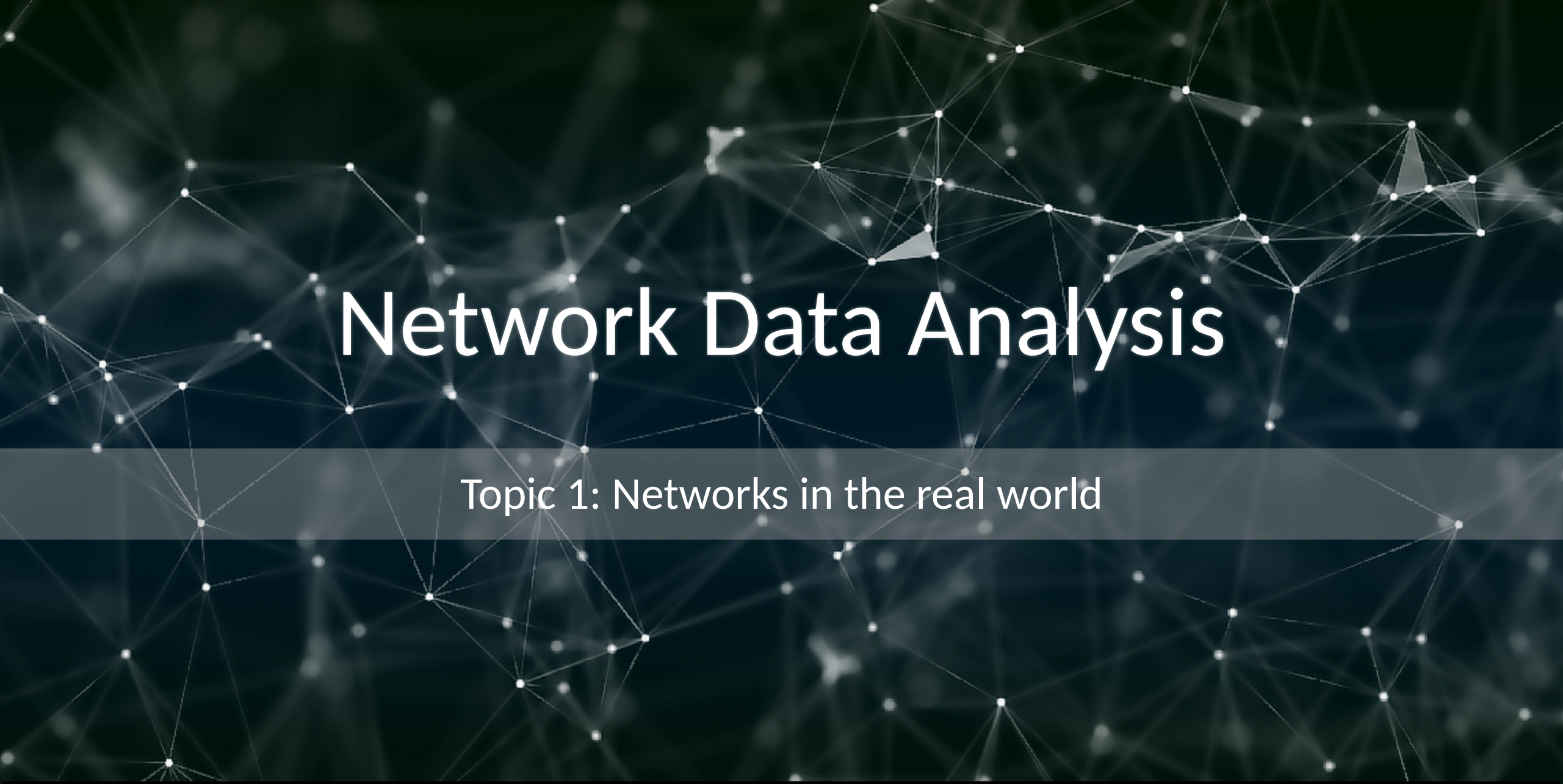
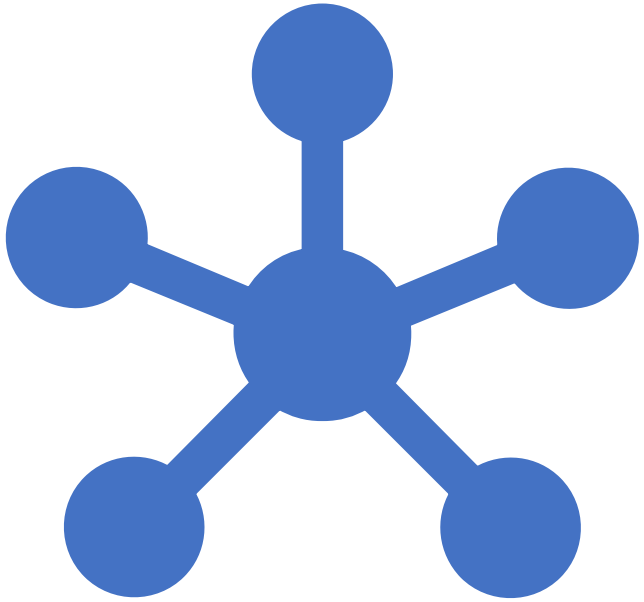


Network Data Analysis

Topic 1: Networks in the real world

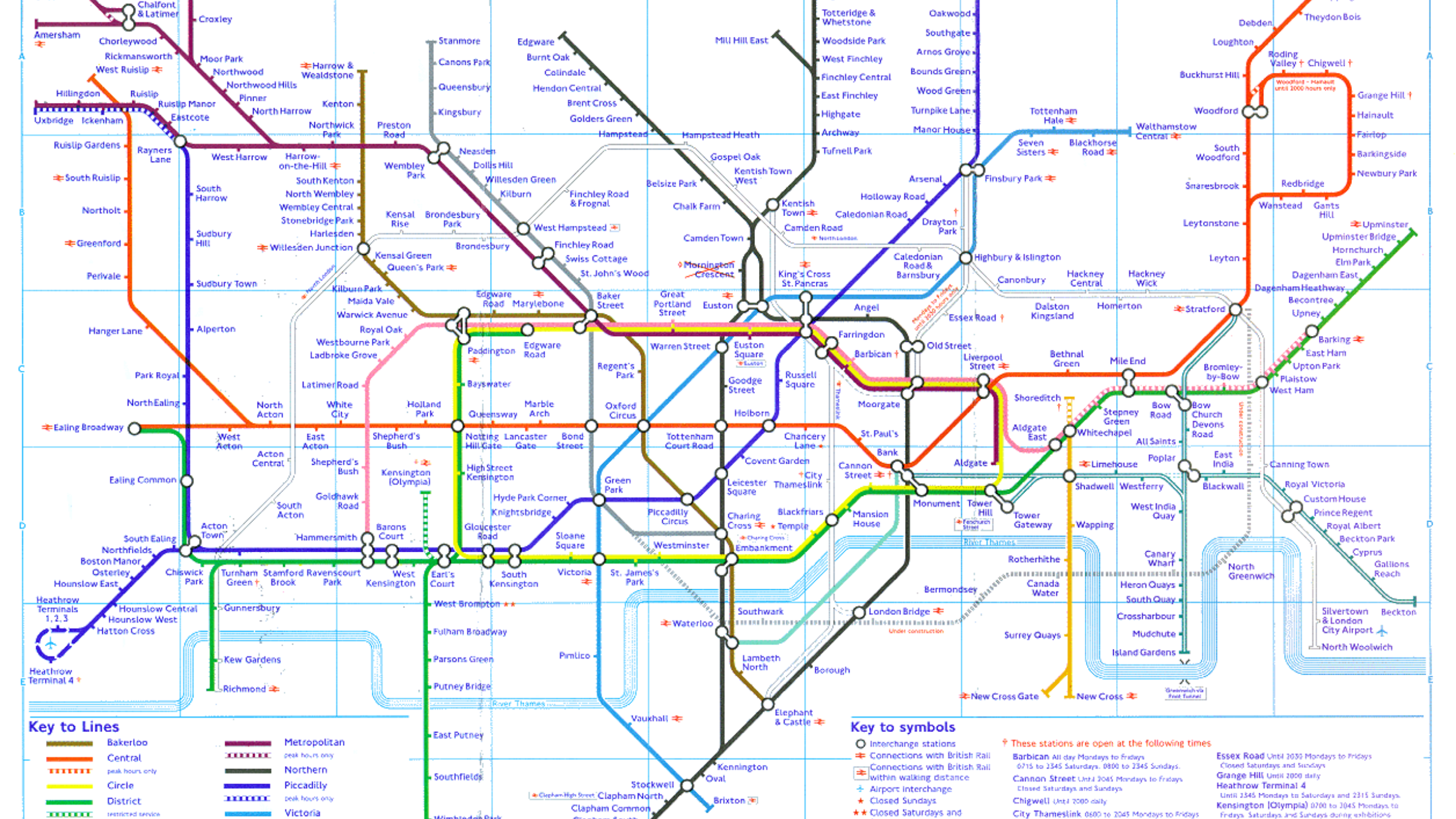
KING'S College LONDON

Albert Meroño-Peñuela
Jacopo de Berardinis



What is a graph?

- A graph, or network, is a set of items (***nodes*** or *vertices*) connected by their relations (***edges*** or *links*).
- Networks are present in the everyday life of all people.
  - Social: Friends & family, society
  - Transport: Rail, road, public transport
  - Information: World economy
  - Biology: Brain cells
  - Communication: Internet
- Consciously or not, we use networks and their properties as a universal language for describing complex data.
  - Networks from science, nature, and technology are more similar than one would expect.
  - Shared vocabulary between fields, including computer science, social science, physics, economics, statistics, biology.


Key to Lines
Bakerloo
Central
peak hours only
Circle
District
restricted service
Metropolitan
peak hours only
Northern
Piccadilly
peak hours only
Victoria
Key to symbols
Interchange stations
Connections with British Rail
Connections with British Rail within walking distance
Airport interchange
Closed Sundays
Closed Saturdays and
These stations are open at the following times
Barbican All day Mondays to Fridays 0715 to 2345 Saturdays. 0800 to 2345 Sundays.
Cannon Street Until 2045 Mondays to Fridays Closed Saturdays and Sundays
Chigwell Until 2000 daily
City Thameslink 0600 to 2045 Mondays to Fridays
Essex Road Until 2030 Mondays to Fridays Closed Saturdays and Sundays
Grange Hill Until 2000 daily
Heathrow Terminal 4 Until 2345 Mondays to Saturdays and 2315 Sundays.
Kensington (Olympia) 0700 to 2045 Mondays to Fridays. Saturdays and Sundays during exhibitions
Amersham
Chalfont & Latimer
Chorleywood
Rickmansworth
West Ruislip
Croxley
Moor Park
Northwood
Northwood Hills
Pinner
North Harrow
Harrow & Wealdstone
Hillingdon
Ruislip
Ruislip Manor
Eastcote
Uxbridge
Ickenham
Ruislip Gardens
Rayners Lane
West Harrow
Harrow-on-the-Hill
Kenton
Northwick Park
Preston Road
South Ruislip
South Harrow
Northolt
Sudbury Hill
Greenford
Perivale
Sudbury Town
Hanger Lane
Alperton
Park Royal
North Ealing
Ealing Broadway
Ealing Common
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Heathrow Terminals 1, 2, 3
Hounslow Central
Hounslow West
Hatton Cross
Heathrow Terminal 4
Chiswick Park
Acton Town
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
Stanmore
Canons Park
Queensbury
Kingsbury
Wembley Park
North Wembley
Wembley Central
Stonebridge Park
Harlesden
Willesden Junction
Kensal Green
Queen's Park
Kilburn Park
Maida Vale
Warwick Avenue
Edgware
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Neasden
Dollis Hill
Willesden Green
Kilburn
Finchley Road & Frognal
Kensal Rise
Brondesbury Park
Brondesbury
West Hampstead
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Swiss Cottage
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Edgware Road
Marylebone
Baker Street
Westbourne Park
Ladbroke Grove
Latimer Road
White City
Shepherd's Bush
Holland Park
Royal Oak
Paddington
Edgware Road
Bayswater
Queensway
Notting Hill Gate
Lancaster Gate
Marble Arch
Bond Street
North Acton
West Acton
East Acton
Acton Central
South Acton
Shepherd's Bush
Goldhawk Road
Kensington (Olympia)
High Street Kensington
Hyde Park Corner
Knightsbridge
Barons Court
Hammersmith
Gloucester Road
Earl's Court
West Kensington
South Kensington
Sloane Square
Victoria
St. James's Park
Green Park
Oxford Circus
Regent's Park
Warren Street
Great Portland Street
Euston
Euston Square
Goodge Street
Tottenham Court Road
Holborn
Chancery Lane
Russell Square
Piccadilly Circus
Leicester Square
Covent Garden
Charing Cross
Embankment
Westminster
Waterloo
Southwark
Lambeth North
Borough
Pimlico
Vauxhall
Elephant & Castle
Kennington Oval
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Clapham High Street
Brixton
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Totteridge & Whetstone
Woodside Park
West Finchley
Finchley Central
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Gospel Oak
Kentish Town West
Belsize Park
Chalk Farm
Camden Town
Mornington Crescent
Kentish Town
Camden Road
Caledonian Road
Holloway Road
Arsenal
Finsbury Park
Oakwood
Southgate
Arnos Grove
Bounds Green
Wood Green
Turnpike Lane
Manor House
Tottenham Hale
Seven Sisters
Blackhorse Road
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Caledonian Road & Barnsbury
Highbury & Islington
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Bethnal Green
Mile End
Stratford
Shoreditch
Aldgate East
Aldgate
Whitechapel
Stepney Green
Bow Road
Bow Church
Devons Road
Bromley-by-Bow
All Saints
Poplar
East India
Limehouse
Shadwell
Westferry
Blackwall
Canning Town
Royal Victoria
Custom House
Prince Regent
Royal Albert
Beckton Park
Cyprus
Gallions Reach
Beckton
Silvertown & London City Airport
North Woolwich
Monument
Tower Hill
Fenchurch Street
Tower Gateway
Bank
Cannon Street
St. Paul's
Mansion House
Blackfriars
City Thameslink
Charing Cross
Temple
Wapping
Rotherhithe
Canada Water
Bermondsey
London Bridge
Surrey Quays
New Cross Gate
New Cross
Canary Wharf
North Greenwich
Heron Quays
South Quay
Crossharbour
Mudchute
Island Gardens
West India Quay
River Thames
Woodford
South Woodford
Buckhurst Hill
Loughton
Debden
Theydon Bois
Roding Valley
Chigwell
Woodford - Hainault until 2000 hours only
Grange Hill
Hainault
Fairlop
Barkingside
Newbury Park
Redbridge
Wanstead
Gants Hill
Snaresbrook
Leytonstone
Leyton
Upminster
Upminster Bridge
Hornchurch
Elm Park
Dagenham East
Dagenham Heathway
Becontree
Upney
Barking
East Ham
Upton Park
Plaistow
West Ham
Pudding to Fulham until 2030 hours only
Under construction
Greenwich via Pivot Tunnel
Richmond
Kew Gardens
Gunnersbury
Turnham Green
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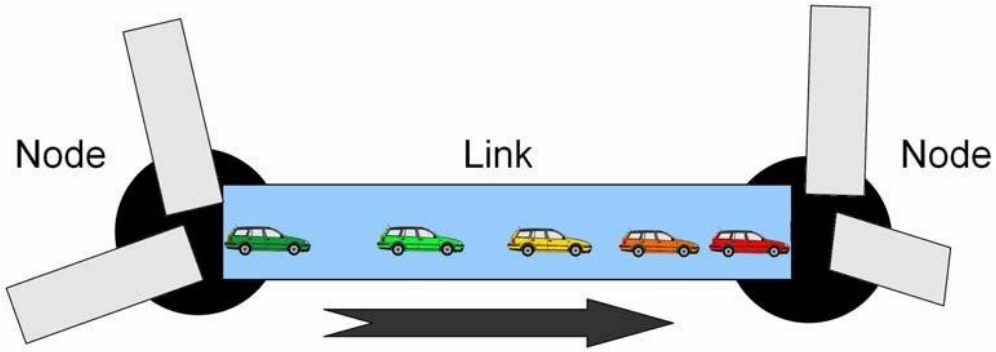
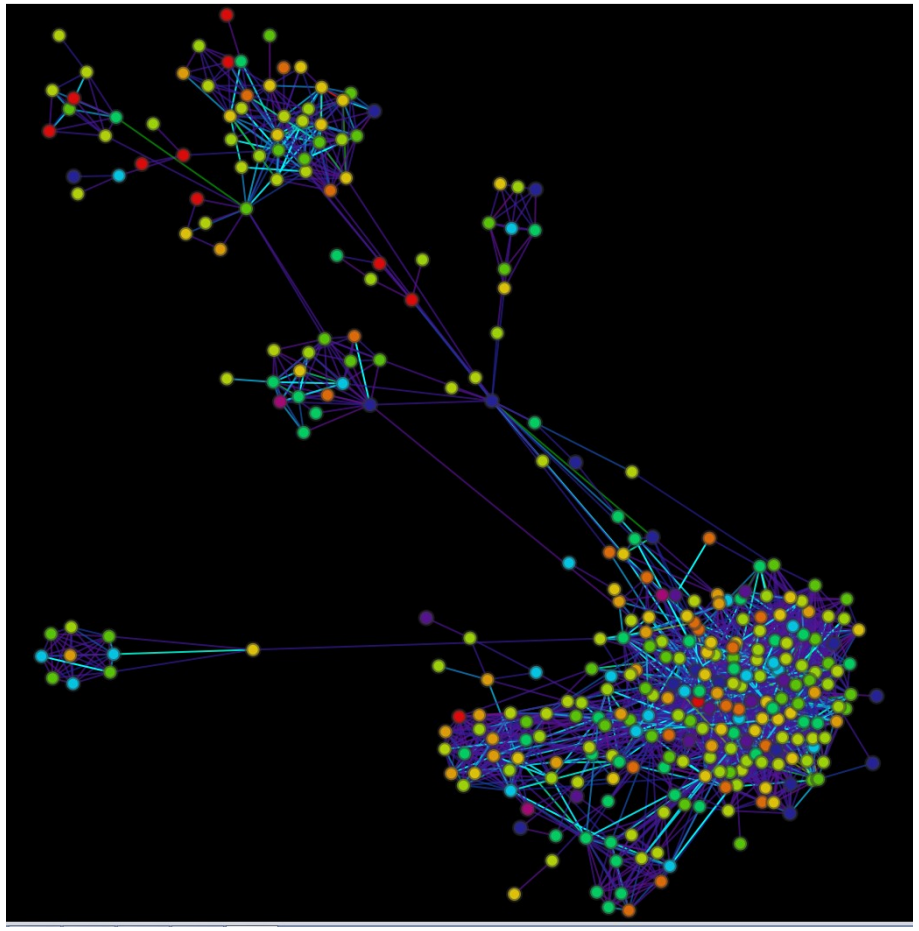
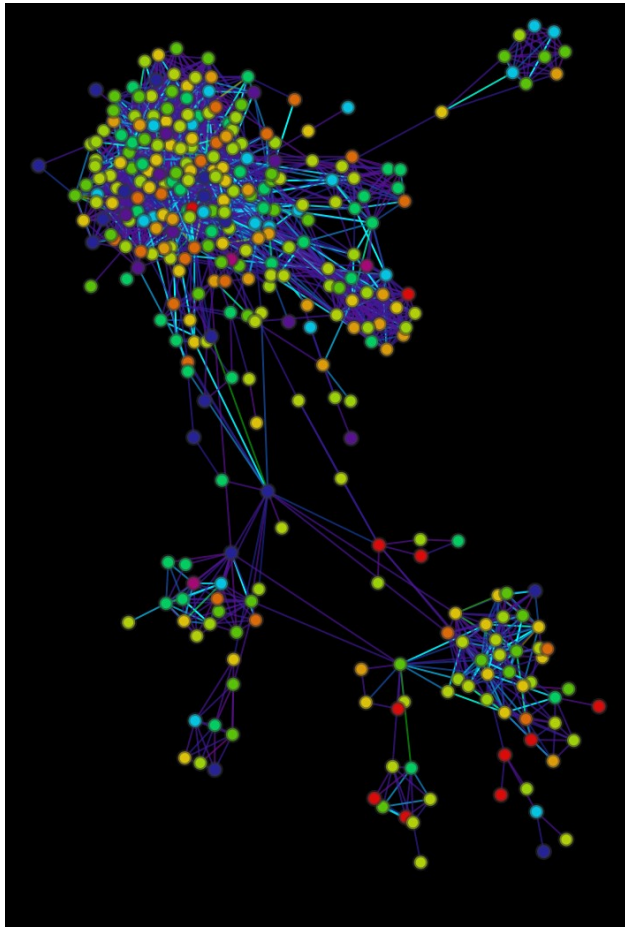
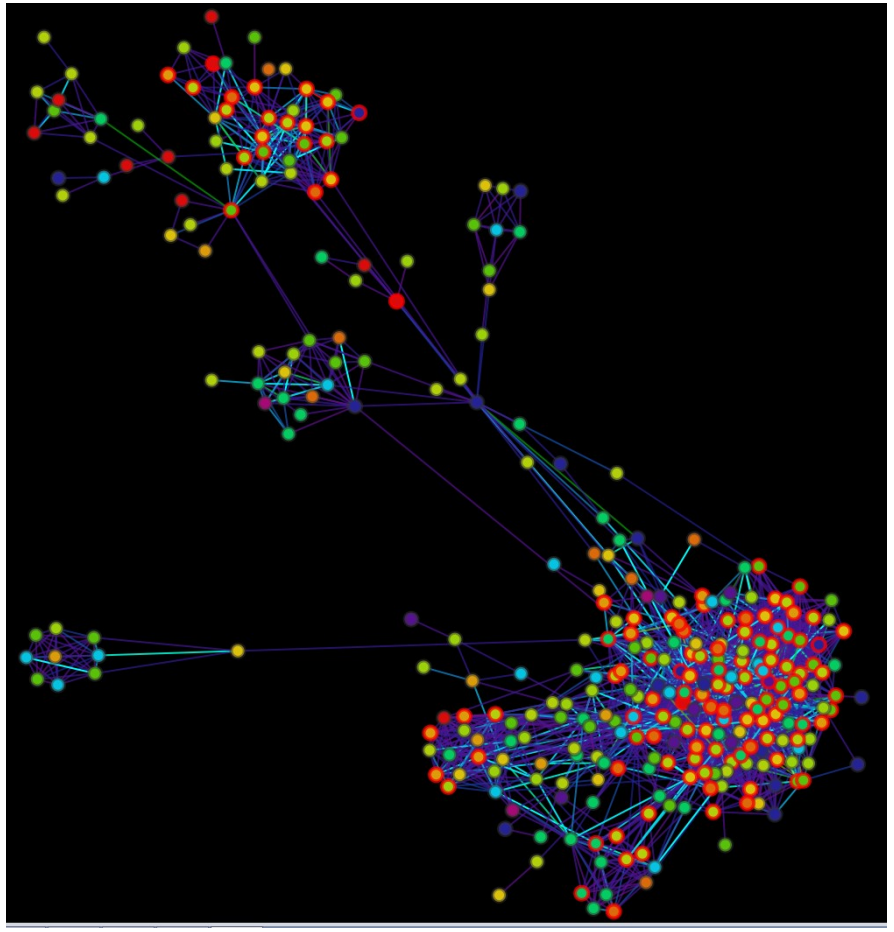
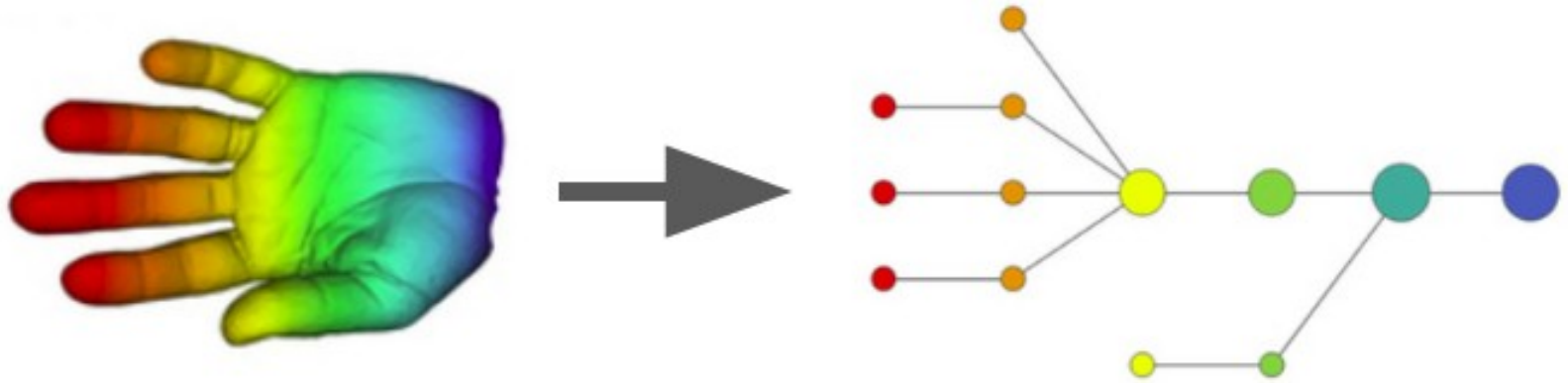
Upney
Barking
East Ham
Upton Park
Plaistow
West Ham
Pudding to Fulham until 2030 hours only
Under construction
Greenwich via Pivot Tunnel

Agent-based modelling of traffic



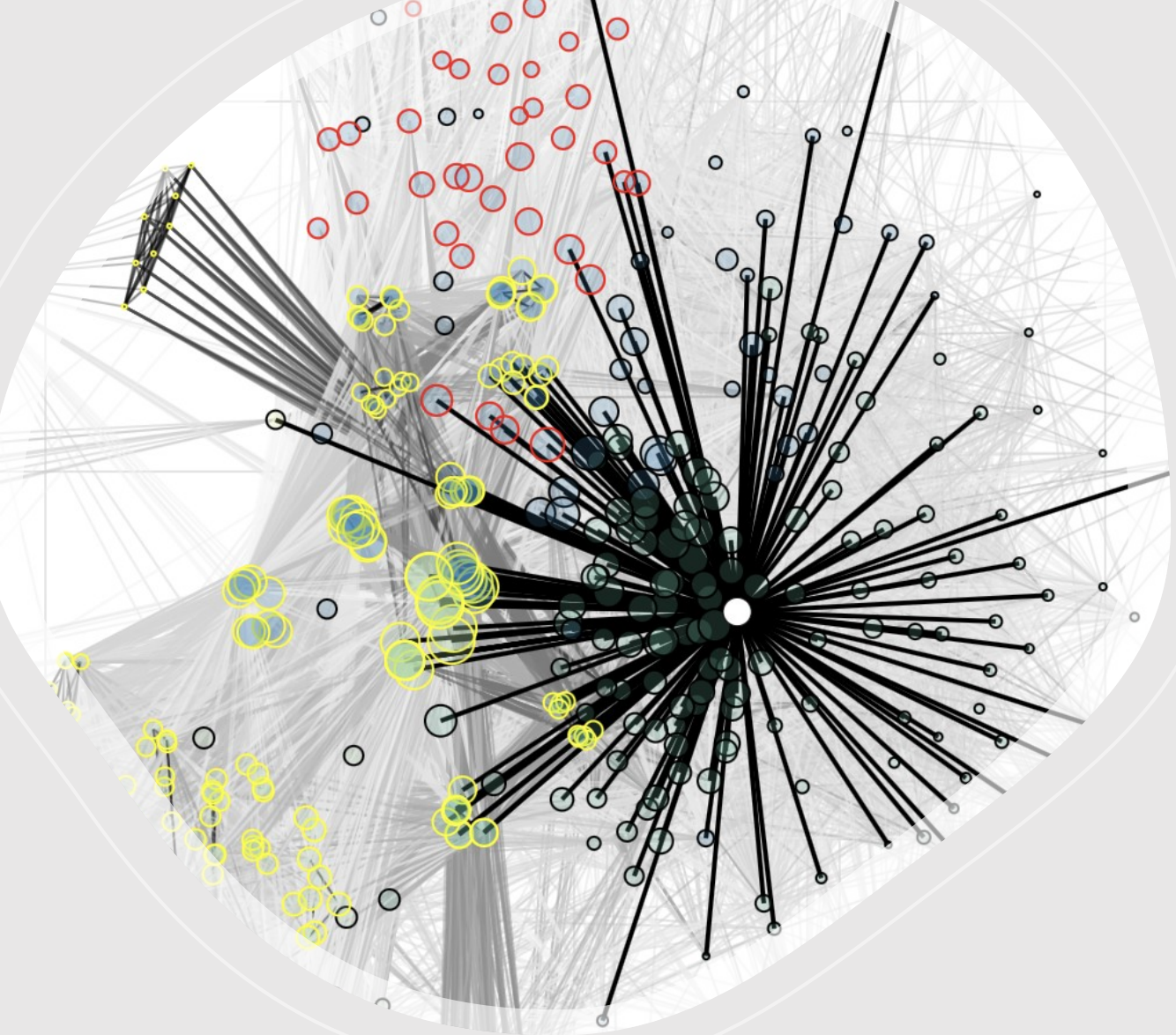




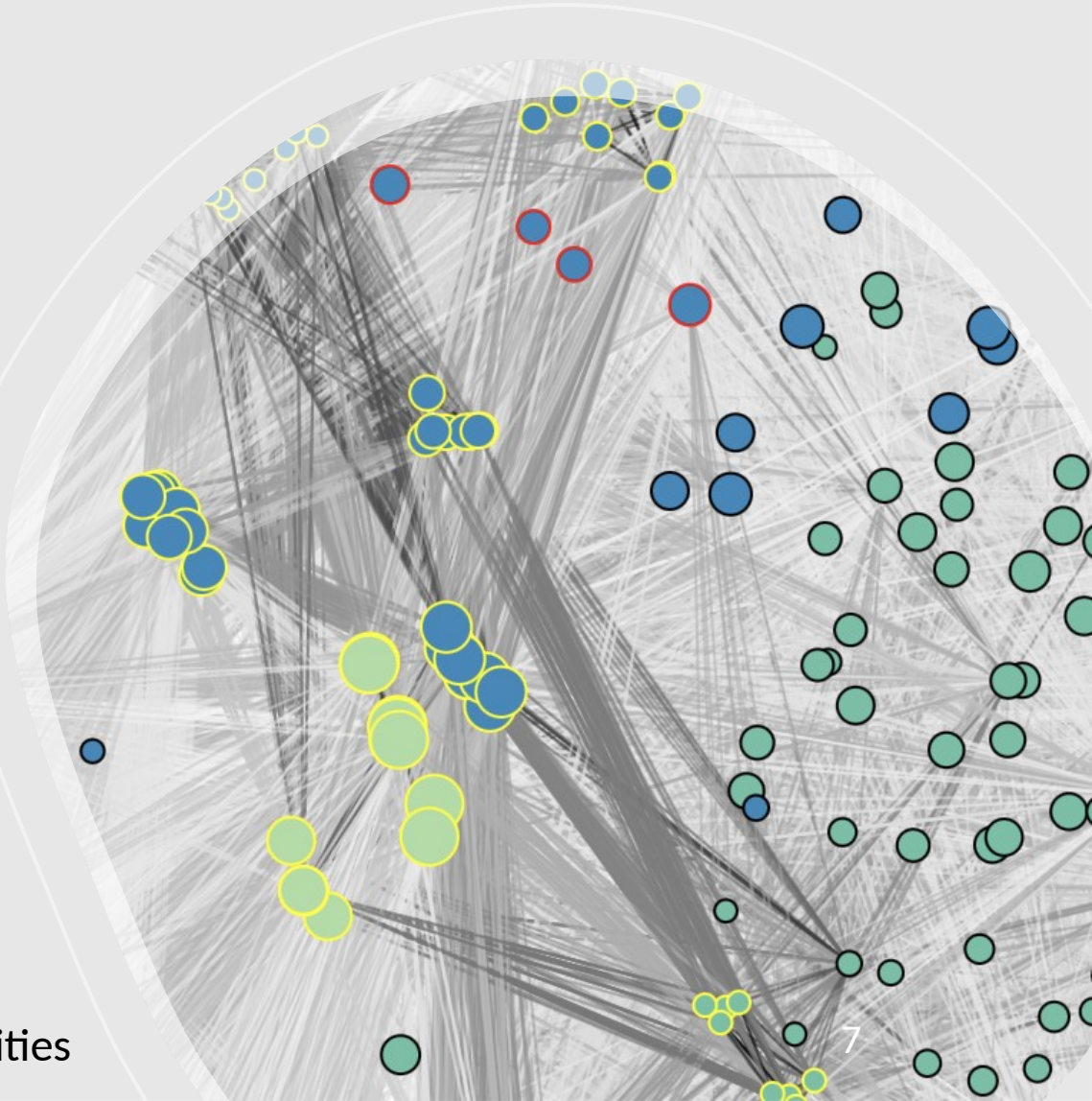
Information spread in social networks



Topological Data Analysis: looking at “the shape of data”

Polifonia





The Harmonic Network

Nodes are tracks, Edges express harmonic similarities

Networks in the world

Social and economic networks: People or groups with contacts or interactions between them

- Friendship networks, business relations between companies, intermarriages between families, labour markets
- **Example**: To understand disease spread, you need to know social networks.
- **Questions**: Degree of connectedness, small-world effects

Information networks: Connections of information objects

- Academic paper citations, network of webpages with links to others
- **Example**: To understand news dissemination, you need to know information networks.
- **Questions**: Ranking, navigation

Spatial networks: Networks where nodes and edges have location

- Road networks, migration destinations, river systems
- **Example**: To understand whether crime is distributed according to space, you need to know street networks
- **Questions**: Network-based clustering or correlations

Study of networks

Empirical: Study network data to find organisational principles

How do we measure and quantify networks?

Mathematical models: Graph theory, statistical models

Models allow us to understand behaviors and distinguish surprising from expected phenomena

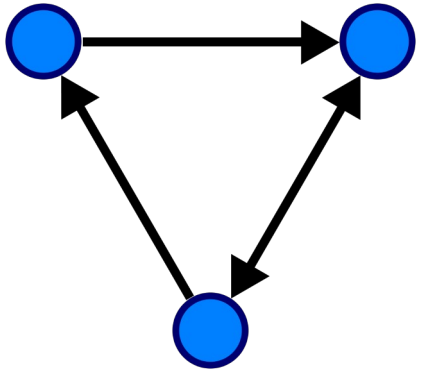
Algorithms for analysing graphs

Hard computational challenges!

Historical study of networks with mathematical graph theory

Graphs and their properties

Graphs



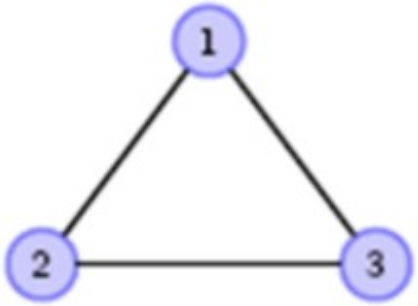
- A **graph** is a mathematical representation of a network, and the basic data structure for analysing network data
- A graph consists of a set of **nodes** and a set of **edges**

$$G = (V, E)$$

Graph representations

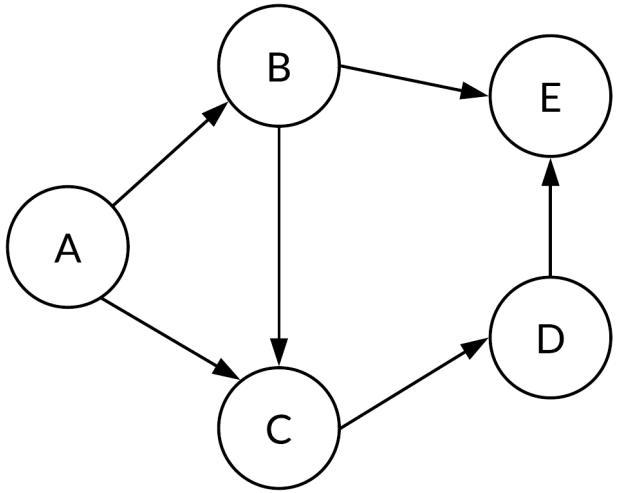
- **Adjacency matrix:** the graph is represented by a tuple **(N, g)**, where **N = {1, 2, …, n}** is a set of **n** nodes, and **g** is a **n x n** matrix (the adjacency matrix). Each element of the matrix $\mathbf{g_{ij}}$ has value **1** if there is an edge between nodes **i** and **j**, and **0** if there is not.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow$$



Graph representations

- **Adjacency list:** represented by a tuple **(N, E)**, where **N = {1, 2, ..., n}** is a set of **n** nodes as before and **E** is a set of edges, where each edge is a tuple of the two nodes connected by the edge, e.g. **E = {(1, 2), (1, 3), (2, 3)}**.

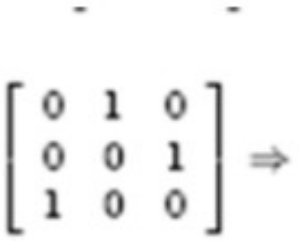


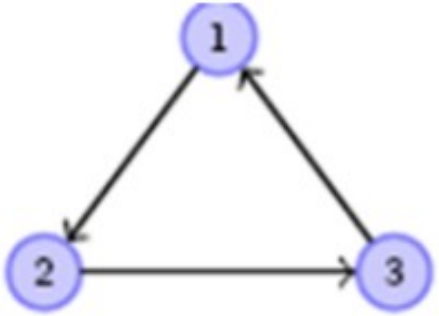

```
Adjacency list:

graph = {
'A': ['B', 'C'],
'B': ['C', 'E'],
'C': ['D'],
'D': ['E'],
}
```

Directed and undirected graphs

- Directed graph: each edge has a direction, going from the **source** node to the **target** (or sink) node. For example, in the directed graph with edges **E = {(1, 2), (1, 3), (2, 3)}**, there is an edge from node 1 (the source) to node 2 (the target) but no edge from node 2 to node 1
- Undirected graph: edges have no direction, e.g. in an undirected graph with edges **E = {(1, 2), (1, 3), (2, 3)}**, there is a connection between nodes 1 and 2 but no notion of whether that goes ‘from’ or ‘to’ each node
- **Underlying undirected graph** of a directed graph is the undirected graph with the same nodes and edges but ignoring any direction



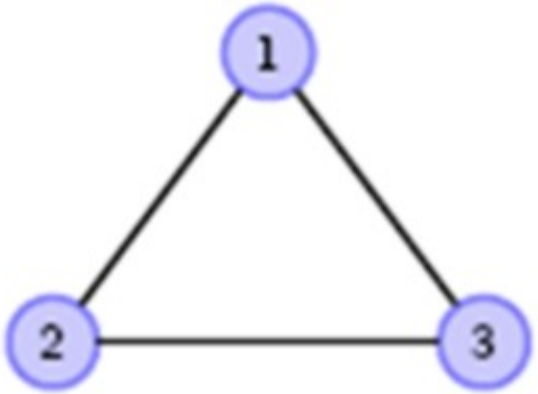


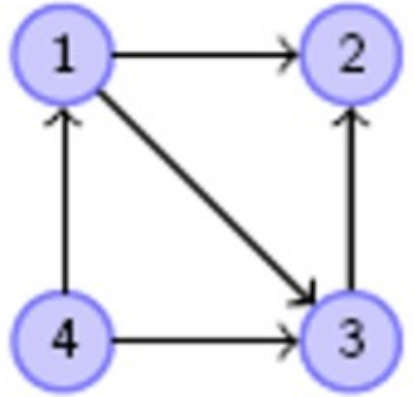
Graph walks

- **Graph walk:** a series of connected edges representing a journey through the graph from one node via other nodes to a final node following the edges. If the graph is directed, then the walk must follow the direction of the edges
- **Graph path:** a graph walk where no node is repeated except possibly the start node if it is the same as the end node

Neighbourhood and degree

- The **neighbourhood** of a node is the set of nodes it is connected to
- The **degree** of a node is the number of edges of that node
  - For undirected graphs, the **degree** of a node is the number of edges that involve that node, i.e. the size of its neighbourhood
  - For directed graphs, the **in-degree** of a node is the number of edges for which the node is the target, while the **out-degree** of a node is the number of edges for which the node is the source





Properties of graphs

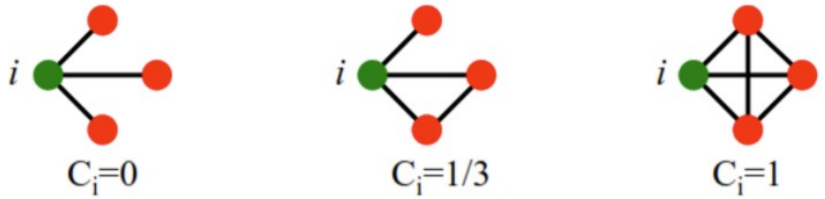
- **Degree distribution: P(k) = N(k)/n** probability of a random node having degree k, for a graph of **n** nodes and **N(k)** is the number of nodes in the graph with degree **k**
- Path **length**: number of edges that the path contains. The **shortest path** between two nodes is the path with the shortest length
- **Distance** between two nodes: length of the shortest path between them
- **Diameter:** longest distance between nodes in a connected graph.

Properties of graphs

- **Average path length:** mean average distance between any two nodes in the network

$$l_G = \frac{1}{n \cdot (n-1)} \cdot \sum_{i \neq j} d(v_i, v_j)$$

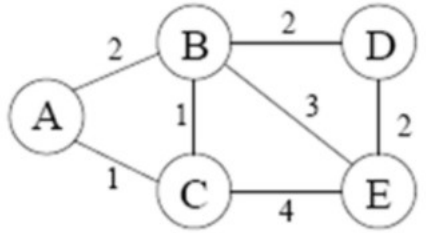
- **Clustering coefficient** of a node is the proportion of its neighbours that are connected to each other. $\mathbf{C_i = e / m}$ is the clustering coefficient of node **I** (m = total possible edges between neighbours; e = actual number of edges between neighbours)

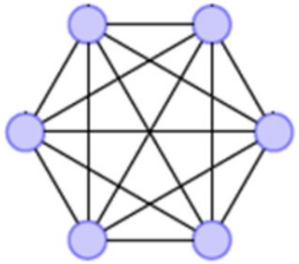


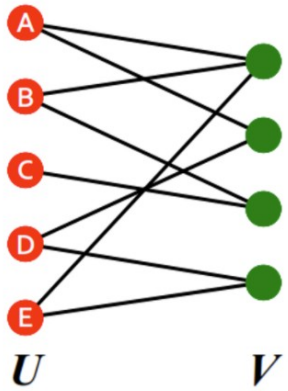
- **Average clustering**: mean average of the clustering coefficients of its nodes

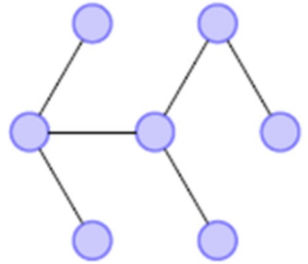
Types of graphs

- **Weighted graph:** labelled graph where edges have numeric labels, e.g. weights
- **Complete graph:** graph where every node is connected to every other node (maximum number of edges, i.e. n(n - 1) / 2)
- **Bipartite graph**: nodes can be divided into two disjoint sets U and V, where every edge connects a node in U to a node in V
- **Tree:** connected undirected graph (with n – 1 edges)

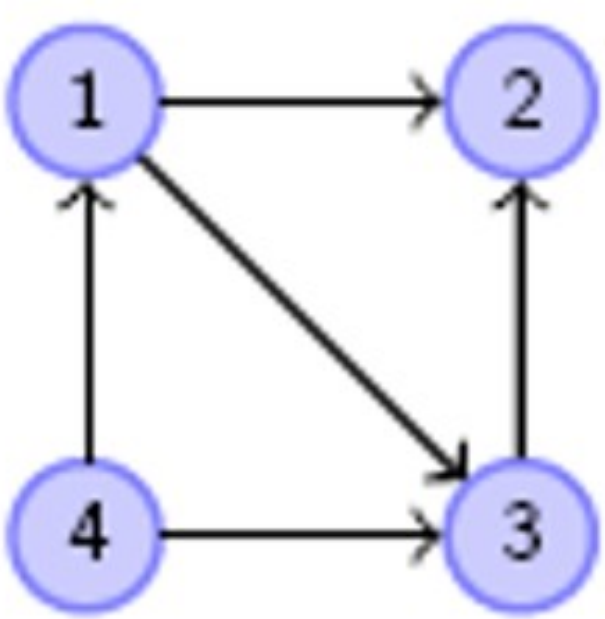






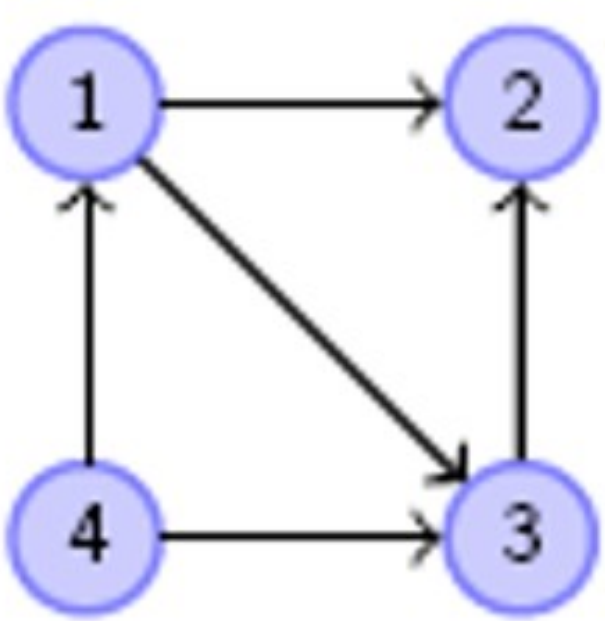


TUTORIAL EXERCISE 1



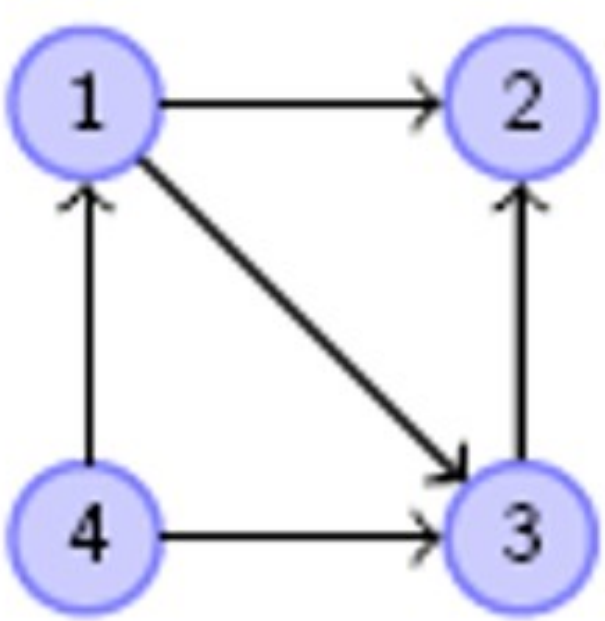
In the out-degree distribution P(k), what is the value of P(2)?

TUTORIAL EXERCISE 2



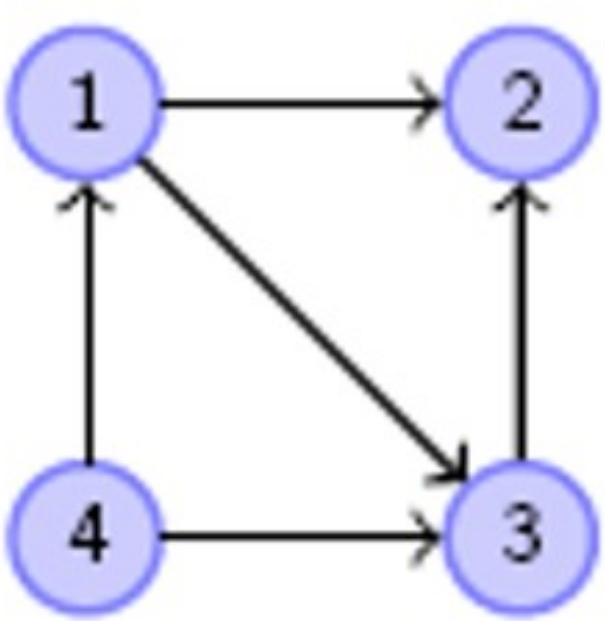
What is the diameter of the underlying undirected graph?

TUTORIAL EXERCISE 3



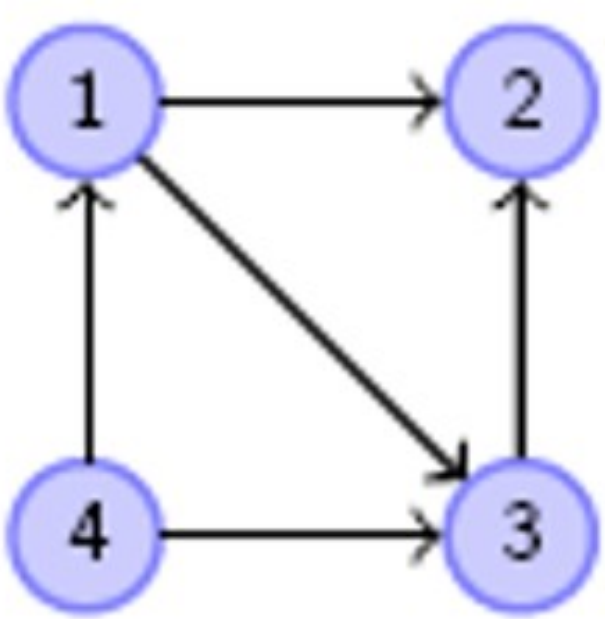
What is the average path length of the underlying undirected graph?

TUTORIAL EXERCISE 4



What is the clustering coefficient of node 1 in the underlying undirected graph?

TUTORIAL EXERCISE 5



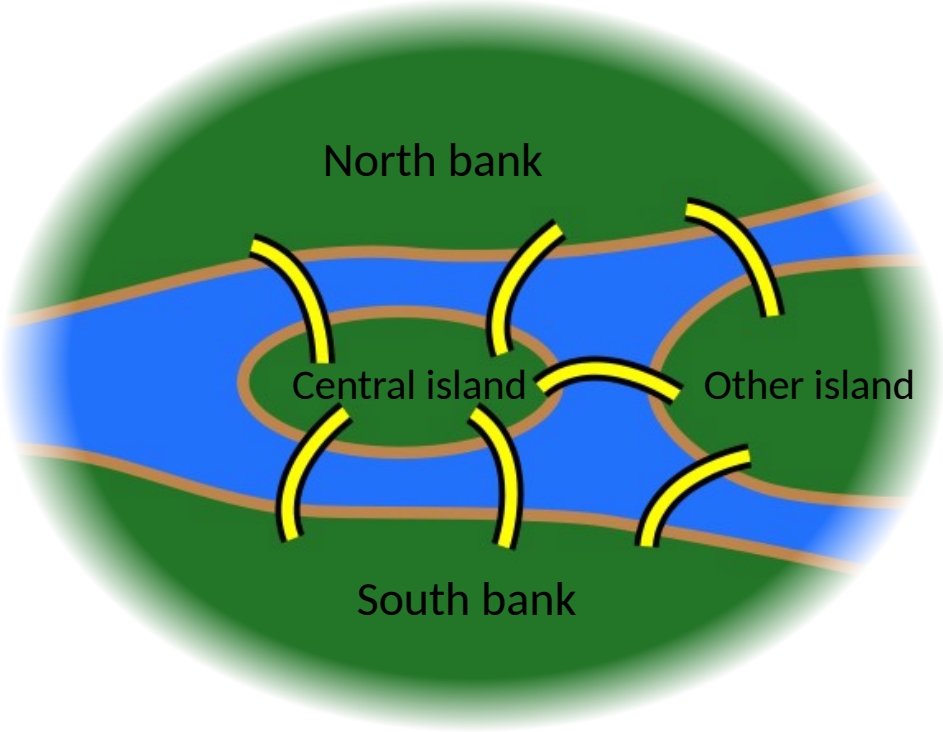
How many of the following sequences could be returned by a depth-first search from node 4?

[4, 1, 2, 3]	[4, 1, 3, 2]
[4, 2, 1, 3]	[4, 2, 3, 1]
[4, 3, 1, 2]	[4, 3, 2, 1]

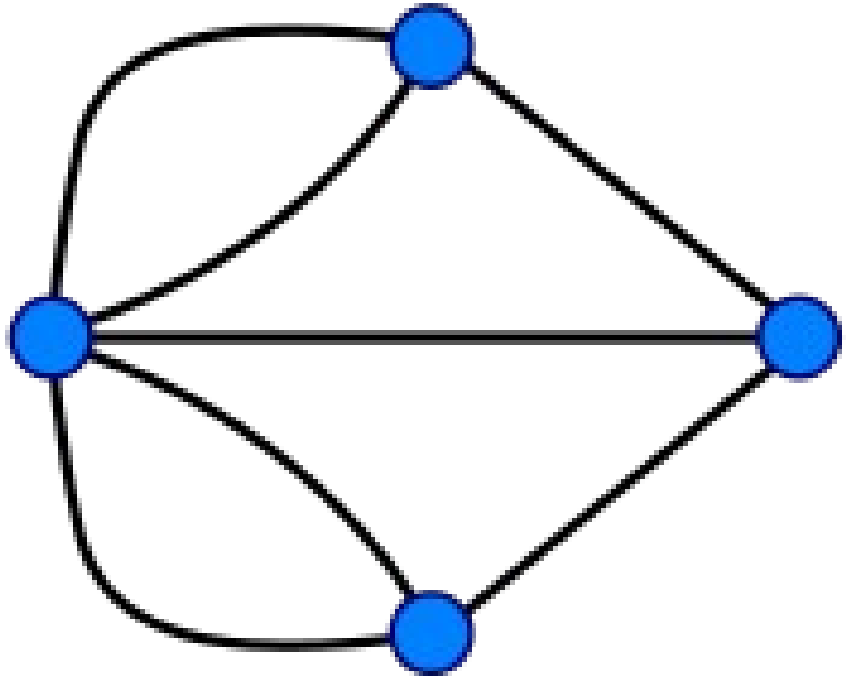
Origins of graph theory

Königsberg bridge problem

- The city of *Königsberg* in *Prussia* (now *Kaliningrad*, *Russia*) was set on both sides of the *Pregel* River
  - two large islands which were connected to each other and the mainland by seven bridges.
- The problem was to find a walk through the city that would cross each bridge once and only once.
  - The islands could not be reached other than by the bridges, and every bridge must have been crossed completely every time.
- Euler proved that the problem has **no** solution



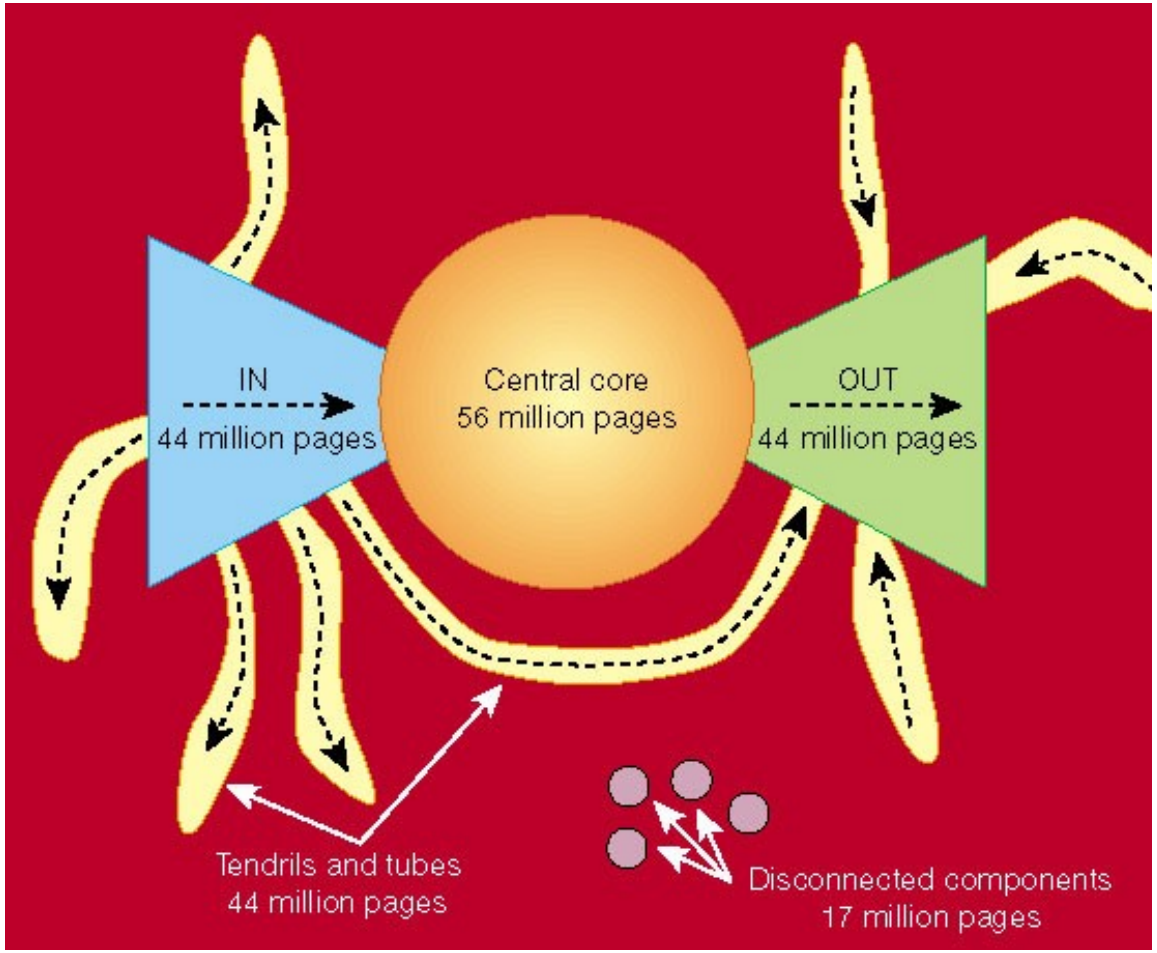
Königsberg bridge problem



Web as a Graph

- Web as a directed graph:
  - Nodes = web pages & Edges = hyperlinks
- Directed version of the Web graph:
  - Largest SCC: 28% of the nodes (56 M)
  - Taking a random node v
    - Out(v) ≈ 50%
    - In(v) ≈ 50%

"The web is a bowtie", Nature, vol. 405, May 2000



Weekly discussion topic

All students: post your thoughts on the module's discussion forum in KEATS

Imagine we are going to analyse how the housing market in a city functions.

- What networks could be relevant to consider in analysing this topic?
- What kinds of graph, as studied in this lesson, represent these networks?
- What might the properties studied in this lesson, such as diameter or cluster coefficient, applied to these networks tell you about the city?
- What public data sets can you find that provide you with data on the networks you've identified?
- Who might hold private network data sets that would also be valuable in this analysis?