Synthetic Division:

Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In fact, synthetic division is simply a shortcut of long division method.

Exercise 2.6

1. Use synthetic division to find the quotient and the remainder, when

(i) $\left(x^2-7x-1\right)\div\left(x+1\right)$

Solution:

$\left(x^2-7x-1\right)\div\left(x+1\right)$

As $\quad x+1=x-\left(-1\right), so\ a=-1$

Now write the coefficients of dividend in a row and a= −1 on the left side.,

$$\begin{array}{r|rrr} & 1 & 7 & -1 \\ -1 & \downarrow & -1 & -6 \\ \hline & 1 & 6 & |-7 \end{array}$$

Quotient $\quad = Q(x) = x+6$

Remainder $= R = -7$

(ii) $\left(4x^3-5x+15\right)\div\left(x+3\right)$

Solution:

As $\quad x+3=x-\left(-3\right), so\ a=-3$

Now write the coefficients of dividend in a row and a= −3 on the left side.

	1	5	2	-8	0
-4	↓	-4	-4	8	
	1	1	-2	0	

The depressed equation is

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x+2) - 1(x+2) = 0$

$(x-1)(x+2) = 0$

Either $x - 1 = 0$ *or* $x + 2 = 0$

$x = 1$ $\quad$ $x = -2$

Thus, $-4, -2, 1$ and 3 are the roots of the given equation.

$$\begin{array}{r|rrrr} & 4 & 0 & -5 & 15 \\ -3 & \downarrow & -12 & 36 & -93 \\ \hline & 4 & -12 & 31 & |-78 \end{array}$$

Quotient $= Q(x) = 4x^2 - 12x + 31$

Remainder $= R = -78$

(iii) $\left(x^3 + x^2 - 3x + 2\right) \div (x - 2)$

Solution:

$\left(x^3 + x^2 - 3x + 2\right) \div (x - 2)$

As $x - 2$, *so* $a = 2$

Now write the coefficients of dividend in a row and $a = 2$ on the left side.

$$\begin{array}{r|rrrr} & 1 & 1 & -3 & 2 \\ 2 & \downarrow & 2 & 6 & 6 \\ \hline & 1 & 3 & 3 & |8 \end{array}$$

Quotient $= Q(x) = x^2 + 3x + 3$

Remainder $= R = 8$

2. Find the value of h using synthetic division, if

(i) 3 is the zero of the polynomial $2x^3 - 3hx^2 + 9$

Solution:

$P(x) = 2x^3 - 3hx^2 + 9$ and its zero is 3.

Then by synthetic division.

$$\begin{array}{r|rrrr} & 2 & -3h & 0 & 9 \\ 3 & \downarrow & 6 & 18-9h & 54-27h \\ \hline & 2 & 6-3h & 18-9h & |63-27h \end{array}$$

Remainder $= 63 - 27h$

Since 3 is the zero of the polynomial, therefore

Remainder $= 0$, that is

$$63 - 27h = 0$$
$$27h = 63$$
$$h = \frac{63}{27} = \frac{7}{3}$$

(ii) 1 is the zero of the polynomial $x^3 - 2hx^2 + 11$

Solution:

$P(x) = x^3 - 2hx^2 + 11$ and its zero is 1.

Then by synthetic division.

$$\begin{array}{r|cccc}
 & 1 & -2h & 0 & 11 \\
1 & \downarrow & 1 & 1-2h & 1-2h \\
\hline
 & 1 & 1-2h & 1-2h & |\,12-2h
\end{array}$$

Remainder $= 12 - 2h$

Since 1 is the zero of the polynomial, therefore

Remainder $= 0$, that is

$$12 - 2h = 0$$
$$2h = 12$$
$$h = \frac{12}{2} = 6$$

(iii) -1 is the zero of the polynomial $2x^3 + 5hx - 23$.

Solution:

$P(x) = 2x^3 + 5hx - 23$ and its zero is -1.

Then by synthetic division.

$$\begin{array}{r|cccc}
 & 2 & 0 & 5h & -23 \\
-1 & \downarrow & -2 & 2 & -5h-2 \\
\hline
 & 2 & -2 & 5h+2 & |\,-5h-25
\end{array}$$

Remainder $= -5h - 25$

Since -1 is the zero of the polynomial, therefore

Remainder $= 0$, that is

$$-5h - 25 = 0$$
$$-5h = 25$$
$$h = \frac{25}{-5} = -5$$

3. Use synthetic division to find the values of l and m, if

(i) (x+3) and (x-2) are the factors of the polynomial

$$x^3 - 4x^2 + 2lx + m$$

Solution:

Since $(x+3)$ and $(x-2)$ are the factors of $\mathrm{p}(x) = x^3 + 4x^2 + 2lx + m$.

Therefore -3 and 2 are zeroes of polynomial $\mathrm{p}(x)$.

Now by synthetic division,

$$\begin{array}{r|cccc} & 1 & 4 & 2l & m \\ -3 & \downarrow & -3 & -3 & -6l+9 \\ \hline & 1 & 1 & 2l-3 & |\, m-6l+9 \end{array}$$

Remainder $= m - 6l + 9$

Since -3 is the zero of the polynomial, therefore,

Remainder $= 0$, that is,

$m - 6l + 9 = 0 \qquad ....(i)$

and

$$\begin{array}{r|cccc} & 1 & 4 & 2l & m \\ 2 & \downarrow & 2 & 12 & 4l+24 \\ \hline & 1 & 6 & 2l+12 & |\, m+4l+24 \end{array}$$

Remainder $= m + 4l + 24$

Since 2 is the zero of polynomial, therefore,

Remainder $= 0$, that is ,

$m + 4l + 24 = 0 \qquad ....(ii)$

Now, subtact eq(ii) from eq(i), we get

$$
\begin{aligned}
m-6l+9&=0\\
\underline{\pm m\pm 4l\pm 24}&\underline{=0}\\
-10l-15&=0\\
-10l&=15\\
l&=-\frac{15}{10}\\
l&=-\frac{3}{2}
\end{aligned}
$$

Put $l=-\frac{3}{2}$ in eq.(i), we get

$$
\begin{aligned}
&m-6\left(-\frac{3}{2}\right)+9=0\\
&m+9+9=0\\
&m+18=0\\
&m=-18
\end{aligned}
$$

Thus, $l=-\frac{3}{2},\ m=-18$

(ii) **(x-1) and (x+1) are the factors of the polynomial** $x^3-3lx^2+2mx+6$.

Solution:

Since $(x-1)$ and $(x+1)$ are the factors of $\mathrm{p}(x)=x^3-3lx^2+2mx+6$.

Therefor 1 and -1 are zeroes of polynomial $\mathrm{p}(x)$.

Now by synthetic division,

$$
\begin{array}{r|cccc}
 & 1 & -3l & 2m & 6\\
1 & \downarrow & 1 & 1-3l & 2m-3l+1\\
\hline
 & 1 & 1-3l & 2m-3l+1 & |\,2m-3l+7
\end{array}
$$

Remainder $=2m-3l+7$

Since 1 is the zero of the polynomial, therefore,

Remainder $=0$, that is,

$2m-3l+7=0 \quad ....(i)$

and

$$
\begin{array}{c|cccc}
 & 1 & -3l & 2m & 6 \\
-1 & \downarrow & -1 & 3l+1 & -2m-3l-1 \\
\hline
 & 1 & -3l-1 & 2m+3l+1 & |-2m-3l+5
\end{array}
$$

Remainder $=-2m-3l+5$

Since -1 is the zero of polynomial, therefore,

Remainder $=0$, that is ,

$-2m-3l+5=0 \qquad \ldots\ldots(ii)$

Add eq(ii) in eq(i), we get

$$
\begin{aligned}
2m-3l+7&=0\\
\underline{-2m-3l+5}&\underline{=0}\\
-6l+12&=0\\
-6l&=-12\\
l&=\frac{12}{6}\\
l&=2
\end{aligned}
$$

Put $l=2$ in eq.(i), we get

$$
\begin{aligned}
2m-3(2)+7&=0\\
2m-6+7&=0\\
2m+1&=0\\
2m&=-1\\
m&=-\frac{1}{2}
\end{aligned}
$$

Thus, $l=2, m=-\frac{1}{2}$

4. Solve by using synthetic division, if

(i) 2 is the root of the equation $x^3-28x+48=0$

Solution:

Since 2 is the root of the equation $x^3-28x+48=0$

Then by synthetic division, we get

$$
\begin{array}{r|rrrr}
 & 1 & 0 & -28 & 48 \\
2 & \downarrow & 2 & 4 & -48 \\
\hline
 & 1 & 2 & -24 & |0
\end{array}
$$

The depressed equation is

$x^2 + 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$

$x(x+6) - 4(x+6) = 0$

$(x-4)(x+6) = 0$

Either $x - 4 = 0$ *or* $x + 6 = 0$

$x = 4$ $\qquad$ $x = -6$

Hence, $-6, 2$ and 4 are the roots of the given equation.

(ii) 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$.

Solution:

Since 3 is the root of the equation $2x^3 - 3x^2 - 11x + 6 = 0$

Then by synthetic division, we get

$$
\begin{array}{r|rrrr}
 & 2 & -3 & -11 & 6 \\
3 & \downarrow & 6 & 9 & -6 \\
\hline
 & 2 & 3 & -2 & |0
\end{array}
$$

The depressed equation is

$2x^2 + 3x - 2 = 0$

$2x^2 + 4x - x - 2 = 0$

$2x(x+2) - 1(x+2) = 0$

$(2x-1)(x+2) = 0$

Either $2x - 1 = 0$ *or* $x + 2 = 0$

$2x = 1$ $\qquad$ $x = -2$

$x = \frac{1}{2}$

Hence, $-2, \frac{1}{2}$ and 3 are the roots of the given equation.

(iii) -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$.

Solution:

Since -1 is the root of the equation $4x^3 - x^2 - 11x - 6 = 0$

Then by synthetic division, we get

$$\begin{array}{r|rrrr} & 4 & -1 & -11 & -6 \\ -1 & \downarrow & -4 & 5 & 6 \\ \hline & 4 & -5 & -6 & |0 \end{array}$$

The depressed equation is

$4x^2 - 5x - 6 = 0$

$4x^2 - 8x + 3x - 6 = 0$

$4x(x-2) + 3(x+2) = 0$

$(4x+3)(x-2) = 0$

Either $4x + 3 = 0$ *or* $x - 2 = 0$

$4x = -3$ $\quad x = 2$

$x = -\dfrac{3}{4}$

Hence, $-\dfrac{3}{4}, -1$ and 2 are the roots of the given equation.

5. Solve by using synthetic division, if

(i) 1 and 3 the roots of the equation $x^4 - 10x^2 + 9 = 0$.

Solution:

Since 1 and 3 are the root of the equation

$x^4 - 10x^2 + 9 = 0$

Then by synthetic division, we get

	1	0	-10	0	9	
1	$\downarrow$	1	1	-9	-9	
	1	1	-9	-9	0	
3	$\downarrow$	3	12	9		
	1	4	3	0		

The depressed equation is

$x^2+4x+3=0$

$x^2+3x+x+3=0$

$x(x+3)+1(x+3)=0$

$(x+1)(x+3)=0$

Either $x+1=0$ *or* $x+3=0$

$x=-1$ $\quad$ $x=-3$

Thus, $-3,-1,1$ and 3 are the roots of the given equation.

(ii) 3 and -4 are the roots of the equation $x^4+2x^3-13x^2-14x+24=0$.

Solution:

Since 3 and -4 are the root of the equation

$x^4+2x^3-13x^2-14x+24=0$

Then by synthetic division, we get

	1	2	-13	-14	24
3	$\downarrow$	3	15	6	-24