a. $\lim_{x\to\infty} \frac{\ln(x^{2020})}{x}$

$= \lim_{x\to\infty} \frac{2020\ln(x)}{x}$

$= 2020 \lim_{x\to\infty} \frac{\ln(x)}{x} = \frac{\infty}{\infty}$ (bertemu indeterminate form)

Menggunakan aturan L'Hopital

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

sehingga didapat:

$2020 \lim_{x\to\infty} \frac{\ln(x)}{x} = 2020 \lim_{x\to\infty} \frac{\frac{d}{dx}\ln(x)}{\frac{d}{dx}x}$

$= 2020 \lim_{x\to\infty} \frac{1/x}{1} = 2020 \lim_{x\to\infty} \frac{1}{x} = 2020 \cdot 0 = 0$

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b. $\lim_{x\to\infty} \frac{x^{2020}}{e^x} = \lim_{x\to\infty} \left(x^{2020}e^{-x}\right) = \infty \cdot 0$ (bertemu indeterminate form)

Menggunakan aturan L'Hopital

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

sehingga didapat:

$\lim_{x\to\infty} \frac{x^{2020}}{e^x} = \lim_{x\to\infty} \frac{\frac{d}{dx}x^{2020}}{\frac{d}{dx}e^x}$

$= \lim_{x\to\infty} \frac{2020 \cdot x^{2019}}{e^x} = \frac{\infty}{\infty}$ (masih bertemu indeterminate form, maka diturunkan lagi)

$= \lim_{x\to\infty} \frac{2020 \cdot 2019 \cdot x^{2018}}{e^x} = \frac{\infty}{\infty}$ (masih bertemu indeterminate form, maka diturunkan lagi)

$= \lim_{x\to\infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot x^{2017}}{e^x} = \frac{\infty}{\infty}$ (masih bertemu indeterminate form, maka diturunkan lagi)

$= \lim_{x\to\infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot 2017 \cdot x^{2016}}{e^x} = \frac{\infty}{\infty}$ (masih bertemu indeterminate form, maka diturunkan lagi)

$= \lim_{x\to\infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot 2017 \cdot 2016 \cdot x^{2015}}{e^x} = \frac{\infty}{\infty}$ (masih bertemu indeterminate form, maka diturunkan lagi)

...

diturunkan sebanyak 2020 kali, sehingga tidak menghasilkan indeterminate form lagi

...

... (proses disingkat, hasilnya membentuk pola yang bagus, mudah dibaca)

...

setelah diturunkan 2020 kali menjadi pola factorial berikut

$= \lim_{x\to\infty} \frac{2020 \cdot 2019 \cdot 2018 \cdot 2017 \cdot 2016 \cdot 2015 \cdot 2014 \cdot 2013 \cdot ... \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{e^x}$

$= \lim_{x\to\infty} \frac{2020!}{e^x} = \frac{2020!}{\infty} = 0$

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c. $\lim\limits_{x \to \pi/2} \dfrac{3\sec(x)+5}{\tan(x)}$

Catatan: $\tan(x) = \dfrac{\sin(x)}{\cos(x)}, \quad \sec(x) = \dfrac{1}{\cos(x)}$

$$= \lim_{x \to \pi/2} \frac{3\sec(x)+5}{\sin(x)\,/\cos(x)}$$

$$= \lim_{x \to \pi/2} \left(\frac{3\sec(x)}{\sin(x)\,/\cos(x)} + \frac{5}{\sin(x)\,/\cos(x)} \right)$$

$$= \lim_{x \to \pi/2} \left(\frac{3(1/\cos(x))}{\sin(x)\,/\cos(x)} + \frac{5}{\sin(x)\,/\cos(x)} \right)$$

$$= \lim_{x \to \pi/2} \left(\frac{3\cancel{\cos(x)}}{\cancel{\cos(x)}\sin(x)} + \frac{5\cos(x)}{\sin(x)} \right)$$

$$= \lim_{x \to \pi/2} \left(\frac{3}{\sin(x)} + \frac{5\cos(x)}{\sin(x)} \right)$$

$$= \left(\frac{3}{\sin(\pi/2)} + \frac{5\cos(\pi/2)}{\sin(\pi/2)} \right)$$

$$= \frac{3}{1} + \frac{5 \cdot 0}{1} = \frac{3}{1} + 0 = 3$$

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d. $\lim\limits_{x \to \infty} [x - \ln(x)]$

$$= \lim_{x \to \infty} x \left[1 - \frac{\ln(x)}{x}\right] \quad \text{(dipecah menjadi 2 limit)}$$

$$= \left(\lim_{x \to \infty} x\right) \cdot \left(\lim_{x \to \infty} \left[1 - \frac{\ln(x)}{x}\right]\right) \quad \text{(bertemu indeterminate form pada limit ke-2)}$$

Menggunakan aturan L'Hopital

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$= \left(\lim_{x \to \infty} x\right) \cdot \left(\lim_{x \to \infty} \left[1 - \frac{\frac{d}{dx}\ln(x)}{\frac{d}{dx}x}\right]\right)$$

$$= \left(\lim_{x \to \infty} x\right) \cdot \left(\lim_{x \to \infty} \left[1 - \frac{1/x}{1}\right]\right)$$

$$= \left(\lim_{x \to \infty} x\right) \cdot \left(\lim_{x \to \infty} \left[1 - \frac{1}{x}\right]\right)$$

$$= \left(\lim_{x \to \infty} x\right) \cdot (1 - 0)$$

$$= \infty \cdot 1 = \infty$$

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e. $\lim\limits_{x\to 0}\left[\csc^2(x) - \dfrac{1}{x^2}\right]$

$$\boxed{\text{Catatan: } \csc(x) = \frac{1}{\sin(x)}, \quad \sin(2x) = 2\sin(x)\cos(x)}$$

$$= \lim_{x\to 0}\left[\frac{1}{\sin^2(x)} - \frac{1}{x^2}\right]$$

$$= \lim_{x\to 0}\left[\frac{x^2}{x^2\sin^2(x)} - \frac{\sin^2(x)}{x^2\sin^2(x)}\right] \text{ (disamakan penyebutnya)}$$

$$= \lim_{x\to 0}\frac{x^2-\sin^2(x)}{x^2\sin^2(x)} = \frac{0^2-\sin^2(0)}{0^2\sin^2(0)} = \frac{0}{0} \text{ (bertemu indeterminate form)}$$

$$\boxed{\begin{array}{l}\text{Menggunakan aturan L'Hopital}\\ \displaystyle\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}\end{array}}$$

$$\text{sehingga } \lim_{x\to 0}\frac{x^2-\sin^2(x)}{x^2\sin^2(x)} = \lim_{x\to 0}\frac{\frac{d}{dx}\left(x^2-\sin^2(x)\right)}{\frac{d}{dx}\left(x^2\sin^2(x)\right)}$$

$$= \lim_{x\to 0}\frac{2x-\sin(2x)}{2x\sin^2(x)+\sin(2x)\,x^2} = \frac{0}{0} \text{ (masih bertemu indeterminate form, diturunkan lagi)}$$

$$= \lim_{x\to 0}\frac{\frac{d}{dx}(2x-\sin(2x))}{\frac{d}{dx}(2x\sin^2(x)+\sin(2x)\,x^2)}$$

$$= \lim_{x\to 0}\frac{2-\cos(2x)\cdot 2}{2x^2\cos(2x)+4x\sin(2x)+2\sin^2(x)}$$

$$= \lim_{x\to 0}\frac{1-\cos(2x)}{x^2\cos(2x)+2x\sin(2x)+\sin^2(x)} = \frac{0}{0} \text{ (masih bertemu indeterminate form, diturunkan lagi)}$$

$$= \lim_{x\to 0}\frac{\frac{d}{dx}(1-\cos(2x))}{\frac{d}{dx}(x^2\cos(2x)+2x\sin(2x)+\sin^2(x))}$$

$$= \lim_{x\to 0}\frac{2\sin(2x)}{2x\cos(2x)-2x^2\sin(2x)+2\sin(2x)+4x\cos(2x)+\sin(2x)}$$

$$= \lim_{x\to 0}\frac{2\sin(2x)}{6x\cos(2x)+3\sin(2x)-2x^2\sin(2x)} = \frac{0}{0} \text{ (masih bertemu indeterminate form, diturunkan lagi)}$$

$$= \lim_{x\to 0}\frac{\frac{d}{dx}(2\sin(2x))}{\frac{d}{dx}(6x\cos(2x)+3\sin(2x)-2x^2\sin(2x))}$$

$$= \lim_{x\to 0}\frac{4\cos(2x)}{6\cos(2x)-12x\sin(2x)+6\cos(2x)-2\left(2x\sin(2x)+\cos(2x)\cdot 2x^2\right)}$$

$$= \lim_{x\to 0}\frac{4\cos(2x)}{12\cos(2x)-4x^2\cos(2x)-16x\sin(2x)}$$

$$= \lim_{x\to 0}\frac{\cancel{4}\cos(2x)}{\cancel{4}(3\cos(2x)-x^2\cos(2x)-4x\sin(2x))}$$

$$= \lim_{x\to 0}\frac{\cos(2x)}{3\cos(2x)-x^2\cos(2x)-4x\sin(2x)}$$

$$= \frac{\cos(2(0))}{3\cos(2(0))-0^2\cos(2(0))-4(0)\sin(2(0))}$$

$$= \frac{\cos(0)}{3\cos(0)-0-0} = \frac{1}{3}$$

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f. $\lim_{x\to 0}(\cos(x))^{\csc(x)}$

$\boxed{\text{Menggunakan aturan pangkat: } a^x = e^{\ln(a^x)} = e^{x\cdot\ln(a)}}$

sehingga didapatkan $\cos(x) =$

$$\lim_{x\to 0}(\cos(x))^{\csc(x)} = \lim_{x\to 0}\left(e^{\ln\left((\cos(x))^{\csc(x)}\right)}\right)$$
$$= \lim_{x\to 0}\left(e^{\csc(x)\ln(\cos(x))}\right)$$

Menggunakan aturan rantai pada limit

Jika $\lim_{u\,\to\, b} f\,(u) = L,$ dan $\lim_{x\,\to\, a} g\,(x) = b,$ dan $f\,(x)$ kontinu pada $x = b$, maka $\lim_{x\,\to\, a} f\,(g\,(x)) = L$

Dimisalkan $g\,(x) = \csc\,(x)\ln\,(\cos\,(x))$ dan $f\,(u) = e^u$

$\lim_{x\to 0}\csc\,(x)\ln\,(\cos\,(x)) = \infty\cdot 0$ (bertemu indeterminate form)

Menggunakan aturan L'Hopital

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0}\csc\,(x)\ln\,(\cos\,(x)) = \lim_{x\to 0}\frac{\ln\,(\cos\,(x))}{\frac{1}{\csc(x)}} = \lim_{x\to 0}\frac{\frac{d}{dx}\ln\,(\cos\,(x))}{\frac{d}{dx}\frac{1}{\csc(x)}}$$
$$= \lim_{x\to 0}\frac{-\tan(x)}{\cos(x)} = \lim_{x\to 0}\frac{-\sin(x)}{\cos(x)\cos(x)} = \frac{-\sin(0)}{\cos(0)\cos(0)} = \frac{0}{1\cdot 1} = 0$$

$$\lim_{u\to\,0}\,(e^u) = e^0 = 1$$

$$\boxed{\therefore \lim_{x\to 0}(\cos(x))^{\csc(x)} = 1}$$

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g. $\lim_{x\to\pi/2}[\tan(x) - \sec(x)]$

$$= \lim_{x\to\pi/2}[\tan(x) - \sec(x)]\,\frac{\tan(x)+\sec(x)}{\tan(x)+\sec(x)}$$
$$= \lim_{x\to\pi/2}\frac{(\tan(x)-\sec(x))(\tan(x)+\sec(x))}{\tan(x)+\sec(x)}$$
$$= \lim_{x\to\pi/2}\frac{\tan^2(x)-\sec^2(x)}{\tan(x)+\sec(x)}$$

$\boxed{\text{Identitas Trigonometri: } \tan^2(x) - \sec^2(x) = -1}$

$$= \lim_{x\to\pi/2}\frac{-1}{\tan(x)+\sec(x)}$$
$$= \frac{-1}{-\infty} = \frac{1}{\infty}$$
$$= 0$$

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h. $\lim_{x\to 0^+} \dfrac{\cot(x)}{\sqrt{-\ln(x)}} = \dfrac{\infty}{\infty}$ (bertemu indeterminate form)

$$\boxed{\begin{array}{l}\text{Menggunakan aturan L'Hopital}\\ \lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}\end{array}}$$

$$= \lim_{x\to 0^+} \frac{\frac{d}{dx}\cot(x)}{\frac{d}{dx}\sqrt{-\ln(x)}}$$

$$= \lim_{x\to 0^+} \frac{-\csc^2(x)}{-\frac{1}{2x\sqrt{-\ln(x)}}}$$

$$= \lim_{x\to 0^+} \left(2x\csc^2\left(x\right)\sqrt{-\ln\left(x\right)}\right)$$

$$= 2\cdot \lim_{x\to\ 0^+} \left(x\csc^2\left(x\right)\sqrt{-\ln\left(x\right)}\right)$$

$$= 2\cdot \lim_{x\to\ 0^+} \left(\frac{x\sqrt{-\ln\left(x\right)}}{\sin^2\left(x\right)}\right) = \frac{0}{0} \text{ (bertemu dengan indeterminate form)}$$

$$= 2\cdot \lim_{x\to\ 0^+} \left(\frac{\frac{d}{dx}\left(x\sqrt{-\ln\left(x\right)}\right)}{\frac{d}{dx}(\sin^2\left(x\right))}\right)$$

$$= 2\cdot \lim_{x\to\ 0^+} \left(\frac{\sqrt{-\ln\left(x\right)} - \frac{1}{2\sqrt{-\ln(x)}}}{2\sin(x)\cos(x)}\right)$$

$$= 2\cdot \lim_{x\to\ 0^+} \left(\frac{\sqrt{-\ln\left(x\right)} - \frac{1}{2\sqrt{-\ln(x)}}}{\sin\left(2x\right)}\right)$$

$$= 2\cdot \lim_{x\to\ 0^+} \left(\frac{-2\ln\left(x\right)-1}{2\sin\left(2x\right)\sqrt{-\ln\left(x\right)}}\right)$$

$$= 2\cdot \frac{1}{2}\cdot \lim_{x\to\ 0^+} \left(\frac{-2\ln\left(x\right)-1}{\sin\left(2x\right)\sqrt{-\ln\left(x\right)}}\right)$$

$$= 2\cdot \frac{1}{2}\cdot \frac{\infty}{\text{tidak terdefinisi}} = \infty \text{ (limit divergen/tidak ada)}$$

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