

$$\begin{aligned} \text{a. } \lim_{x \rightarrow \infty} \frac{\ln(x^{2020})}{x} \\ &= \lim_{x \rightarrow \infty} \frac{2020 \ln(x)}{x} \\ &= 2020 \lim_{x \rightarrow \infty} \frac{\ln(x)}{x} = \frac{\infty}{\infty} \text{ (bertemu indeterminate form)} \end{aligned}$$

Menggunakan aturan L'Hopital

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

sehingga didapat:

$$\begin{aligned} 2020 \lim_{x \rightarrow \infty} \frac{\ln(x)}{x} &= 2020 \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} \ln(x)}{\frac{d}{dx} x} \\ &= 2020 \lim_{x \rightarrow \infty} \frac{1/x}{1} = 2020 \lim_{x \rightarrow \infty} \frac{1}{x} = 2020 \cdot 0 = 0 \end{aligned}$$

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$$\text{b. } \lim_{x \rightarrow \infty} \frac{x^{2020}}{e^x} = \lim_{x \rightarrow \infty} (x^{2020} e^{-x}) = \infty \cdot 0 \text{ (bertemu indeterminate form)}$$

Menggunakan aturan L'Hopital

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

sehingga didapat:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^{2020}}{e^x} &= \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} x^{2020}}{\frac{d}{dx} e^x} \\ &= \lim_{x \rightarrow \infty} \frac{2020 \cdot x^{2019}}{e^x} = \frac{\infty}{\infty} \text{ (masih bertemu indeterminate form, maka diturunkan lagi)} \\ &= \lim_{x \rightarrow \infty} \frac{2020 \cdot 2019 \cdot x^{2018}}{e^x} = \frac{\infty}{\infty} \text{ (masih bertemu indeterminate form, maka diturunkan lagi)} \\ &= \lim_{x \rightarrow \infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot x^{2017}}{e^x} = \frac{\infty}{\infty} \text{ (masih bertemu indeterminate form, maka diturunkan lagi)} \\ &= \lim_{x \rightarrow \infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot 2017 \cdot x^{2016}}{e^x} = \frac{\infty}{\infty} \text{ (masih bertemu indeterminate form, maka diturunkan lagi)} \\ &= \lim_{x \rightarrow \infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot 2017 \cdot 2016 \cdot x^{2015}}{e^x} = \frac{\infty}{\infty} \text{ (masih bertemu indeterminate form, maka diturunkan lagi)} \end{aligned}$$

...

diturunkan sebanyak 2020 kali, sehingga tidak menghasilkan indeterminate form lagi

...

... (proses disingkat, hasilnya membentuk pola yang bagus, mudah dibaca)

...

setelah diturunkan 2020 kali menjadi pola factorial berikut

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot 2017 \cdot 2016 \cdot 2015 \cdot 2014 \cdot 2013 \cdot \dots \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{e^x} \\ &= \lim_{x \rightarrow \infty} \frac{2020!}{e^x} = \frac{2020!}{\infty} = 0 \end{aligned}$$

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c. $\lim_{x \rightarrow \pi/2} \frac{3 \sec(x) + 5}{\tan(x)}$

Catatan: $\tan(x) = \frac{\sin(x)}{\cos(x)}, \quad \sec(x) = \frac{1}{\cos(x)}$

$$\begin{aligned}
 &= \lim_{x \rightarrow \pi/2} \frac{3 \sec(x) + 5}{\sin(x) / \cos(x)} \\
 &= \lim_{x \rightarrow \pi/2} \left(\frac{3 \sec(x)}{\sin(x) / \cos(x)} + \frac{5}{\sin(x) / \cos(x)} \right) \\
 &= \lim_{x \rightarrow \pi/2} \left(\frac{3(1/\cos(x))}{\sin(x) / \cos(x)} + \frac{5}{\sin(x) / \cos(x)} \right) \\
 &= \lim_{x \rightarrow \pi/2} \left(\frac{3 \cancel{\cos(x)}}{\cancel{\cos(x)} \sin(x)} + \frac{5 \cos(x)}{\sin(x)} \right) \\
 &= \lim_{x \rightarrow \pi/2} \left(\frac{3}{\sin(x)} + \frac{5 \cos(x)}{\sin(x)} \right) \\
 &= \left(\frac{3}{\sin(\pi/2)} + \frac{5 \cos(\pi/2)}{\sin(\pi/2)} \right) \\
 &= \frac{3}{1} + \frac{5 \cdot 0}{1} = \frac{3}{1} + 0 = 3
 \end{aligned}$$

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d. $\lim_{x \rightarrow \infty} [x - \ln(x)]$

$$= \lim_{x \rightarrow \infty} x \left[1 - \frac{\ln(x)}{x} \right] \quad (\text{dipecah menjadi 2 limit})$$

$$= \left(\lim_{x \rightarrow \infty} x \right) \cdot \left(\lim_{x \rightarrow \infty} \left[1 - \frac{\ln(x)}{x} \right] \right) \quad (\text{bertemu indeterminate form pada limit ke-2})$$

Menggunakan aturan L'Hopital

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

$$= \left(\lim_{x \rightarrow \infty} x \right) \cdot \left(\lim_{x \rightarrow \infty} \left[1 - \frac{\frac{d}{dx} \ln(x)}{\frac{d}{dx} x} \right] \right)$$

$$= \left(\lim_{x \rightarrow \infty} x \right) \cdot \left(\lim_{x \rightarrow \infty} \left[1 - \frac{1/x}{1} \right] \right)$$

$$= \left(\lim_{x \rightarrow \infty} x \right) \cdot \left(\lim_{x \rightarrow \infty} \left[1 - \frac{1}{x} \right] \right)$$

$$= \left(\lim_{x \rightarrow \infty} x \right) \cdot (1 - 0)$$

$$= \infty \cdot 1 = \infty$$

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e. $\lim_{x \rightarrow 0} \left[\csc^2(x) - \frac{1}{x^2} \right]$

Catatan: $\csc(x) = \frac{1}{\sin(x)}$, $\sin(2x) = 2 \sin(x) \cos(x)$

$$= \lim_{x \rightarrow 0} \left[\frac{1}{\sin^2(x)} - \frac{1}{x^2} \right]$$

$$= \lim_{x \rightarrow 0} \left[\frac{x^2}{x^2 \sin^2(x)} - \frac{\sin^2(x)}{x^2 \sin^2(x)} \right] \text{ (disamakan penyebutnya)}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 - \sin^2(x)}{x^2 \sin^2(x)} = \frac{0^2 - \sin^2(0)}{0^2 \sin^2(0)} = \frac{0}{0} \text{ (bertemu indeterminate form)}$$

Menggunakan aturan L'Hopital

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

sehingga $\lim_{x \rightarrow 0} \frac{x^2 - \sin^2(x)}{x^2 \sin^2(x)} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(x^2 - \sin^2(x))}{\frac{d}{dx}(x^2 \sin^2(x))}$

$$= \lim_{x \rightarrow 0} \frac{2x - \sin(2x)}{2x \sin^2(x) + \sin(2x) x^2} = \frac{0}{0} \text{ (masih bertemu indeterminate form, diturunkan lagi)}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(2x - \sin(2x))}{\frac{d}{dx}(2x \sin^2(x) + \sin(2x) x^2)}$$

$$= \lim_{x \rightarrow 0} \frac{2 - \cos(2x) \cdot 2}{2x^2 \cos(2x) + 4x \sin(2x) + 2 \sin^2(x)}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2 \cos(2x) + 2x \sin(2x) + \sin^2(x)} = \frac{0}{0} \text{ (masih bertemu indeterminate form, diturunkan lagi)}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(1 - \cos(2x))}{\frac{d}{dx}(x^2 \cos(2x) + 2x \sin(2x) + \sin^2(x))}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin(2x)}{2x \cos(2x) - 2x^2 \sin(2x) + 2 \sin(2x) + 4x \cos(2x) + \sin(2x)}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin(2x)}{6x \cos(2x) + 3 \sin(2x) - 2x^2 \sin(2x)} = \frac{0}{0} \text{ (masih bertemu indeterminate form, diturunkan lagi)}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{d}{dx}(2 \sin(2x))}{\frac{d}{dx}(6x \cos(2x) + 3 \sin(2x) - 2x^2 \sin(2x))}$$

$$= \lim_{x \rightarrow 0} \frac{4 \cos(2x)}{6 \cos(2x) - 12x \sin(2x) + 6 \cos(2x) - 2(2x \sin(2x) + \cos(2x) \cdot 2x^2)}$$

$$= \lim_{x \rightarrow 0} \frac{4 \cos(2x)}{12 \cos(2x) - 4x^2 \cos(2x) - 16x \sin(2x)}$$

$$= \lim_{x \rightarrow 0} \frac{4 \cos(2x)}{4(3 \cos(2x) - x^2 \cos(2x) - 4x \sin(2x))}$$

$$= \lim_{x \rightarrow 0} \frac{\cos(2x)}{3 \cos(2x) - x^2 \cos(2x) - 4x \sin(2x)}$$

$$= \frac{\cos(2(0))}{3 \cos(2(0)) - 0^2 \cos(2(0)) - 4(0) \sin(2(0))}$$

$$= \frac{\cos(0)}{3 \cos(0) - 0 - 0} = \frac{1}{3}$$

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$$\text{f. } \lim_{x \rightarrow 0} (\cos(x))^{\csc(x)}$$

Menggunakan aturan pangkat: $a^x = e^{\ln(a^x)} = e^{x \cdot \ln(a)}$

sehingga didapatkan $\cos(x) =$

$$\begin{aligned} \lim_{x \rightarrow 0} (\cos(x))^{\csc(x)} &= \lim_{x \rightarrow 0} \left(e^{\ln((\cos(x))^{\csc(x)})} \right) \\ &= \lim_{x \rightarrow 0} \left(e^{\csc(x) \ln(\cos(x))} \right) \end{aligned}$$

Menggunakan aturan rantai pada limit

Jika $\lim_{u \rightarrow b} f(u) = L$, dan $\lim_{x \rightarrow a} g(x) = b$, dan $f(x)$ kontinu pada $x = b$, maka $\lim_{x \rightarrow a} f(g(x)) = L$

Dimisalkan $g(x) = \csc(x) \ln(\cos(x))$ dan $f(u) = e^u$

$\lim_{x \rightarrow 0} \csc(x) \ln(\cos(x)) = \infty \cdot 0$ (bertemu indeterminate form)

Menggunakan aturan L'Hopital

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \csc(x) \ln(\cos(x)) &= \lim_{x \rightarrow 0} \frac{\ln(\cos(x))}{\frac{1}{\csc(x)}} = \lim_{x \rightarrow 0} \frac{\frac{d}{dx} \ln(\cos(x))}{\frac{d}{dx} \frac{1}{\csc(x)}} \\ &= \lim_{x \rightarrow 0} \frac{-\tan(x)}{\cos(x)} = \lim_{x \rightarrow 0} \frac{-\sin(x)}{\cos(x) \cos(x)} = \frac{-\sin(0)}{\cos(0) \cos(0)} = \frac{0}{1 \cdot 1} = 0 \end{aligned}$$

$$\lim_{u \rightarrow 0} (e^u) = e^0 = 1$$

$$\therefore \lim_{x \rightarrow 0} (\cos(x))^{\csc(x)} = 1$$

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$$\text{g. } \lim_{x \rightarrow \pi/2} [\tan(x) - \sec(x)]$$

$$\begin{aligned} &= \lim_{x \rightarrow \pi/2} [\tan(x) - \sec(x)] \frac{\tan(x) + \sec(x)}{\tan(x) + \sec(x)} \\ &= \lim_{x \rightarrow \pi/2} \frac{(\tan(x) - \sec(x))(\tan(x) + \sec(x))}{\tan(x) + \sec(x)} \\ &= \lim_{x \rightarrow \pi/2} \frac{\tan^2(x) - \sec^2(x)}{\tan(x) + \sec(x)} \end{aligned}$$

Identitas Trigonometri: $\tan^2(x) - \sec^2(x) = -1$

$$\begin{aligned} &= \lim_{x \rightarrow \pi/2} \frac{-1}{\tan(x) + \sec(x)} \\ &= \frac{-1}{-\infty} \\ &= 0 \end{aligned}$$

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$$\text{h. } \lim_{x \rightarrow 0^+} \frac{\cot(x)}{\sqrt{-\ln(x)}}$$

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