HW4

February 16, 2023

1 Thermal Physics - Homework 4

Alfaifi, Ammar - 201855360

[3]:
```python
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from math import comb, factorial, log
import scipy.constants as con
from matplotlib_inline.backend_inline import set_matplotlib_formats

set_matplotlib_formats('pdf')
plt.rcParams |= {
    'text.usetex': True,
    'figure.figsize': (10, 4)
}
sns.set_theme()
set_matplotlib_formats('svg', 'pdf')
```

1.1 Question 1

We note that the multiplicity of an idea gas of one molecule in 2D is propational to the area of available momentum space as well as in the position space, thus

$$\Omega_1 \propto A \times A_p$$

Further, molecule's kientic energy must equal U

$$U = \frac{1}{2}m(v_x^2 + v_y^2) = \frac{1}{2m}(p_x^2 + p_y^2)$$

or

$$p_x^2 + p_y^2 = 2mU$$

so $\sqrt{2mU}$ defines circumference of hypercircle in momentum space.

From Heisenberg uncertainity principle $\Delta x \, \Delta p_x \approx h$ so we have the multiplicity as

$$\Omega_1 = \frac{AA_p}{h^2}$$

and for two indistinguisable molecules, the number of distinct states is

$$\Omega_1 = \frac{1}{2}\frac{A^2}{h^4} \times area$$

In general, for N particles

$$\Omega_N = \frac{1}{N!}\frac{A^N}{h^{2N}} \times (\text{area})$$

area of hypercircle in d

$$circ = \frac{2\pi^{d/2}}{(\frac{d}{2}-1)!} r^{d-1}$$

Then, $d = 2N$ and $r = \sqrt{2mU}$

$$\Omega_N = \frac{1}{N!}\frac{A^N}{h^{2N}}\frac{2\pi^N}{(N-1)!} r^{2N-1} \approx \frac{1}{N!}\frac{A^N}{h^{2N}}\frac{2\pi^N}{N!} r^{2N}$$

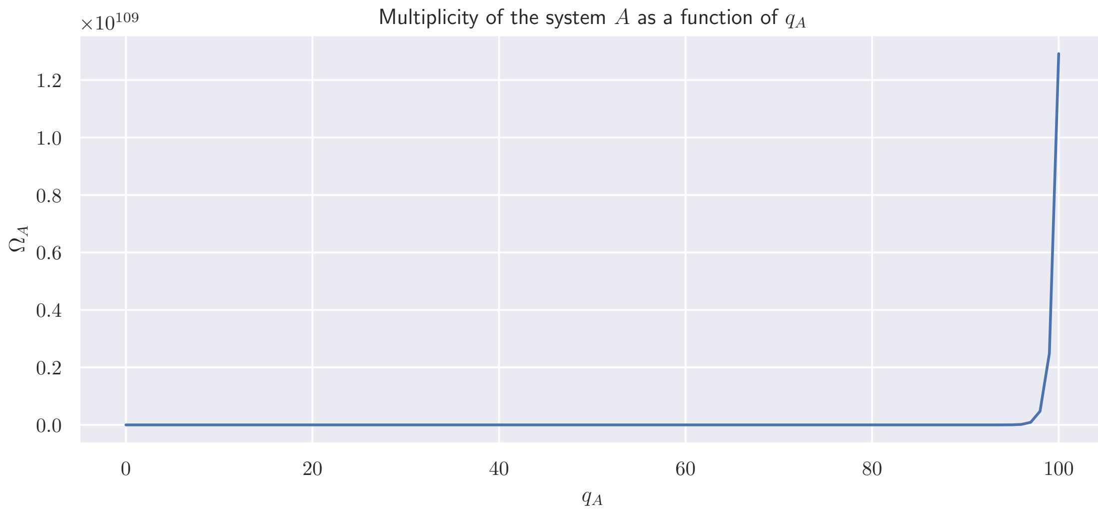
The the entropy will be

$$S = k \ln \Omega_N =$$

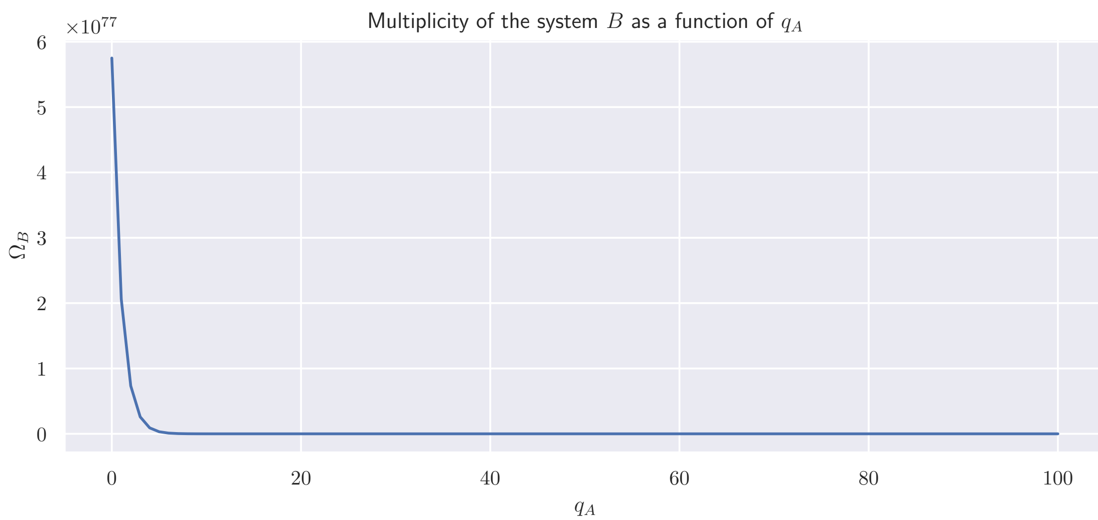
1.2 Question 3

$N_A = 420$, $N_B = 180$, and $q = 100$

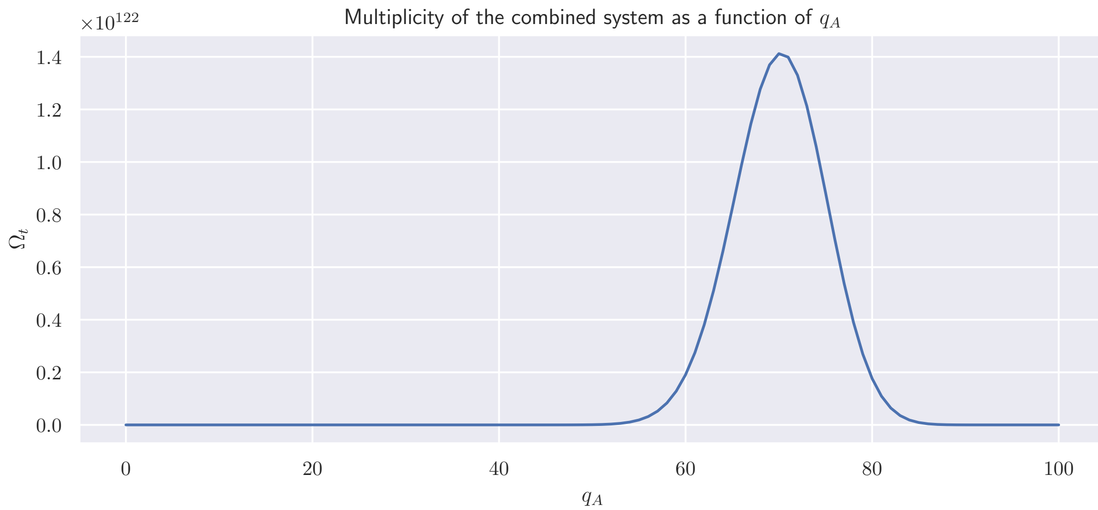
[4]:
```
N_A = 420; N_B = 180
N = N_A + N_B; q_total = 100

q_A = np.arange(0, q_total+1)
q_B = q_A[::-1]

omega_A = np.array([float(comb(q + N_A - 1, q)) for q in q_A])
omega_B = np.array([float(comb(q + N_B - 1, q)) for q in q_B])
omega_total = omega_A * omega_B
prob = omega_total / np.sum(omega_total)

plt.title('Multiplicity of the system $A$ as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$\Omega_A$')
plt.plot(q_A, omega_A)
plt.show()
```



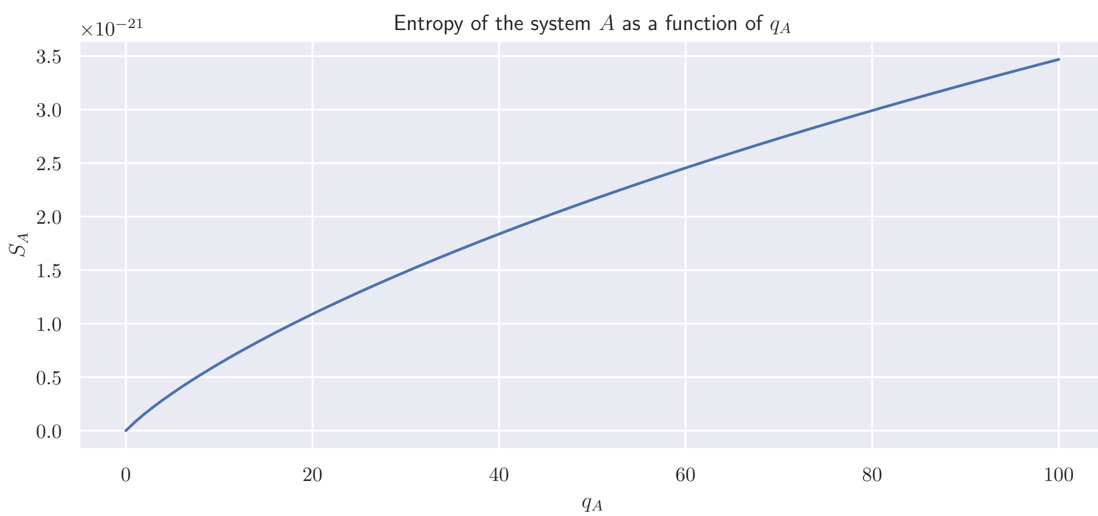
[5]:
```
plt.title('Multiplicity of the system $B$ as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$\Omega_B$')
plt.plot(q_A, omega_B)
plt.show()
```



[6]:
```
plt.title('Multiplicity of the combined system as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$\Omega_t$')
plt.plot(q_A, omega_total)
plt.show()
```

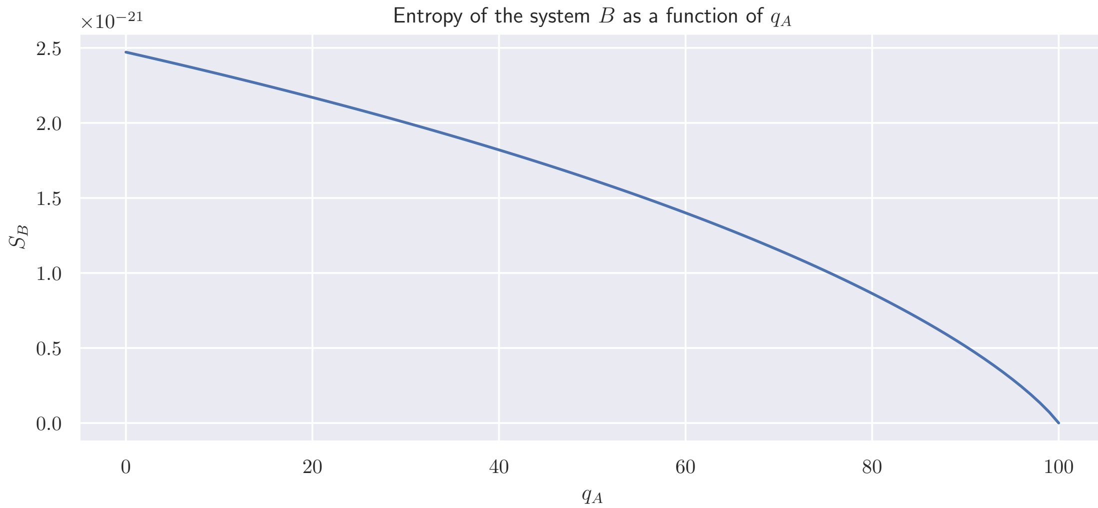



```
[7]: S_A = con.k * np.log(omega_A)
plt.title('Entropy of the system $A$ as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$S_A$')
plt.plot(q_A, S_A)
plt.show()
```

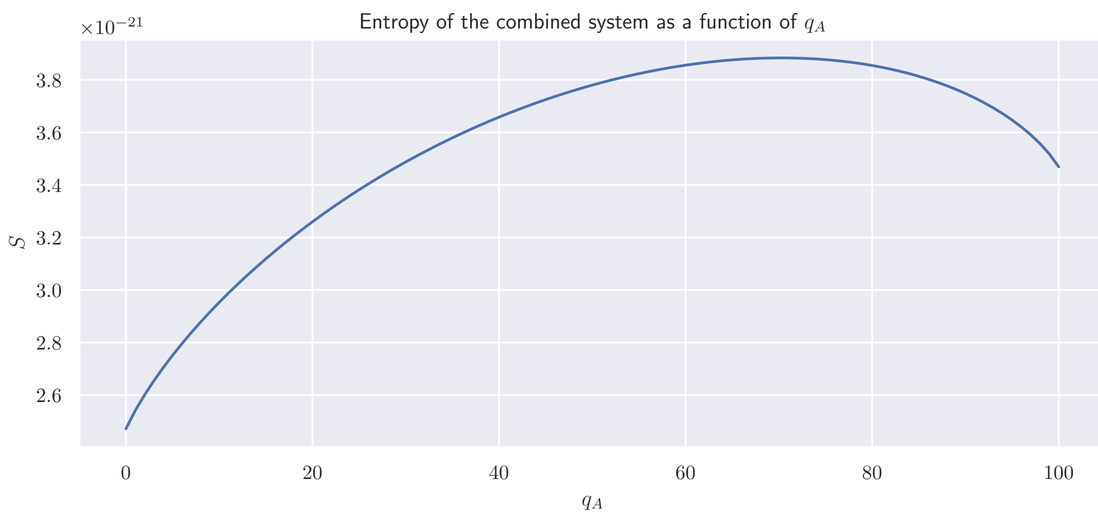



```
[8]: S_B = con.k * np.log(omega_B)
plt.title('Entropy of the system $B$ as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$S_B$')
plt.plot(q_A, S_B)
```

```
plt.show()
```



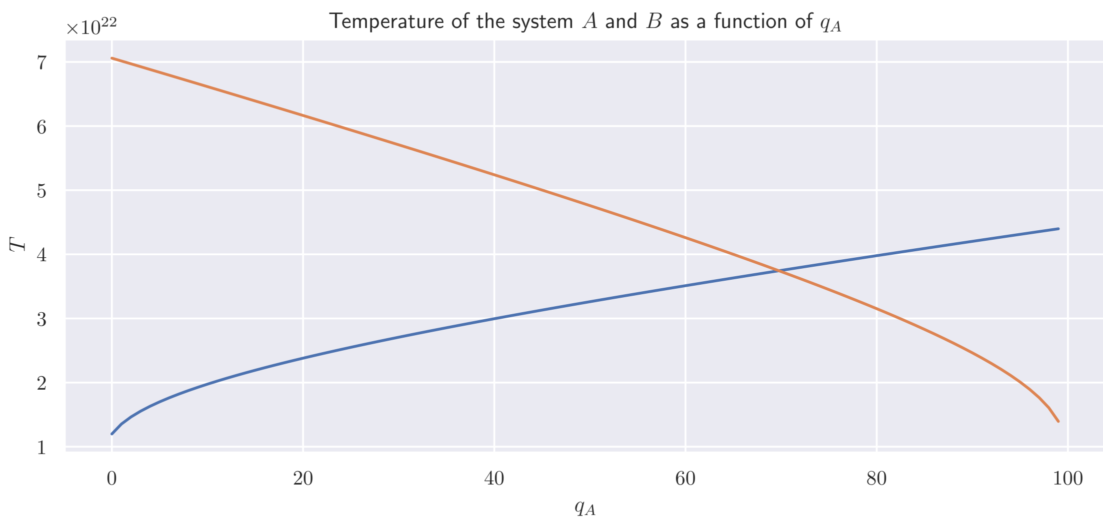
[9]:
```
S_total = con.k * np.log(omega_total)
plt.title('Entropy of the combined system as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$S$')
plt.plot(q_A, S_total)
plt.show()
```



We see that the entropy of the combined systemm is maximum at $q_A = 70$.

1.3 Problem 4

[10]:
```
temp_A = np.diff(q_A) / np.diff(S_A)
temp_B = np.diff(q_B) / np.diff(S_B)

plt.title('Temperature of the system $A$ and $B$ as a function of $q_A$')
plt.xlabel('$q_A$')
plt.ylabel('$T$')
plt.plot(q_A[:-1], temp_A)
plt.plot(q_A[:-1], temp_B)
plt.show()
```



From above, the entropy is max at 70. At this values both systems rached that same temperature which is called **thermal equilibrium**