$$\frac{\partial}{\partial x^1}\left(D^{11}\frac{\partial f}{\partial x^1}+D^{12}\frac{\partial f}{\partial x^2}\right)+\frac{\partial}{\partial x^2}\left(D^{21}\frac{\partial f}{\partial x^1}+D^{22}\frac{\partial f}{\partial x^2}\right)=-S$$

weak-form:

$$\int_{y_{j-1/2}}^{y_{j+1/2}}\psi\left(D^{11}\frac{\partial f}{\partial x^1}+D^{12}\frac{\partial f}{\partial x^2}\right)\Big|_{x_{i-1/2}}^{x_{i+1/2}}dx_2-\iint_I\frac{\partial\psi}{\partial x^1}\left(D^{11}\frac{\partial f}{\partial x^1}+D^{12}\frac{\partial f}{\partial x^2}\right)dx_1dx_2$$

$$+\int_{x_{i-1/2}}^{x_{i+1/2}}\psi\left(D^{21}\frac{\partial f}{\partial x^1}+D^{22}\frac{\partial f}{\partial x^2}\right)\Big|_{y_{j-1/2}}^{y_{j+1/2}}dx_1-\iint_I\frac{\partial\psi}{\partial x^2}\left(D^{21}\frac{\partial f}{\partial x^1}+D^{22}\frac{\partial f}{\partial x^2}\right)dx_1dx_2$$

$$=-\iint_I S\psi\,dx_1dx_2$$

$$\det\begin{vmatrix}D^{11}&D^{12}\\D^{12}&D^{22}\end{vmatrix}=D^{11}D^{22}-D^{12}D^{22}$$

$$=\frac{(D_{11}y+D_{12}x)(D_{11}x+D_{12}y)}{x^2+y^2}-\frac{x^2y^2(D_{11}-D_{22})^2}{x^2+y^2}$$

$$=D_{12}D_{11}$$

CROSS-DERIVATIVE TERMS:

$$\int_{y_{j-1/2}}^{y_{j+1/2}}\psi\,D^{12}\frac{\partial f}{\partial x^2}\Big|_{x_{i-1/2}}^{x_{i+1/2}}dx_2-\iint_I\frac{\partial\psi}{\partial x^1}D^{12}\frac{\partial f}{\partial x^2}\,dx_1dx_2$$

$$+\int_{x_{i-1/2}}^{x_{i+1/2}}\psi\,D^{21}\frac{\partial f}{\partial x^1}\Big|_{y_{j-1/2}}^{y_{j+1/2}}dx_1-\iint_I\frac{\partial\psi}{\partial x^2}D^{21}\frac{\partial f}{\partial x^1}\,dx_1dx_2$$