Coherent States

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Comparison

Stationary States vs. Coherent States

Stationary States

Stationary States

Stationary states are eigenstates of the Hamiltonian $\hat{H}$

For quantum harmonic oscillator the Hamiltonian is

$$\hat{H} = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

Coherent States

Coherent States

Mathematically, a coherent state is defined to be the (unique) eigenstates of the annihilation operator $\hat{a}$ with corresponding eigenvalue α.

What are Coherent States?

Coherent states are defined as eigenstates of the annihilation operator. Specifically, they are defined as:

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle \tag{1}$$

since $\hat{a}$ is not hermitian, α is a complex number and $|\alpha\rangle$ is the coherent state.

Derive Coherent States

We write the coherent states in the basis of the Hamiltonian.

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n(\alpha)|n\rangle; \qquad c_n(\alpha) = \langle n|\alpha\rangle$$

$$\begin{aligned}\hat{a}\,|\alpha\rangle &= \sum_{n=0}^{\infty} c_n(\alpha)\,\hat{a}\,|n\rangle = \sum_{n=1}^{\infty} c_n(\alpha)\sqrt{n}\;|n-1\rangle \\ &= \sum_{n=0}^{\infty} c_{n+1}(\alpha)\sqrt{n+1}\;|n\rangle\end{aligned}$$

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle \Rightarrow \sum_{n=0}^{\infty} c_{n+1}(\alpha)\sqrt{n+1}\;|n\rangle = \alpha\sum_{n=0}^{\infty} c_n(\alpha)\;|n\rangle$$

Derive Coherent States

$$c_n(\alpha) = \frac{\alpha^n}{\sqrt{n!}}\, c_0$$

The coherent states are

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} c_0(\alpha)\ |n\rangle$$

Normalization

$$1 = \langle\alpha|\alpha\rangle = |c_0(\alpha)|^2 \overbrace{\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!}}^{e^{|\alpha|^2}}$$

$$|c_0(\alpha)|^2 = e^{-|\alpha|^2} \Rightarrow c_0(\alpha) = e^{-|\alpha|^2/2}$$

Derive Coherent States

Finally, the coherent state in the energy basis is

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \, |n\rangle$$

Properties of Coherent States

Coherent states have several important properties which make them useful for many applications.

- Coherent states are minimum-uncertainty states: they have the minimum possible uncertainty in both position and momentum.
- Coherent states have a Gaussian wavefunction, which makes them easy to work with.
- Coherent states are stable: they remain unchanged over time.
- Coherent states can be used to approximate classical behavior.

Applications of Coherent States

- coherent states are used in quantum optics to describe light.
- coherent states can be used to simulate the behavior of classical systems.
- coherent states can be used to encode quantum information.
- coherent states are used in quantum computing to implement quantum algorithms.

Time Evolution

Time Evolution

$$|\psi(0)\rangle = |\alpha_0\rangle = e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} \, |n\rangle$$

Harmonic oscillator

$\hat{H}$ is time independent: conservative system.

$$|\psi(t)\rangle = e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} \, e^{-iE_n t\hbar} \, |n\rangle$$

Time Evolution

$$|\psi(t)\rangle \, e^{-i\omega t/2} e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha_0 e^{-i\omega t})^n}{\sqrt{n!}} \, |n\rangle$$

let $\alpha \equiv \alpha_0 e^{-i\omega t}$

$$|\psi(0)\rangle = |\alpha_0\rangle \rightarrow |\psi(t)\rangle = e^{-i\omega t/2} \, |\alpha\rangle \, ;$$

Conclusion

A coherent state stays coherent at all times.

Creation Operator

About Creation Opreator

Note

Creation operator has no eigenstates.

$$\hat{a}^\dagger |\lambda\rangle = \lambda |\lambda\rangle$$

$$\sum_{n=0}^{\infty} c_n \sqrt{n+1}\, |n+1\rangle = \lambda \sum_{n=0}^{\infty} c_n\, |n\rangle$$

$$\begin{aligned}
|0\rangle &: \quad 0 = \lambda c_0 \Rightarrow c_0 = 0 \\
|1\rangle &: \quad c_0 = \lambda c_0 \Rightarrow c_1 = 0 \\
&\vdots \\
|n\rangle &: \quad c_{n-1}\sqrt{n} = \lambda c_n \Rightarrow c_n = 0
\end{aligned}$$

Conclusion

In conclusion, coherent states are special types of quantum states which have several useful properties and applications. They are widely used in quantum physics, quantum optics, and quantum information.

Thank you for listening!