Physics Undergraduate Research (PHYS497)

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1 Instroduction

This paper shows the idea of relizaion of braiding of Majorana fermions but as series of measurement istead. And then using Jordan-Wigner transformation to write Majorana operators in terms of spin (fermionic) system. Hence, we can use methods provided by `qiskit` package to simulate such an idea and system.

2 As Series of Measurement

As a demostration for the idea, we'll start with a system of 4 Majorana fermions, corresponding to two fermions. The configuration of Majorana fermions is shown in fig. The true braiding operator between γ_0 and γ_3 is given by

$$U = e^{\frac{\pi}{4}\gamma_0\gamma_3} \tag{1}$$

Then, to relize this braiding operator as just series of measurement we do this in four steps:

1. $(1 + i\gamma_1\gamma_2)$
2. $(1 + i\gamma_1\gamma_0)$
3. $(1 + i\gamma_3\gamma_1)$
4. $(1 + i\gamma_1\gamma_2)$

3 Jordan-Wigner Transformation

We shall redefine the our γ s in term of fermionic spin operators, giving us a way to model this system in much more fimiliar systems, such as qubits in quantum computing information. So we'll have:

- $\gamma_0 = Z_0$
- $\gamma_1 = X_0 Z_1$
- $\gamma_2 = X_0 Y_1$
- $\gamma_3 = Y_0$

Note: tensor product is understod, if there is one gate, tensor product with indentity of that subsystem is implicit. Then 4-step series of measurement on the system becomes

1. $(1 + iX_0 Z_1 X_0 Y_1) = (1 + X_1)$
2. $(1 + iX_0 Z_1 Z_0) = (1 + Y_0 Z_1)$
3. $(1 + iY_0 X_0 Z_1) = (1 + Z_0 Z_1)$
4. $(1 + X_1)$

Also for true braiding operator we get

$$e^{i\frac{\pi}{4}X_0} = \frac{1}{\sqrt{2}}(1 + iX_0) \quad \text{or} \quad e^{-i\frac{\pi}{4}X_0} = \frac{1}{\sqrt{2}}(1 - iX_0) \tag{2}$$

4 Applying all Measurements

Let's understand the possible outcomes from the general case of the measurement operator, that is,

$$(1 + S_3 X_1)(1 + S_2 Z_0 Z_1)(1 + S_1 Y_0 Z_1)(1 + S_0 X_1) \tag{3}$$

Expanding the middle two factors as

$$(1 + S_3 X_1)(1 + S_2 Z_0 Z_1 + S_1 Y_0 Z_1 + S_2 S_1 Z_0 Z_1 Y_0 Z_1)(1 + S_0 X_1) \tag{4}$$

Utilizing the Pauli gates anitcommutation relations, we move the LHS factor to RHS, as for first term we get

$$(1 + S_3 X_1)(1 + S_0 X_1) = \delta_{S_0,S_3}\,(1 + S_0 X_1)$$

For second term,

$$(1 + S_3 X_1) S_2 Z_0 Z_1 (1 + S_0 X_1) = \delta_{S_0,-S_3}\, S_2 Z_0 Z_1 (1 + S_0 X_1)$$

For the 3rd term,

$$(1 + S_3 X_1) S_1 Y_0 Z_1 (1 + S_0 X_1) = \delta_{S_0,-S_3}\, S_1 Y_0 Z_1 (1 + S_0 X_1)$$

For the 3rd term,

$$(1 + S_3 X_1) S_2 S_1 Z_0 Z_1 Y_0 Z_1 (1 + S_0 X_1) = \delta_{S_0,S_3}\, - iX_0 S_2 S_1 (1 + S_0 X_1)$$

5 Constructing Protocol

Now, we'll investigate the protocol classic outcomes, then we shall decide based on it whether we did relize a braiding between γ_0 & γ_3, if not, what operators to apply to fix it. From Section 4, we simplify it to

$$[\delta_{S_0,S_3} + \delta_{S_0,-S_3}\, S_2 Z_0 Z_1 + \delta_{S_0,-S_3}\, S_1 Y_0 Z_1 + \delta_{S_0,S_3}\, - iX_0 S_2 S_1](1 + S_0 X_1)$$

Let's study different cases:

Case 1: $S_0 = S_3$ We get

$$[1 - iX_0S_2S_1](1 + S_0X_1)$$

Note, the right factor just acts on subsystem 1 that we don't care about it outcomes.

Case 1.1: $S_1 = -S_2$

$$[1 + iX_0](1 + S_0X_1)$$

relizing counterclockwise braiding operator in Equation 2.

Case 1.2: $S_1 = S_2$

$$[1 - iX_0](1 + S_0X_1)$$

relizing clockwise braiding operator in Equation 2.

Case 2: $S_0 \neq S_3$ We get

$$[S_2Z_0Z_1 + S_1Y_0Z_1]\,(1 + S_0X_1)$$

let's factor out Z_0Z_1

$$Z_0Z_1[S_2 - iS_1X_0]\,(1 + S_0X_1)$$

In this case we always want to multiply by Z_0, then we'll have

Case 2.1: $S_1 = S_2$

$$S_1Z_0Z_1[1 - iX_0](1 + S_0X_1)$$

relizing the inverse braiding operator

Case 2.2: $S_1 = -S_2$

$$S_1Z_0Z_1[S_1S_2 - iX_0](1 + S_0X_1)$$

But $S_1S_2 = -1$, then

$$-S_1Z_0Z_1[1 + iX_0](1 + S_0X_1)$$

relizing the braiding operator