

# Coherent States

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## Stationary States vs. Coherent States



# Stationary States

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## Stationary States

Stationary states are eigenstates of the Hamiltonian  $\hat{H}$

For quantum harmonic oscillator the Hamiltonian is

$$\hat{H} = \hbar\omega \left( a^\dagger a + \frac{1}{2} \right)$$

$$\hat{H}|n\rangle = E_n|n\rangle$$

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

# Coherent States

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## Coherent States

Mathematically, a coherent state is defined to be the (unique) eigenstates of the annihilation operator  $\hat{a}$  with corresponding eigenvalue  $\alpha$ .

## What are Coherent States?

Coherent states are defined as eigenstates of the annihilation operator. Specifically, they are defined as:

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle \quad (1)$$

since  $\hat{a}$  is not hermitian,  $\alpha$  is a complex number and  $|\alpha\rangle$  is the coherent state.



## Derive Coherent States

We write the coherent states in the basis of the Hamiltonian.

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n(\alpha) |n\rangle; \quad c_n(\alpha) = \langle n|\alpha\rangle$$

$$\begin{aligned} \hat{a} |\alpha\rangle &= \sum_{n=0}^{\infty} c_n(\alpha) \hat{a} |n\rangle = \sum_{n=1}^{\infty} c_n(\alpha) \sqrt{n} |n-1\rangle \\ &= \sum_{n=0}^{\infty} c_{n+1}(\alpha) \sqrt{n+1} |n\rangle \end{aligned}$$

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle \Rightarrow \sum_{n=0}^{\infty} c_{n+1}(\alpha) \sqrt{n+1} |n\rangle = \alpha \sum_{n=0}^{\infty} c_n(\alpha) |n\rangle$$

## Derive Coherent States

$$c_n(\alpha) = \frac{\alpha^n}{\sqrt{n!}} c_0$$

The coherent states are

$$|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} c_0(\alpha) |n\rangle$$

Normalization

$$1 = \langle \alpha | \alpha \rangle = |c_0(\alpha)|^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \overbrace{e^{|\alpha|^2}}$$
$$|c_0(\alpha)|^2 = e^{-|\alpha|^2} \Rightarrow c_0(\alpha) = e^{-|\alpha|^2/2}$$

## Derive Coherent States

Finally, the coherent state in the energy basis is

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

## Properties of Coherent States

Coherent states have several important properties which make them useful for many applications.

- Coherent states are minimum-uncertainty states: they have the minimum possible uncertainty in both position and momentum.
- Coherent states have a Gaussian wavefunction, which makes them easy to work with.
- Coherent states are stable: they remain unchanged over time.
- Coherent states can be used to approximate classical behavior.

## Applications of Coherent States

- coherent states are used in quantum optics to describe light.
- coherent states can be used to simulate the behavior of classical systems.
- coherent states can be used to encode quantum information.
- coherent states are used in quantum computing to implement quantum algorithms.

# **Time Evolution**

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## Time Evolution

$$|\psi(0)\rangle = |\alpha_0\rangle = e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} |n\rangle$$

### Harmonic oscillator

$\hat{H}$  is time independent: conservative system.

$$|\psi(t)\rangle = e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} e^{-iE_n t \hbar} |n\rangle$$

## Time Evolution

$$|\psi(t)\rangle e^{-i\omega t/2} e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha_0 e^{-i\omega t})^n}{\sqrt{n!}} |n\rangle$$

let  $\alpha \equiv \alpha_0 e^{-i\omega t}$

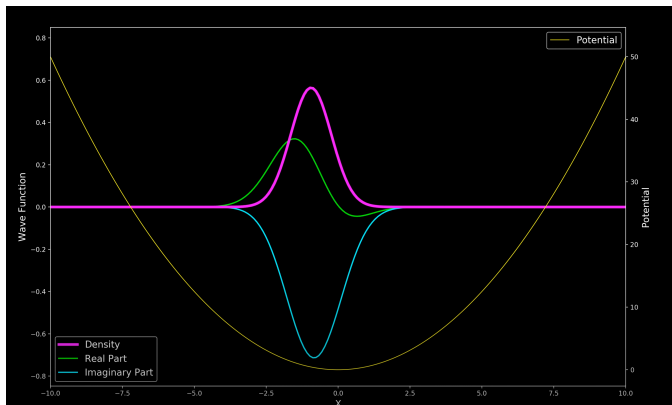
$$|\psi(0)\rangle = |\alpha_0\rangle \rightarrow |\psi(t)\rangle = e^{-i\omega t/2} |\alpha\rangle ;$$

### Conclusion

A coherent state stays coherent at all times.



# Time Evolution



<https://www.youtube.com/watch?v=60OD3fahkHU>

# Creation Operator

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## About Creation Operator

### Note

Creation operator has no eigenstates.

$$\hat{a}^\dagger |\lambda\rangle = \lambda |\lambda\rangle$$

$$\sum_{n=0}^{\infty} c_n \sqrt{n+1} |n+1\rangle = \lambda \sum_{n=0}^{\infty} c_n |n\rangle$$

$$|0\rangle : \quad 0 = \lambda c_0 \Rightarrow c_0 = 0$$

$$|1\rangle : \quad c_0 = \lambda c_0 \Rightarrow c_1 = 0$$

$$\vdots$$

$$|n\rangle : \quad c_{n-1} \sqrt{n} = \lambda c_n \Rightarrow c_n = 0$$

## Conclusion

In conclusion, coherent states are special types of quantum states which have several useful properties and applications. They are widely used in quantum physics, quantum optics, and quantum information.

**Thank you for listening!**

