term we get

$$(1 + S_3X_1)(1 + S_0X_1) = \delta_{S_0,S_3} (1 + S_0X_1)$$

For second term,

$$(1 + S_3X_1)S_2Z_0Z_1(1 + S_0X_1) = \delta_{S_0,-S_3} S_2Z_0Z_1(1 + S_0X_1)$$

For the 3rd term,

$$(1 + S_3X_1)S_1Y_0Z_1(1 + S_0X_1) = \delta_{S_0,-S_3} S_1Y_0Z_1(1 + S_0X_1)$$

For the 4th term,

$$(1 + S_3X_1)S_2S_1Z_0Z_1Y_0Z_1(1 + S_0X_1) = \delta_{S_0,S_3} -iX_0S_2S_1(1 + S_0X_1)$$

## 5 Constructing Protocol

Now, we'll investigate the protocol classic outcomes, then we shall decide based on it whether we did realize a braiding between  $\gamma_0$  &  $\gamma_3$ , if not, what operators to apply to fix it. From Section 4, we simplify it to

$$[\delta_{S_0,S_3} + \delta_{S_0,-S_3} S_2Z_0Z_1 + \delta_{S_0,-S_3} S_1Y_0Z_1 + \delta_{S_0,S_3} -iX_0S_2S_1](1 + S_0X_1)$$

Let's study different cases:

**Case 1:**  $S_0 = S_3$  We get

$$[1 - iX_0 S_2 S_1](1 + S_0 X_1)$$

Note, the right factor just acts on subsystem 1 that we don't care about its outcomes.

**Case 1.1:**  $S_1 = -S_2$

$$[1 + iX_0](1 + S_0 X_1)$$

relizing counterclockwise braiding operator in Equation 2.

**Case 1.2:**  $S_1 = S_2$

$$[1 - iX_0](1 + S_0 X_1)$$

relizing clockwise braiding operator in Equation 2.

**Case 2:**  $S_0 \neq S_3$  We get

$$[S_2 Z_0 Z_1 + S_1 Y_0 Z_1](1 + S_0 X_1)$$

let's factor out  $Z_0 Z_1$

$$Z_0 Z_1 [S_2 - iS_1 X_0](1 + S_0 X_1)$$

In this case we always want to multiply by  $Z_0$ , then we'll have

**Case 2.1:**  $S_1 = S_2$

$$S_1 Z_0 Z_1 [1 + iX_0](1 + S_0 X_1)$$

relizing the braiding operator

**Case 2.2:**  $S_1 = -S_2$

$$S'_1 Z_0 Z_1 [1 - iX_0](1 + S_0 X_1)$$

relizing the opposite braiding operator