Exercise 1.4

Solve the following equations.

(1) $2x+5=\sqrt{7x+16}$

Solution:

$$2x+5=\sqrt{7x+16} \qquad ......(i)$$

Squaring both sides, we get

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$

$$4x^2+20x+25=7x+16$$

$$4x^2+20x+25-7x-16=0$$

$$4x^2+20x-7x+25-16=0$$

$$4x^2+13x+9=0$$

$$4x^2+9x+4x+9=0$$

$$x(4x+9)+1(4x+9)=0$$

$$(x+1)(4x+9)=0$$

Either $x+1=0$ *or* $4x+9=0$

$x=-1$ $\qquad$ $4x=-9$

$$x=-\frac{9}{4}$$

Check :

Put $\quad x=-1$ in eq.(i), *we get*

$x=\frac{12}{6}$ $\qquad$ $x=\frac{164}{6}$

$x=2$ $\qquad$ $x=\frac{82}{3}$

Check :

Put $x=2$ in eq.(i), *we get*

$\sqrt{11-2}-\sqrt{6-2}=\sqrt{27-2}$

$\sqrt{9}-\sqrt{4}=\sqrt{25}$

$3-2=5$

$1=5$ $\qquad$ (which is not true)

Put $x=\frac{82}{3}$ *in eq* (i), *we get*

$\sqrt{11-\frac{82}{3}}-\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$

$\sqrt{-\frac{49}{3}}-\sqrt{-\frac{64}{3}}=\sqrt{-\frac{1}{3}}$

$\frac{7i}{\sqrt{3}}-\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$

$\frac{-i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$ $\qquad$ (which is not true)

Thus, solution set = { }

(8) $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution:

$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ $\qquad$ ......(i)

Squaring both sides, we get

$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$

$(4a+x)+(a-x)-2\sqrt{(4a+x)(a-x)}=a$

$4a+x+a-x-2\sqrt{4a^2-3ax-x^2}=a$

$5a-2\sqrt{4a^2-3ax-x^2}=a$

$-2\sqrt{4a^2-3ax-x^2}=a-5a$

$$-2\sqrt{4a^2-3ax-x^2}=-4a$$
$$\Rightarrow \sqrt{4a^2-3ax-x^2}=2a$$

Squaring both sides, we get

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$
$$4a^2-3ax-x^2=4a^2$$
$$4a^2-4a^2-3ax-x^2=0$$
$$-3ax-x^2=0$$
$$-x(x+3a)=0$$
$$x(x+3a)=0$$

$Either\ \ x=0\ \ or\ \ x+3a=0$

$x=-3a$

Check :

$Put\ \ \ \ x=0$ in eq.(i), $we\ get$

$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$
$$\sqrt{4a}-\sqrt{a}=\sqrt{a}$$
$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$
$$\sqrt{a}=\sqrt{a}\qquad\text{(which is true)}$$

$Put\ \ \ \ x=-3a$ in eq.(i), $we\ get$

$$\sqrt{4a+(-3a)}-\sqrt{a-(-3a)}=\sqrt{a}$$
$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$
$$\sqrt{a}-\sqrt{4a}=\sqrt{a}$$
$$\sqrt{a}-2\sqrt{a}=\sqrt{a}$$
$$-\sqrt{a}=\sqrt{a}\qquad\text{(which is not true)}$$

Thus, solution set = $\{0\}$

(9) $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution:

$$\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1 \quad ......(i)$$

Let $x^2+x=y$

So eq.(i) becomes

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides, we get

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=1$$

$$(y+1)+(y-1)-2\sqrt{(y+1)(y-1)}=1$$

$$y+1+y-1-2\sqrt{y^2-1}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$$-2\sqrt{y^2-1}=1-2y$$

Squaring both sides, we get

$$\left(-2\sqrt{y^2-1}\right)^2=(1-2y)^2$$

$$4\left(y^2-1\right)=1-4y+4y^2$$

$$4y^2-4=1-4y+4y^2$$

$$4y^2-4-1+4y-4y^2=0$$

$$4y-5=0$$

$$y=\frac{5}{4}$$

Put $y=\frac{5}{4}$ *in* $x^2+x=y$, *we get*

$$x^2+x=\frac{5}{4}$$

$$4x^2+4x=5$$

$$4x^2+4x-5=0$$

Here a = 4, b = 4, c = -5

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm4\sqrt{6}}{8}=\frac{4\left(-1\pm\sqrt{6}\right)}{8}=\frac{-1\pm\sqrt{6}}{2}$$

Thus, solution set = $\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

(10) $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution:

$\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3 \qquad ......(i)$

Let $x^2+3x=y$

So eq.(i) becomes

$\sqrt{y+8}+\sqrt{y+2}=3$

Squaring both sides, we get

$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$

$(y+8)+(y+2)+2\sqrt{(y+8)(y+2)}=9$

$y+8+y+2+2\sqrt{y^2+10y+16}=9$

$2y+10+2\sqrt{y^2+10y+16}=9$

$2\sqrt{y^2+10y+16}=9-2y-10$

$2\sqrt{y^2+10y+16}=-2y-1$

$2\sqrt{y^2+10y+16}=-(2y+1)$

Squaring both sides, we get

$$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$$

$$4\left(y^2+10y+16\right)=4y^2+4y+1$$

$$4y^2+40y+64=4y^2+4y+1$$

$$4y^2-4y^2+40y-4y+64-1=0$$

$$36y=-63$$

$$y=-\frac{63}{36}$$

Put $y=-\frac{63}{36}$ *in* $x^2+3x=y$, *we get*

$$x^2+3x=-\frac{63}{36}$$

$$\Rightarrow 36x^2+108x=-63$$

$$36x^2+108x+63=0$$

Here a=36, b=108, c=63

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$x=\frac{-108\pm\sqrt{11664+9072}}{72}$$

$$x=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{-108\pm 36\sqrt{2}}{72}$$

$$x=\frac{36\left(-3\pm\sqrt{2}\right)}{72}$$

$$x=\frac{-3\pm\sqrt{2}}{2}$$

Thus, Solution set = $\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

(11) $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution:

$\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5 \qquad ......(i)$

Let $x^2+3x=y$

So eq.(i) becomes

$\sqrt{y+9}+\sqrt{y+4}=5$

Squaring both sides, we get

$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=(5)^2$

$(y+9)+(y+4)+2\sqrt{(y+9)(y+4)}=25$

$y+9+y+4+2\sqrt{y^2+13y+36}=25$

$2y+13+2\sqrt{y^2+13y+36}=25$

$2\sqrt{y^2+13y+36}=25-2y-13$

$2\sqrt{y^2+13y+36}=-2y+12$

$2\sqrt{y^2+13y+36}=-2(y-6)$

$\Rightarrow \sqrt{y^2+13y+36}=-(y-6)$

Squaring both sides, we get

$\left(\sqrt{y^2+13y+36}\right)^2=\left[-(y-6)\right]^2$

$y^2+13y+36=y^2-12y+36$

$y^2-y^2+13y+12y+36-36=0$

$25y=0$

$\Rightarrow y=0$

Put $y=0$ *in* $x^2+3x=y$, *we get*

$x^2+3x=y$

$x^2+3x=0$

$x(x+3)=0$

Either $x=0$ *or* $x+3=0$

$$x = -3$$

Thus, Solution set $= \{-3, 0\}$

$2(-1)+5=\sqrt{7(-1)+16} \quad \Rightarrow -2+5=\sqrt{-7+16}$

$3=\sqrt{9} \Rightarrow 3=3$ (which is true)

Put $x=-\frac{9}{4}$ *in eq* (i), *we get*

$2\left(-\frac{9}{4}\right)+5=\sqrt{7\left(-\frac{9}{4}\right)+16}$

$-\frac{9}{2}+5=\sqrt{-\frac{63}{4}+16}$

$\frac{1}{2}=\sqrt{\frac{1}{4}}$

$\frac{1}{2}=\frac{1}{2}$ (which is true)

Thus, solution set $=\left\{-1,-\frac{9}{4}\right\}$

(2) $\sqrt{x+3}=3x-1$

Solution:

$\sqrt{x+3}=3x-1 \;......(i)$

Squaring both sides, we get

$\left(\sqrt{x+3}\right)^2=(3x-1)^2$

$x+3=9x^2-6x+1$

$9x^2-6x+1-x-3=0$

$9x^2-7x-2=0$

$9x^2-9x+2x-2=0$

$9x(x-1)+2(x-1)=0$

$(9x+2)(x-1)=0$

Either $9x+2=0$ *or* $x-1=0$

$9x=-2 \qquad x=1$

$x=-\frac{2}{9} \qquad x=1$

Check :

Put $x = -\frac{2}{9}$ in eq.(i), *we get*

$$\sqrt{-\frac{2}{9}+3} = 3\left(-\frac{2}{9}\right)-1$$

$$\sqrt{\frac{25}{9}} = -\frac{2}{3}-1$$

$$\sqrt{\frac{25}{9}} = -\frac{5}{3}$$

$$\frac{5}{3} \neq -\frac{5}{3} \ \text{(which is not true)}$$

Put $x = 1$ *in* $eq(i)$, *we get*

$$\sqrt{1+3} = 3(1)-1$$

$$\sqrt{4} = 3-1$$

$$2 = 2 \ \text{(which is true)}$$

Thus, solution set = $\{1\}$

(3) $4x = \sqrt{13x+14}-3$

Solution:

$$4x = \sqrt{13x+14}-3 \qquad ......(i)$$

$$4x+3 = \sqrt{13x+14}$$

Squaring both sides, we get

$$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$$

$$16x^2+24x+9 = 13x+14$$

$$16x^2+24x-13x+9-14 = 0$$

$$16x^2+11x-5 = 0$$

$$16x^2+16x-5x-5 = 0$$

$$16x(x+1)-5(x+1) = 0$$

$$(16x-5)(x+1) = 0$$

$$\textit{Either } 16x-5=0 \quad \textit{or} \quad x+1=0$$
$$16x=5 \qquad x=-1$$
$$x=\frac{5}{16} \qquad x=-1$$

Check :

$$\textit{Put} \quad x=\frac{5}{16} \text{ in eq.}(i), \textit{ we get}$$

$$4\left(\frac{5}{16}\right)=\sqrt{13\left(\frac{5}{16}\right)+14}-3$$
$$\frac{5}{4}=\sqrt{\frac{65}{16}+14}-3$$
$$\frac{5}{4}=\sqrt{\frac{289}{16}}-3$$
$$\frac{5}{4}=\frac{17}{4}-3$$
$$\frac{5}{4}=\frac{5}{4} \qquad (\text{which is true})$$

$$\textit{Put } x=-1 \textit{ in eq}(i), \textit{ we get}$$
$$4(-1)=\sqrt{13(-1)+14}-3$$
$$-4=\sqrt{-13+14}-3$$
$$-4=\sqrt{1}-3$$
$$-4=1-3$$
$$-4\neq-2 \quad (\text{which is not true})$$

$$\text{Thus, solution set} = \left\{\frac{5}{16}\right\}$$

(4) $\sqrt{3x+100}-x=4$

Solution:

$$\sqrt{3x+100}-x=4 \qquad ......(i)$$
$$\sqrt{3x+100}=4+x$$

Squaring both sides, we get

$$\left(\sqrt{3x+100}\right)^2=(x+4)^2$$

$$3x+100=x^2+8x+16$$

$$x^2+8x+16-3x-100=0$$

$$x^2+5x-84=0$$

$$x^2+12x-7x-84=0$$

$$x(x+12)-7(x+12)=0$$

$$(x-7)(x+12)=0$$

$$\textit{Either } x-7=0 \quad \textit{or} \quad x+12=0$$

$$x=7 \qquad\qquad x=-12$$

Check :

Put $x=7$ in eq.(i), *we get*

$$\sqrt{3(7)+100}-7=4 \qquad \Rightarrow \sqrt{21+100}-7=4$$

$$\sqrt{121}-7=4 \qquad \Rightarrow 11-7=4$$

$$4=4 \quad (\text{which is true})$$

Put $x=-12$ *in eq*(i), *we get*

$$\sqrt{3(-12)+100}-(-12)=4 \qquad \Rightarrow \sqrt{-36+100}+12=4$$

$$\sqrt{64}+12=4 \qquad \Rightarrow 8+12=4$$

$$20\neq 4 \quad (\text{which is not true})$$

Thus, solution set = $\{7\}$

(5) $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution:

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$(x+5)+(x+21)+2\sqrt{(x+5)(x+21)}=x+60$$

$$x+5+x+21+2\sqrt{x^2+26x+105}=x+60$$

$$2x+26+2\sqrt{x^2+26x+105}=x+60$$

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60} \quad ......(i)$$

Squaring both sides, we get

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$
$$2\sqrt{x^2+26x+105}=-x+34$$
$$2\sqrt{x^2+26x+105}=-(x-34)$$

Squaring both sides, we get

$$\left(2\sqrt{x^2+26x+105}\right)^2=\left[-(x-34)\right]^2$$
$$4\left(x^2+26x+105\right)=x^2-68x+1156$$
$$4x^2+104x+420=x^2-68x+1156$$
$$4x^2-x^2+104x+68x+420-1156=0$$
$$3x^2+172x-736=0$$

Here a=3, b=172, c=–736

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$
$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$
$$x=\frac{-172\pm\sqrt{38416}}{6}$$
$$x=\frac{-172-196}{6} \quad or \quad x=\frac{-172+196}{6}$$
$$x=-\frac{368}{6} \qquad x=\frac{24}{6}$$
$$x=-\frac{184}{3} \qquad x=4$$

Check :

Put $x=-\frac{184}{3}$ in eq.(i), *we get*

$$\sqrt{-\frac{184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{-\frac{169}{3}}+\sqrt{-\frac{121}{3}}=\sqrt{-\frac{4}{3}}$$
$$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$
$$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}\ (\text{which is not true})$$

$Put\ x=4\ \ in\ eq\ (i),\ we\ get$
$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$
$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$
$$3+5=8$$
$$8=8\qquad(\text{which is true})$$

Thus, solution set = $\{4\}$

(6) $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Solution:

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}\qquad\ldots\ldots(i)$$

Squaring both sides, we get
$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$
$$(x+1)+(x-2)+2\sqrt{(x+1)(x-2)}=x+6$$
$$x+1+x-2+2\sqrt{x^2-x-2}=x+6$$
$$2x-1+2\sqrt{x^2-x-2}=x+6$$
$$2\sqrt{x^2-x-2}=-x+7$$
$$2\sqrt{x^2-x-2}=-(x-7)$$

Squaring both sides, we get
$$\left(2\sqrt{x^2-x-2}\right)^2=\left[-(x-7)\right]^2$$
$$4\left(x^2-x-2\right)=x^2-14x+49$$
$$4x^2-4x-8=x^2-14x+49$$
$$4x^2-x^2-4x+14x-8-49=0$$
$$3x^2+10x-57=0$$

Here a=3, b=10, c= -57

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-10\pm\sqrt{(10)^2-4(3)(-57)}}{2(3)}$$

$$x=\frac{-10\pm\sqrt{100+684}}{6}$$

$$x=\frac{-10\pm\sqrt{784}}{6}$$

$$x=\frac{-10\pm 28}{6}$$

$$x=\frac{-10-28}{6}\ or \qquad x=\frac{-10+28}{6}$$

$$x=\frac{-38}{6} \qquad\qquad x=\frac{18}{6}$$

$$x=-\frac{19}{3} \qquad\qquad x=3$$

Check :

Put $x=-\frac{19}{3}$ in eq.(i), *we get*

$$\sqrt{-\frac{19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{-\frac{19}{3}+6}$$

$$\sqrt{-\frac{16}{3}}+\sqrt{\frac{-25}{3}}=\sqrt{-\frac{1}{3}}$$

$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$$\frac{9i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad (\text{which is not true})$$

Put $x=3$ *in eq*(i), *we get*

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$

$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

$$2+1=3$$

$$3=3 \quad (\text{which is true})$$

Thus, solution set = $\{3\}$

(7) $\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$

Solution:

$\sqrt{11-x}-\sqrt{6-x}=\sqrt{27-x}$ ......(i)

Squaring both sides, we get

$\left(\sqrt{11-x}-\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$

$(11-x)+(6-x)-2\sqrt{(11-x)(6-x)}=27-x$

$11-x+6-x-2\sqrt{(11-x)(6-x)}=27-x$

$17-2x-2\sqrt{x^2-17x+66}=27-x$

$-2\sqrt{x^2-17x+66}=27-x-17+2x$

$-2\sqrt{x^2-17x+66}=10+x$

Squaring both sides, we get

$\left(-2\sqrt{x^2-17x+66}\right)^2=(10+x)^2$

$4\left(x^2-17x+66\right)=100+20x+x^2$

$4x^2-68x+264=x^2+20x+100$

$4x^2-x^2-68x-20x+264-100=0$

$3x^2-88x+164=0$

Here a = 3, b = -88, c = 164

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$

$x=\dfrac{88\pm\sqrt{7744-1968}}{6}$

$x=\dfrac{88\pm\sqrt{5776}}{6}$

$x=\dfrac{88\pm 76}{6}$

$x=\dfrac{88-76}{6}$ *or* $x=\dfrac{88+76}{6}$