Exercise 1.1

1. Write the following quadratic equations in the standard form and point out pure quadratic equations.

i. **(x+7) (x-3) = -7**

Solution:

$$(x+7)(x-3)=-7$$
$$x(x-3)+7(x-3)=-7$$
$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The above equation is a quadratic equation.

ii. $\frac{x^2+4}{3}-\frac{x}{7}=1$

Solution:

$$\frac{x^2+4}{3}-\frac{x}{7}=1$$

Multiply both sides by 21, we get

$$21\times\frac{x^2+4}{3}-21\times\frac{x}{7}=1\times 21$$
$$7(x^2+4)-3x=21$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The above equation is a quadratic equation.

$$\left(x-\frac{34}{22}\right)^2=\frac{-132+1156}{484}$$
$$\left(x-\frac{34}{22}\right)^2=\frac{1024}{484}$$

Taking square root on both sides we get

$$\left(x-\frac{34}{22}\right)=\pm\sqrt{\frac{1024}{484}}$$
$$x-\frac{34}{22}=\pm\frac{32}{22}$$

$$x=\frac{34}{22}\pm\frac{32}{22}$$
$$x=\frac{34\pm 32}{22}$$
$$x=\frac{34+32}{22},\quad x=\frac{34-32}{22}$$
$$=\frac{66}{22}\qquad =\frac{2}{22}$$
$$=3\qquad =\frac{1}{11}$$

Thus solution set $\left\{3,\frac{1}{11}\right\}$

iv. $lx^2+mx+n=0\,,\ l\neq 0$

Solution:

$$lx^2+mx+n=0$$
$$lx^2+mx=-n$$
$$\frac{lx^2}{l}+\frac{mx}{l}=-\frac{n}{l}$$
$$x^2+\frac{mx}{l}=-\frac{n}{l}$$
$$(x)^2+2(x)\left(\frac{m}{2l}\right)+\left(\frac{m}{2l}\right)^2=-\frac{n}{l}+\left(\frac{m}{2l}\right)^2$$

$$\left(x+\frac{m}{2l}\right)^2=-\frac{n}{l}+\frac{m^2}{4l^2}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{-4l\,n+m^2}{4l^2}$$

$$\left(x+\frac{m}{2l}\right)^2=\frac{m^2-4l\,n}{4l^2}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{m}{2l}\right)^2}=\pm\sqrt{\frac{m^2-4\ln}{4l^2}}$$

$$x+\frac{m}{2l}=\pm\sqrt{\frac{m^2-4l\,n}{4l^2}}$$

$$x=-\frac{m}{2l}\pm\sqrt{\frac{m^2-4l\,n}{4l^2}}$$

$$x=\frac{-m\pm\sqrt{m^2-4l\,n}}{2l}$$

Thus solution set $=\left\{\dfrac{-m\pm\sqrt{m^2-4l\,n}}{2l}\right\}$

v. $3x^2+7x=0$

Solution:

$$3x^2+7x=0$$

$$\frac{3x^2}{3}+\frac{7x}{3}=\frac{0}{3}$$

$$x^2+\frac{7}{3}x=0$$

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=0+\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{6}\right)^2} = \pm\sqrt{\left(\frac{7}{6}\right)^2}$$

$$x+\frac{7}{6} = \pm\frac{7}{6}$$

$$x = -\frac{7}{6}\pm\frac{7}{6}$$

$$x = -\frac{7}{6}+\frac{7}{6} \quad or \quad x = -\frac{7}{6}-\frac{7}{6}$$

$$x = 0 \qquad\qquad x = -\frac{14}{6}$$

$$x = -\frac{7}{3}$$

Thus solution set $= \left\{0, -\frac{7}{3}\right\}$

vi. $x^2-2x-195=0$

Solution:

$$x^2-2x-195=0$$

$$x^2-2x=195$$

$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root on both sides

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$

$$x-1=\pm 14$$

$$x=1\pm 14$$

$$x=1+14 \quad or \quad x=1-14$$

$$=15 \qquad\qquad =-13$$

Thus, solution set $= \{-13, 15\}$

vii. $-x^2+\frac{15}{2}=\frac{7}{2}x$

Solution:

$$-x^2-\frac{7}{2}x=-\frac{15}{2}$$

$$-\left(x^2+\frac{7}{2}x\right)=-\frac{15}{2}$$

$$x^2+\frac{7}{2}x=\frac{15}{2}$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=-\frac{7}{4}\pm\frac{13}{4}$$

$$x=-\frac{7}{4}+\frac{13}{4} \quad or \quad x=-\frac{7}{4}-\frac{13}{4}$$

$$x=\frac{3}{2} \qquad x=-5$$

Thus, solution set $=\left\{\frac{3}{2},-5\right\}$

viii. $x^2+17x+\frac{33}{4}=0$

Solution:

$$x^2+17x=-\frac{33}{4}$$

$$(x)^2+2(x)\left(\frac{17}{2}\right)+\left(\frac{17}{2}\right)^2=-\frac{33}{4}+\left(\frac{17}{2}\right)^2$$

$$\left(x+\frac{17}{2}\right)^2=-\frac{33}{4}+\frac{289}{4}$$

$$\left(x+\frac{17}{2}\right)^2=\frac{256}{4}$$

Taking square root on both sides,

$$\sqrt{\left(x+\frac{17}{2}\right)^2}=\pm\sqrt{\frac{256}{4}}$$

$$x+\frac{17}{2}=\pm\frac{16}{2}$$

$$x=-\frac{17}{2}\pm\frac{16}{2}$$

$$x=-\frac{17}{2}+\frac{16}{2} \quad or \quad x=-\frac{17}{2}-\frac{16}{2}$$

$$x=-\frac{1}{2} \qquad x=-\frac{33}{2}$$

Thus solution set $=\left\{-\frac{1}{2},-\frac{33}{2}\right\}$

ix. $4-\dfrac{8}{3x+1}=\dfrac{3x^2+5}{3x+1}$

Solution:

$$4-\frac{8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{4(3x+1)-8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{12x+4-8}{3x+1}=\frac{3x^2+5}{3x+1}$$
$$\frac{12x-4}{3x+1}=\frac{3x^2+5}{3x+1}$$

Multiplying both sides by $(3x+1)$, we get

$$12x-4=3x^2+5$$
$$3x^2+5-12x+4=0$$
$$3x^2-12x+9=0$$
$$3(x^2-4x+3)=0$$
$$\Rightarrow x^2-4x+3=0$$
$$x^2-4x=-3$$
$$(x)^2-2(x)(2)+(2)^2=-3+(2)^2$$
$$(x-2)^2=1$$

Taking square root on both sides,

$$\sqrt{(x-2)^2}=\pm\sqrt{1}$$
$$x-2=\pm 1$$
$$x=2\pm 1$$
$$x=2+1 \quad or \quad x=2-1$$
$$x=3 \qquad\qquad x=1$$

Thus solution set $=\{1,3\}$

x. $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution:

$$7(x+2a)^2+3a^2=5a(7x+23a)$$
$$7(x^2+4ax+4a^2)+3a^2=35ax+115a^2$$
$$7x^2+28ax+28a^2+3a^2=35ax+115a^2$$
$$7x^2+28ax-35ax+28a^2+3a^2-115a^2=0$$
$$7x^2-7ax-84a^2=0$$
$$7(x^2-ax-12a^2)=0$$
$$x^2-ax-12a^2=0$$
$$x^2-ax=12a^2$$
$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$
$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$
$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root on both sides, we get

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$
$$x-\frac{a}{2}=\pm\frac{7a}{2}$$
$$x=\frac{a}{2}\pm\frac{7a}{2}$$
$$x=\frac{a}{2}+\frac{7a}{2} \quad or \quad x=\frac{a}{2}-\frac{7a}{2}$$
$$=\frac{8a}{2} \qquad\qquad =-\frac{6a}{2}$$
$$=4a \qquad\qquad =-3a$$

Thus solution set = $\{-3a, 4a\}$

iii. $\frac{x}{x+1}+\frac{x+1}{x}=6$

Solution:

$$\frac{x}{x+1}+\frac{x+1}{x}=6$$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$

$$x^2+x^2+2x+1=6x(x+1)$$

$$2x^2+2x+1=6x^2+6x$$

$$2x^2-6x^2+2x-6x+1=0$$

$$-4x^2-4x+1=0$$

$$-(4x^2+4x-1)=0$$

$$\Rightarrow 4x^2+4x-1=0$$

The above equation is a quadratic equation.

iv. $\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$

Solution:

$$\frac{x+4}{x-2}-\frac{x-2}{x}+4=0$$

$$\frac{x(x+4)-(x-2)^2+4x(x-2)}{x(x-2)}=0$$

$$\left(x^2+4x\right)-\left(x^2-4x+4\right)+4\left(x^2-2x\right)=0$$

$$x^2+4x-x^2+4x-4+4x^2-8x=0$$

$$x^2-x^2+4x^2+4x+4x-8x-4=0$$

$$4x^2+8x-8x-4=0$$

$$4x^2-4=0$$

$$4\left(x^2-4\right)=0$$

$$x^2-1=0$$

The above equation is a pure quadratic equation.

v. $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

Solution:

$$\frac{x+3}{x+4}-\frac{x-5}{x}=1$$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$

$$\left(x^2+3x\right)-x(x-5)-4(x-5)=x(x+4)$$

$$x^2+3x-x^2+5x-4x+20=x^2+4x$$

$$x^2-x^2+3x+5x-4x+20=x^2+4x$$

$$4x+20=x^2+4x$$

$$-x^2+4x-4x+20=0$$

$$-x^2-20=0$$

$$-\left(x^2-20\right)=0$$

$$x^2-20=0$$

The above equation is a pure quadratic equation.

vi. $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$

Solution:

$$\frac{x+1}{x+2}+\frac{x+2}{x+3}=\frac{25}{12}$$

$$\frac{(x+1)(x+3)+(x+2)^2}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{x(x+3)+1(x+3)+\left(x^2+4x+4\right)}{x(x+3)+2(x+3)}=\frac{25}{12}$$

$$\frac{x^2+3x+x+3+x^2+4x+4}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$
$$25\left(x^2+5x+6\right)=12\left(2x^2+8x+7\right)$$
$$25x^2+125x+150=24x^2+96x+84$$
$$x^2+29x+66=0$$

The above equation is a quadratic equation.

Q2. Solve by factorization:

i. $x^2-x-20=0$

Solution:

$$x^2-x-20=0$$
$$x^2-5x+4x-20=0$$
$$x(x-5)+4(x-5)=0$$
$$(x+4)(x-5)=0$$

Either $x+4=0$ *or* $x-5=0$

$x=-4$ $\quad x=5$

Thus, solution set $=\{-4,5\}$

ii. $3y^2=y(y-5)$

Solution:

$$3y^2=y(y-5)$$
$$3y^2=y^2-5y$$
$$3y^2-y^2+5y=0$$
$$2y^2+5y=0$$
$$y(2y+5)=0$$

Either $y=0$ or $2y+5=0$

$$2y = -5$$
$$y = \frac{-5}{2}$$

Thus, solution set $= \left\{0, -\frac{5}{2}\right\}$

iii. $4 - 32x = 17x^2$

Solution:

$$4 - 32x = 17x^2$$

or $17x^2 + 32x - 4 = 0$

$$17x^2 + 34x - 2x - 4 = 0$$
$$17x(x+2) - 2(x+2) = 0$$
$$(17x-2)(x+2) = 0$$

Either $17x - 2 = 0$ *or* $x + 2 = 0$

$17x = 2$ $x = -2$

$$x = \frac{2}{17}$$

Thus, solution set $= \left\{\frac{2}{17}, -2\right\}$

iv. $x^2 - 11x = 152$

Solution:

$x^2 - 11x = 152$

$x^2 - 11x - 152 = 0$

$x^2 - 19x + 8x - 152 = 0$

$x(x-19) + 8(x-19) = 0$

$(x+8)(x-19) = 0$

Either $(x+8) = 0$ *or* $(x-19) = 0$

$x = -8 \qquad x = 19$

Thus, solution set = {-8,19}

v. $\dfrac{x+1}{x} + \dfrac{x}{x+1} = \dfrac{25}{12}$

Solution:

$\dfrac{x+1}{x} + \dfrac{x}{x+1} = \dfrac{25}{12}$

$\dfrac{(x+1)^2 + x^2}{x(x+1)} = \dfrac{25}{12}$

$\dfrac{x^2 + 2x + 1 + x^2}{x^2 + x} = \dfrac{25}{12}$

$\dfrac{2x^2 + 2x + 1}{x^2 + x} = \dfrac{25}{12}$

$25(x^2 + x) = 12(2x^2 + 2x + 1)$

$25x^2 + 25x = 24x^2 + 24x + 12$

$25x^2 - 24x^2 + 25x - 24x - 12 = 0$

$x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$x(x+4) - 3(x+4) = 0$

$(x-3)(x+4) = 0$

Either $x - 3 = 0$ *or* $x + 4 = 0$

$x = 3 \qquad x = -4$

Thus, solution set = $\{3, -4\}$

vi. $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

Solution:

$$\frac{2}{x-9}=\frac{1}{x-3}-\frac{1}{x-4}$$
$$\frac{2}{x-9}=\frac{(x-4)-(x-3)}{(x-3)(x-4)}$$
$$\frac{2}{x-9}=\frac{x-4-x+3}{x^2-7x+12}$$
$$\frac{2}{x-9}=\frac{-1}{x^2-7x+12}$$
$$2\left(x^2-7x+12\right)=-1(x-9)$$
$$2x^2-14x+24=-x+9$$
$$2x^2-14x+x+24-9=0$$
$$2x^2-13x+15=0$$
$$2x^2-10x-3x+15=0$$
$$2x(x-5)-3(x-5)=0$$
$$(2x-3)(x-5)=0$$

Either $2x-3=0$ *or* $x-5=0$

$2x=3 \qquad x=5$

$x=\dfrac{3}{2} \qquad x=5$

Thus solution set $=\{5,\dfrac{3}{2}\}$

Q3. Solve the following equations by completing square:

i. $7x^2+2x-1=0$

Solution:

$$7x^2+2x-1=0$$
$$7x^2+2x=1$$
$$\frac{7x^2}{7}+\frac{2x}{7}=\frac{1}{7}$$
$$x^2+\frac{2x}{7}=\frac{1}{7}$$
$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$
$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$
$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Takin square root on both sides, we get

$$x+\frac{1}{7}=\pm\sqrt{\frac{8}{49}}$$
$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$
$$x=-\frac{1}{7}\pm\frac{2\sqrt{2}}{7}$$
$$x=\frac{-1\pm2\sqrt{2}}{7}$$

Thus, solution set = $\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

ii. $ax^2+4x-a=0$, $a\neq 0$

Solution:

$$ax^2+4x=a$$

$$\frac{ax^2}{a}+\frac{4x}{a}=\frac{a}{a}$$

$$x^2+\frac{4x}{a}=1$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root on both sides, we get

$$x+\frac{2}{a}=\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x=-\frac{2}{a}\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

Thus, solution set $=\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

iii. $11x^2-34x+3=0$

Solution:

$$11x^2-34x+3=0$$

$$11x^2-34x=-3$$

$$\frac{11x^2}{11}-\frac{34x}{11}=-\frac{3}{11}$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{34}{22}\right)+\left(\frac{34}{22}\right)^2=-\frac{3}{11}+\left(\frac{34}{22}\right)^2$$

$$\left(x-\frac{34}{22}\right)^2=-\frac{3}{11}+\frac{1156}{484}$$