PHYS213

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Chapter 1

The Quantum Theory of Light

1.1 Heartz's Experiments; Light as EM Wave

• Between 1865 and 1873, James Clerk Maxwell, and according to his theory, an alternating current would set up fluctuating electric and magnetic fields in the region surrounding the original disturbance.

• These waves were predicted to have a frequency equal to the frequency of the current oscillations.

• And more importnatly, they would behave in every way like *light*.

• Naturally this led to the unifying and simplifying postulate that light was also a type of Maxwell wave or electromagnetic disturbance, created by extremely high frequency electric oscillators in matter.

• it was apparent that this model of light emission was incapable of direct experimental verification, because the highest electrical frequencies then attainable were about $10^9 Hz$.

• Heinrich Hertz showed that Maxwell's theory was correct and that an oscillating electric current does indeed radiate electromagnetic waves that possess every characteristic of light except the same wavelength as light.

• By using a simple spark gap oscillator in Fig.(1.1).

• Hertz next proceeded to show that these electromagnetic waves could be reflected, refracted, focused, polarized, and made to interfere—- in short, he convinced physicists of the period that Hertzian waves and light waves were one and the same.

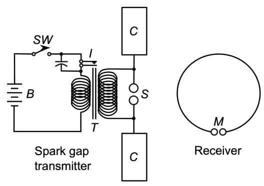

Figure 1.1: Heartz's Transmitter.

1.2 Blackbody Radiation

The problem is to predict the radiation intensity at a given wavelength emitted by a hot glowing solid at a specific temperature.

• In 1792, Thomas Wedgwood observed that all the objects in his ovens, regard- less of their chemical nature, size, or shape, became red at the same tempera- ture.

• In 1859, Gustav Kirchhoff proved a theorem when he showed by arguments based on thermodynamics that for any body in thermal equilibrium with radiation the emitted power is proportional to the power absorbed. More specifically,

$$e_f = J(f,T)A_f \tag{1.1}$$

- e_f is the power emitted per unit area per unit frequenct by a heated object.
- A_f is the absorption power (fraction of the incident power absorbed per unit area per unit frequency by the heated object).
- $J(f,T)$ is a universal function (the same for all bodies) that depends only on f.

- A **blackbody** is defined as an object that absorbs all the electromagnetic radiation falling on it and consequently appears black. It has $a_f = 1$ for all frequencies and so Kirchhoff's theorem for a blackbody becomes,

$$e_f = J(f,T) \tag{1.2}$$

- Eq.(1.1) shows that the power emitted only per unit area per unit frequency by a blackbody depends only on the light frequency and the temperature.

- Because absorption and emission are connected by Kirchhoff's theorem, we see that a *blackbody* or perfect absorber is also an ideal *radiator*.
In practice, a small opening in any heated *cavity*, such as a port in an oven, behaves like a blackbody because such an opening traps all incident radiation.

- In 1879, Josef Stefan found experimentally that the total power per unit area emitted at all frequencies by a hot solid, was proportional to the fourth power of its absolute temperature. Therefore, *Stefan's law* may be written as

$$e_{total} = \int_0^\infty e_f \, df = \sigma T^4 \tag{1.3}$$

  - e_{total} is the power per unit area emitted at the surface of the blackbody at all frequencies.
  - e_f is the power per unit area per unit frequency emitted by the blackbody.
  - T is the absolute temperature of the body.
  - σ is the Stefan – Boltzmann constant $\sigma = 3 \times 10^5 \, \mathrm{W/(m^2\,K^4)}$.

A body that is not an ideal radiator will obey the same general law but with a coefficient, a, less than 1:

$$e_{total} = a\sigma T^4 \tag{1.4}$$

- Notice that in Fig.(1.2) λ_{max} shifts to shorter wavelengths as the blackbody gets hotter.
In 1893, Wilhelm Wien proposed a general form for the blackbody distribution law $J(f,T)$ that gave the correct experimental behavior of λ_{max} with temperature. This law is called **Wien's displacement law** and may be written

$$\lambda_{max} T = 2.898 \times 10^{-3} \, \mathrm{m\,K} \tag{1.5}$$

  - λ_{max} is the the blackbody's maximum intensity.
  - T is the absolute temperature of the surface.

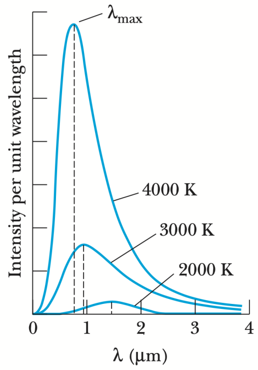

Figure 1.2: Emission from a glowing solid.

- Let's consider the energy per unit volume per unit frequency of the radiation within the blackbody cavity, $u(f,T)$. For light in equilibrium with the walls, the power emitted per square centimeter of opening is simply proportional to the energy density of the light in the cavity.

- An important guess as to the form of the universal function $u(f,T)$ was made in 1893 by Wien and had the form

$$u(f,T) = Af^3 e^{-\beta f/T} \tag{1.6}$$

  - A and β are constants

- Within a year Friedrich Paschen, had confirmed Wien's guess by working in the then difficult infrared range of 1 to 4 μm and at temperatures of 400 to 1600 K.Paschen had made most of his measurements in the maximum energy region of a body heated to 1500K and had found good agreement with Wien's exponential law. In 1900, however, Lummer and Pringsheim extended the measurements to 18 μm, and Rubens and Kurlbaum went even farther– to 60 μm. Both teams concluded that Wien's law failed in this region.

Enter Planck On a Sunday evening early in October of 1900, Max Planck discovered the famous blackbody formula.

Planck knew that Wien's law agreed well with the data at high frequency and indeed had been working hard for several years to derive Wien's exponential law from the principles of statistical mechanics and Maxwell's laws. See Eq.(1.17)

The Quantum Energy Planck was convinced that blackbody radiation was produced by vibrating submicroscopic electric charges, which he called *resonators*. He assumed that the walls of a glowing cavity were composed of literally billions of these *resonators* (whose exact nature was unknown at the time), all vibrating at different frequencies. Hence, according to Maxwell, each *oscillator* should emit radiation with a frequency corresponding to its vibration frequency. **Also according to *classical Maxwellian theory*, an oscillator of frequency f could have any value of energy and could change its amplitude continuously as it radiated any fraction of its energy.**

- This is where Planck made his revolutionary proposal. To secure agreement with experiment, Planck had to assume that the total energy of a resonator with mechanical frequency f could only be an integral multiple of hf or

$$E_{resonator} = nhf \quad n = 1, 2, 3, \ldots \tag{1.7}$$

he concluded that emission of radiation of frequency f occurred when a resonator dropped to the next lowest energy state.
Thus the resonator can change its energy only by the difference ΔE according to

$$\Delta E = hf \tag{1.8}$$

That is, it cannot lose just any amount of its total energy, but only a finite amount, hf, the so-called ***quantum*** *of energy.*

- Planck explained the continuous spectrum of the blackbody by assuming that *the heated walls contained resonators vibrating at many different frequencies*, each emitting light at the same frequency as its vibration frequency.

- considering the conditions leading to equilibrium between the *wall resonators* and the *radiation* in the blackbody cavity, he found that the spectral energy density $u(f,T)$ could be expressed as

$$u(f,T)\,df = \bar{E}N(f)\,df \tag{1.9}$$

- $N(f)$ number of oscillators having frequency between f and $f + df$.
- $\bar{E}$ average energy emitted per oscillator.

Furthermore, Planck showed that the number of oscillators with frequency between f and $f+df$ was proportional to f^2 or

$$N(f)\,df = \frac{8\pi f^2}{c^3}\,df \tag{1.10}$$

1.3 The Rayleigh-Jeans Law and Planck's Law

Both Planck's law and the Rayleigh–Jeans law[1] may be derived using the idea that the blackbody radiation energy per unit volume can expressed as in Eq.(1.9). We will consider **Rayleigh–Jeans** and **Planck** calculations to see the effect on $u(f,T)$ of calculating $\bar{E}$ from a *continuous* distribution of classical oscillator energies (Rayleigh–Jeans) as opposed to a *discrete* set of quantum oscillator energies (Planck).

[1] the classical theory of blackbody radiation formulated by Lord Rayleigh, John William Strutt, 1842–1919, English physicist, and James Jeans, 1887–1946, English astronomer and physicist

Rayleigh-Jeans Law *While Planck concentrated on the thermal equilibrium of cavity radiation with oscillating electric charges in the cavity walls, Rayleigh concentrated directly on the electromagnetic waves in the cavity.*

- Rayleigh and Jeans reasoned that the *standing electromagnetic waves* in the cavity could be considered to have a `temperature` T, because they constantly exchanged energy with the walls and caused a thermometer within the cavity to reach the same temperature as the walls. Further, they considered a *standing polarized electromagnetic wave* to be equivalent to a one-dimensional oscillator (1.3).

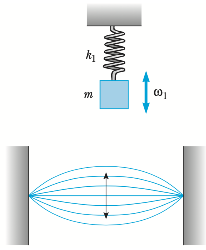

Figure 1.3: A one-dimensional harmonic oscillator.

- Using the same general idea as Planck, they expressed the energy density as a product of the number of standing waves (oscillators) and the average energy per oscillator. They found the average oscillator energy $\bar{E}$ to be independent of frequency and equal to $k_B T$ from the *Maxwell-Boltzmann distribution law* (see Chapter 10). According to this distribution law,

$$P(E) = P_0 \, e^{-(E-E_0)/k_B T} \tag{1.11}$$

- the probability P of finding an individual system (such as a molecule or an atomic oscillator)
- with energy E above some
- minimum energy(E_0) in a large group of systems at
- temperature T
- where P0 is the probability that a system has the minimum energy.

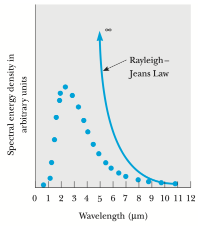

Figure 1.4: The failure of the classical Rayleigh–Jeans law Eq.(1.15) to fit the observed spectrum of a blackbody heated to 1000 K.

- In the case of a discrete set of allowed energies, the average energy($\bar{E}$) is given by

$$\bar{E} = \frac{\Sigma E \, P(E)}{\Sigma P(E)} \tag{1.12}$$

where division by the sum in the denominator serves to normalize the total probability to 1.

- *In the classical case considered by Rayleigh*, an oscillator can have any energy E in a continuous range from 0 to ∞. Thus the Eq.(1.12) muse be replaced with integrals, so

$$\bar{E} = \frac{\int_0^\infty E e^{-E/k_B T} \, dE}{\int_0^\infty e^{-E/k_B T} \, dE} = k_B T \tag{1.13}$$

The derivation of the density of modes, $N(f)\, df$, gives

$$N(f)\, df = \frac{8\pi f^3}{c^3}\, df \qquad \text{or in wavelength} \qquad N(\lambda)\, d\lambda = \frac{8\pi}{\lambda^4}\, d\lambda \tag{1.14}$$

The spectral energy density is simply the *density of modes* multiplied by $k_B T$, or

$$u(f,T)\, df = \frac{8\pi f^2}{c^3} k_B T \, df \qquad \text{or in wavelength} \qquad u(\lambda, T)\, d\lambda = \frac{8\pi}{\lambda^4} k_B T \, d\lambda \tag{1.15}$$

- However, as in Figure 1.4, this classical expression, known as the *Rayleigh–Jeans law*, does not agree with the experimental results in the short wavelength region. Eq.(1.15) diverges as $\lambda \to 0$, predicting unlimited energy emission in the ultraviolet region, which was dubbed the "ultraviolet catastrophe". To conclude that the classical theory fails to explain blackbody radiation.

Planck Law Planck concentrated on the energy *states of resonators* in the cavity walls and used the condition that the `resonators and cavity radiation were inequilibrium` to determine the spectral quality of the radiation. By thermodynamic reasoning (and apparently unaware of Rayleigh's derivation), he arrived at the same expression for $N(f)$ as Rayleigh. However, Planck arrived at a different form for E by allowing only *discrete* values of energy for his resonators. He found, using the Maxwell-Boltzmann distribution law,

$$\bar{E} = \frac{hf}{e^{hf/k_bT} - 1} \tag{1.16}$$

Multiplying $\bar{E}$ by $N(f)$ gives the Planck distribution law,

$$\begin{aligned} & u(f,T)\,df = \frac{8\pi f^2}{c^3}\left(\frac{hf}{e^{hf/k_BT} - 1}\right) df \\ \text{or in wavelength} \qquad & u(\lambda,T)\,d\lambda = \frac{8\pi hc}{\lambda^5(e^{hc/\lambda k_BT} - 1)}\,d\lambda \end{aligned} \tag{1.17}$$

- h Planck's constant 6.626×10^{-34} J s
- k_B Boltzmann's constant 1.380×10^{-23} J K^{-1}

In summary, Planck arrived at his blackbody formula by making two startling assumptions:

1. The energy of a charged oscillator of frequency f is limited to discrete values nhf.
2. During emission or absorption of light, the change in energy of an oscillator is hf.

★ By integrating Eq.(1.17) for $0 \to \infty$ we will get Stefan's law in Eq.(1.3).

1.4 Light Quantization and The Photoelectric Effect

Einstein recognized an inconsistency between Planck's quantization of oscillators in the walls of the blackbody and Planck's insistence that the cavity radiation consisted of classical electromagnetic waves.

★ By showing that the change in entropy of blackbody radiation was like the change in entropy of an ideal gas consisting of independent particles, Einstein reached the conclusion that light itself is composed of "grains," irreducible finite amounts, or quanta of energy.

• Hertz first established that clean metal surfaces emit charges when exposed to ultraviolet light. In 1888 Hallwachs discovered that the emitted charges were negative, and in 1899 J. J. Thomson showed that the emitted charges were electrons, now called photoelectrons.

• In 1902, Philip Lenard was studying the photoelectric effect with intense carbon arc light sources. He found that electrons are emitted from the metal with a range of velocities and that the maximum kinetic energy of photoelectrons, K_{max}, does not depend on the *intensity* of the exciting light. He lso indicated that K_{max} increases with light *frequency*.

• In the photoelectric setup, K_{max} is easily measured by applying a retarding voltage and gradually increasing it until the most energetic electrons are stopped and the photocurrent becomes zero. At this point,

$$K_{max} = \frac{1}{2}m_e v_{max}^2 = eV_s \tag{1.18}$$

As shown in Fig.(1.5) Lenard found that K_{max} or V_s is independent of light intensity I.

The classical electromagnetic theory has major difficulties explaining:

1. The linear dependence of K_{max} on light frequency, shown in Fig.(1.6).
2. There is no *time lag* between the start of illumination and the start of the photocurrent.

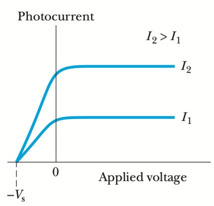

Figure 1.5: Photocurrent versus applied voltage

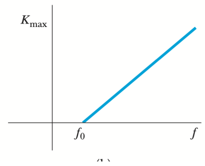

Figure 1.6: K_{max} versus frequency

★ Einstein's theory of the photoelectric effect: **He maintained that the energy of light is not distributed evenly over the classical wavefront, but is concentrated in discrete regions (or in "bundles"), called quanta, each containing energy, hf.**

- Therefore, according to Einstein, the maximum kinetic energy for emitted electrons is

$$K_{max} = hf - \phi \tag{1.19}$$

where ϕ is the **work function** of the metal, which corresponds to the minimum energy with which an electron is bound in the metal. Eq.(1.19) explained the independence of K_{max} and intensity found by Lenard. For a fixed light frequency f, an increase in light intensity means more photons and more photoelectrons per second, although K_{max} remains unchanged according to this equation.

- By setting $K_{max} = 0$ we get the characteristic threshold frequency,

$$f_0 = \frac{\phi}{h} \tag{1.20}$$

1.5 X-Rays

X-rays were discovered in 1895 by the German physicist Wilhelm Roentgen. He found that a beam of high-speed electrons striking a metal target produced a new and extremely penetrating type of radiation. Max von Laue in Germany and William Henry Bragg and William Lawrence Bragg (a father and son team) in England suggested using single crystals such as calcite as natural three-dimensional gratings, the periodic atomic arrangement in the crystals constituting the *grating rulings*.

- A particularly simple method of analyzing the scattering of x-rays from parallel crystal planes was proposed by W. L. Bragg in 1912. Consider two successive planes of atoms as shown in Fig.(1.7).

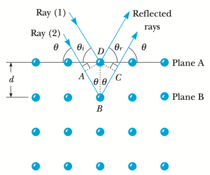

Figure 1.7: Bragg scattering of x-rays from successive planes of atoms.

- The adjacent atoms in a single plane, A, will scatter constructively if the angle of incidence, θ_i, equals angel of reflaction, θ_r.

Atoms in *successive planes* (A & B), will scatter constructively if the the path length difference for rays (1) & (2) is a ehole number of wavelength, thus $n\theta$, so

$$\overline{AB} + \overline{BC} = n\theta, \qquad n = 1, 2, 3, \ldots \tag{1.21}$$

Thus Bragg equation is

$$\Delta L = 2d \sin\theta = n\lambda \tag{1.22}$$