PHYS210

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Contents

Chapter 4

Vector Analusis

4.1 Theorems

Line Integral The line integral for $f(x, y)$; $x = x(t)$ & $y = y(t)$

$$\int_C f(x, y)\, ds = \int_a^b f(x(t), y(t))L\, dt$$

The integral of '$f(x, y)$' along 'C' with respect to 'x'. Or,

$$\int_C f(x, y)\, dx\, dy = \int_a^b f(x(t), y(t))x'(t)\, dt$$

where $x'(t)dt = dx$.

★ $\vec{r}(t) = (1 - t)\vec{r_1} + \vec{r_2}$

Line integral of vector fields If $\vec{F} = P\hat{\imath} + Q\hat{\jmath} + R\hat{k}$, where Q, P, and R are scalra functions of (x, y, z).

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)dt = \int_C \vec{F} \cdot \vec{T}\, ds$$
$$\Rightarrow \int_C P\, dx + Q\, dy + R\, dz$$

where $\vec{T}$ is the unit tangent vector.

Fundamental theorem for line integral

$$\int_a^b F'(x)\, dx = F(b) - F(a)$$

$$\int_C \nabla f \cdot d\vec{r} = \int_C df = \vec{r}(b) - \vec{r}(a)$$

which says that the line integral of ∇f is the <u>net</u> change in f. Note that $\oint \vec{F} \cdot d\vec{r} = 0$

Green's Theorem

$$\oint_C \vec{F} \cdot d\vec{r} = \oint_R P\, dx + Q\, dy = \int_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

let $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$ to get the area A.

In vector form

$$\oint_R \vec{F} \cdot d\vec{r} = \int_D \nabla\vec{F} \cdot \hat{k}\, dA$$

As an special case of Stoke's theorem The integral of the tangential component of $\vec{F}$ along C is the the double integral of the vertical component of curl$\vec{F}$ over the region D enclosed by C. Also we have

$$\oint_R \vec{F} \cdot \hat{n}\, ds = \int_D \nabla \cdot \vec{F}\, dA$$

The line integral of the normal component of $\vec{F}$ along C is the double integral of the divergence of $\vec{F}$ over region D enclosed by C.

★ tangent $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$

★ normal $\hat{n}ds = dy\hat{\imath} - dx\hat{\jmath}$

Stoke's Theorem

$$\oint_C \vec{F} \cdot d\vec{r} = \int_S \nabla \times \vec{F} \cdot d\vec{S}$$

The line integral around the boundry curve of S of the tangential component of $\vec{F}$ is the the surface integral over S of the normal component of curl$\vec{F}$.

Divergence Theoprem

$$\oint_A \vec{F} \cdot d\vec{s} = \int_E \nabla \cdot \vec{F} dV$$

The flux of F across the boundry surface of E is the triple integral of the divergence of F over E.