Exercise 1.3

Q1. Solve the following equations.

(1) $2x^4 - 11x^2 + 5 = 0$

Solution:

$2x^4 - 11x^2 + 5 = 0 \qquad \ldots\ldots(i)$

Let $x^2 = y$ *then* $x^4 = y^2$

So eq.(i) becomes

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(2y-1)(y-5) = 0$

Either $(2y-1) = 0$ *or* $(y-5) = 0$

$2y = 1 \qquad y = 5$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get

$x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

Put $y = 5$ *in* $x^2 = y$, *we get*

$x^2 = 5$

$\sqrt{x^2} = \pm\sqrt{5}$

$x = \pm\sqrt{5}$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Solution:

$x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$

Dividing each term by x^2, *we get*

$\frac{x^4}{x^2} - 2\frac{x^3}{x^2} - 2\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = 0$

$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$

$x^2 + \frac{1}{x^2} - 2x + \frac{2}{x} - 2 = 0$

$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x - \frac{1}{x}\right) - 2 = 0 \quad .....(i)$

Let $\quad x - \frac{1}{x} = y$

$\left(x - \frac{1}{x}\right)^2 = y^2$

$x^2 + \frac{1}{x^2} - 2 = y^2$

$x^2 + \frac{1}{x^2} = y^2 + 2$

So eq. (i) becomes

$y^2 + 2 - 2y - 2 = 0$

$y^2 - 2y = 0$

$y(y-2) = 0$

Either $y = 0$ *or* $y - 2 = 0, \Rightarrow y = 2$

Put $\quad y = 0 \quad$ in $x - \frac{1}{x} = y$, we get

$x - \frac{1}{x} = y$

$x - \frac{1}{x} = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

Put $y = 2$ in $x - \frac{1}{x} = y$, we get

$x - \frac{1}{x} = y$

$x - \frac{1}{x} = 2$

$x^2 - 1 = 2x$

$x^2 - 2x - 1 = 0$

$x^2 - 2x - 1 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2\left(1 \pm \sqrt{2}\right)}{2}$$

$$x = 1 \pm \sqrt{2}$$

Thus, solution set = $\left\{\pm 1, 1 \pm \sqrt{2}\right\}$

(11) $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Solution:

$2x^4 + x^3 - 6x^2 + x + 2 = 0$

Dividing each term by x^2, *we get*

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6x^2}{x^2} + \frac{x}{x^2} + \frac{2}{x^2} = 0$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad .....(i)$$

Let $\quad x + \frac{1}{x} = y$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So eq. (i) becomes

$$2\left(y^2-2\right)+y-6=0$$

$$2y^2-4+y-6=0$$
$$2y^2+y-10=0$$
$$2y^2+5y-4y-10=0$$
$$y(2y+5)-2(2y+5)=0$$
$$(y-2)(2y+5)=0$$

Either $y-2=0$ *or* $2y+5=0$

$y=2 \qquad 2y=-5$

$y=2 \qquad y=\frac{-5}{2}$

Put $y=2$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=2$$
$$x^2+1=2x$$
$$x^2-2x+1=0$$
$$(x-1)^2=0$$
$$\Rightarrow \quad x-1=0$$
$$\Rightarrow \quad x=1$$

Put $y=-\frac{5}{2}$ in $x+\frac{1}{x}=y$, we get

$$x+\frac{1}{x}=y$$
$$x+\frac{1}{x}=-\frac{5}{2}$$
$$2x^2+2=-5x$$
$$2x^2+5x+2=0$$
$$2x^2+4x+x+2=0$$
$$2x(x+2)+1(x+2)=0$$

Either $2x+1=0$ or $x+2=0$

$2x=-1 \qquad x=-2$

$x=-\frac{1}{2}$

Thus, solution set $=\left\{1,-2,-\frac{1}{2}\right\}$

(12) $4.2^{2x+1}-9.2^x+1=0$

Solution:

$4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x}.2 - 9.2^x + 1 = 0$ ....(i)

Let $2^x = y$ Then $2^{2x} = y^2$

So eq. (i) becomes

$4.y^2.2 - 9.y + 1 = 0$

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(8y-1)(y-1) = 0$

Either $8y - 1 = 0$ *or* $y - 1 = 0$

$8y = 1$ $\quad$ $y = 1$

$y = \frac{1}{8}$ $\quad$ $y = 1$

Put $y = \frac{1}{8}$ in $2^x = y$, we get

$2^x = y$

$2^x = \frac{1}{8}$

$2^x = \frac{1}{2^3}$

$2^x = 2^{-3}$

$x = -3$

Put $y = 1$ in $2^x = y$, we get

$2^x = y$

$2^x = 1$

$2^x = 2^0$

$x = 0$

Thus, solution set $= \{-3, 0\}$

(13) $3^{2x+2} = 12.3^x - 3$

Solution:

$3^{2x+2} = 12.3^x - 3$

$3^{2x}.3^2 - 12.3^x + 3 = 0$ ....(i)

Let $3^x = y$ Then $3^{2x} = y^2$

So eq. (i) becomes

$y^2.9-12y+3=0$

$9y^2-12y+3=0$

$9y^2-9y-3y+3=0$

$9y(y-1)-3(y-1)=0$

$(9y-3)(y-1)=0$

Either $9y-3=0$ *or* $y-1=0$

$$
\begin{array}{ll}
9y=3 & y=1 \\
y=\dfrac{3}{9} & y=1 \\
y=\dfrac{1}{3} & y=1
\end{array}
$$

Put $y=\dfrac{1}{3}$ in $3^x=y$, we get *Put* $y=1$ in $3^x=y$, we get

$$
\begin{array}{ll}
3^x=y & 3^x=y \\
3^x=\dfrac{1}{3} & 3^x=1 \\
3^x=3^{-1} & 3^x=3^0 \\
x=-1 & x=0
\end{array}
$$

Thus, solution set $=\{-1,0\}$

(14) $2^x+64.2^{-x}-20=0$

Solution:

$2^x+64.2^{-x}-20=0 \quad ....(i)$

Let $2^x=y$, Then $2^{-x}=\dfrac{1}{y}$

So eq. (i) becomes

$y-64.\dfrac{1}{y}-20=0$

$y^2-64-20y=0$

$y^2-20y-64=0$

$y^2-16y-4y-64=0$

$y(y-16)-4(y-16)=0$

$(y-4)(y-16)=0$

Either $y-4=0$ *or* $y-16=0$

$y=4 \qquad y=16$

Put $y=4$ in $2^x=y$, we get

$2^x=y$

$2^x=4$

$2^x=2^2$

$x=2$

Put $y=16$ in $2^x=y$, we get

$2^x=y$

$2^x=16$

$2^x=2^4$

$x=4$

Thus, solution set $=\{2,4\}$

(15) $(x+1)(x+3)(x-5)(x-7)=192$

Solution:

$(x+1)(x+3)(x-5)(x-7)=192$

As $1-5=3-7$

$4=4$

So $[(x+1)(x-5)][(x+3)(x-7)]=192$

$[x^2-5x+x-5][x^2-7x+3x-21]=192$

$(x^2-4x-5)(x^2-4x-21)=192 \qquad ......(i)$

Let $x^2-4x=y$

$$(y-5)(y-21)=192$$
$$y^2-21y-5y+105=192$$
$$y^2-26y+105-192=0$$
$$y^2-26y-87=0$$
$$y^2-29y+3y-87=0$$
$$y(y-29)+3(y-29)=0$$
$$(y+3)(y-29)=0$$

Either $y+3=0$ *or* $y-29=0$

$y=-3$ $\quad$ $y=29$

Put $y=-3$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=-3$$
$$x^2-4x+3=0$$
$$x^2-3x-x+3=0$$
$$x(x-3)-1(x-3)=0$$
$$(x-1)(x-3)=0$$

Either $x-1=0$ *or* $x-3=0$

$x=1$ $\quad$ $x=3$

Put $y=29$ in $x^2-4x=y$, we get

$$x^2-4x=y$$
$$x^2-4x=29$$
$$x^2-4x-29=0$$

Here $a=1, b=-4, c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{16+116}}{2}$$
$$x=\frac{4\pm\sqrt{132}}{2}$$
$$x=\frac{4\pm2\sqrt{33}}{2}$$
$$x=\frac{2\left(2\pm\sqrt{33}\right)}{2}$$
$$x=2\pm\sqrt{33}$$

Thus, solution set $=\left\{1,3,2\pm\sqrt{33}\right\}$

(2) $2x^4 = 9x^2 - 4$

Solution:

$2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0 \qquad ......(i)$

Let $x^2 = y$ *then* $x^4 = y^2$

So eq.(i) becomes

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(2y-1)(y-4) = 0$

Either $(2y-1) = 0$ *or* $(y-4) = 0$

$2y = 1 \qquad\qquad y = 4$

Put $y = \frac{1}{2}$ in $x^2 = y$, we get

$x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\frac{1}{\sqrt{2}}$

Put $y = 4$ *in* $x^2 = y$, *we get*

$x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{4}$

$x = \pm 2$

Thus, solution set $= \left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

(3) $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution:

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5x^{\frac{1}{2}} - 7x^{\frac{1}{4}} + 2 = 0 \qquad ......(i)$

$Let\ x^{\frac{1}{4}} = y\ \ then\ x^{\frac{1}{2}} = y^2$

So eq.(i) becomes

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(5y-2)(y-1) = 0$

$Either\ (5y-2) = 0 \quad or \quad (y-1) = 0$

$5y = 2 \qquad y = 1$

$y = \frac{2}{5} \qquad y = 1$

$Put \quad y = \frac{2}{5}$ in $x^{\frac{1}{4}} = y$, we get

$$x^{\frac{1}{4}} = y$$

$$x^{\frac{1}{4}} = \frac{2}{5}$$

Taking power '4' on both siedes we get

$$\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$$

$$x = \frac{2^4}{5^4}$$

$$x = \frac{16}{625}$$

$Put\ \ y = 1\ \ in\ x^{\frac{1}{4}} = y, we\ get$

$$x^{\frac{1}{4}} = y$$

$$x^{\frac{1}{4}} = 1$$

Taking power '4' on both siedes we get

$$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$$

$$\left(x^{\frac{1}{4}}\right)^4 = 1$$

$$x = 1$$

Thus, solution set $= \left\{\frac{16}{625}, 1\right\}$

(4) $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution:

$x^{\frac{2}{3}}+54=15x^{\frac{1}{3}}$

$x^{\frac{2}{3}}-15x^{\frac{1}{3}}+54=0 \qquad ......(i)$

$Let\ x^{\frac{1}{3}}=y,\ then\ x^{\frac{2}{3}}=y^2$

So eq.(i) becomes

$y^2-15y+54=0$

$y^2-9y-6y+54=0$

$y(y-9)-6(y-9)=0$

$(y-6)(y-9)=0$

$Either\ (y-9)=0\ or\ (y-6)=0$

$y=9 \qquad y=6$

$Put\quad y=9$ in $x^{\frac{1}{3}}=y,$ we get

$x^{\frac{1}{3}}=y$

$x^{\frac{1}{3}}=9$

Taking cube on both siedes we get

$\left(x^{\frac{1}{3}}\right)^3=(9)^3$

$x=729$

$Put\ y=6\ in\ x^{\frac{1}{3}}=y, we\ get$

$x^{\frac{1}{3}}=y$

$x^{\frac{1}{3}}=6$

Taking cube on both siedes we get

$\left(x^{\frac{1}{3}}\right)^3=(6)^3$

$x=216$

Thus, solution set $=\{729,216\}$

(5) $3x^{-2}+5=8x^{-1}$

Solution:

$3x^{-2}+5=8x^{-1}$

$3x^{-2}-8x^{-1}+5=0 \qquad ......(i)$

Let $x^{-1} = y$, *then* $x^{-2} = y^2$

So eq.(i) becomes

$$3y^2 - 8y + 5 = 0$$
$$3y^2 - 5y - 3y + 5 = 0$$
$$y(3y-5) - 1(3y-5) = 0$$
$$(y-1)(3y-5) = 0$$

Either $y - 1 = 0$ *or* $3y - 5 = 0$

$y = 1 \qquad 3y = 5$

$y = 1 \qquad y = \frac{5}{3}$

Put $y = 1$ in $x^{-1} = y$, we get

$$x^{-1} = y$$
$$x^{-1} = 1$$
$$\frac{1}{x} = 1$$
$$x = 1$$

Put $y = \frac{5}{3}$ *in* $x^{-1} = y$, *we get*

$$x^{-1} = y$$
$$x^{-1} = \frac{5}{3}$$
$$\frac{1}{x} = \frac{5}{3}$$
$$x = \frac{3}{5}$$

Thus, solution set $= \left\{1, \frac{3}{5}\right\}$

(6) $\left(2x^2+1\right) + \dfrac{3}{2x^2+1} = 4$

Solution:

$$\left(2x^2+1\right) + \frac{3}{2x^2+1} = 4 \quad ......(i)$$

Let $2x^2 + 1 = y$,

So eq.(i) becomes

$$y+\frac{3}{y}=4$$

Multiplying both sides by 'y', we get

$$y^2+3=4y$$
$$y^2-4y+3=0$$
$$y^2-3y-y+3=0$$
$$y(y-3)-1(y-3)=0$$
$$(y-1)(y-3)=0$$

Either $y-1=0$ *or* $y-3=0$

$y=1$ $\quad$ $y=3$

Put $y=1$ in $2x^2+1=y$, we get

$$2x^2+1=1$$
$$2x^2=1-1$$
$$2x^2=0$$
$$x^2=0$$
$$x=0$$

Put $y=3$ *in* $2x^2+1=y$, *we get*

$$2x^2+1=3$$
$$2x^2=3-1$$
$$2x^2=2$$
$$x^2=1$$
$$x=\pm1$$

Thus, solution set $=\{-1,0,1\}$

(7) $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$

Solution:

$$\frac{x}{x-3}+4\left(\frac{x-3}{x}\right)=4 \quad ......(i)$$

Let $\dfrac{x}{x-3}=y$,

So eq.(i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by 'y', we get

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$(y)^2-2(y)(2)+(2)^2=0$$
$$(y-2)^2=0$$
$$(y-2)=0$$

Put $y=2$ in $\frac{x}{x-3}=y$, we get

$$\frac{x}{x-3}=y$$
$$\frac{x}{x-3}=2$$
$$2(x-3)=x$$
$$2x-6=x$$
$$2x-x=6$$
$$x=6$$

Thus, solution set $=\{6\}$

(8) $\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$

Solution:

$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=2\frac{1}{6}$$
$$\frac{4x+1}{4x-1}+\frac{4x-1}{4x+1}=\frac{13}{6} \quad ......(i)$$

Let $\frac{4x+1}{4x-1}=y$.

So eq.(i) becomes

$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by '6y', we get

$6y^2+6=13y$

$6y^2-13y+6=0$

$6y^2-9y-4y+6=0$

$3y(2y-3)-2(2y-3)=0$

$(3y-2)(2y-3)=0$

Either $3y-2=0$ *or* $2y-3=0$

$3y=2 \qquad 2y=3$

$y=\frac{2}{3} \qquad y=\frac{3}{2}$

Put $y=\frac{2}{3}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$
$$\frac{4x+1}{4x-1}=\frac{2}{3}$$
$$3(4x+1)=2(4x-1)$$
$$12x+3=8x-2$$
$$12x-8x=-2-3$$
$$4x=-5$$
$$x=-\frac{5}{4}$$

Put $y=\frac{3}{2}$ in $\frac{4x+1}{4x-1}=y$, we get

$$\frac{4x+1}{4x-1}=y$$
$$\frac{4x+1}{4x-1}=\frac{3}{2}$$
$$3(4x-1)=2(4x+1)$$
$$12x-3=8x+2$$
$$12x-8x=2+3$$
$$4x=5$$
$$x=\frac{5}{4}$$

Thus, solution set $=\left\{\pm\frac{5}{4}\right\}$

(9) $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

Solution:

$\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12} \qquad \ldots\ldots(i)$

$Let\ \frac{x-a}{x+a}=y,$

So eq.(i) becomes

$$y-\frac{1}{y}=\frac{7}{12}$$

Multiplying both sides by '12y', we get

$$12y^2-12=7y$$

$$12y^2-7y-12=0$$

$$12y^2-16y+9y-12=0$$

$$4y(3y-4)+3(3y-4)=0$$

$$(4y+3)(3y-4)=0$$

Either $4y+3=0$ *or* $3y-4=0$

$$4y=-3 \qquad 3y=4$$

$$y=-\frac{3}{4} \qquad y=\frac{4}{3}$$

Put $y=-\frac{3}{4}$ in $\frac{x-a}{x+a}=y$, we get

$$\frac{x-a}{x+a}=y$$

$$\frac{x-a}{x+a}=-\frac{3}{4}$$

$$4(x-a)=-3(x+a)$$

$$4x-4a=-3x-3a$$

$$4x+3x=4a-3a$$

$$7x=a$$

$$x=\frac{a}{7}$$

Put $y=\frac{4}{3}$ in $\frac{x-a}{x+a}=y$, we get

$$\frac{x-a}{x+a}=y$$

$$\frac{x-a}{x+a}=\frac{4}{3}$$

$$4(x+a)=3(x-a)$$

$$4x+4a=3x-3a$$

$$4x-3x=-4a-3a$$

$$x=-7a$$

Thus, solution set = $\left\{-7a,\frac{a}{7}\right\}$

(10) $x^4-2x^3-2x^2+2x+1=0$