Physics Undergraduate Research (PHYS497)

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1 Instroduction

This paper shows the idea of relizaion of braiding of Majorana fermions but as series of measurement istead. And then using Jordan-Wigner transformation to write Majorana operators in terms of spin (fermionic) system. Hence, we can use methods provided by `qiskit` package to simulate such an idea and system.

2 As Series of Measurement

As a demostration for the idea, we'll start with a system of 4 Majorana fermions, corresponding to two fermions. The configuration of Majorana fermions is shown in fig. The true braiding operator between γ_0 and γ_3 is given by

$$U = e^{\frac{\pi}{4}\gamma_0\gamma_3} \tag{1}$$

Then, to relize this braiding operator as just series of measurement we do this in four steps:

1. $(1 + i\gamma_1\gamma_2)$
2. $(1 + i\gamma_0\gamma_1)$
3. $(1 + i\gamma_3\gamma_1)$
4. $(1 + i\gamma_1\gamma_2)$

3 Jordan-Wigner Transformation

We shall redefine the our γ s in term of fermionic spin operators, giving us a way to model this system in much more fimiliar systems, such as qubits in quantum computing information. So we'll have:

- $\gamma_0 = Z_0$
- $\gamma_0 = X_0Z_1$

- $\gamma_0 = X_0 Y_1$
- $\gamma_0 = Y_0$

Note: tensor product is understod, if there is one gate, tensor product with indentity of that subsystem is implicit. Then 4-step series of measurement on the system becomes

1. $(1 + X_1)$
2. $(1 - Y_0 Z_1)$
3. $(1 + Z_0 Z_1)$
4. $(1 + X_1)$

Also for true braiding operator we get

$$U = e^{i\frac{\pi}{4}X_0} = \frac{(1 + iX_0)}{\sqrt{2}} \tag{2}$$

4 Applying all Measurements

Let's understand the possible outcomes from the general case of the measurement operator, that is,

$$(1 + S_0 X_1)(1 + S_1 Z_0 Z_1)(1 + S_2 Y_0 Z_1)(1 + S_3 X_1) \tag{3}$$

Expanding the middle two factors as

$$(1 + S_0 X_1)(1 + S_1 Z_0 Z_1 + S_2 Y_0 Z_1 + S_1 S_2 Z_0 Z_1 Y_0 Z_1)(1 + S_3 X_1) \tag{4}$$

Utilizing the Pauli gates anitcommutation relations, we move the LHS factor to RHS, as for first term we get

$$(1 + S_0 X_1)(1 + S_3 X_1) = \delta_{S_0, S_3}(1 + S_3 X_1) \tag{5}$$

For second term,

$$(1 + S_0 X_1) S_1 Z_0 Z_1 (1 + S_3 X_1) = \delta_{-S_0, S_3} S_1 Z_0 Z_1 (1 + S_3 X_1) \tag{6}$$