MATH208

Alfaifi, Ammar

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Chapter 1

First-Order Differential Equations

1.1 Differential Equations & Mathmatical Models

- An equation relating an unknown function and one or more of its derivatives is called a differential equation, such $\frac{dx}{dt} = x^2 + t^2$.
- The order of a differential equation is the order of the highest derivative that appears in it.
- The most general form of an **nth-order** differential equation with independent variable 5 and unknown function or dependent variable $y = y(x)$ is

$$F(x, y', y'', \ldots, y^{(n)}) = 0 \tag{1.1}$$

- The continuous function $u = u(x)$ is a solution of the differential equation in (1.1) on the interval I provided that the derivatives $u', u'', \ldots, u^{(n)}$ exist on I and

$$F(x, u', u'', \ldots, u^{(n)}) = 0 \tag{1.2}$$

  for all x in I.
- Then $u = u(x)$ **satisfies** the differential equation in (1.1) on I.
- An nth-order differential equation ordinarily has an n-parameter family of solutions.
- An **ordinary** *differential equations* where the unknown function (dependent variable) depends on only a single independent variable.
- We concentrate on first-order differential equations with initial condition to **solve**

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0 \tag{1.3}$$

  meameans to find a differentiable function $y = y(x)$ that satisfies both conditions in Eq.(1.3) on some interval containing x_0.

1.2 Integrals as General & Particular Solutions

- As an especial case, if f in Eq.(1.3) does not actually involve the dependent variable y, so

$$\boxed{\frac{dy}{dx} = f(x)} \tag{1.4}$$

  then we need only to take the integral of Eq.(1.4), so

$$\boxed{y(x) = \int f(x)\, dx + C} \tag{1.5}$$

- Eq.(1.5) is a **general solution** of Eq.(1.4).
- When applying an initial condition ($y_0 = y(x_0)$), we get a **particular solution**.
- This idea extends to second-order differential equations of the special form:

$$\frac{d^2y}{dx^2} = g(x) \tag{1.6}$$

in which the function g on the right-hand side involves neither the dependent variable y nor its derivative dy/dx.

1.3 Separable Equations

- In general, a first-order differential equation is called *separable* provided that $f(x, y)$ can be written as the product of a function of x and a function of y:

$$\frac{dy}{dx} = f(x, y) = g(x)k(y) = \frac{g(x)}{h(y)} \tag{1.7}$$

where $h(y) = 1/k(y)$

- So we can write informally

$$h(y)dy = g(x)dx \tag{1.8}$$

- Which we can integrate both side as

$$\int h(y)\, dy = \int g(x)\, dx \tag{1.9}$$

provided that both antiderivatives of $h(y)$ and $g(x)$ can be found.

Chapter 2

Linear Systems and Matrices

2.1 Introduction to Linear Systems

Consider the following equation

$$ax + by + cz = d \qquad \text{or } ax + by = c$$

each one is called a **linear equation**; because the **variables** are power of one.

• A **system** of linear equations (or **linear system**) is a finite collections of linear equations involving ceratin variables (*unknowns*). for examole,

$$a_1x + b_1y = c_1a_2x + b_2y = c_2$$

which is a linear system in two unkowns. A **solution** is a pair of (x, y) that satisfies *both* equations *simultaneously*.

- A **consistent** systems have at least one solution.
- A **inconsistent** systems have no solution.

For a linear system, we havr three possiblities:

1. exactly on solution.
2. no solution.
3. infinitely many solutions.

The Method of Eliminations

1. Multiply one equation with nonzero number.
2. Interchange two equations.
3. Add a constant multiple of one equation to another equation.

A Differential Equation Consider this equation

$$y(x) = Ae^{nx} + Be^{-nx} \tag{2.1}$$

after differentiation we get

$$y'(x) = nAe^{nx} - nBe^{-nx}$$

also

$$y''(x) = n^2Ae^{nx} + n^2Be^{-nx} = n^2y(x)$$

Thus Eq.(2.1) is a solution to the differential equation

$$y'' - n^2y = 0 \tag{2.2}$$

to find the constant A and B we need to solve a linear systme after applying the initial condition.

2.2 Matrices and Gaussian Elimination

A general system of m linear equation in n variables $x_1, x_2, \dots, x_n$ written in form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 +, \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 +, \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 +, \dots + a_{mn}x_n &= b_m \end{aligned} \tag{2.3}$$

The **coefficient matrix** of this linea system is written in $m \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

For large number of entries we can denote a_{ij} as $a(i, j)$. The **constant matrix** is

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

This is called a **vector** (**column**) matrix. Combining both matrices we get a $m \times (n+1)$ **augmented matrix**

$$[A\, b] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{bmatrix}$$

Elemnetary Row Operations As we did before for solving a linear sustem, we can generalize that for augmented matrices as

Elemnetary Row Operations (2.4)

1. Multiply any row by a nonzero number.
2. Interchange two rows.
3. Add a constant multiple of one row to another row.

Row-Equivalent Matrices (2.5)
Two matrices are called **tow eauivalent** if one can be obtained from the pther by a finite sequence of elementary row opearations

THEOREM1: Equivalent Systems and Equivalent Matrices (2.6)

If the augmneted coefficient matrices of two systems are row equivalent, then the two systems have the same solution set.

Echelon Matrices Let us see how sould the augmented matrix look after some elementry row operations,

DEFINITION: Echelon Matrix

The matrix E is called an **echelon matrix** provided it has the following two properties: (2.7)

1. Every row that consists of entirely zeros (if any) liew *beneath* every row that contains a nonzero element.

2. In each row contains nonzero element, the *first* nonzero element lies strictly to the right of the first nonzero element in the preceding row (if there is a preseding row).