# MILP-Based Charging and Route Selection of Electric Vehicles in Smart Grid

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## ABSTRACT

Widely accepted as an eco-friendly alternative to conventional vehicles, electric vehicles (EVs), however, have a limitation, such as a charging schedule is necessary for its journey since overloading at a charging station may cause grid failure. Also, despite the current advancement in technology, the battery capacity of EVs is still limited, which affects the cruise range of the vehicles, and it can be solved by en route charging of EVs. However, the charging rate may vary across different public charging stations. This may motivate electric vehicle owners to follow a route that is different from the traditional shortest route. In this paper, we consider a joint charging and route optimization problem, where a transport operator has a number of EVs at a warehouse, and he/she is supposed to deliver certain goods or services to different delivery locations. We have proposed two mixed-integer linear programming (MILP) models, where the delivery locations are first distributed among the EVs, and second, routes for the EVs are determined that minimizes the total travel time, while charging on the route. We prove that the problem is NP-complete. Detailed simulation has been carried out on a realistic dataset [4][19], and solved using the commercial solver CPLEX, and IBM's drop-solved platform. The results show that an even distribution of delivery locations among EVs along with their partial charging at different charging stations en route proves to be a useful model for fast delivery of services/goods while minimizing their total travel time.

## CCS CONCEPTS

• **Mathematics of computing**; • **Applied computing** → **Transportation**;

## KEYWORDS

Electric vehicles, Charging, Routing, Scheduling

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## 1 INTRODUCTION

Electric vehicles (EVs) have been receiving considerable attention in recent years as an environment-friendly and cost-effective alternative over conventional vehicles. EVs can be charged at different fast charging stations by plugging their charge cables into the charge outlets. EVs have several important applications. In the service sector, EVs have been extensively used for several important services such as postal services, goods delivery, emergency services (police, fire brigade, ambulance), and so on [6]. For efficient implementation of these services, prompt deliveries are necessary within a stipulated deadline, and they can be done by selecting optimal routes for EVs while taking into consideration the state-of-charge of EVs' batteries, waiting times at public charging stations located along the routes. Limited battery capacity of an EV may cause the EV to be stranded in the middle of a long distance trip, which motivates drivers to select a route which is different from the shortest route. Therefore, it is necessary to develop an intelligent transportation model that ensures a flexible charging-aware navigation of EV fleets through a city network, while considering charging constraints, battery capacities and so on [20].

In smart grid, a central entity, called the aggregator, is responsible for selecting optimal routes while scheduling the EVs for charging at different public charging stations. In this paper, we assume that a transport company has a set of delivery locations where it has to deliver the goods before coming back to the warehouse. The company owns a set of EVs that are used for delivering goods. Initially, the company communicates several information such as the set of EVs, their delivery locations, EVs' average speeds, maximum battery capacities, initial state-of-charges, energy consumption rates for travel etc. to the aggregator through some communication network. The aggregator has the knowledge of the city road map, the locations of the charging stations, and different charging station parameters such as charging rate, number of charge outlets. The aggregator, then decides the navigation as well as charging schedule of EVs (EV, route, charging station, start and end time of charging, charging energy) based on several mobility and timing constraints in advance and communicates back to the company.

In this paper, we have formulated a mixed integer linear programming (MILP) based joint charging and routing optimization problem, called the *Minimum Travel with EV Charging (MinTravelEVC)* problem, where the objective of the aggregator is to first distribute the delivery locations among the EVs, and subsequently minimize the total travel and charging time while deciding the routes of the EVs and selecting the charging stations that will be used by the EVs to charge their batteries during delivering goods/services. The performance of the proposed scheme has been shown on a city map containing 131 roads and 71 intersections [6], with charging stations placed in some of the intersections. We have used a real dataset [4][19] to implement our model, and solved using the commercial solver CPLEX, and IBM's drop-solved platform. Specifically, the paper has the following contributions.

1. MILP models are proposed for an even distribution of delivery locations among EVs, and subsequently finding routes for them to deliver goods at delivery points while considering their en route charging.
2. The problem is proved to be NP-complete.
3. The proposed model addresses the effect of congestion at different charging stations. In particular, we show how partial charging at different charging stations proves to be effective for tackling congestion while ensuring fast delivery of services.
4. Detailed simulation has been performed on a real dataset to show the efficacy of our model in delivering services/goods at various locations in a city network.

The rest of this paper is organized as follows. Related works are discussed in Section 2. The *(MinTravelEVC)* problem is formulated in Section 3, and proved to be NP-complete in Section 4. Section 5 describes the linearization of different non-linear constraints. Section 6 presents the simulation setup and evaluates the performance of the proposed models. Finally, Section 7 concludes the paper.

## 2 RELATED WORKS

In the literature, some works have only considered the routing of EVs. An energy-optimal routing problem is formulated by Sachenbacher et al. [27], where one-charge distance rather than en route charging is considered with constraints of recuperation and battery capacity. Routing problems is also studied for long distance trips with different objectives like minimizing charging delays [10], energy-efficient route [5], [28], or a combination objective of travel time, charging time, and energy consumption [3]. The aforementioned works on en route charging, however, implicitly assumes constant electricity price, and do not consider uncertainties in navigation.

| Optimization Method | Reference |
|---|---|
| Branch and Bound | [8], [14] |
| Mixed Integer Linear Programming | [7], [16], [4], [6], [18], [32] |
| Genetic Algorithm | [29], [28], [17] |
| Heuristic Method | [26], [22], [19], [21] |
| Fuzzy Optimization | [31], [33] |
| Linear Programing | [35], [25] |
| Stochastic Optimization | [10], [11] |
| Evolutionary Algorithm | [13] |
| Dynamic Programming | [9], [12], [30] |

**Table 1: Optimization Technique Used**

Research works has also been done in the field of delivery optimization. Jin et al. propose [15] a branch-and-bound algorithm to maximize the profit of a firm by quoting different delivery times for all consumers. Fuzzy-based clustering methods are used in [34] to divide delivery jobs among drivers for different locations, and then the optimal sequence of deliveries has been decided with different timing assignments. In [32], a bi-criteria mixed integer program is formulated that minimizes the total delivery costs from manufacturers to consumers while minimizing lateness of deliveries. Yu et al. [35] employ based-on-set-partitioning formulation to minimize carbon emissions of pickup and delivery through integrated scheduling. Cerna et al. [6] present a mixed integer linear programming model to optimize the costs of maintenance and extra hours for scheduling a fleet of EVs so that the products are delivered to given destinations. However, they do not considered time-differentiated pricing. Marandi et al. [18] propose a simulated annealing approach to transport a set of goods among factories through batches of limited capacity to minimize the total delivery cost, tardiness and holding cost. Nait et al. [22] proposed a predictive model for evaluating the charging request for electric vehicles. The model also predicts the charging time and rate for multiple vehicles by considering their arrival time and state of charge. Web services and geo-positioning techniques are used by Ruzmetov et al. [26] to increase the coordination between the electric vehicles, charging stations, and energy providers, and an optimization technique is used to assign EVs to the charging station. Michael et al. [29] proposed a genetic algorithm technique is used to solve the charging demand of the electric vehicles, and the shortest distance between two nodes is calculated by dynamic Dijkstra.

A mixed-integer linear programming formulation is discussed by Merve et al. [16] for solving a time window based vehicle routing problem; also, the newly arrived electric vehicles to the charging station have to wait in the queue before getting their turn to start the charging. CPLEX solver is used for solving the MILP model, and large instances of the problems are solved by combining the MILP model with Adaptive Large Neighborhood Search. A unified framework is developed by Chen et al. [7] using a mixed-integer programming model. The developed considers the real-time charging demand with a time window of the electric vehicles and choose an optimal departure time at the charging station to reduce the congestion, and the departure time is considered as a decision variable. Christofides et al. [8] proposed two tree search algorithms, the shortest spanning k-degree center tree and q-routes, to solve a generalized version of the multiple traveling salesman problem. A tabular summary of the literature review is shown in table 1 with optimization techniques and their respective references.

To the best of our knowledge, there does not exist any work that considers partial charging of EVs on their way while addressing the effect of congestion at multiple charging stations.

## 3 PROBLEM FORMULATION

Consider a city transportation map as a directed graph $G = (V, E)$ where $V$ is the set of road-intersections, and $E \subseteq V \times V$ represents the set of road segments. Each road in the city is modelled as an ordered pair $(i, j) \in E$ that denotes a directed edge from node $i$ to node $j$ having distance $d_{i,j}$.

A transport operator has a set of $H$ EVs ($|H| = n$) that starts from a warehouse $wh$, delivers goods (or services) at different locations in the city, and finally comes back to the warehouse. In this paper, we address two important challenges at this point.

(1) How the transport operator distributes the delivery locations among the EVs?

| Variables | Meaning |
|---|---|
| | **City Parameters** |
| $V$ | Set of intersections (nodes) |
| $E$ | Set of directed edges |
| $d_{ij}$ | Distance from node $i$ to node $j$ |
| | **Charging Station Parameters** |
| $CS$ | Set of charging stations |
| $m$ | Total number of charging stations |
| $\rho_s$ | Charge transfer per unit time from a charge outlet of a charging station $s$ |
| | **EV Parameters** |
| $H$ | Set of EVs |
| $n$ | Number of EVs |
| $D$ | Set of delivery locations |
| $D_h$ | Set of delivery locations for EV $h$ |
| $sl_h$ | Start location of the journey for EV $h$ |
| $el_h$ | End location of the journey for EV $h$ |
| $v_h$ | Average speed of EV $h$ |
| $mc_h$ | Maximum battery capacity of EV $h$ |
| $ISOC_h$ | Initial state-of-charge of EV $h$ |
| $CSOC_{i,h}$ | State-of-charge of EV $h$ at node $i$ |
| $e_h$ | Energy consumed by $h$ per unit distance of travel |
| $at_{i,h}$ | Arrival time of $h$ at node $i$ |
| $dt_{i,h}$ | Departure time of $h$ from node $i$ |
| $st_{i,h}$ | Time spend at node $i$ by EV $h$ (Service Time) |
| $wt_{i,h}$ | Waiting time at charging station $i$ by EV $h$ |
| | **Other Parameters** |
| $b_{i,j,h}$ | A boolean decision variable indicating whether the EV $h$ travels through road $ij$ |
| $w_{i,h}$ | A boolean decision variable indicating whether the EV $h$ charging at node $i$ |
| $x_{h,h',i}$ | A boolean decision variable indicating whether the EV $h$ is charging before $h'$ at charging station $i$ |
| $x_{i,j}$ | A boolean decision variable indicating whether node $i$ has node $j$ as a centroid |

**Table 2: Variables in Problem Formulation**

(2) How the routes of the EVs will be determined?

## 3.1 Distribution of Delivery Locations among EVs

We assume that an EV can deliver services to at most $L$ delivery locations in a single tour. The length of the shortest path between two delivery points $i$ and $j$ is represented by $ds_{i,j}$. Since, each EV starts and ends its tour in $wh$, and their tour are mutually exclusive in the sense that each delivery location will be visited at most once by an EV, we create $n$ copies of the warehouse $wh$ as a set $W = \{wh_1, wh_2, \dots wh_n\}$, where mathematically they represent different locations, but in practice, they are the same warehouse $wh$. To deliver to all the delivery locations, $L$ must be greater than or equal to $\frac{|D'|}{n}$, where $D' = D \cup W$. To ensure short length tours for the EVs, our objective is to minimize the distances among the delivery locations and its centroids, such that the set of delivery locations for an EV is as compact as possible. The objective function is written as follows.

$$\min \sum_{(i,j)\in D'\times D'} ds_{i,j} \cdot x_{i,j} \tag{1}$$

subject to the following constraints.

Each delivery location can only be associated with exactly one centroid, which is expressed as follows.

$$\forall i \in D' \quad \sum_{j\in D'} x_{i,j} = 1 \tag{2}$$

The tour of an EV must have a warehouse associated with it. Also, we need to ensure that no two warehouses belong to the same EV, which is expressed as follows.

$$\forall j \in D' \quad \sum_{i\in W} x_{i,j} \leq 1 \tag{3}$$

The constraint below ensures that the total number of delivery locations allocated to an EV doesn't exceed $L$.

$$\forall j \in D' \quad \sum_{i\in D'} x_{i,j} \leq L \tag{4}$$

Since the total number of EV available to the transport operator is $n$, we need to ensure that there are only $n$ tours generated by this model, which is expressed as follows.

$$\sum_{j\in D'} \bigvee_{i\in D'} x_{i,j} = n \tag{5}$$

The solution to this model gives an even distribution of delivery locations among EVs, i.e., an EV $h \in H$ starts from a warehouse, delivers goods to a set of delivery locations $D_h$, and finally returns back to a warehouse.

## 3.2 Finding the Routes for EVs

In this section, we have to decide the route of an EV $h$ through its delivery locations $D_h$.

The charging stations are placed in some of the road-intersections of the city. The set of charging stations is denoted by $CS$ that provide charging services to the electric vehicles (EVs) ($|CS| = m$). Without loss of generality, it is assumed that there is only one charging outlet at each charging station such that only one EV can be charged at a time. Each charging station $s \in CS$ has a charging rate (energy transfer in unit time) $\rho_s$.

Each EV $h$ has a maximum battery capacity $mc_h$, initial state of charge $ISOC_h$. An EV $h$ moves with an average speed $v_h$, and its energy consumption due to travel a unit distance is $e_h$. In the problem formulation, we have defined two boolean decision variable $b_{i,j,h}$ and $w_{j,h}$ as follows.

$\forall h \in H, \forall i, j \in V,$

$$b_{i,j,h} = \begin{cases} 1, & \text{if EV } h \in H \text{ is using edge } (i, j) \\ 0, & \text{otherwise}. \end{cases}$$

$\forall h \in H, \forall j \in CS,$

$$w_{j,h} = \begin{cases} 1, & \text{if EV } h \text{ is charging at charging station } j \\ 0, & \text{otherwise}. \end{cases}$$

During the journey of an EV $h$, it has to pass through different intersections, where it may stop at an intersection for delivery of goods, or for charging, or simply it may pass through an intersection without stopping. Let, $at_{j,h}$ and $dt_{j,h}$ denote the arrival time and departure time at intersection $j$ respectively. When an EV $h$ arrived at node $j$ from node $i$, the arrival time of $h$ at node $j$ ($at_{j,h}$) is calculated by adding the time to travel distance $d_{i,j}$ to the departure time at node $i$ ($dt_{i,h}$) (Equation 6).

$$\forall h \in H, \forall j \in V, \quad at_{j,h} = \sum_{\forall i \in V} b_{i,j,h} \cdot \left( dt_{i,h} + \frac{d_{i,j}}{v_h} \right) \tag{6}$$

Based on whether an intersection is a delivery point for $h$, or a charging station that is used to charge $h$, or an intersection through which $h$ simply passes through, three cases are possible for calculation of $dt_{j,h}$.

Case I. *The intersection j is a delivery point for EV h*: It is assumed that when an EV $h$ reaches a delivery point $j$, it delivers the goods immediately without waiting. The EV $h$ takes $st_{j,h}$ time (service time) to deliver goods at $j$. The departure time $dt_{j,h}$ is calculated by adding service time ($st_{j,h}$) to its arrival time ($at_{j,h}$).

$$\forall h \in H, \forall j \in D_h, \quad dt_{j,h} = at_{j,h} + st_{j,h} \tag{7}$$

Goods are also assumed to be delivered by EV $h$ at $j$ within a stipulated deadline $dl_{j,h}$, i.e., $dt_{j,h} \leq dl_{j,h}$.

Case II. *The intersection j is a charging station for EV h* When an EV $h$ passes by a road segment $(i,j)$ in its route without being charged at $j$, its current state of charge at node $j$ will be equal to its state of charge at node $i$ minus charge lost in traveling distance $d_{i,j}$, i.e., $e_h \cdot d_{i,j}$. The current state of charge ($CSOC_{j,h}$) of an EV $h$ at node $j$ is given as follows.

$$\forall h \in H, \forall j \in V,$$
$$CSOC_{j,h} = \sum_{\forall i \in V} b_{i,j,h} \cdot \left( CSOC_{i,h} - \frac{e_h \cdot d_{i,j}}{mc_h} \right) \quad \text{if } w_{j,h} = 0 \tag{8}$$

It may so happen that when an EV $h$ has reached a charging station $j$, it may not start its charging immediately, because some other EV $h'$ is charging at that time. In such cases, it may have to wait for $h'$ to complete its charging. The waiting time for an EV $h$ at a charging station $j$ is denoted by $wt_{j,h}$. We assume that $wt_{i,h} \geq 0$ only at a charging station, and zero at other intersections. When an EV $h$ is charged at $j$, its current state of charge will be equal to 1, i.e., $CSOC_{j,h} = 1$. The charging time ($ch_{j,h}$) of an EV $h$ at charging station $j$ is given as follows.

$$\forall h \in H, \forall j \in CS, \quad ch_{j,h} =$$
$$\sum_{\forall i \in V} b_{i,j,h} \cdot w_{j,h} \cdot \frac{mc_h \cdot (1 - CSOC_{i,h}) - e_h \cdot d_{i,j}}{\rho_j} \tag{9}$$

If the EV $h$ is actually scheduled for charging at a charging station $j$, then $dt_{j,h}$ is calculated by adding its charging time and waiting time at $j$ to its arrival time.

$$\forall h \in H, \forall j \in CS,$$
$$dt_{j,h} = at_{j,h} + wt_{j,h} + ch_{j,h} \quad \text{if } w_{j,h} = 1 \tag{10}$$

Case III. *EV h passes through a intersection $j \notin \{CS \wedge D_h\}$* In this case, the departure time is equal to the arrival time for $h$, i.e., $dt_{j,h} = at_{j,h}$, and current state of charge will be calculated same as Equation 8.

$$\forall h \in H, \forall j \in V, \quad CSOC_{j,h} = CSOC_{i,h} - \frac{e_h \cdot d_{i,j}}{mc_h} \quad \text{if } b_{i,j,h} = 1 \tag{11}$$

Based on the aforementioned model, an EV $h \in H$ starts its journey from $sl_h$, delivers goods to a set of delivery points $D_h$, and finally returns back to $el_h$. Note that the locations $D_h$ are pre-scheduled, but the order in which these delivery points will be traversed by the EV $h$ has to be decided. Moreover, an EV may require charging at different charging stations in its route while delivering goods, since the total distance travelled by the EV may be long. The total travel time $Trvl_h$ of an EV $h$ is given as follows.

$$Trvl_h = \sum_{\forall i,j \in V} b_{i,j,h} \cdot \frac{d_{i,j}}{v_h} \tag{12}$$

The total time $Chrg_h$ for charging the EV $h$ is given as follows.

$$Chrg_h = \sum_{\forall j \in CS} (dt_{j,h} - at_{j,h}) \tag{13}$$

In this paper, we have formally formulated an optimization problem using a mixed integer linear programming (MINLP) model, where the objective is to minimize the total time spent $Time_h$ in travel as well as charging for all the EVs, i.e., $Time_h = Trvl_h + Chrg_h$. The objective is given as follows.

$$Minimize \sum_{\forall h \in H} Time_h \tag{14}$$

subject to the following constraints.

*3.2.1 Path constraints.* Constraints 15, 16 ensures that there is no cycle in the route for EV $h$ and there is at most one incoming and outgoing edge to any node. In the above constraints, $\sum_{\forall i \in V} b_{i,j,h}$ represents the total number of incoming edges to node $j$ for EV $h$. Similarly $\sum_{\forall j \in V} b_{i,j,h}$ represents the total number of outgoing edges from node $i$ for EV $h$.

$$\forall h \in H, \forall j \in V \quad \sum_{\forall i \in V} b_{i,j,h} \leq 1 \tag{15}$$

$$\forall h \in H, \forall i \in V \quad \sum_{\forall j \in V} b_{i,j,h} \leq 1 \tag{16}$$

We also assign $b_{i,j,h} = 0$, if there is no edge between $i, j$. This eliminates many redundant binary decision variables from the solution space.

Constraints 17, 18, 19 ensures that there is a path selected from $sl_h$ to $el_h$ for an EV $h$. Constraint 17 indicates that there is no incoming edge to the start location $sl_h$, whereas only one outgoing edge is required.

$$\forall h \in H \quad \sum_{\forall i \in V} b_{i,sl_h,h} - \sum_{\forall j \in V} b_{sl_h,j,h} = -1 \tag{17}$$

Constraint 18 ensures that for all intermediate nodes (nodes except $sl_h$ and $el_h$) in the route of $h$, the number of incoming edges needs to be equal to the number of outgoing edges.

$$\forall h \in H, \forall k \in V \setminus \{sl_h, el_h\} \quad \sum_{\forall i \in V} b_{i,k,h} - \sum_{\forall j \in V} b_{k,j,h} = 0 \tag{18}$$

Constraint 19 indicates that there is no outgoing edge from the end location $el_h$, whereas only one incoming edge is required.

$$\forall h \in H \quad \sum_{\forall i \in V} b_{i,el_h,h} - \sum_{\forall j \in V} b_{el_h,j,h} = 1 \tag{19}$$

Constraints 20, 21 use Miller-Tucker-Zemlin formulation [24], where it ensures that there are no sub-tours in the route selected for EV $h$.

$$\forall h \in H, \forall i,j \in V \quad u_{i,h} - u_{j,h} + |V| \cdot b_{i,j,h} \leq |V| - 1 \quad (i \neq j) \tag{20}$$

$$\forall h \in H, \forall i \in V \quad 1 \leq u_{i,h} \leq |V| \tag{21}$$

*3.2.2 Charging constraints.* Constraint 22 ensures that an EV $h$ can only charge at charging station $j$ that is in its route.

$$\forall h \in H, \forall j \in CS \quad \sum_{\forall i \in V} b_{i,j,h} \geq w_{j,h} \tag{22}$$

When an EV $h$ arrives at charging station $j$ for charging, it needs to wait $wt_{j,h}$ unit time, if another EV $h'$ is charging at charging station $j$, otherwise it can start charging without any waiting time. A binary decision variable $x_{h',h,j}$ is used to indicate that EV $h'$ is charging before EV $h$ at charging station $j$.

$$\forall h \in H, \forall j \in CS, \quad x_{h',h,j} = \begin{cases} 1, & \text{if EV } h' \text{ is charging before } h \text{ at station } j \\ 0, & \text{otherwise.} \end{cases}$$

Constraint 23 and 24 ensures that the waiting time $wt_{j,h}$ for EV $h$ at charging station $j$ is at least $(dt_{j,h'} - at_{j,h})$ if $x_{h',h,j} = 1$, i.e., if EV $h'$ is charging at charging station $j$ and EV $h$ has also arrived at $j$.

$$\forall h, h' \in H, \forall j \in CS, \quad wt_{j,h} \geq (dt_{j,h'} - at_{j,h}) \cdot x_{h',h,j} \tag{23}$$

$$\forall h \in H, \forall j \in CS \quad wt_{j,h} \geq 0 \tag{24}$$

Constraint 25 ensures mutual exclusion, i.e., only one EV can be charged at a time at a charging station $j$. Constraint 26 ensures that when two EVs are scheduled to charge at charging station $j$ i.e., $w_{j,h} = 1$ and $w_{j,h'} = 1$, then at least one of them should start charging at charging station $j$.

$$\forall h \in H, \forall h' \in H, \forall j \in CS \quad x_{h',h,j} + x_{h,h',j} \leq 1 \tag{25}$$

$$\forall h \in H, \forall h' \in H, \forall j \in CS \quad \begin{aligned} x_{h',h,j} + x_{h,h',j} &\geq w_{j,h} + w_{j,h'} - 1 \\ x_{h',h,j} + x_{h,h',j} &\leq w_{j,h} \\ x_{h',h,j} + x_{h,h',j} &\leq w_{j,h'} \end{aligned} \tag{26}$$

## 4 PROOF OF NP-COMPLETENESS

To prove the *MinTravelEVC* problem as NP-complete, we consider a simplified version of the problem, where the transport operator has a single EV $h$ with an infinite battery capacity ($mc_h = \infty$), which eliminates its use of any of the charging stations. Now, the objective is to minimize the total travel time of $h$, while delivering goods to $D_h$ delivery locations. The equivalent decision version of the *MinTravelEVC* problem (*D-MinTravelEVC*) can be defined as follows. Given a transportation network $G = (V, E)$ , an EV $h$ with unlimited battery capacity, a set $D_h$ of delivery locations, and a positive integer $\delta$, decide whether it is possible to find a path such that all $D_h$ delivery locations are covered exactly once with total travel time $Time_h \leq \delta$?

To prove that the *D-MinTravelEVC* problem is NP-complete, we first show that it is in NP and then, reduce the Travelling Salesperson Problem [23] to it in deterministic polynomial time to prove that *D-MinTravelEVC* is NP-hard.

The decision version of the Travelling Salesperson Problem is defined as follows. Given a complete weighted graph $G'$, a set $Q$ of cities, and a positive integer $\alpha$, decide whether there is a tour through all the cities with the length of the total travel path is $\leq \alpha$?

Theorem 1. *The D-MinTravelEVC problem is NP-complete.*

Proof. Given a navigation sequence of EV $h$ through $D_h$, which is a polynomial size certificate for *D-MinTravelEVC*, one can calculate the total travel time ($Time_h$) for the EV $h$ and verify whether $Time_h \leq \delta$ along with the constraints in deterministic polynomial time, and Hence D-MinTravelEVC $\in$ NP.

For proof of NP-hardness of D-MinTravelEVC, we will reduce an instance of Travelling Salesperson Problem to the D-MinTravelEVC problem in deterministic polynomial time. Given a complete weighted graph $G'$, a set $Q$ of cities, and a positive integer $\alpha$ as input to Travelling Salesperson Problem, we reduce it to an equivalent input for the D-MinTravelEVC using the following steps.

(1) The transportation network G is identical to the complete weighted graph $G'$.
(2) City $q_1 \in Q$ is the starting location (Warehouse location) for EV $h$, i.e., $sl_h = q_1$ and $el_h = q_1$, with battery capacity of infinity ($mc_h = \infty$).
(3) $D_h = Q$.
(4) Finally, $\delta = \alpha$.

From the above reduction, it is clear that both Travelling Salesperson Problem and D-MinTravelEVC have a one-to-one relationship, and hence, it is NP-Hard. Since D-MinTravelEVC is in NP and it is NP-hard, therefore, D-MinTravelEVC is NP-complete. □

# 5 LINEARIZATION

In Section 3, Equation 6, 8, 9, 10, 11 and 23 are non-linear, since some of the terms are expressed as a product of one or more binary variables with a continuous variable, and therefore, they require to be linearized. In this section, we perform *Big-M Linearization* method to linearize the aforesaid equations before solving them through CPLEX solver.

## 5.1 The Big-M Linearization Method

In this section, the *Big-M Linearization* method is illustrated with two general cases. In Big-M method, value of $M$ is vital; taking $M$ too small can end up producing wrong results.

*5.1.1 Linearization Method for a Product of a Binary Variable and a Continuous Variable.* Consider the following equation $x = C \cdot b$, where $b \in \{0,1\}$ is a binary variable, and $C$ is a continuous variable. The equation implies that if $b = 1$, then $x = c$; otherwise $x = 0$. Assume that $0 \le C \le M$, where is $M$ is a large value. The equation is linearized as follows.

$$x \le M \cdot b$$

$$x \le C$$

$$x \ge C - (1-b)M$$

$$x \ge 0$$

Case I. *b is zero:* The first two equations become $x \le 0$ and $x \le C$, which implies $x \le 0$, and the last two equations become $x \ge C - M$ where $C < M$, and $x \ge 0$, which implies $x \ge 0$. Therefore, $x = 0$.

Case II. *b is one:* The first two equations become $x \le M$ and $x \le C$, which implies $x \le C$, and the last two equations become $x \ge C$ and $x \ge 0$, which implies $x \ge C$. Therefore, $x = C$.

*5.1.2 Linearization Method for a Product of two Binary Variables.* Consider the following equation $x = a \cdot b$, where $b, a \in \{0,1\}$ are binary variables. The equation implies that if $a$ and $b$ is 1 then $x = 1$, otherwise value of $x$ is zero. The equation is linearized as follows.

$$x \ge a + b - 1$$

$$x \le a$$

$$x \le b$$

Case I. *a and b both are one:*
From first inequality we get $x \ge 1$, from second and third we get $x \le 1$. Therefore, $x = 1$.

Case II. *a and b both are zero:*
From first inequality we get $x \ge -1$, from second and third we get $x \le 0$. Therefore, $x = 0$.

Case III. *otherwise:*
From first inequality we get $x \ge 0$, from second and third inequality we will get $x \le 0$ and $x \le 1$. Therefore, $x = 0$.

## 5.2 Using Big-M Linearization Method

Using strategies discussed in Section 5.1, we linearize Equation 6, 8, 9, 10, 11 and 23 as follows.

(1) *Equation 6 is linearized as follows:*

$\forall h \in H, \forall i, j \in V$

$$\begin{aligned} at_{j,h} &\ge \left(dt_{i,h} + \frac{d_{i,j}}{v_h}\right) - M \cdot (1 - b_{i,j,h}) \\ at_{j,h} &\le \left(dt_{i,h} + \frac{d_{i,j}}{v_h}\right) + M \cdot (1 - b_{i,j,h}) \\ at_{j,h} &\le M \cdot \sum_{\forall i \in V} b_{i,j,h} \\ at_{j,h} &\ge 0 \end{aligned} \tag{27}$$

Here, the arrival time at node $j$ can only be decided after checking $b_{i,j,h}$ for all values of $i$. The first two inequalities ensure that when an edge $(i,j)$ is taken by EV $h$, i.e., $b_{i,j,h} = 1$, then arrival time will be equal to $dt_{i,h} + \frac{d_{i,j}}{v_h}$. Otherwise, the first two inequalities implies $at_{j,h} \in [-M, +M]$. Again, the last inequality implies that $at_{j,h} \ge 0$. Therefore, $at_{j,h} \in [0, +M]$. If EV $h$ does not pass through node $j$, then the last two inequalities ensure that $at_{j,h} = 0$.

(2) *Equation 8 and 11 are linearized as follows:*
An EV can take one of the three possible action at an intersection, we have provided linearization for each case.

Case I. *The intersection j is a charging station and EV h is scheduled to charge at j:*

$\forall h \in H, \forall j \in CS$

$$\begin{aligned} CSOC_{j,h} &\ge 1 - M \cdot (1 - w_{j,h}) \\ CSOC_{j,h} &\le 1 + M \cdot (1 - w_{j,h}) \end{aligned} \tag{28}$$

In above linearization when the value of $w_{j,h}$ is 1, i.e., EV $h$ is charging at charging station $j$, both equations ensure that $CSOC_{j,h}$ is 1, i.e., 100% charged.

Case II. *The intersection j is a charging station and EV h is* **not** *scheduled to charge at j:*

$\forall h \in H, \forall i \in V, \forall j \in CS$

$$\begin{aligned} &k_{i,j,h} \ge b_{i,j,h} - w_{j,h} \\ &k_{i,j,h} \le b_{i,j,h} \\ &k_{i,j,h} \le 1 - w_{j,h} \\ &CSOC_{j,h} \ge CSOC_{i,h} - \frac{e_h \cdot d_{i,j}}{mc_h} - M \cdot (1 - k_{i,j,h}) \\ &CSOC_{j,h} \le CSOC_{i,h} - \frac{e_h \cdot d_{i,j}}{mc_h} + M \cdot (1 - k_{i,j,h}) \end{aligned} \tag{29}$$

To ensure that EV $h$ has a route through charging station $j$ where it is not scheduled for charging, we used a binary variable $k_{i,j,h}$ and in last two equations, we used binary variable $k_{i,j,h}$ to get the updated state of charge at charging station $j$.

Case III. *The intersection j is* **not** *a charging station*:

$$\forall h \in H, \forall i \in V, \forall j \in V \setminus CS$$
$$CSOC_{j,h} \geq CSOC_{i,h} - \frac{e_h \cdot d_{i,j}}{mc_h} - M \cdot (1 - b_{i,j,h}) \quad (30)$$
$$CSOC_{j,h} \leq CSOC_{i,h} - \frac{e_h \cdot d_{i,j}}{mc_h} + M \cdot (1 - b_{i,j,h})$$

Above linearization ensures that if path $(i, j)$ is in the route of EV $h$ and $j$ is a non-charging intersection, then $CSOC_{i,h}$ is reduced by $\frac{e_h \cdot d_{i,j}}{mc_h}$ to obtain $CSOC_{j,h}$.

(3) *Equation 9 is linearized as follows:*

$$\forall h \in H, \forall i \in V, \forall j \in CS$$
$$z_{i,j,h} \geq b_{i,j,h} + w_{j,h} - 1$$
$$z_{i,j,h} \leq b_{i,j,h}$$
$$z_{i,j,h} \leq w_{j,h}$$
$$ch_{j,h} \geq \frac{mc_h \cdot (1 - CSOC_{i,h}) - e_h \cdot d_{i,j}}{\rho_j} - M \cdot (1 - z_{i,j,h}) \quad (31)$$
$$ch_{j,h} \leq \frac{mc_h \cdot (1 - CSOC_{i,h}) - e_h \cdot d_{i,j}}{\rho_j} + M \cdot (1 - z_{i,j,h})$$
$$ch_{j,h} \leq M \cdot w_{j,h}$$
$$ch_{j,h} \geq 0$$

To ensure that EV $h$ has a route through charging station $j$ and scheduled for charging at $j$ we used a binary variable $z_{i,j,h}$ and we used binary variable $z_{i,j,h}$ to obtain charging time at the charging station $j$.

(4) *Equation 10 is linearized as follows:*

$$\forall h \in H, \forall i \in V, \forall j \in CS$$
$$dt_{j,h} \geq at_{j,h} + ch_{j,h} + wt_{j,h} - M \cdot (1 - w_{j,h})$$
$$dt_{j,h} \leq at_{j,h} + ch_{j,h} + wt_{j,h} + M \cdot (1 - w_{j,h}) \quad (32)$$
$$dt_{j,h} \geq at_{j,h}$$

In the above linearzation, we ensure that when EV $h$ is charging at the charging station $j$, the charging time and the waiting time are added to the arrival time of $j$ to calculate the departure time of $j$. For other cases, the departure time of $j$ is greater than the arrival time of $j$.

(5) *Equation 23 is linearized as follows:*

$$wt_{i,h} \geq (dt_{i,h'} - at_{i,h}) - M \cdot (1 - x_{h',h,i}) \quad (33)$$
$$wt_{i,h} \geq 0$$

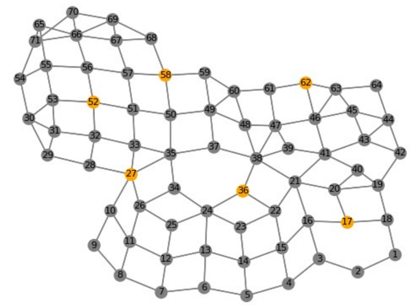

**Figure 1: Map of the City**

## 6 SIMULATION RESULTS

The performance of the proposed scheme has been shown on a city map [6] containing 131 roads and 71 intersections [6], with charging stations placed in some of the intersections. In the city map [6] (shown in Figure 1), gray nodes represent road intersections, and orange nodes represent charging stations. The charging power of these stations is set as per Indian Power Ministry standards and shown in table 3[1].

We have used a real dataset [4][19] to implement our model, and solved using the commercial solver CPLEX [2], and IBM's drop-solved platform. Each instance consists of 5 EVs, and the delivery locations are different across different instances. We study how our proposed model affects the distribution of delivery locations among EVs, the travel time, and charging time of EVs. Also, we analyze how the partial charging of EVs at multiple charging stations minimizes the overall travel time of the EVs through their charging profile/behavior.

| Station ID | power (Kw) |
|---|---|
| 36 | 55 |
| 52 | 22 |
| 62 | 55 |
| 58 | 22 |
| 27 | 22 |
| 17 | 55 |

**Table 3: Charging Stations Power**

Each EV has an initial state of charge of 15.75 kWh and maximum capacity of the battery $mc_h$ is 35 kWh. The current energy drawn from the battery of an EV may vary from 3.5 kWh to 35 kWh during its journey. We assume that EVs have started their journey at the same time with an average speed 40 km/h. The energy consumption rate of an EV is 0.4 kWh/km [31].

First, we discuss in detail about one of the instances from the dataset, and analyse our results. The transport operator has five EVs parked initially at the warehouse, which is located at node 34. The delivery locations are situated at the following nodes: 2, 5, 7, 10, 19, 22, 25, 29, 33, 38, 41, 46, 50, 51, 57, 60, 61, 65, 68, and 70. In the first stage, the delivery locations is distributed among five EVs through solving our model 3.1 in CPLEX, and their travel paths are determined in the second stage 3.2. The allocation of delivery locations is shown in Table 4. Table 5 shows the shortest routes for EVs to complete their journey, node ids in bold represent the delivery locations, and node ids with an asterisk sign (*) represent charging stations.

| EV | Delivery-1 | Delivery-2 | Delivery-3 | Delivery-4 |
|---|---|---|---|---|
| 1 | 25 | 10 | 7 | 5 |
| 2 | 22 | 2 | 19 | 41 |
| 3 | 38 | 50 | 61 | 46 |
| 4 | 29 | 51 | 68 | 60 |
| 5 | 33 | 57 | 65 | 70 |

**Table 4: Delivery Schedule of EVs**

| EV | Route |
|---|---|
| EV1 | **34** → 24 → **25** → 12 → 13 → 14 → 15 → 16 → **17**$^*$ → 16 → 3 → 4 → **5** → 6 → **7** → 8 → 9 → **10** → 11 → 26 → **34** |
| EV2 | **34** → 24 → **36**$^*$ → 22 → 21 → **41** → 40 → **19** → 18 → **17**$^*$ → 18 → 1 → **2** → 3 → 4 → 15 → **22** → 23 → 24 → **34** |
| EV3 | **34** → 24 → **36**$^*$ → **38** → 47 → **46** → **62**$^*$ → **61** → 60 → 49 → **50** → 35 → **34** |
| EV4 | **34** → 26 → 27 → 28 → **29** → 28 → **27**$^*$ → 33 → **51** → 50 → **58**$^*$ → **68** → 58 → 59 → **60** → 48 → 38 → **36**$^*$ → 24 → **34** |
| EV5 | **34** → 35 → **33** → 32 → **52**$^*$ → 56 → 66 → **70** → 69 → 68 → **58**$^*$ → **57** → 56 → 55 → **65** → 55 → 53 → 31 → 32 → 33 → 35 → **34** |

**Table 5: Routes of EVs**

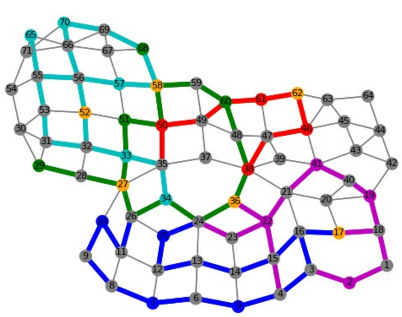

**Figure 2: Journey of all EVs**

In Table 6, the total travel time, the charging time across different charging stations, the total waiting time at different charging station en route is given. The last column represents the total time spent for an EV in its journey. It is important to note that EV2 has a waiting time of 0.41 hr, since both EV2 and EV3 came at the charging station 36 at 0.31 and EV3 is scheduled earlier than EV2. The charging time for EV3 is 0.41, and therefore, its departure time becomes $0.31 + 0.41 = 0.72$. EV2 has to wait till EV3's charging is over, and it starts its charging at 0.72, where it departs at time $0.72 + 0.41 = 1.13$.

| EV ID | Travel Time | Charging Time | Waiting Time | Total Time |
|---|---|---|---|---|
| EV1 | 2.79 | 0.62 | 0.00 | 3.41 |
| EV2 | 3.44 | 0.83 | 0.41 | 4.67 |
| EV3 | 3.38 | 0.80 | 0.00 | 4.18 |
| EV4 | 4.17 | 2.65 | 0.00 | 6.81 |
| EV5 | 4.41 | 2.66 | 0.00 | 7.07 |

**Table 6: Total Time taken by EVs**

It is observed that the delivery locations are distributed evenly among the EVs. By even distribution, we not only mean that each EV has almost an equal number of delivery locations, but also the total length of a path traversed by an EV is almost identical. This is evident also from the Figure 2. Figure 2 is the graphical representation of entire table 5. The path of each EV is shown with a different color. EV1, EV2, EV3, EV4, and EV5 are assigned blue, magenta, red, green, and cyan respectively.

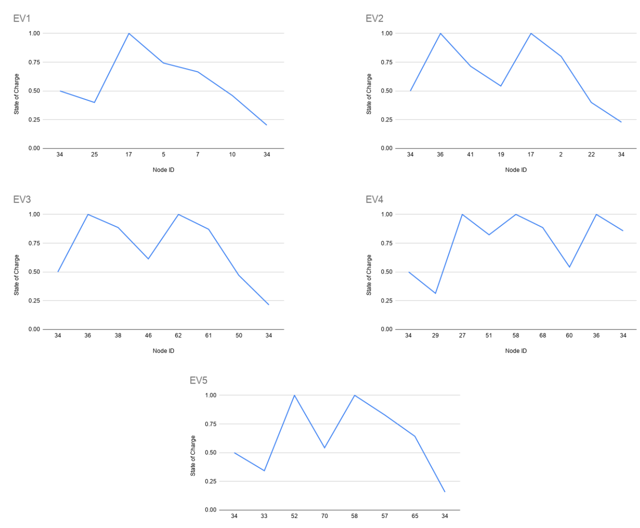


**Figure 3: Charge profile of EVs**

Figure 3 shows the charge profile of an EV. By charge profile, we mean the state of charge (SOC) of an EV battery at different point in time: 0 indicates an empty battery, whereas 1 represents that the battery is full charged. EV1 begins its journey from node 34 with SOC 0.50. The SOC reduces to 0.40 when it reaches its first delivery location at node 25. EV1 charges for the first time at the charging station 17 and its SOC becomes 1.00. Next it delivers goods to node 5, 7, 10, and finally returns back to 34. The SOCs of EV1 at node 5, 7, 10 and 34 are 0.74, 0.66, 0.46 and 0.20 respectively. EV2 requires two charging stations 36 and 17 to complete its journey and the

SOC at different delivery locations and charging stations are as follows. It starts with an SOC 0.50 from node 34, and charges at node 36 with SOC = 1.00. It then delivers goods at node 41 (SOC = 0.71), and at node 19 (SOC = 0.54) before it charges for the second time at charging station 17 (SOC = 1.00). After that, EV2 delivers at node 2 (SOC = 0.80) and node 22 (SOC = 0.40) before coming back to warehouse at node 34 (SOC = 0.23).

So far, we have discussed and analyzed our results on a single instance. The behavior is almost identical in other instances. Here, we give a comparative study of some of the instances from the dataset. As mentioned before, each instance consists of 5 EVs, and the delivery locations are different across different instances. In Table 7, a detailed comparison between full charging and partial charging is carried out. Table 7 has five columns, namely, the name of the instance, the total travel time while opting partial charging, the total travel time while opting full charging, average solver execution time for partial charge mode, and average solver execution time for full charge mode. Average execution time is generated by taking an average of 50 runs of the model. The results show that partial charging helps EVs to minimize the total travel time, but the software takes longer to decide a better charging plan.

| Instance | Travel Time (Partial charging) | Travel Time (Full charging) | Execution Time (Partial charging) | Execution Time (Full charging) |
|---|---|---|---|---|
| tc0c10s2cf1 | 18.85 | 19.53 | 2.44 | 1.28 |
| tc0c10s2ct1 | 14.23 | 15.34 | 4.28 | 1.54 |
| tc0c10s3cf1 | 18.85 | 19.53 | 5.14 | 4.50 |
| tc0c10s3ct1 | 12.83 | 13.53 | 18.34 | 6.28 |
| tc1c10s2cf2 | 14.25 | 14.80 | 8.48 | 6.26 |
| tc1c10s2cf3 | 18.27 | 23.27 | 21.65 | 24.05 |
| tc1c10s2ct2 | 14.48 | 15.15 | 325.06 | 16.38 |
| tc1c10s2ct3 | 18.97 | 23.93 | 68.00 | 53.22 |
| tc1c10s3cf2 | 14.25 | 14.80 | 23.95 | 23.50 |
| tc1c10s3cf3 | 18.27 | 23.27 | 455.65 | 203.42 |
| tc1c10s3cf4 | 17.75 | 19.66 | 100.38 | 6.65 |
| tc2c10s2cf0 | 15.58 | 17.60 | 42.70 | 12.02 |
| tc2c10s2ct0 | 12.74 | 15.62 | 30.54 | 113.29 |
| tc2c10s3cf0 | 15.58 | 17.60 | 159.12 | 24.06 |

**Table 7: Full charging vs. partial charging**

In summary, the proposed MILP model helps to distribute the delivery locations evenly across the EVs so that the average travel time is reduced. Also, partial charging of an EV at multiple charging stations proves to be useful for fast delivery of services/goods. However, if a transport operator owns a large number of EVs, the software may take longer time to decide the itinerary for the EVs.

## 7 CONCLUSION

In this paper, we have proposed two Mixed Integer Linear Programming models for the allocation of delivery locations among EVs, and minimization of the travel and charging time of EVs to ensure a fast delivery. The model evenly distributes the delivery locations to the EVs such that no one has to travel a longer route. By solving the second model, the route for an EV has been determined such that products are delivered in minimum possible time. The model considers various operational constraints such as delivery locations, limited battery capacity of the EV, congestion at charging stations, and other charging as well as mobility parameters to schedule the optimal route for the EVs. We have proved that the problem is an NP-complete problem. The analysis and evaluation of the proposed model on the real dataset has shown how an even distribution of delivery points among the EVs, and their partial charging minimizes the total travel time of the EVs while comparing to full charging scenarios. The proposed model may act as a useful tool in the analysis, evaluation, and scheduling of EVs in a city network, thereby enticing several delivery as well as emergency services to use this intelligent transport technology.

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