PHYS213 Paper

Alfaifi, Ammar

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Abstract

Up to now, we only considered, in prevoius chapters, the quantum mechanics in one dimensional space. We need to extend the quantumn idea in more degree of freedom, i.e., for 3D space, as well as the centeral force for atoms. Further, we aim to distinguish the *sharp* observables.

1 PARTICLE IN A THREE-DIMENSIONAL BOX

We consider a Cubic box whose length L. Inside this box a free particle collide with *smooth* elastically with box' walls, presrving the magnitude of each momentum component, in addition to the total energy. This means that the particle occupies the the region $0 < x, y, z < L$.

Since this is a three dimensional coordinate, the wavefunction Ψ is a function of the position vector $\vec{r} = (x, y, z)$. The probability density $|\Psi(\vec{r})|^2$ is a *probability per volume*. Thus, the **Schrödinger's equation** is given by

$$-\frac{\hbar}{2m}\nabla^2\Psi + U(\vec{r}) = i\hbar\frac{\partial\Psi}{\partial t} \tag{1}$$

2 CENTERAL FORCES AND ANGULAR MOMENTUM

In contrast of the Cartesain coordinates, we turn our attention to the spherical coordinates, since it is more convenient while dealing with **centeral forces**. In atoms situation, the centeral force is the Columb attractive force originating from the nucleus, affecting on electrons. Now the vector position becomes $\vec{r} = (r, \theta, \phi)$, as in Figure 1. It is important to note that in orbiting electron, its angular momentum $\vec{L}$ is only *shart* along a chosen axis while the others components being "fuzzy". Thus, **it is impossible to specify simultaneously any two components of angular momentum**, so we take L_z as the sharp component.

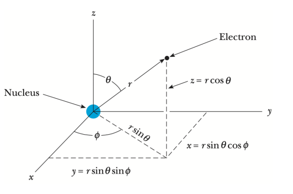

Figure 1: Spherical coordinates for an electron.

Using separation of variables so that the spatial wavefunction becomes $\psi(\vec{r}) = \psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$ so we can find an easy solution for the wavefunction. Then we can find that the sharp observable $|\vec{L}|$, L_z, and E for centeral force. We have

$$\begin{aligned} |\vec{L}| &= \sqrt{\ell(\ell+1)}\hbar \qquad & \ell &= 0, 1, \ldots, (n-1) \\ L_z &= m_\ell \hbar \qquad & m_\ell &= 0, \pm 1, \ldots, \pm \ell \end{aligned}$$

3 SPACE QUANTIZATION

Last time we know that the orentaion of the angular momentum vector $\vec{L}$ is specified by the numbers ℓ and m_ℓ. The fact that the direction of $\vec{L}$ is quantized with respect to an axis is called **space quantization**. This naming is because of **this property does not originate from the law of force but derives from the space structure itself**.

4 ATOMIC HYDROGEN

Now we want to consider the easiest real atom, that is hydrogen. Using the Columb's law to find the radial potential and the wavefunction for centeral force, we can conclude the following result for **hydrogen-like ions**

$$\Psi(r, \theta, \phi, t) = R(r)\, Y_\ell^{m_\ell}(\theta, \phi)\, e^{-i\omega t} \tag{2}$$

The number n is called the **priciple quantum number**. It specifies the **shell**, and for each number $n = 1, 2, 3, \ldots$ has a corresponding name K, L, M, $\ldots$ so on. Likewise, the **orbital quantum number** $\ell = 0, 1, 2, \ldots$ represents the subshells, named with s, p, d, f, $\ldots$ so on. In **optical transations**, photons carry off corresponding energy difference bewtween the states. But not all transations can occur. To conserve the *total* angular momentum, the angular momentum of the electron must differ by exactly one unit, that is

$$\Delta\ell = \pm 1 \quad \textit{and} \quad \Delta m_\ell = 0, \pm 1$$