# Exercise 2.3

**1. Without solving, find the sum and the product of the following quadratic equations.**

**(i)** $x^2 - 5x + 3 = 0$

**Solution:**

$x^2 - 5x + 3 = 0$

Here a=1, b=-5, c=3

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{(-5)}{1} = 5$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

**(ii)** $3x^2 + 7x - 11 = 0$

**Solution:**

$3x^2 + 7x - 11 = 0$

Here a=3, b=7, c=-11

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{7}{3}$

And product of roots = $\alpha\beta = \frac{c}{a} = -\frac{11}{3}$

Put $\alpha=-10$ in eq.$(i)$, we get

$$\beta=7+10$$
$$\beta=17$$

Put $\alpha=-10, \beta=17$ in eq.$(ii)$, we get

$$(-10)(17)=3m-5$$
$$5-170=3m$$
or $$3m=-165$$
$$m=-55$$

**(ii) The roots of the equation $x^2+7x+3m-5=0$ satisfy the relation $3\alpha-2\beta=4$.**

**Solution:**

$x^2+7x+3m-5=0$

Here a=1, b=7, c=$3m-5$

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha+\beta=-\dfrac{b}{a}=-\dfrac{(7)}{1}=-\dfrac{7}{1}=-7$

And product of roots = $\alpha\beta=\dfrac{c}{a}=\dfrac{3m-5}{1}=3m-5$

Now $\alpha+\beta=-7$

$\beta=-7-\alpha$ ........$(i)$ *and* $\alpha\beta=3m-5$ .......$(ii)$

Since $3\alpha-2\beta=4$ ........$(iii)$

*Put* $\beta$ in eq$(iii)$, we have

$$3\alpha-2(-7-\alpha)=4$$
$$3\alpha+14+2\alpha=4$$
$$3\alpha+2\alpha=4-14$$
$$5\alpha=-10$$
$$\alpha=-2$$

Put $\alpha=-2$ in eq.$(i)$, we get

$$\beta = -7 - (-2)$$
$$\beta = -7 + 2$$
$$\beta = -5$$

Put $\alpha = -2$ *and* $\beta = -5$ in eq.$(iii)$, we get

$$(-2)(-5) = 3m - 5$$
$$10 = 3m - 5$$
*or* $$3m = 10 + 5$$
$$3m = 15$$
$$m = 5$$

**(iii) The roots of the equation $3x^2 - 2x + 7m + 2 = 0$ satisfy the relation** $7\alpha - 3\beta = 18$.

**Solution:**

$3x^2 - 2x + 7m + 2 = 0$

Here a=3, b=-2, c=$7m + 2$

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\dfrac{b}{a}$ *and* $\alpha\beta = \dfrac{c}{a}$

$$\alpha + \beta = -\frac{(-2)}{3} \qquad \alpha\beta = \frac{7m+2}{3} \quad ........(ii)$$

$$\alpha + \beta = \frac{2}{3}$$

$$\beta = \frac{2}{3} - \alpha \quad ........(i)$$

Since $7\alpha - 3\beta = 18$ ........$(iii)$

*Put* $\beta = \dfrac{2}{3} - \alpha$ in eq$(iii)$, we have

$$7\alpha - 3\left(\frac{2}{3} - \alpha\right) = 18$$
$$7\alpha - 2 + 3\alpha = 18$$
$$7\alpha + 3\alpha = 18 + 2$$

$$10\alpha=20$$
$$\alpha=2$$

Put $\alpha = 2$ in eq.$(i)$, we get

$$\beta = \frac{2}{3} - 2$$
$$\beta = \frac{2-6}{3}$$
$$\beta = -\frac{4}{3}$$

Put $\alpha = 2$ *and* $\beta = -\frac{4}{3}$ in eq.$(ii)$, we get

$$(2)\left(-\frac{4}{3}\right) = \frac{7m+2}{3}$$
$$-\frac{8}{3} = \frac{7m+2}{3}$$
$$-\frac{8}{3} \times 3 = 7m+2$$
$$-8 = 7m+2$$
$$7m = -8-2$$

*or* $$7m = -10$$
$$m = -\frac{10}{7}$$

**6. Find m, if sum and product of the roots of the following equations is equal to a given number λ.**

**(i)** $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

**Solution:**

$(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

Here a=2m+3, b=7m − 5, c=3$m$ − 10

Let $\alpha$ and $\beta$ be the roots of the given equation

Then $\alpha + \beta = -\frac{b}{a}$ *and* $\alpha\beta = \frac{c}{a}$

**(iii)** $px^2 - qx + r = 0$

**Solution:**

$px^2 - qx + r = 0$

Here a=p, b=-q, c=r

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{(-q)}{p} = \frac{q}{p}$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{r}{p}$

**(iv)** $(a+b)x^2 - ax + b = 0$

**Solution:**

$(a+b)x^2 - ax + b = 0$

Here a=$a+b$, b=-a, c=b

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{(-a)}{a+b} = \frac{a}{a+b}$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{b}{a+b}$

**(v)** $(l + \mathrm{m})x^2 + (m+n)x + n - l = 0$

**Solution:**

$(l+\mathrm{m})x^2+(m+n)x+n-l=0$

Here a=$l+\mathrm{m}$, b=$m+n$, c=$n-l$

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha+\beta=-\dfrac{b}{a}=-\dfrac{m+n}{l+\mathrm{m}}$

And product of roots = $\alpha\beta=\dfrac{c}{a}=\dfrac{n-l}{l+\mathrm{m}}$

**(vi)** $7x^2-5mx+9n=0$

**Solution:**

$7x^2-5mx+9n=0$

Here a=7, b=$-5m$, c=9n

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha+\beta=-\dfrac{b}{a}=-\dfrac{(-5m)}{7}=\dfrac{5m}{7}$

And product of roots = $\alpha\beta=\dfrac{c}{a}=\dfrac{9n}{7}$

**2. Find the value of k, if**

**(i) Sum of the roots of the equation 2kx²-3x+4k=0 is twice the product of the roots.**

**Solution:**

$2kx^2-3x+4k=0$

Here a=2k, b=-3, c=4k

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha+\beta=-\dfrac{b}{a}=-\dfrac{(-3)}{2k}=\dfrac{3}{2k}$

And product of roots $= \alpha\beta = \frac{c}{a} = \frac{4k}{2k} = 2$

As sum of the roots is twice the product of the roots, so

$\alpha + \beta = 2\alpha\beta$

$\frac{3}{2k} = 2(2)$

$\frac{3}{2k} = 4$

$or\; k = \frac{3}{8}$

**(ii) Sum of the roots of the equation** $x^2 + (3k-7)x + 5k = 0$ is $\frac{3}{2}$ **times the product of the roots.**

**Solution:**

$x^2 + (3k-7)x + 5k = 0$

Here a=1, b=$3k-7$, c=5k

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{(3k-7)}{1} = -3k+7$

And product of roots $= \alpha\beta = \frac{c}{a} = \frac{5k}{1} = 5k$

As sum of the roots is $\frac{3}{2}$ of the product of the roots, so

$\alpha + \beta = \frac{3}{2}\alpha\beta$

$-3k + 7 = \frac{3}{2}(5k)$

$-3k + 7 = \frac{15k}{2}$

$-6k + 14 = 15k$

$15k + 6k = 14$

$21k = 14$

And product of roots $= \alpha\beta = \frac{c}{a} = \frac{4k}{2k} = 2$

As sum of the roots is twice the product of the roots, so

$\alpha + \beta = 2\alpha\beta$

$\frac{3}{2k} = 2(2)$

$\frac{3}{2k} = 4$

$or\ k = \frac{3}{8}$

**(ii) Sum of the roots of the equation $x^2 + (3k-7)x + 5k = 0$ is $\frac{3}{2}$ times the product of the roots.**

**Solution:**

$x^2 + (3k-7)x + 5k = 0$

Here a=1, b=3$k$ − 7, c=5k

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{(3k-7)}{1} = -3k + 7$

And product of roots $= \alpha\beta = \frac{c}{a} = \frac{5k}{1} = 5k$

As sum of the roots is $\frac{3}{2}$ of the product of the roots, so

$\alpha + \beta = \frac{3}{2}\alpha\beta$

$-3k + 7 = \frac{3}{2}(5k)$

$-3k + 7 = \frac{15k}{2}$

$-6k + 14 = 15k$

$15k + 6k = 14$

$21k = 14$

$k = \frac{14}{21}$

$k = \frac{2}{3}$

**3. Find k, if**

**(i) Sum of the squares of the roots of the equation $4kx^2 + 3kx - 8 = 0$ is 2.**

**Solution:**

$4kx^2 + 3kx - 8 = 0$

Here a=4k, b=3$k$, c=-8

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{3k}{4k} = -\frac{3}{4}$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{-8}{4k}$

As sum of the square of roots is twice the product of the roots, so

$\alpha^2 + \beta^2 = 2$

$(\alpha + \beta)^2 - 2\alpha\beta = 2 \qquad \because (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\left(-\frac{3}{4}\right)^2 - 2\left(\frac{-8}{4k}\right) = 2 \qquad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\frac{9}{16} + \frac{16}{4k} = 2$

$\frac{16}{4k} = 2 - \frac{9}{16} \quad \rightarrow \quad \frac{16}{4k} = \frac{32 - 9}{16}$

$\frac{16}{4k} = \frac{23}{16} \quad \rightarrow \quad 23 \times 4k = 16 \times 16$

$k = \frac{16 \times 16}{23 \times 4} \quad \rightarrow \quad k = \frac{64}{23}$

**(ii) Sum of the squares of the roots of the equation $x^2$-2kx+(2k+1) =0 is 6.**

**Solution:**

$x^2 - 2kx + (2k+1) = 0$

Here a=1, b=-2k, c=2k+1

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha + \beta = -\frac{b}{a} = -\frac{(-2k)}{1} = 2k$

And product of roots = $\alpha\beta = \frac{c}{a} = \frac{2k+1}{1} = 2k+1$

As sum of the square of roots is 6 to the product of the roots, so

$\alpha^2 + \beta^2 = 6$

$(\alpha+\beta)^2 - 2\alpha\beta = 6 \qquad \because (\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$(2k)^2 - 2(2k+1) = 6 \qquad \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$4k^2 - 4k - 2 = 6$

$4k^2 - 4k - 2 - 6 = 0$

$4k^2 - 4k - 8 = 0$

$4(k^2 - k - 2) = 0$

$\Rightarrow k^2 - k - 2 = 0$

$k^2 - 2k + k - 2 = 0$

$k(k-2) + 1(k-2) = 0$

$(k-1)(k-2) = 0$

Either $k+1=0$ *or* $k-2=0$

$k=-1 \qquad k=2$

**4. Find p, if**

**(i) The roots of the equation $x^2 - x + p^2 = 0$ differ by unity.**

**Solution:**

$x^2 - x + p^2 = 0$

Here a=1, b=-1, c=$p^2$

Let $\alpha$ and $\alpha$-1 be the roots of the given equation

$$\text{Then } \alpha+\alpha\text{-}1 = -\frac{b}{a} \quad \textit{and} \quad \alpha(\alpha\text{-}1) = \frac{c}{a}$$

$$2\alpha - 1 = -\frac{(-1)}{1} \qquad \alpha^2\text{-}1 = \frac{p^2}{1}$$

$$2\alpha - 1 = 1 \qquad \alpha^2\text{-}1 = p$$

$$2\alpha = 1+1 \qquad \textit{put } \alpha = 1 \text{ in above eq.,we get}$$

$$2\alpha = 2 \qquad (1)^2 - 1 = p$$

$$\alpha = 1 \qquad 1-1 = p$$

$$p = 0$$



**(ii) The roots of the equation $x^2 + 3x + p - 2 = 0$ differ by 2.**

**Solution:**

$x^2 + 3x + p - 2 = 0$

Here a=1, b=3, c=$p - 2$

Let $\alpha$ and $\alpha - 2$ be the roots of the given equation

$$\text{Then } \alpha+\alpha-2 = -\frac{b}{a} \quad \textit{and} \quad \alpha(\alpha-2) = \frac{c}{a}$$

$$2\alpha - 2 = -\frac{3}{1} \qquad \alpha^2 - 2\alpha = \frac{p-2}{1}$$

$$2\alpha - 2 = -3 \qquad \alpha^2 - 2\alpha = p - 2$$

$$2\alpha = -3+2 \qquad \textit{put } \alpha = -\frac{1}{2} \text{ in above eq.,we get}$$

$$\alpha = -\frac{1}{2} \qquad \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = p - 2$$

$$\frac{1}{4}+1=p-2$$
$$\frac{1}{4}=p-2-1$$
$$\frac{1}{4}=p-3$$
$$4p-12=1$$
$$4p=1+12$$
$$4p=13$$
$$p=\frac{13}{4}$$

**5. Find m, if**

**(i) The roots of the equation x² -7x+3m-5=0 satisfy the relation** $3\alpha+2\beta=4$ .

**Solution:**

$x^2-7x+3m-5=0$
Here a=1, b= $-7$, c= $3m-5$

Let $\alpha$ and $\beta$ be the roots of the given equation

Then Sum of roots = $\alpha+\beta=-\frac{b}{a}=-\frac{(-7)}{1}=7$

And product of roots = $\alpha\beta=\frac{c}{a}=\frac{3m-5}{1}=3m-5$

Now $\alpha+\beta=7$

$\beta=7-\alpha$ ........$(i)$ *and* $\alpha\beta=3m-5$ .......$(ii)$

Since $3\alpha+2\beta=4$ ........$(iii)$

*Put* $\beta$ in eq$(iii)$, we have

$3\alpha+2(7-\alpha)=4$

$3\alpha+14-2\alpha=4$

$3\alpha-2\alpha+14=4$

$\alpha=4-14$

$\alpha=-10$