HW5

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1 Thermal Physics - Homework 5

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[3]:
```python
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
import scipy.constants as con
from scipy.special import comb
from matplotlib_inline.backend_inline import set_matplotlib_formats

set_matplotlib_formats('pdf')
plt.rcParams |= {
    'text.usetex': True,
    'figure.figsize': (10, 4)
}
sns.set_theme()
set_matplotlib_formats('svg', 'pdf')
```

1.1 Question 1

Consider two-state paramagnet of $N = 150$ identical dipoles in a magentic field of strength B, each of which has a magnetic dipole moment of μ

1.1.1 Numeriacal Solution

$$\frac{U}{\mu B} = N - 2N_\uparrow \qquad \frac{S}{k} = \Omega(N_\uparrow) = \binom{N}{N_\uparrow}$$

1.1.2 Analytical Solution

We take (3.28)

$$\frac{S}{k} = N \ln N - N_\uparrow \ln N_\uparrow - (N - N_\uparrow) \ln (N - N_\uparrow)$$

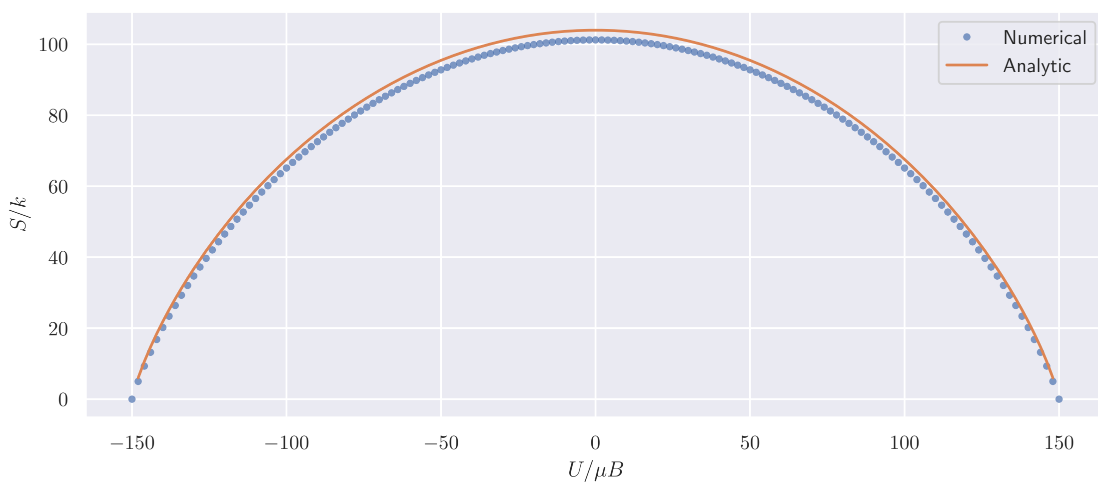
and for substituting (3.30) in (3.31) we have

$$\frac{U}{\mu B} = -N \tanh \frac{1}{2} \ln 2 \frac{N_\uparrow}{N - N_\uparrow}$$

[39]:
```
N = 150
N_up = np.arange(N + 1)

# unitless energy: U/mu B
energy = N - 2 * N_up
# unitless entropy: S/k
entropy = np.log(comb(N, N_up))
# remove first and last val, invlid
N_up_ = N_up[1:-1]
# analytic unitless energy: U/mu B
energy_anal = - N * np.tanh(0.5 * np.log((N - energy[1:-1]) / (N + energy[1:
↪-1])))
# analytic unitless entropy: S/k
entropy_anal = N * np.log(N) - N_up_ * np.log(N_up_) - (N - N_up_) * np.log(N -␣
↪N_up_)

plt.plot(energy, entropy, '.', label='Numerical', alpha=0.7)
plt.plot(energy_anal, entropy_anal, label='Analytic')
plt.xlabel(r'$U/\mu B$')
plt.ylabel(r'$S/k$')
plt.legend()
plt.show()
```



1.2 Question 1

1.2.1 Numerical Solution

We use the following realtion between entropy, energy and temperatur

$$T = \frac{\partial U}{\partial S} \qquad \frac{kT}{\mu B} = -2\frac{\Delta N_{\uparrow}}{\Delta\Omega(N_{\uparrow})}$$

Note is always $\Delta N_\uparrow = 1$

1.2.2 Analytic Solution

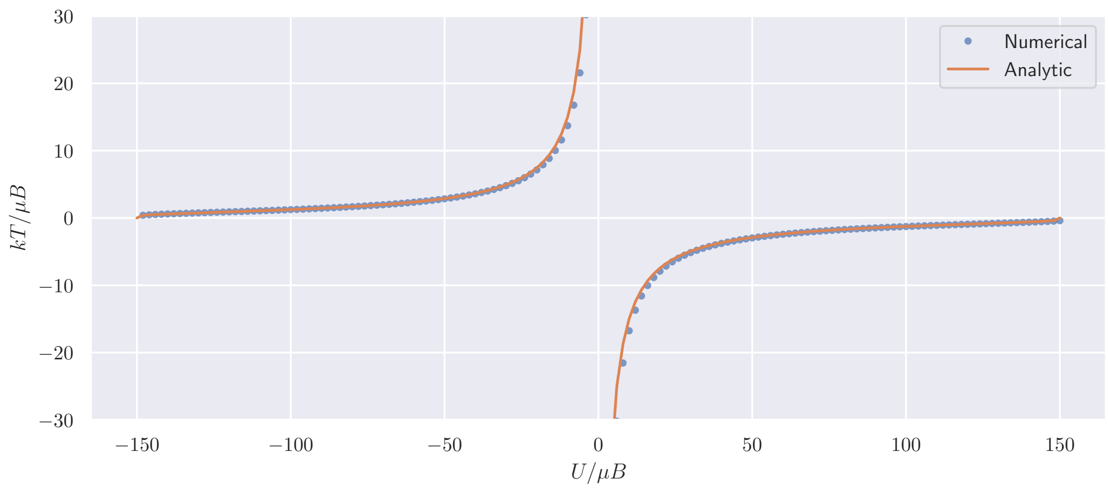
Using

$$\frac{\mu B}{kT} = \frac{1}{2} \ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right)$$

[58]:

```
# Numerical unitless temprature
temp = -2 / np.diff(entropy)
# Analytical unitless temprature
with np.errstate(divide='ignore'):
    # ignore warning of dividing by zero
    temp_anal = 1 / (0.5* np.log((N - energy) / (N + energy)))

plt.plot(energy[:-1], temp, '.', label='Numerical', alpha=0.7)
plt.plot(energy, temp_anal, label='Analytic')
plt.xlabel(r'$U/\mu B$')
plt.ylabel(r'$kT/\mu B$')
plt.ylim([-30, 30])
plt.legend()
plt.show()
```



1.3 Question 3

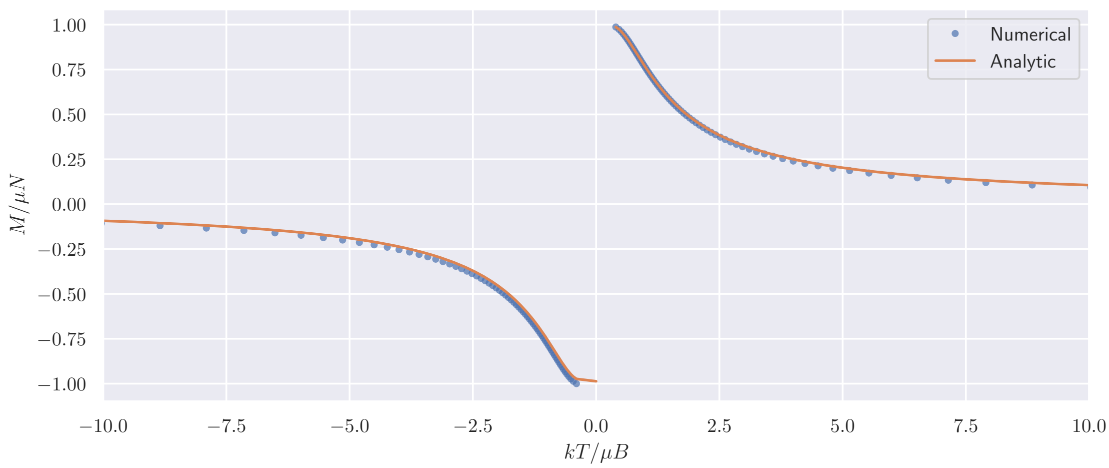
$$\frac{M}{N\mu} = -\frac{U}{B}$$

From (3.32)

$$\frac{M}{N\mu} = \tanh \frac{\mu B}{kT}$$

[78]:
```
# Numerical Magetiztion
x = 1 / temp
# Analytic Magetiztion
Mag = - energy / N

plt.plot(temp, Mag[:-1], '.', label='Numerical', alpha=0.7)
plt.plot(temp_anal[:-1], np.tanh(x), label='Analytic')
plt.xlabel(r'$kT/\mu B$')
plt.ylabel(r'$M/\mu N$')
plt.xlim([-10, 10])
plt.legend()
plt.show()
```

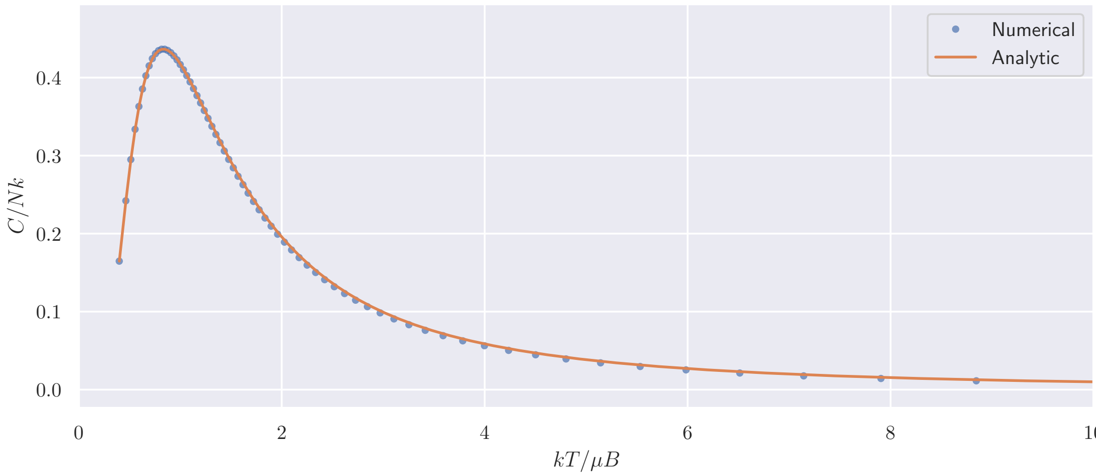


1.4 Question 4

We use (3.33) For analytical solution.

[87]:
```
# Numerical Capacity
cap = 1 / temp
# Analytic Capacity
with np.errstate(divide='ignore'):
    cap_anal = (1 / temp_anal[:-1] / np.cosh(x))**2

plt.plot(temp, Mag, '.', label='Numerical', alpha=0.7)
plt.plot(temp_anal[:-1], cap_anal, label='Analytic')
plt.xlabel(r'$kT/\mu B$')
plt.ylabel(r'$C/Nk$')
plt.xlim([0, 10])
plt.legend()
plt.show()
```



1.5 Question 4

Thermodynamic identity is

$$dU = T\,dS - P\,dV$$

The enthaply is given by

$$H = U + PV$$

At constant volume the identity becomes $dU = T\,dS$, and the definition of capacity at constant volume is $C_V = \frac{\partial U}{\partial T}$

Then we substituse the reduced indentity into the later, we get

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

The differential of H is

$$dH = dU + V\,dP + P\,dV$$

Or from the identity

$$dH = T\,dS + V\,dP$$

But at constant pressure we get, $H = T\,dS$ By dimension analysis, the capacity at constant pressure is

$$C_P = \left(\frac{\partial H}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P$$