Game Playing and Adversarial Search

Chapter 5, Sections 1–5

Outline

◇ Perfect play

◇ Resource limits

◇ α–β pruning

◇ Games of chance

Games vs. search problems

"Unpredictable" opponent $\Rightarrow$ solution is a contingency plan

Time limits $\Rightarrow$ unlikely to find goal, must approximate

Plan of attack:

- algorithm for perfect play (Von Neumann, 1944)
- finite horizon, approximate evaluation (Zuse, 1945; Shannon, 1950; Samuel, 1952–57)
- pruning to reduce costs (McCarthy, 1956)

Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information		bridge, poker, scrabble nuclear war

Representing a game as a search problem

We can formally define a strategic two-player game by:

- initial state
- actions
- terminal test (i.e. win / lose / draw)
- utility function (i.e. numeric reward for outcome)
  chess: +1, 0, -1
  poker: cash won or lost

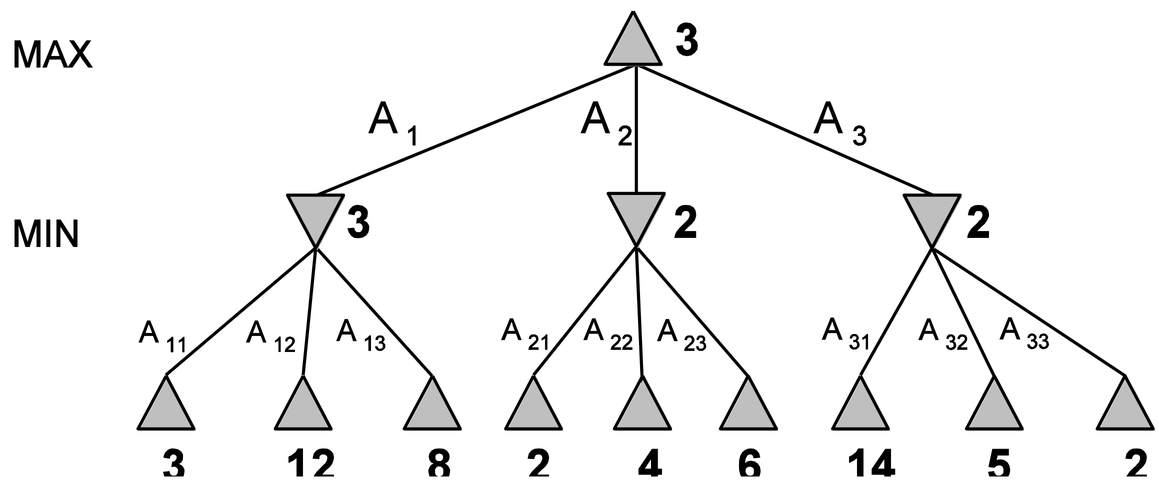
In a zero-sum game with 2 players:
each player's utility for a state are equal and opposite

Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest *minimax value*
= best achievable payoff against best play

E.g., 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(game) returns an operator

   for each op in OPERATORS[game] do
       VALUE[op] ← MINIMAX-VALUE(APPLY(op, game), game)
   end
   return the op with the highest VALUE[op]
```

```
function MINIMAX-VALUE(state, game) returns a utility value

   if TERMINAL-TEST[game](state) then
       return UTILITY[game](state)
   else if  MAX is to move in state then
       return the highest MINIMAX-VALUE of SUCCESSORS(state)
   else
       return the lowest MINIMAX-VALUE of SUCCESSORS(state)
```

Properties of minimax

Complete??

Optimal??

Time complexity??

Space complexity??

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity?? $O(b^m)$

Space complexity?? $O(bm)$ (depth-first exploration)

For chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
$\Rightarrow$ exact solution completely infeasible

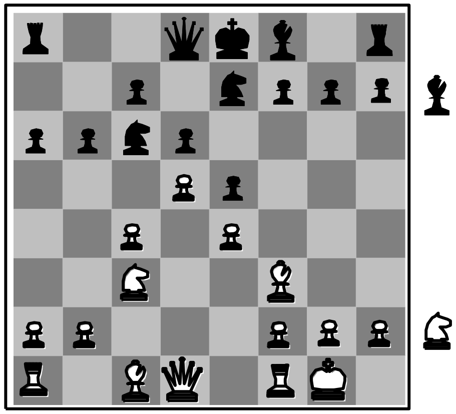
Resource limits

Suppose we have 100 seconds, explore 10^4 nodes/second
$\Rightarrow \underline{10^6}$ nodes per move

Standard approach:

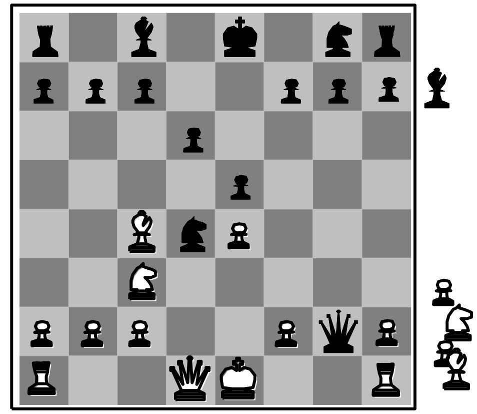
- *cutoff test*
  e.g., depth limit (perhaps add *quiescence search*)
- *evaluation function*
  = estimated desirability of position

Evaluation functions



Black to move

White slightly better



White to move

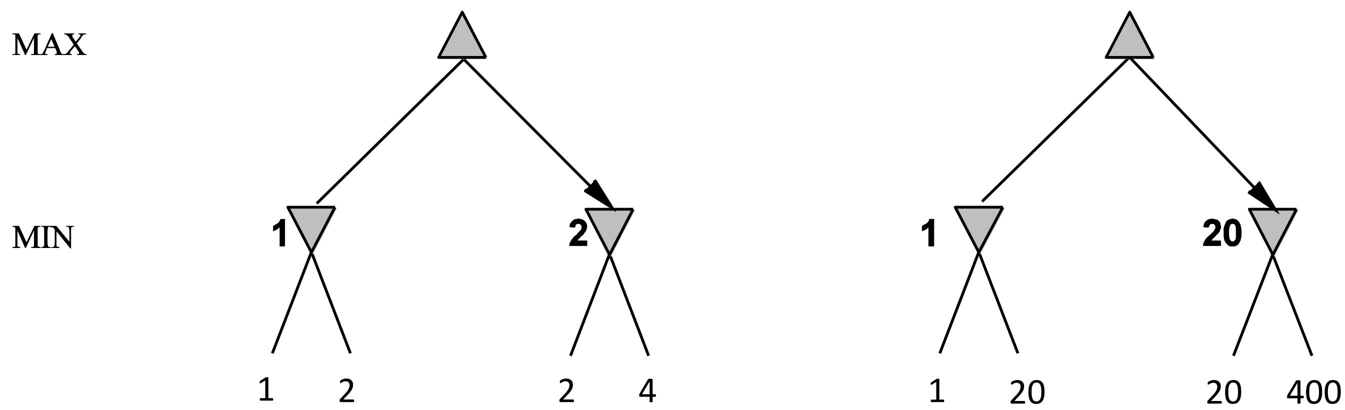
Black winning

For chess, typically *linear* weighted sum of <u>features</u>

$$\mathrm{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens)
etc.

Digression: Exact values don't matter



Behaviour is preserved under any *monotonic* transformation of EVAL

Only the order matters:
payoff in deterministic games acts as an *ordinal utility* function

Cutting off search

MINIMAXCUTOFF is identical to MINIMAXVALUE except

1. TERMINAL? is replaced by CUTOFF?
2. UTILITY is replaced by EVAL

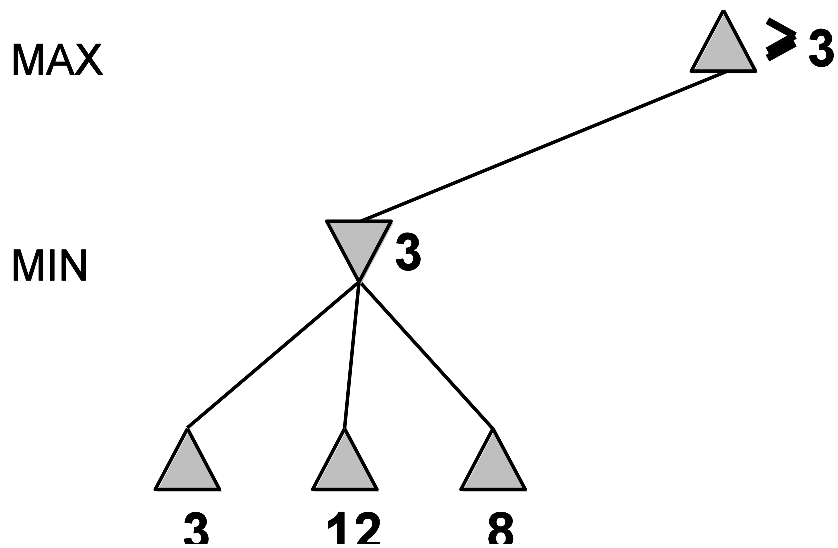
Does it work in practice?

$$b^m = 10^6, \quad b = 35 \quad \Rightarrow \quad m = 4$$

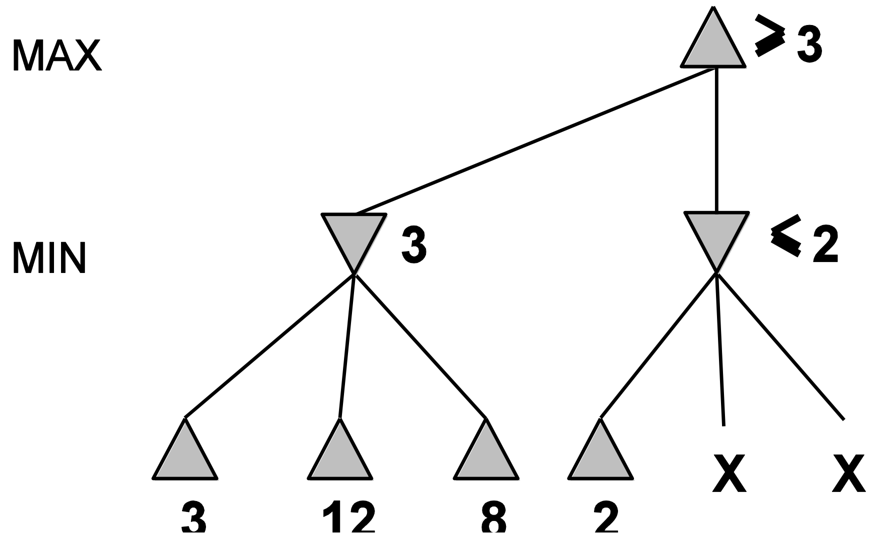
4-ply lookahead is a hopeless chess player!

4-ply $\approx$ human novice
8-ply $\approx$ typical PC, human master
12-ply $\approx$ IBM's Deep Blue, Kasparov

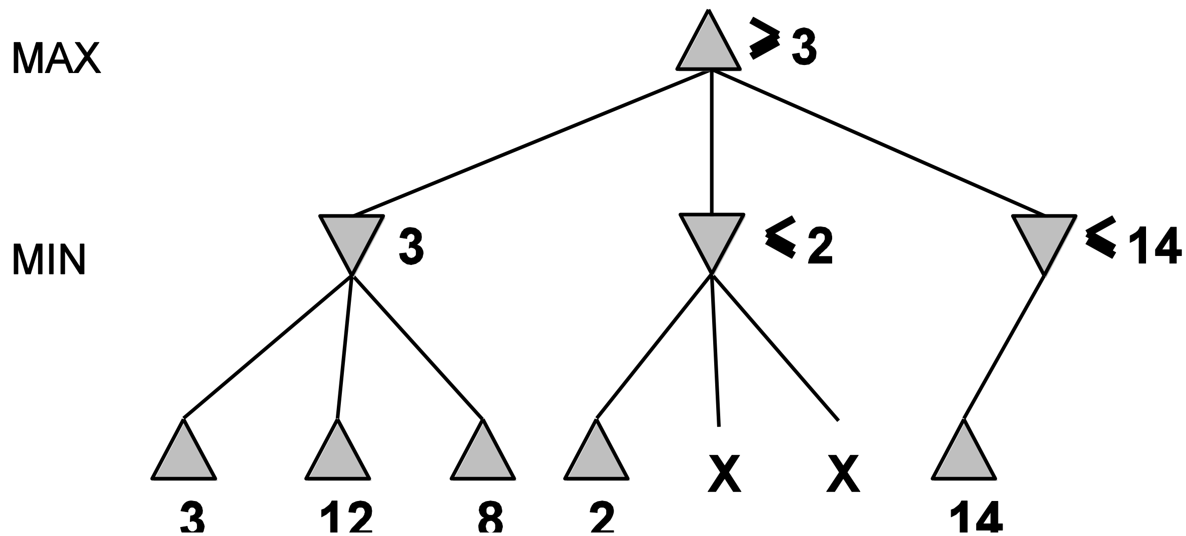
α–β pruning example



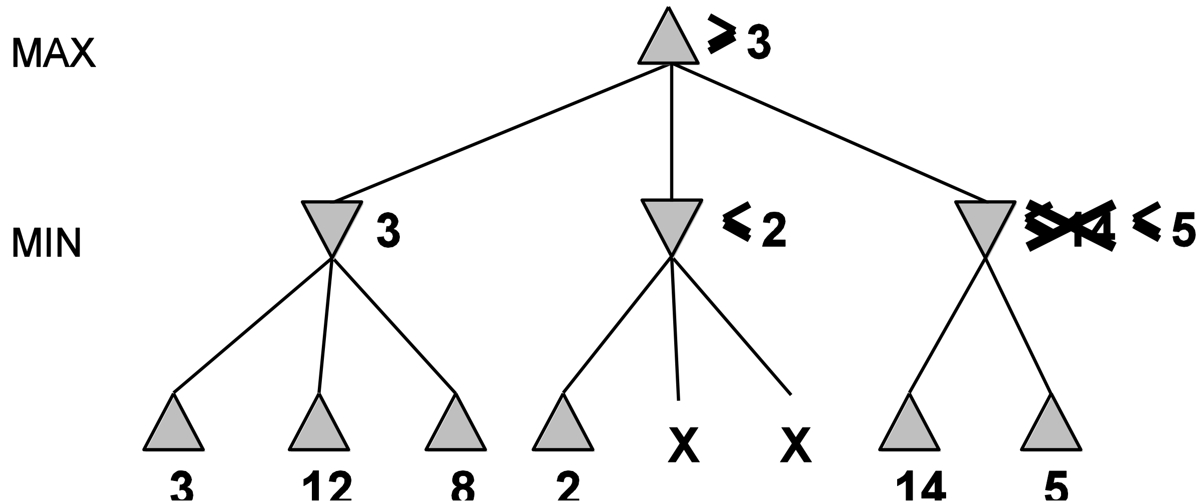
α–β pruning example



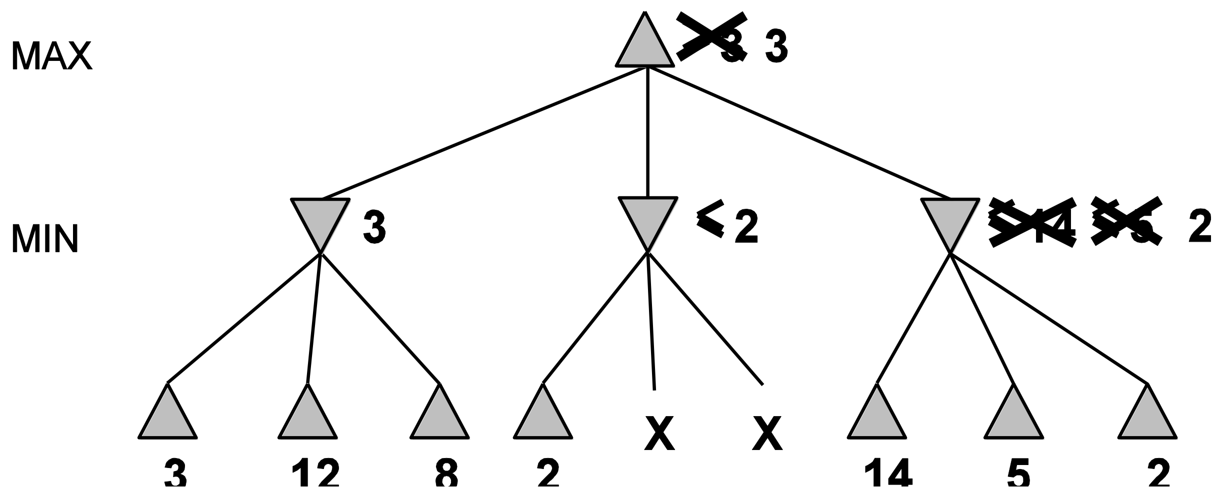
α–β pruning example



α–β pruning example



α–β pruning example



Properties of α–β

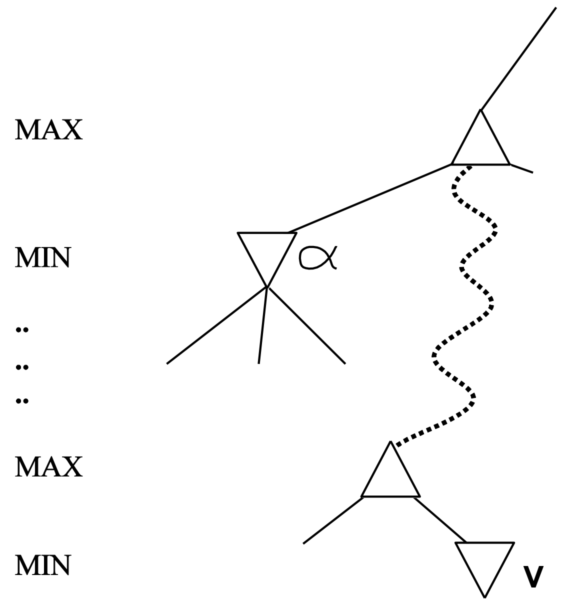
Pruning *does not* affect final result

Good move ordering improves effectiveness of pruning

With "perfect ordering," time complexity $= O(b^{m/2})$
$\Rightarrow$ *doubles* depth of search
$\Rightarrow$ can easily reach depth 8 and play good chess

A simple example of the value of reasoning about which computations are relevant (a form of *metareasoning*)

Why is it called α–β?



α is the best value (to MAX) found so far off the current path

If V is worse than α, MAX will avoid it $\Rightarrow$ prune that branch

Define β similarly for MIN

The α–β algorithm

Basically MINIMAX + keep track of α, β + prune

function MAX-VALUE(*state, game*, α, β) **returns** the minimax value of *state*
  inputs: *state*, current state in game
    game, game description
    α, the best score for MAX along the path to *state*
    β, the best score for MIN along the path to *state*

  if CUTOFF-TEST(*state*) **then return** EVAL(*state*)
  for each s **in** SUCCESSORS(*state*) **do**
    $\alpha \leftarrow$ MAX(α, MIN-VALUE(s, *game*, α, β))
    if $\alpha \geq \beta$ **then return** β
  end
  return α

---

function MIN-VALUE(*state, game*, α, β) **returns** the minimax value of *state*

  if CUTOFF-TEST(*state*) **then return** EVAL(*state*)
  for each s **in** SUCCESSORS(*state*) **do**
    $\beta \leftarrow$ MIN(β, MAX-VALUE(s, *game*, α, β))
    if $\beta \leq \alpha$ **then return** α
  end
  return β

Deterministic games in practice

Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

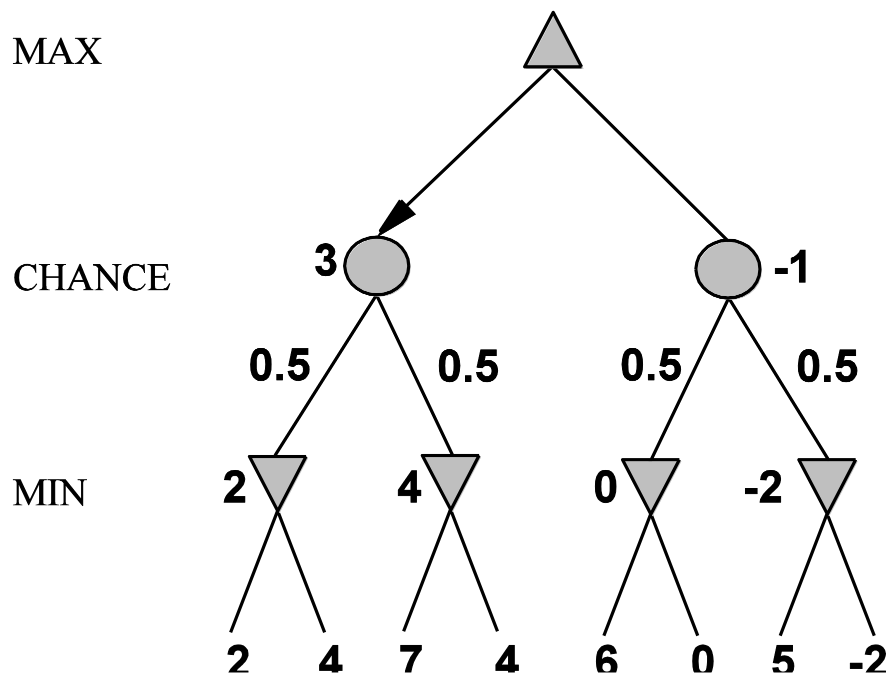
Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go, $b > 300$, so most programs use random moves initially, along with pattern knowledge bases to suggest plausible moves.

Nondeterministic games

E..g, in backgammon, the dice rolls determine the legal moves

Simplified example with coin-flipping instead of dice-rolling:



Algorithm for nondeterministic games

EXPECTIMINIMAX gives perfect play

Just like MINIMAX, except we must also handle chance nodes:

. . .
if *state* is a chance node **then**
  return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)
. . .

A version of α–β pruning is possible
but only if the leaf values are bounded. Why??

Nondeterministic games in practice

Dice rolls increase b: 21 possible rolls with 2 dice
Backgammon $\approx$ 20 legal moves (can be 6,000 with 1-1 roll)

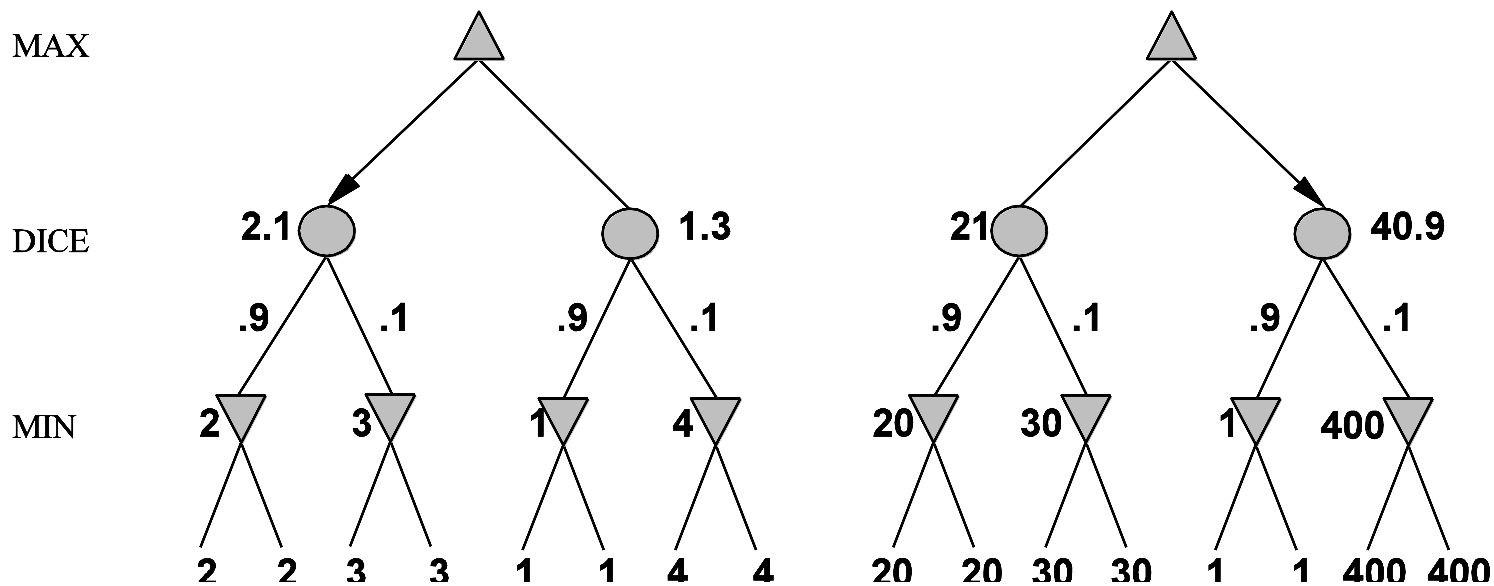
$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks
$\Rightarrow$ value of lookahead is diminished

α–β pruning is much less effective

TDGammon uses depth-2 search + very good Eval
$\approx$ world-champion level

Digression: Exact values DO matter



Behaviour is preserved only by *positive linear* transformation of EVAL

Hence EVAL should be proportional to the expected payoff

Summary

Games illustrate several important points about AI

◇ perfection is unattainable ⇒ must approximate and make trade-offs

◇ uncertainty limits the value of look-ahead

◇ can programs learn for themselves as they play? (stay tuned...)

Examples of skills expected:

◇ Demonstrate operation of game search algorithms

◇ Discuss and evaluate the properties of game search algorithms

◇ Design suitable evaluation functions for a game

◇ Explain how to search in nondeterministic games