HW2

October 15, 2021

1 Homework: numerical methods

PHYS300 made by: Alfaifi, Ammar 201855360

```
[ ]: # import needed packages
import numpy as np

# graph settings
import matplotlib.pyplot as plt
from matplotlib_inline import backend_inline
plt.rcParams['figure.figsize'] = [15, 4]
backend_inline.set_matplotlib_formats('png', 'pdf')
```

2 Stating the funtion

Consider Fourier Series expansion of the following function

$$F(t) = \begin{cases} -1, & -\pi/\omega \leq t \leq 0 \\ +1, & 0 \leq t \leq \pi/\omega \end{cases}$$

take, $\omega = 1 rad/s$

3 Fourier series

For a periodic function $F(t)$ with a period of $T = 2\pi/\omega$, meaning $F(t+T) = (t)$. We can exapnd it in Fourier series as

$$F(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}[a_n cos(n\omega t) + b_n sin(n\omega t)]$$

but since $F(t)$ is antisymetric (odd) about O we can represent $F(t)$ with only sine terms ,i.e., all a_n's vanish. We need to find only b_n

$$b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega t)\,dt$$

4 Expanding

We have the expansion as

$$F(t) = \sum_{n=1}^{\infty} b_n sin(n\omega t)$$

substituting $\omega = 1rad/s$, and $T = 2\pi/\omega$,

$$F(t) = \sum_{n=1}^{\infty} b_n sin(nt)$$

and

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(t) \sin(nt)\, dt$$

Doing the integral of b_n We need to seprate the integral since there are two difetent functions within the integration interval, so

$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} F(t) \sin(nt)\, dt + \int_{0}^{\pi} F(t) \sin(nt)\, dt \right]$$

Now substituting $F(t)$,

$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^{0} -\sin(nt)\, dt + \int_{0}^{\pi} \sin(nt)\, dt \right]$$

After integrating

$$b_n = \frac{1}{\pi} \left[\frac{\cos(nt)}{n} \bigg|_{-\pi}^{0} + \frac{-\cos(nt)}{n} \bigg|_{0}^{-\pi} \right]$$

Calculating the limits

$$b_n = \frac{2}{\pi n} \left[1 - \cos(n\pi) \right]$$

We notice that for $\cos(n\pi)$

$$cos(n\pi) = \begin{cases} -1, & \text{for odd } n \\ 1, & \text{for even } n \text{ and } 0 \end{cases}$$

then we can rewrite it as $(-1)^n$, thus

$$b_n = \frac{2}{\pi n} \left[1 - (-1)^n \right]$$

Thus $F(t)$ can be written as

$$F(t) = \sum_{n=1}^{\infty} \frac{2}{\pi n} \left[1 - (-1)^n \right] sin(nt)$$

Notice that for all even n's, all terms are zero so,

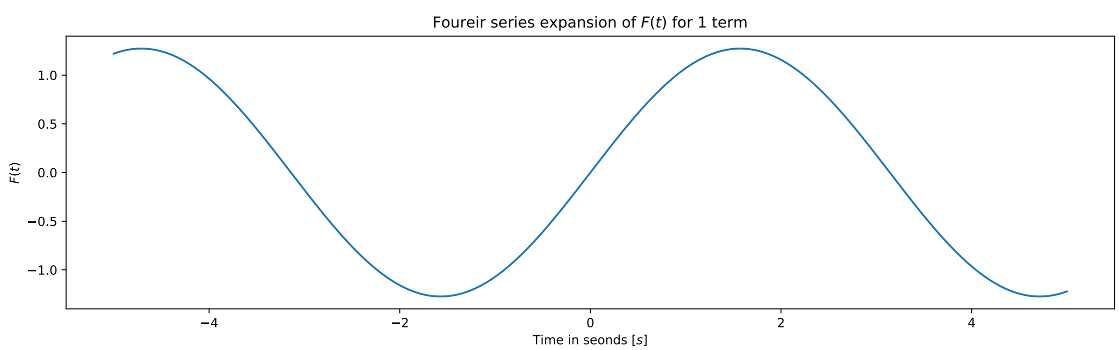
$$\therefore F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin((2n-1)t)}{2n-1}$$

5 Writing the program

We define an approximation of $F(t)$ of N terms as,

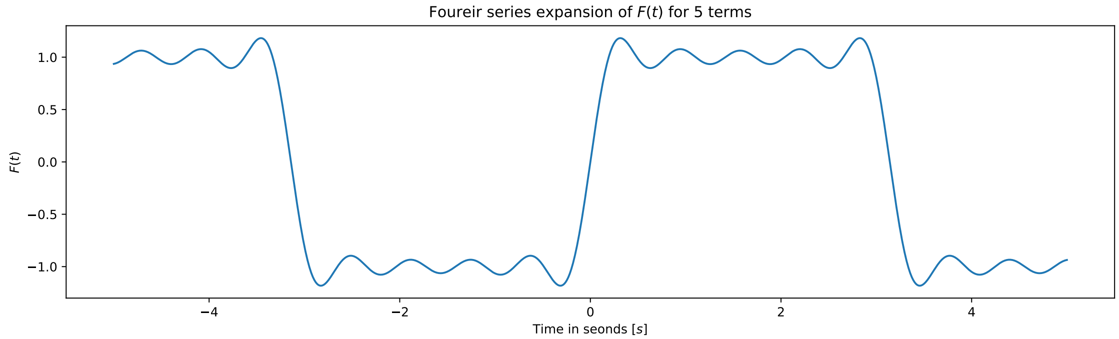
$$F(t) \approx \frac{4}{\pi} \sum_{n=1}^{N} \frac{\sin((2n-1)t)}{2n-1}$$

[ ]:
```
# Define the F(t)
def F(t, N):
    result = 0
    for n in np.arange(1, N+1):
        result += np.sin((2*n-1) * t) / (2*n-1)
    return result * 4 / np.pi

# define time spacing
epsilon = 1e-4
# Define time interval
t = np.arange(-5, 5, epsilon)
# number of terms
N = 1

plt.plot(t, F(t, N))
plt.title(f"Foureir series expansion of $F(t)$ for {N} term")
plt.xlabel('Time in seonds [$s$]')
plt.ylabel('$F(t)$')

plt.show()
```

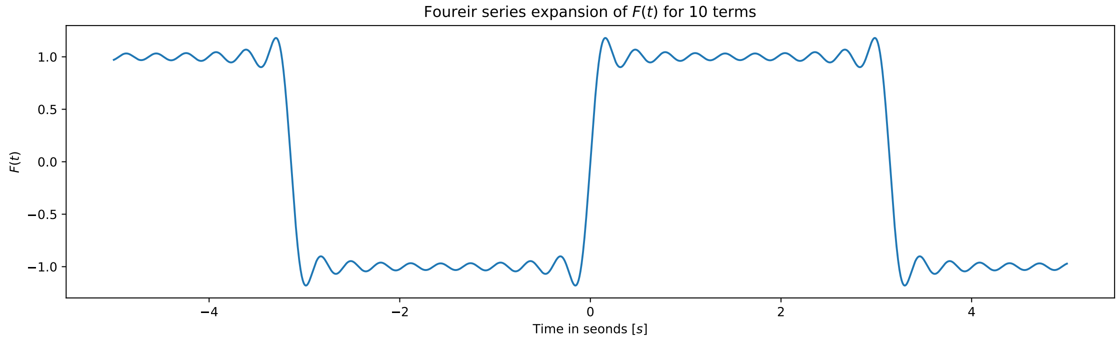


[ ]:
```
N = 5

plt.plot(t, F(t, N))
plt.title(f"Foureir series expansion of $F(t)$ for {N} terms")
plt.xlabel('Time in seonds [$s$]')
plt.ylabel('$F(t)$')
```

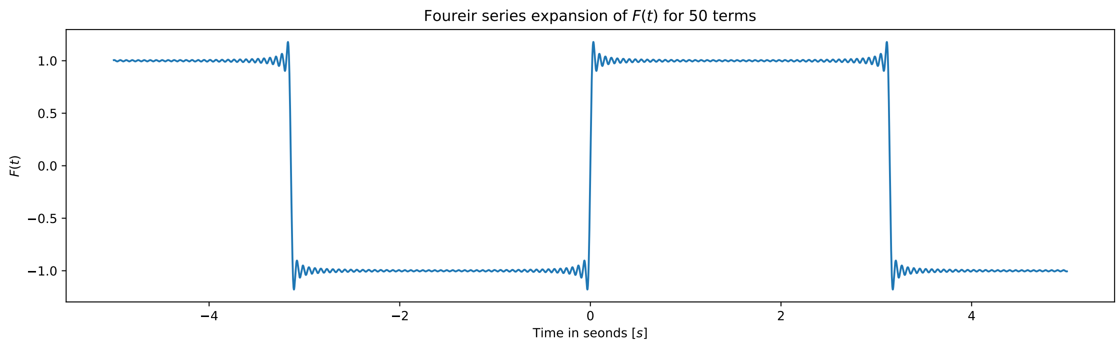
```
plt.show()
```



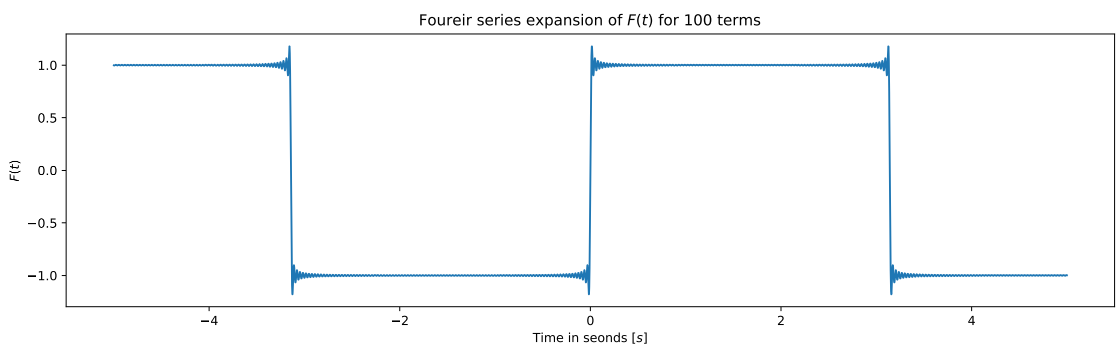
[ ]:
```
N = 10

plt.plot(t, F(t, N))
plt.title(f"Foureir series expansion of $F(t)$ for {N} terms")
plt.xlabel('Time in seonds [$s$]')
plt.ylabel('$F(t)$')

plt.show()
```



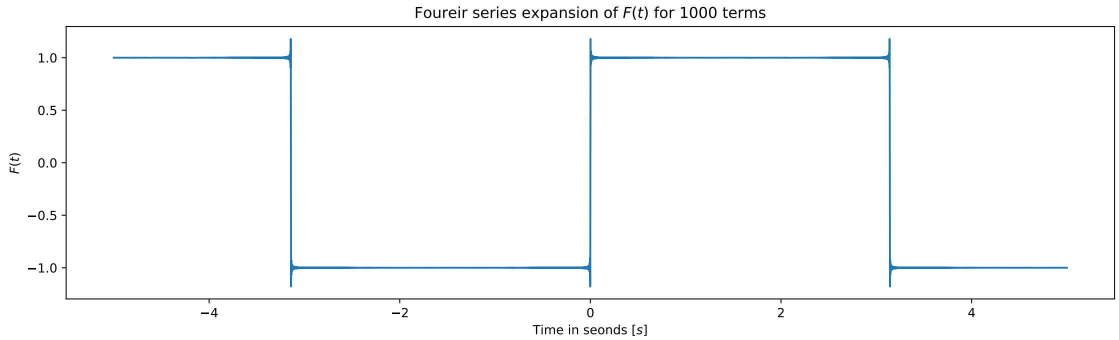
[ ]:
```
N = 50

plt.plot(t, F(t, N))
plt.title(f"Foureir series expansion of $F(t)$ for {N} terms")
plt.xlabel('Time in seonds [$s$]')
plt.ylabel('$F(t)$')

plt.show()
```



[ ]:
```
N = 100

plt.plot(t, F(t, N))
plt.title(f"Foureir series expansion of $F(t)$ for {N} terms")
plt.xlabel('Time in seonds [$s$]')
plt.ylabel('$F(t)$')

plt.show()
```



[ ]:
```
N = 1000

plt.plot(t, F(t, N))
plt.title(f"Foureir series expansion of $F(t)$ for {N} terms")
plt.xlabel('Time in seonds [$s$]')
plt.ylabel('$F(t)$')

plt.show()
```



[ ]:
```
print('Percentage error of 50 terms:', round(np.abs(1 - F(1, 50)) * 100, 2))
print('Percentage error of 1000 terms:', round(np.abs(1 - F(1, 1000)) * 100, 2))
```

5.1 About number of terms needed, N

We see that the as the value of N the the series expainsion is getting more and more accurate to the original $F(t)$. At $N = 1000$ the convergence is at its best, with error of 0.01%. However, I believe that at $N = 50$, i.e., for 50 terms approximation is very stisfiygn for wide application, with error of 0.65%.For high accuaracy applicationa, about 1000 terms is very recommended.

5.2 Weird jump

We notice at the ends of turning point of the graph, there are rabid overshots. This is called **Gibbs effect**. Furthermore, it does not seem it disappears for more and more terms.