PHYS210 - HW2

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This is a solution for the problem 2-22. We want to find the equation of motion of a charged particle in an electromagnetic field using **Lorentz equation**. If the electric field vector $\vec{E}$ and the magnetic field vector $\vec{B}$, the force on a particle of mass m that carries a charge q and has a velocity $\vec{v}$ is given

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{1}$$

where we assume $v \ll c$.

(a) If there is no electeic field and the particle enters the the magnetic field in direction perpendicular to the magnetic field. The particle follows a uniform circular motion with acceleration of $a_c = m\frac{v^2}{r}$ where r is the radius of the path of the circular motion. Thus, by Newton's second law we have

$$F = ma_c \Rightarrow qvBsin(90°) = m\frac{v^2}{r}$$

solving for r

$$r = \frac{mv}{qB} = \frac{v}{\omega_c}$$

where $\omega_c \equiv qB/m$, is the *cyclotron frequency.*

(b) Choose the z-axis to lie in the direction of $\vec{B}$ and let the plane containing $\vec{E}$ and $\vec{B}$ be the yz-plane. Thus,

$$\vec{B} = B\hat{z}, \qquad \vec{E} = E_y\hat{y} + E_z\hat{z}$$

Show that the z component of the motion is given by

$$z(t) = z_0 + \dot{z}_0 t + \frac{qE_z}{2m}t^2$$

where $z(0) \equiv z_0$, and $\dot{z}(0) \equiv \dot{z}_0$.

Deviding both sides Eq.1, by q, we get

$$\frac{\vec{F}}{q} = (E_y\hat{y} + E_z\hat{z}) + (v_x\hat{x} + v_y\hat{y} + v_z\hat{z}) \times B\hat{z} \tag{2}$$