

PHYS210

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## Chapter 4

# Vector Analysis

### 4.1 Theorems

**Line Integral** The line integral for  $f(x, y)$ ;  $x = x(t)$  &  $y = y(t)$

$$\int_C f(x, y) ds = \int_a^b f(x(t), y(t)) L dt$$

The integral of ' $f(x, y)$ ' along ' $C$ ' with respect to ' $x$ '. Or,

$$\int_C f(x, y) dx dy = \int_a^b f(x(t), y(t)) x'(t) dt$$

where  $x'(t)dt = dx$ .

★  $\vec{r}(t) = (1 - t)\vec{r}_1 + \vec{r}_2$

**Line integral of vector fields** If  $\vec{F} = P\hat{i} + Q\hat{j} + R\hat{k}$ , where  $Q$ ,  $P$ , and  $R$  are scalar functions of  $(x, y, z)$ .

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt = \int_C \vec{F} \cdot \vec{T} ds \\ &\Rightarrow \int_C P dx + Q dy + R dz \end{aligned}$$

where  $\vec{T}$  is the unit tangent vector.

**Fundamental theorem for line integral**

$$\begin{aligned} \int_a^b F'(x) dx &= F(b) - F(a) \\ \int_C \nabla f \cdot d\vec{r} &= \int_C df = f(b) - f(a) \end{aligned}$$

which says that the line integral of  $\nabla f$  is the net change in  $f$ . Note that  $\oint \vec{F} \cdot d\vec{r} = 0$

**Green's Theorem**

$$\oint_C \vec{F} \cdot d\vec{r} = \oint_R P dx + Q dy = \int_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

let  $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$  to get the area  $A$ .  
In vector form

$$\oint_R \vec{F} \cdot d\vec{r} = \int_D \nabla \vec{F} \cdot \hat{k} dA$$

As an special case of Stoke's theorem The integral of the tangential component of  $\vec{F}$  along  $C$  is the the double integral of the vertical component of  $\text{curl}\vec{F}$  over the region  $D$  enclosed by  $C$ . Also we have

$$\oint_R \vec{F} \cdot \hat{n} ds = \int_D \nabla \cdot \vec{F} dA$$

The line integral of the normal component of  $\vec{F}$  along  $C$  is the double integral of the divergence of  $\vec{F}$  over region  $D$  enclosed by  $C$ .

★ tangent  $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

★ normal  $\hat{n}ds = dy\hat{i} - dx\hat{j}$

### Stoke's Theorem

$$\oint_C \vec{F} \cdot d\vec{r} = \int_S \nabla \times \vec{F} \cdot d\vec{S}$$

The line integral around the boundry curve of  $S$  of the tangential component of  $\vec{F}$  is the the surface integral over  $S$  of the normal component of  $\text{curl}\vec{F}$ .

### Divergence Theoprem

$$\oint_A \vec{F} \cdot d\vec{s} = \int_E \nabla \cdot \vec{F} dV$$

The flux of  $F$  across the boundry surface of  $E$  is the triple integral of the divergence of  $F$  over  $E$ .