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Project 1 Write-up

Part 1

Create an admissible heuristic, document the exact form of the heuristic and **show examples** that it is consistent. Draw an example of the **best case** and show that your heuristic underestimates the true cost. (6 points per cost function)

Definition of variables:

Height h_P refers to the result returned from getTile(P)

$\Delta h = |h_{Goal} - h_N|$

$\Delta d = \max(|x_{Goal} - x_N|, |y_{Goal} - y_N|)$

a) Cost function: Math.pow(2.0, (getTile(p2) - getTile(p1)))

$$h(N) = \begin{cases} 2 \cdot |h_N - h_G| \quad , & h_N \leq h_G \\ 2^{-255}|h_N - h_G| \ , & h_N > h_G \end{cases}$$

In order to prove that this heuristic is consistent, we need to show that

$$h(N) \leq c(N, P) + h(P)$$

namely, to show that

$$2|h_N - h_G| \leq 2^{h_P - h_N} + 2|h_P - h_G| \ , \quad h_N \leq h_G$$

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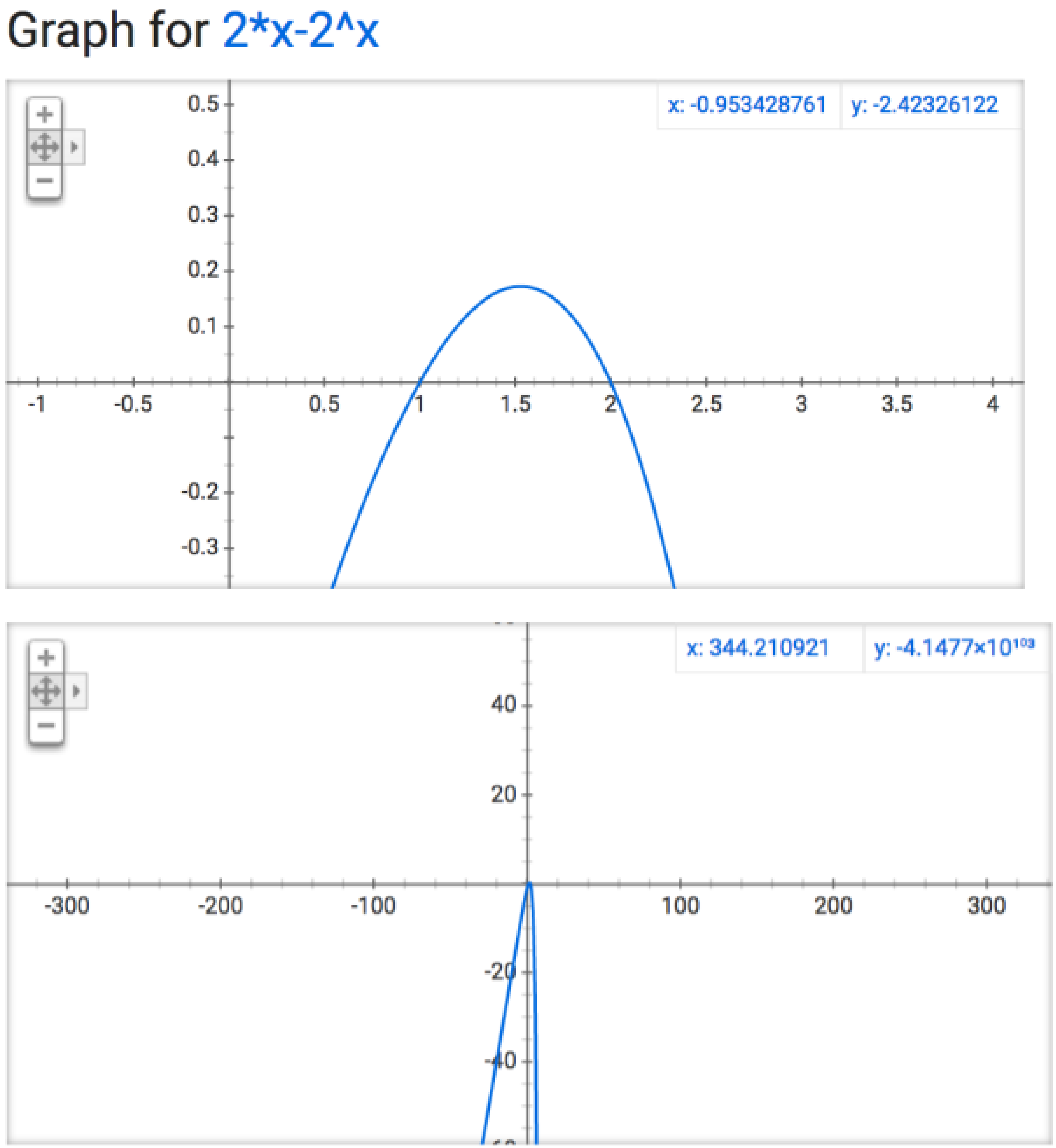
$LHS = 2(h_G - h_N)$
$RHS = 2^{h_P - h_N} + 2(h_G - h_P)$
$RHS - LHS = 2(h_G - h_N) - 2(h_G - h_P) + 2^{h_P - h_N}$
$\qquad\qquad = 2(h_P - h_N) - 2^{h_P - h_N}$
Let $x = h_P - h_N$, then $x \in \mathbb{Z}$ $and - 255 \le x \le 255$.

Based on the plot of $f(x) = 2x - 2^x$, we can see that $2x - 2^x \le 0$ for any integer between -255 and 255.



Therefore, $RHS - LHS \le 0$, and we have proved that $h(N) \le c(N, P) + h(P)$ for $h_N \le h_G$.

Similarly, for $h_N > h_G$, we want to prove

$$2^{-255}|h_N - h_G| \le 2^{h_P - h_N} + 2^{-255}|h_p - h_G|, \quad h_N > h_G$$

$LHS = 2^{-255}(h_N - h_G)$
$RHS = 2^{h_P - h_N} + 2^{-255}|h_p - h_G|$
$RHS - LHS = 2^{h_P - h_N} + 2^{-255}|h_N - h_p|$
So we have, $0 \le 2^{h_P - h_N} + 2^{-255}|h_N - h_p|$

It's clear that the inequality above holds since all the terms on the RHS are positive. Hence, the heuristic is consistent and will never overestimate the true cost.

b) Cost function: 1.0*(getTile(p1) / (getTile(p2) + 1))

We have the following heuristic:

$$h(N) = \begin{cases} (\Delta d - 1) \cdot \dfrac{h_G}{h_G + 1} + \dfrac{h_N}{h_G + 1}, & h_N > h_G \\ (\Delta d - 1) \cdot \dfrac{h_N}{h_N + 1} + \dfrac{h_N}{h_G + 1}, & h_N \le h_G \end{cases}$$

The best case would be when N is adjacent to G, then we would have $\Delta d = 1$, then heuristic $h(N) = \frac{h_N}{h_{G+1}}$ in both cases.
The actual cost to go from N to G is also $\frac{h_N}{h_{G+1}}$ according to the cost function, which is equal to $h(N)$. Therefore, our heuristic does not overestimate the true cost.