Satellite Orbits

Chapter 2

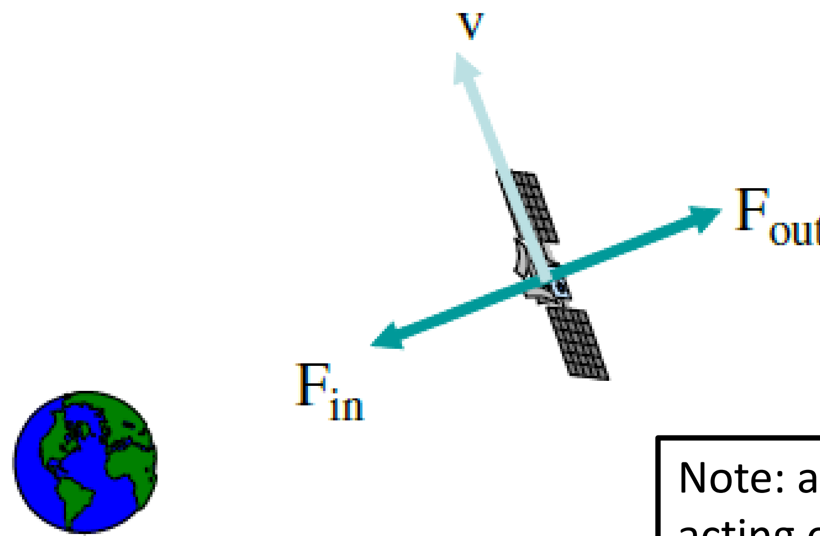
Introduction

- The orbital locations of the spacecraft in a communications satellite system play a major role in determining the coverage and operational characteristics of the services provided by that system.
- The next slides describe the general characteristics of satellite orbits and summarize the characteristics of the most popular orbits for communications applications.

Laws of Motion

- Artificial earth satellites that orbit the earth are governed by the same laws of motion that control the motions of the planets around the sun.
- Satellite orbit determination is based on the Laws of Motion first developed by Johannes Kepler and later refined by Newton in 1665 from his own Laws of Mechanics and Gravitation.
- Competing forces act on the satellite; gravity tends to pull the satellite in toward the earth, whereas its orbital velocity tends to pull the satellite away from the earth.

Figure 2.1 Forces acting on a satellite.



The gravitational force $\bar{F}_{in}$:

$$F_{in} = m\left(\frac{\mu}{r^2}\right)$$

the angular velocity force, F_{out}:

$$F_{out} = m\left(\frac{v^2}{r}\right)$$

Note: all other forces acting on the satellite, such as the gravity forces from the moon, sun and other bodies, are neglected.

m = satellite mass
v = satellite velocity in the plane of orbit
r = distance from the center of the earth (orbit radius)
μ = Kepler's Constant (or Geocentric Gravitational Constant)
$= 3.986004 \times 10^5\ \mathrm{km}^3/\mathrm{s}^2$

Note that for $F_{in} = F_{out}$,

$$v = \left(\frac{\mu}{r}\right)^{\frac{1}{2}}$$

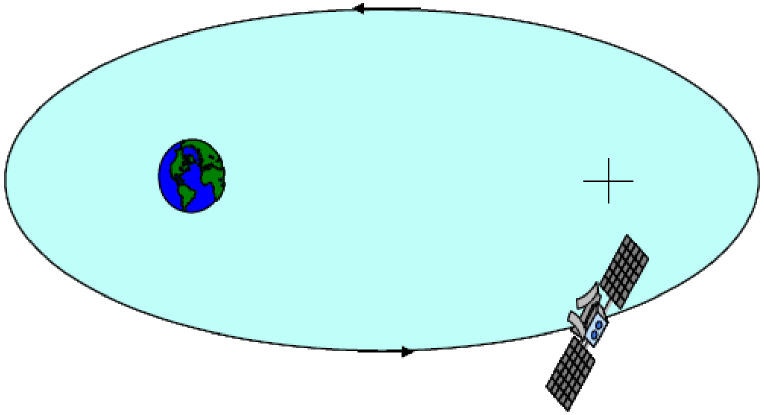
This result gives the velocity required to maintain a satellite at the orbit radius r.

Kepler’s Laws

- Kepler’s laws of planetary motion apply to any two bodies in space that interact through gravitation.
- The laws of motion are described through three fundamental principles.

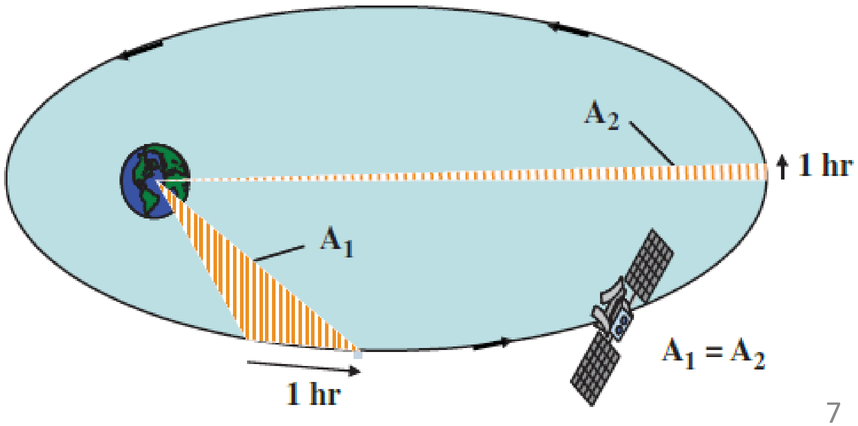
Kepler's First Law

- The path followed by a satellite around the Earth will be an ellipse, with the center of mass of Earth as one of the two foci of the ellipse.
- The "size" of the ellipse will depend on satellite mass and its angular velocity.



Kepler's Second Law

- For equal time intervals, the satellite sweeps out equal areas in the orbital plane.
- The satellite orbital velocity is not constant.
- The satellite is moving much faster at locations near the earth, and slows down as it approaches apogee.



Kepler's Third Law (1/2)

- The square of the periodic time of orbit is proportional to the cube of the mean distance between the two bodies.

$$T^2 = \left[\frac{4\pi^2}{\mu}\right] a^3$$

T = orbital period, in s
a = distance between the two bodies, in km
μ = Kepler's Constant = $3.986004 \times 10^5\ \mathrm{km}^3/\mathrm{s}^2$

- If the orbit is circular, then **a = r**, and $r = \left[\frac{\mu}{4\pi^2}\right]^{\frac{1}{3}} T^{\frac{2}{3}}$
- This demonstrates an important result:

$$\text{Orbit Radius} = [\mathit{Constant}] \times (\text{Orbit Period})^{\frac{2}{3}}$$

Under this condition, a specific orbit period is determined only by proper selection of the orbit radius. This allows the satellite designer to select orbit periods, which best meet particular application requirements by locating the satellite at the proper orbit altitude

Kepler's Third Law (2/2)

The altitudes required to obtain a specific number of repeatable ground traces with a circular orbit.

Table 2.1 Orbit altitudes for specified orbital periods.

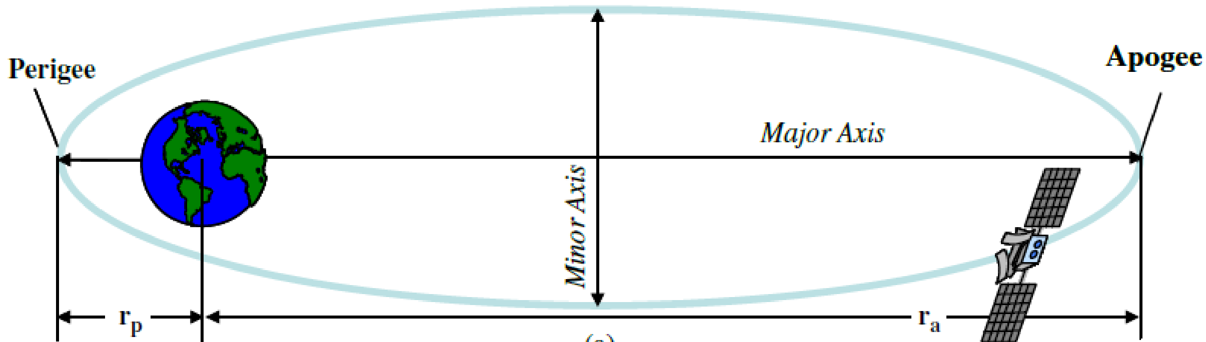
Revolutions/day	Nominal Period (hours)	Nominal Altitude (km)
1	24	36,000
2	12	20,200
3	8	13,900
4	6	10,400
6	4	6,400
8	3	4,200

Orbital Elements

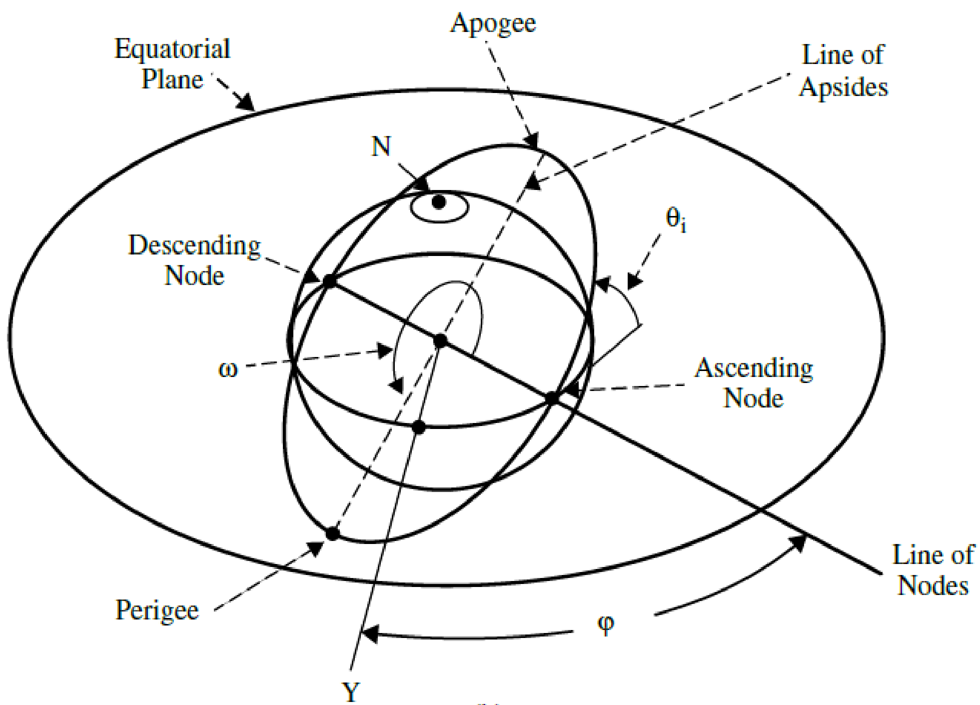
- *Define the set of parameters needed to uniquely* specify the location of an orbiting satellite.
- They are used to determine the satellite track and provide a prediction of satellite location for extended periods beyond the current time.
- The minimum number of parameters required is six.
  - 1) Eccentricity
  - 2) Semi-Major Axis
  - 3) Time of Perigee
  - 4) Right Ascension of Ascending Node
  - 5) Inclination Angle
  - 6) Argument of Perigee

Watch this video: https://www.youtube.com/watch?v=QZrYaKwZwhI

Orbital Parameters



(a)



(b)

Definition of orbital parameters

- Line of Apsides – the line joining the perigee and apogee through the center of the earth.
- Ascending Node – the point where the orbit crosses the equatorial plane, going from south to north.
- Descending Node – the point where the orbit crosses the equatorial plane, going from north to south.
- Line of Nodes – the line joining the ascending and descending nodes through the center of the earth.
- Argument of Perigee, ω – the angle from ascending node to perigee, measured in the orbital plane.
- Right Ascension of the Ascending Node, φ – the angle measured eastward, in the equatorial plane, from the line of the first point of Aries (Y) to the ascending node.

Orbital Parameters: The eccentricity

- It is a measure of the "circularity" of the orbit.

$$e = \frac{r_a - r_p}{r_a + r_p}$$

Elliptical Orbit	$0 < e < 1$
Circular Orbit	$e = 0$

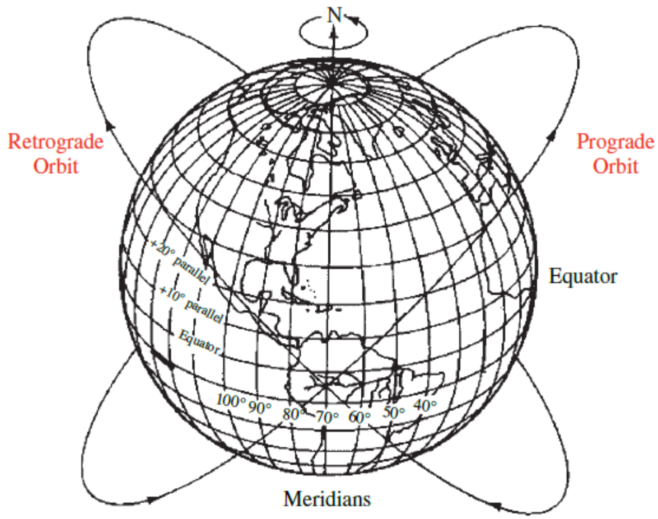
e = the eccentricity of the orbit
r_a = the distance from the center of the earth to the apogee point
r_p = the distance from the center of the earth to the perigee point

Orbital Parameters: Inclination Angle

- **θi** is the angle between the orbital plane and the earth's equatorial plane.
- A satellite that is in an orbit with some inclination angle is in an **inclined orbit.**
- A satellite that is in orbit in the equatorial plane (inclination angle = 0°) is in an **equatorial orbit.**
- A satellite that has an inclination angle of 90° is in a **polar orbit**.

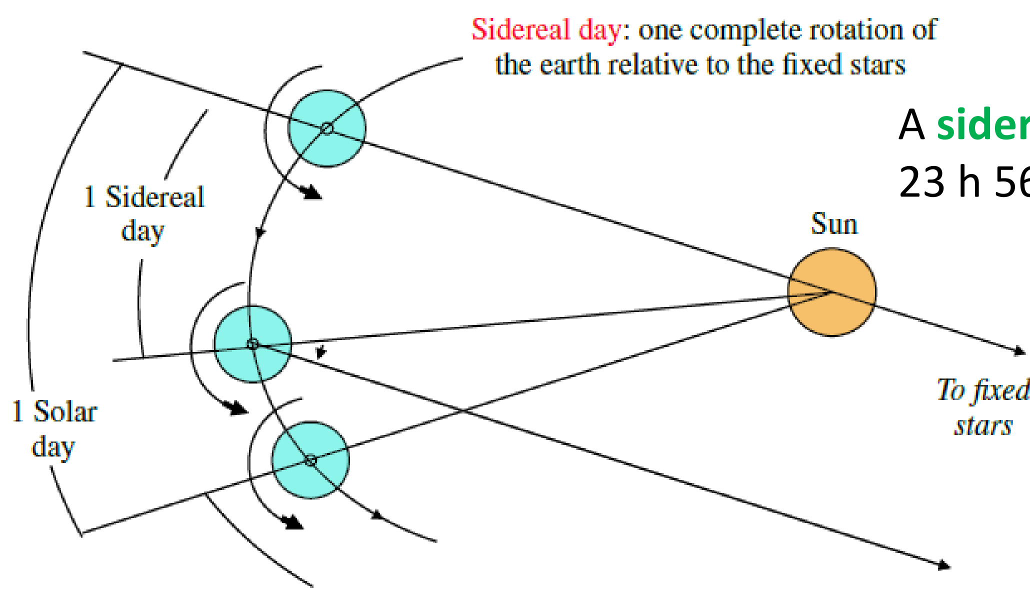
Prograde and Retrograde orbits

- Prograde orbit: an orbit in which the satellite moves in the same direction as the earth's rotation.
  - Its inclination angle is between 0° and 90°.
- Retrograde orbit: moves in a direction opposite (counter to) the earth's rotation.
  - Its inclination angle between 90° and 180° .
- Most satellites are launched in a prograde orbit, since the earth's rotational velocity enhances the satellite's orbital velocity.



Sidereal Time

- Satellite orbits coordinates are specified in *Sidereal Time rather than in solar time.*
- Solar time, which forms the basis of all global time standards, is based on one complete rotation of the earth relative to the sun.
- Sidereal time is based on one complete rotation of the earth relative to a fixed star reference.



A **sidereal day** is approximately
23 h 56 min 4.0905 s or 23.9344696 h

1 mean Solar day = 1.002738 mean Sidereal days

1 mean Sidereal day = 0.9972696 mean Solar days

Orbits in Common Use

- With all of the possible combinations of orbit parameters available to the satellite designer, there is an almost endless list of possible orbits that can be used.
- Experience has narrowed down the list of orbits in common use for communications, sensor and scientific satellites.

Geostationary Earth Orbit (GEO)

- The orbit radius is chosen so that the period of revolution of the satellite is exactly set to the period of the earth's rotation, one mean sidereal day.
- The orbit is circular (eccentricity = 0), and the orbit is in the equatorial plane (inclination angle = 0°).

Geostationary Earth Orbit (GEO)

- From Kepler's Third Law, the orbit radius for the GEO, r_S, is found as

$$r_S = \left[\frac{\mu}{4\pi^2}\right]^{\frac{1}{3}} T^{\frac{2}{3}} = \left[\frac{3.986004 \times 10^5}{4\pi^2}\right]^{\frac{1}{3}} (86164.09)^{\frac{2}{3}}$$
$$= 42,164.17 \text{ km}$$

Where T = 1 mean sidereal day = 86,164.09 s.

- The geostationary height (altitude above the earth's surface) is

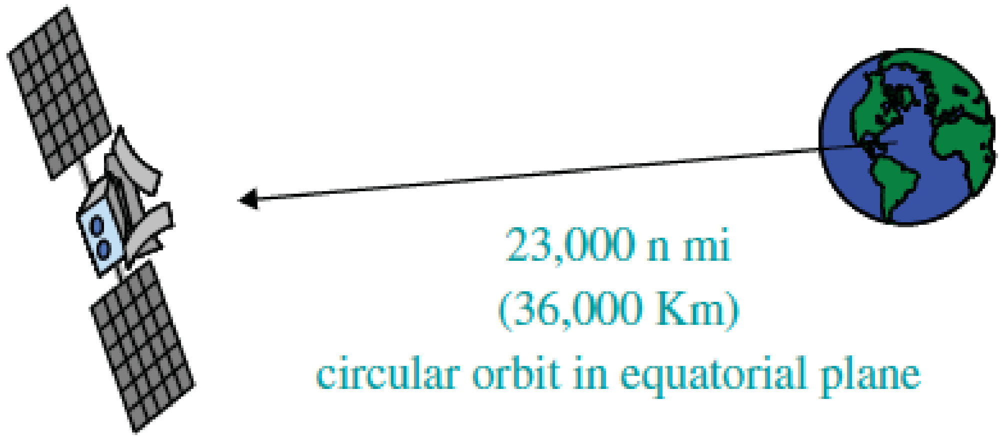
$$h_S = r_S - r_E$$
$$= 42,164 - 6,378$$
$$= 35,786 \text{ km} \longrightarrow$$ **often rounded to 36,000 km**

where r_E = equatorial earth radius = 6, 378 km.

Geosynchronous Earth Orbit (GSO)

- The geostationary orbit is an ideal orbit that cannot be achieved for real artificial satellites because there are many other forces besides the earth's gravity acting on the satellite.
- A "perfect orbit," that is, one with e exactly equal to zero and with **θi** exactly equal to 0° cannot be practically achieved without extensive station keeping and a vast amount of fuel to maintain the precise position required.
- A typical GSO orbit in use today would have an inclination angle slightly greater than 0° and possibly an eccentricity that also exceeds 0.
- The "real world" GSO orbit that results is often referred to as a ***geosynchronous earth orbit (GSO)*** *to differentiate it from the* ***ideal geostationary*** **orbit.**

GSO – geosynchronous earth orbit



- most common
- fixed slant paths
- Little or no ground station tracking required
- 2 or 3 satellites for global coverage (except for poles)

Geostationary (GEO) – ideal orbit (inclination = 0°)

Geosynchronous (GSO) – all real orbits (inclination ≠ 0°)

GSO: Disadvantages

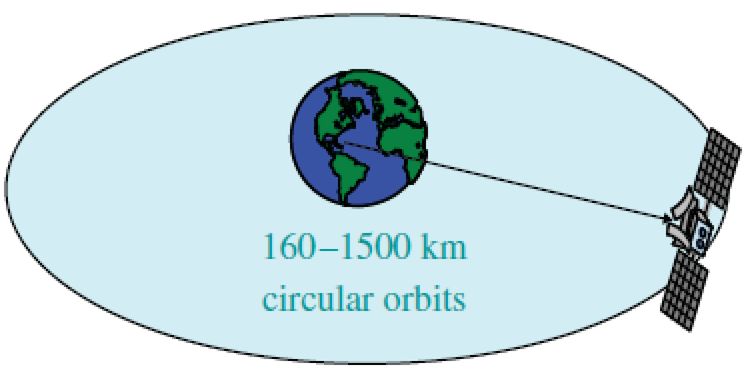
- The long path length produces a large path loss and a significant latency (time delay) for the radiowave signal propagating to and from the satellite.
  - The two way (up to the satellite and back) delay will be approximately 260 ms for a ground station located at a mid-latitude location.
  - This could produce problems, particularly for voice communications or for certain protocols that cannot tolerate large latency.
- The GSO cannot provide coverage to high latitude locations.
  - The highest latitude, at which the GSO satellite is visible, with a 10° earth station elevation angle, is about 70° , North or South latitude.
  - Coverage can be increased somewhat by operation at higher inclination angles, but that produces other problems, such as the need for increased ground antenna tracking, which increases costs and system complexity.
- The number of satellites that can operate in geostationary orbits is obviously limited, since there is only one equatorial plane, and the satellites must be spaced to avoid interference between each other.

Low Earth Orbit (LEO)

- Earth satellites that operate well below the geostationary altitude, typically at altitudes from 160 to 2500 km, and in near circular orbits.
- The earth-satellite links are much shorter, leading to lower path losses, which results in lower power, smaller antenna systems.
- Propagation delay is also less because of shorter path distances.
- LEO satellites, with the proper inclinations, can cover high latitude locations, including polar areas, which cannot be reached by GEO satellites.
- More LEO satellites are required to provide communications services comparable to the GEO case, but LEO satellites are much smaller and require significantly less energy to insert into orbit, hence total life cycle costs may be lower.

Low Earth Orbit (LEO)

- A major disadvantage of the LEO satellite is its restricted operations period, since the satellite is not at a fixed location in the sky, but instead sweeps across the sky for as little as 8 to 10 minutes from a fixed location on earth.
- If continuous global or wide area coverage is desired, a constellation of multiple LEO satellites is required, with links between the satellites to allow for point-to-point communications.
- Some current LEO satellite networks operate with 12, 24 and 66 satellites to achieve the desired coverage.



— requires earth terminal tracking

— approx. 8 to 10 minutes per pass for an earth terminal

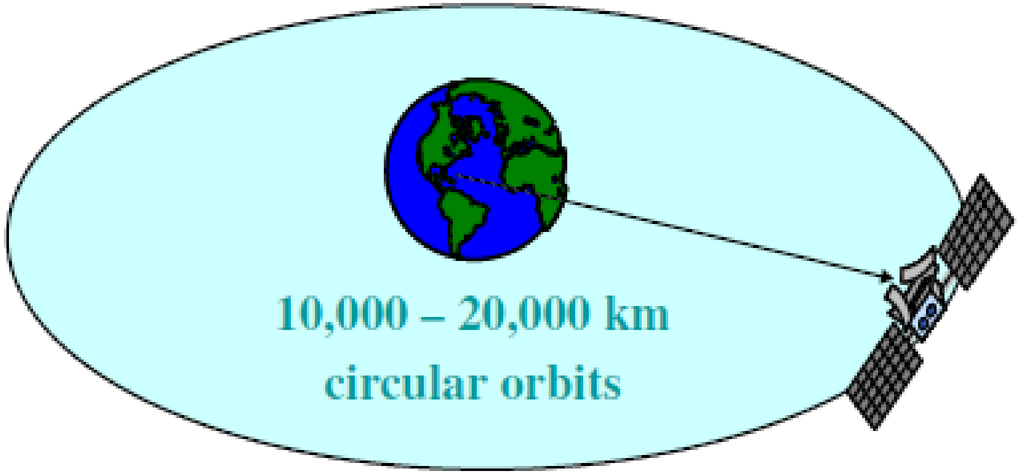
— requires multiple satellites (12, 24, 66, ...) for global coverage

— popular for mobile satellite communications applications

Medium Earth Orbit (MEO)

- Satellites that operate in the range between LEO and GEO, typically at altitudes of 10,000 to 20,000 km.
- The desirable features of the MEO include:
  - repeatable ground traces for recurring ground coverage;
  - selectable number of revolutions per day;
  - adequate relative satellite-earth motion to allow for accurate and precise position measurements.
- A typical MEO would provide one to two hours of observation time for an earth terminal at a fixed location.
- MEO satellites have characteristics that have been found useful for meteorological, remote sensing, navigation and position determination applications.
- The Global Positioning System (GPS), for example, employs a constellation of up to 24 satellites operating in 12 hour circular orbits, at an altitude of 20,184 km.

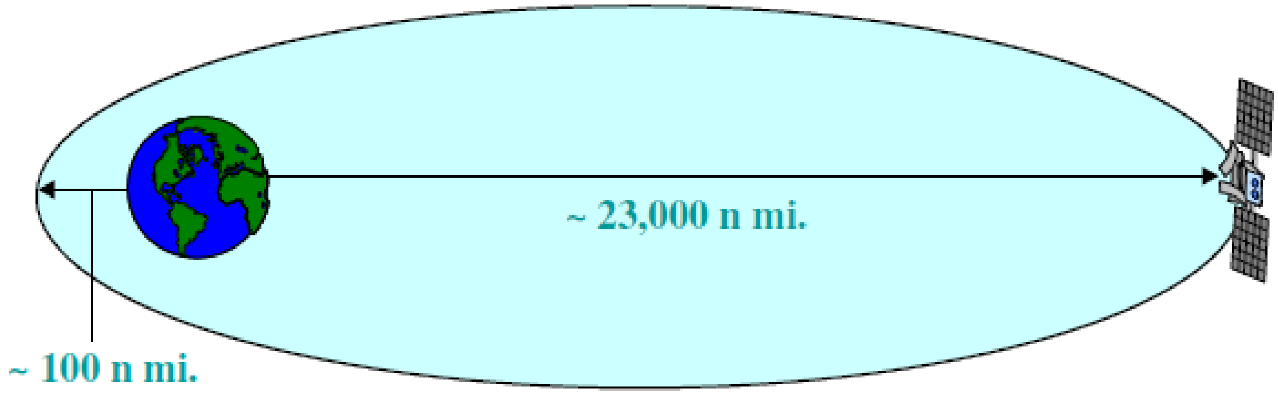
MEO - medium earth orbit.



- similar to LEO, but at higher circular orbits
- 1 to 2 hours per pass for an earth terminal
- used for meteorological, remote sensing and position location applications

Highly Elliptical Orbit

- Satellites operating in ***high elliptical (high eccentricity) orbits*** *are* used to provide coverage to high latitude areas not reachable by GSO, and which require longer contact periods than available with LEO satellites.



— popular for high latitude or polar coverage

— often referred to as the 'MOLNIYA' oribt

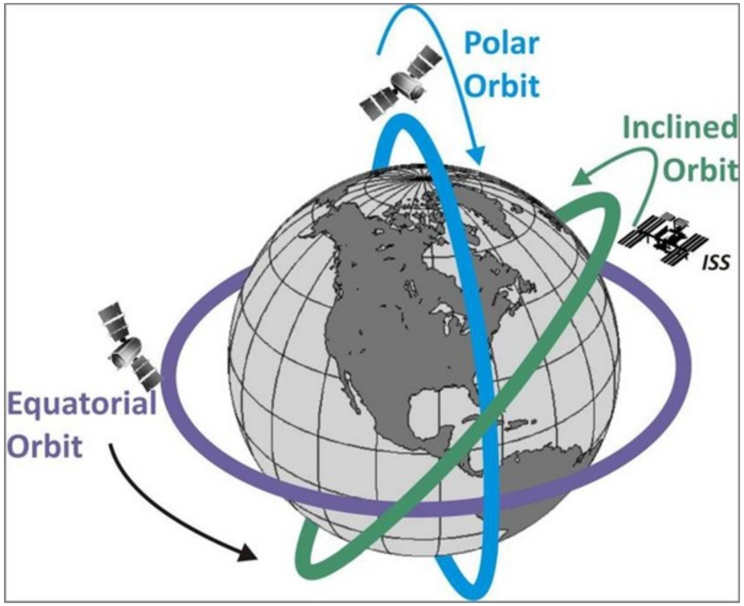
— 8 to 10 hours of 12 hours HEO orbit available for communications from earth terminal, with "GSO like" operations

HEO: Molniya orbit

- It is the most popular HEO orbit used for communications satellites.
- The orbit is designed to provide extended coverage in the high northern latitudes which comprise most of the former Soviet Union’s land mass, where GSO satellites cannot provide coverage.
- A typical Molnyia orbit has a perigee altitude of about 1000 km, and an apogee altitude of nearly 40,000 km.
- This corresponds to an eccentricity of about 0.722.
- The orbit has a nominal period of 12 hours, which means that it repeats the same ground trace twice each day

Polar Orbit

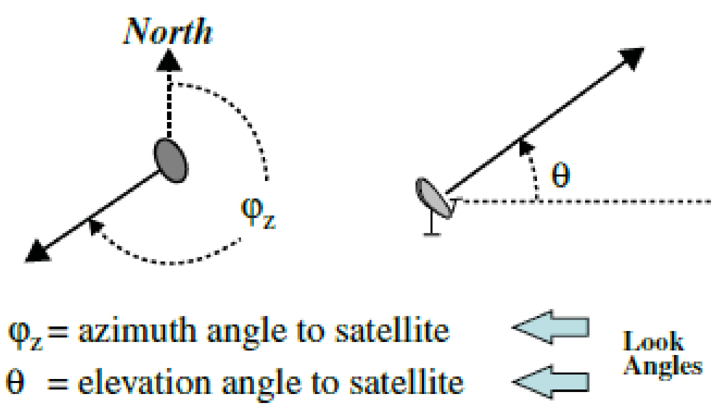
- A circular orbit with an inclination near 90° is referred to as a ***polar orbit.***
- ***Polar orbits*** are very useful for sensing and data gathering services, since their orbital characteristics can be selected to scan the entire globe on a periodic cycle.

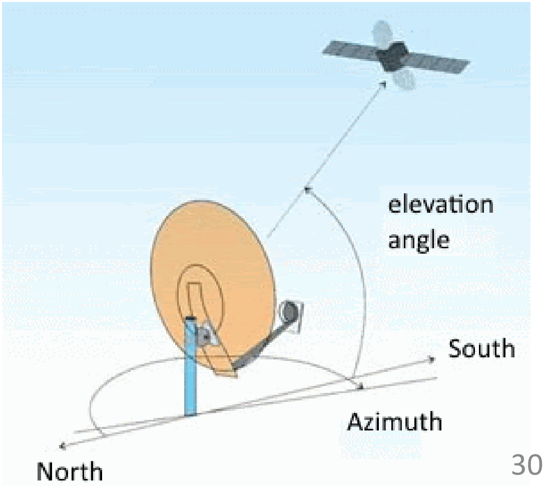


Geometry of GSO Links

- The GSO is the dominant orbit used for communications satellites.
- The three key parameters for the evaluation of the GSO link are:

d = range (distance) from the earth station (ES) to the satellite, in km
φ_z = azimuth angle from the ES to the satellite, in deg.
θ = elevation angle from the ES to the satellite, in deg.





Input parameters

- The are many sources available in the orbital mechanics and satellite literature which describe the detailed development of the calculations for the GSO parameters, range, elevation angle, and azimuth angle.
- There are also several software packages available for the determination of orbital parameters, for both GSO and NGSO satellites networks.
- The input parameters required to determine the GSO parameters are:

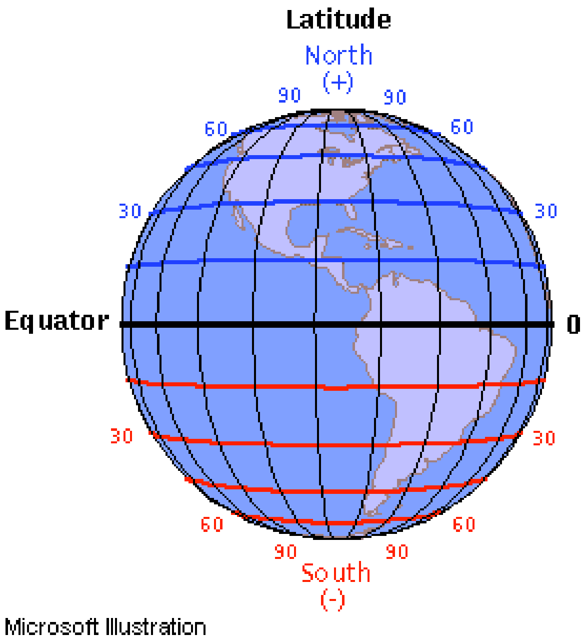
l_E = earth station longitude, in degrees
l_S = satellite longitude, in degrees
L_E = earth station latitude, in degrees
L_S = satellite latitude in degrees (assumed to be 0, i.e., inclination angle = 0)
H = earth station altitude above sea level, in km

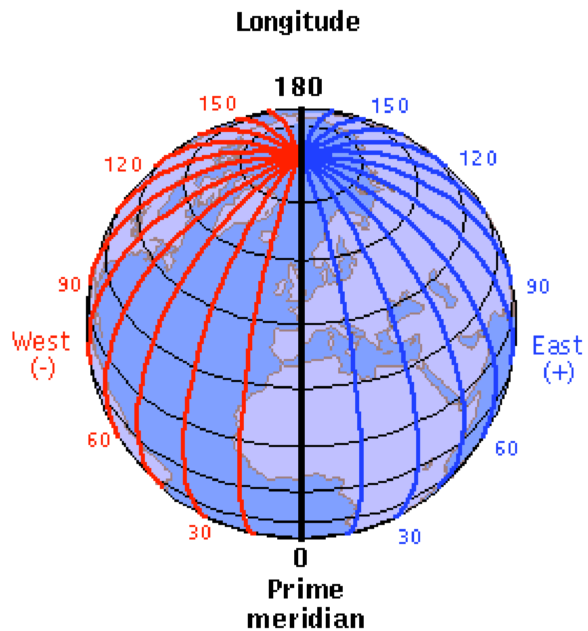
- Additional parameters required for the calculations are:

Equatorial Radius $r_e = 6378.14$ km
Geostationary Radius $r_S = 42,164.17$ km
Geostationary Height (Altitude) $h_{GSO} = rs - re = 35,786$ km
Eccentricity of the Earth $e_e = 0.08182$

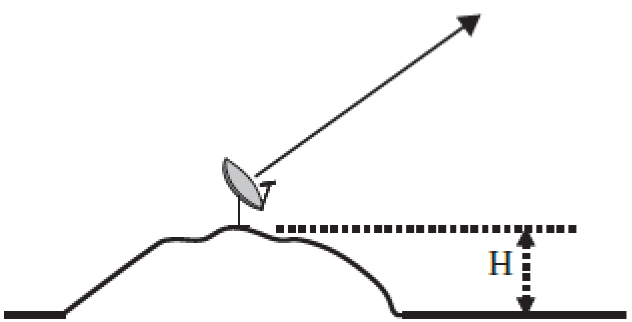
Differential longitude, B, defined as the difference between the earth station and satellite longitudes:

$$B = l_E - l_S$$





Sign Convention: Longitudes east of the Greenwich Meridian and latitudes north of the equator are positive.



Range to Satellite

- This result will be used to determine several important parameters for satellite link analysis, including the free space path loss, which is directly dependent on the complete path length from the earth station antenna to the satellite antenna.

The range $$d = \sqrt{R^2 + {r_s}^2 - 2\,R\,r_s\cos(\Psi_E)\cos(B)}$$

$$R = \sqrt{\ell^2 + z^2}$$ radius of the earth at the earth station latitude and longitude.

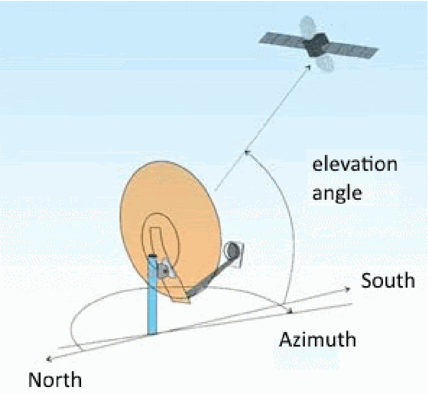
$$\ell = \left(\frac{r_e}{\sqrt{1 - e_e^2\sin^2(L_E)}} + H\right)\cos(L_E) \qquad z = \left(\frac{r_e(1 - e_e^2)}{\sqrt{1 - e_e^2\sin^2(L_E)}} + H\right)\sin(L_E) \qquad \Psi_E = \tan^{-1}\left(\frac{z}{\ell}\right)$$

Elevation Angle to Satellite

- The elevation angle is important since it determines the slant path through the earth's atmosphere, and will be the major parameter in evaluating atmospheric degradations such as rain attenuation, gaseous attenuation, and scintillation on the path.
- Generally, the lower the elevation angle, the more serious the atmospheric degradations will be, since more of the atmosphere will be present to interact with the radiowave on the path to the satellite.
- The elevation angle from the earth station to the satellite is determined from

$$\theta = \cos^{-1}\left(\frac{r_e + h_{GSO}}{d}\sqrt{1 - \cos^2(B)\ \cos^2(L_E)}\right)$$

r_e = equatorial radius = 6378.14 km
h_{GSO} = geostationary altitude = 35,786 km
d = range, in km
B = differential longitude, in degrees
L_E = ES latitude, in degrees



Azimuth Angle to Satellite

Table 2.2 Determination of azimuth angle from intermediate angle.

Condition[a)]	$\varphi_Z =$	Figure (2.14)
SS point is NE of ES	A_i	(a)
SS point is NW of ES	$360 - A_i$	(b)
SS point is SE of ES	$180 - A_i$	(c)
SS point is SW of ES	$180 + A_i$	(d)

$$A_i = \sin^{-1}\left(\frac{\sin(|B|)}{\sin(\beta)}\right)$$

where |B| is the absolute value of the differential longitude,

$$|B| = |l_E - l_S|$$

$$\beta = \cos^{-1}[\cos(B)\cos(L_E)]$$

l_E = earth station longitude, in degrees
l_S = satellite longitude, in degrees
L_E = earth station latitude, in degrees

Azimuth Angle to Satellite

The point on the earth's equator at the satellite longitude is called the ***subsatellite point,*** SS.

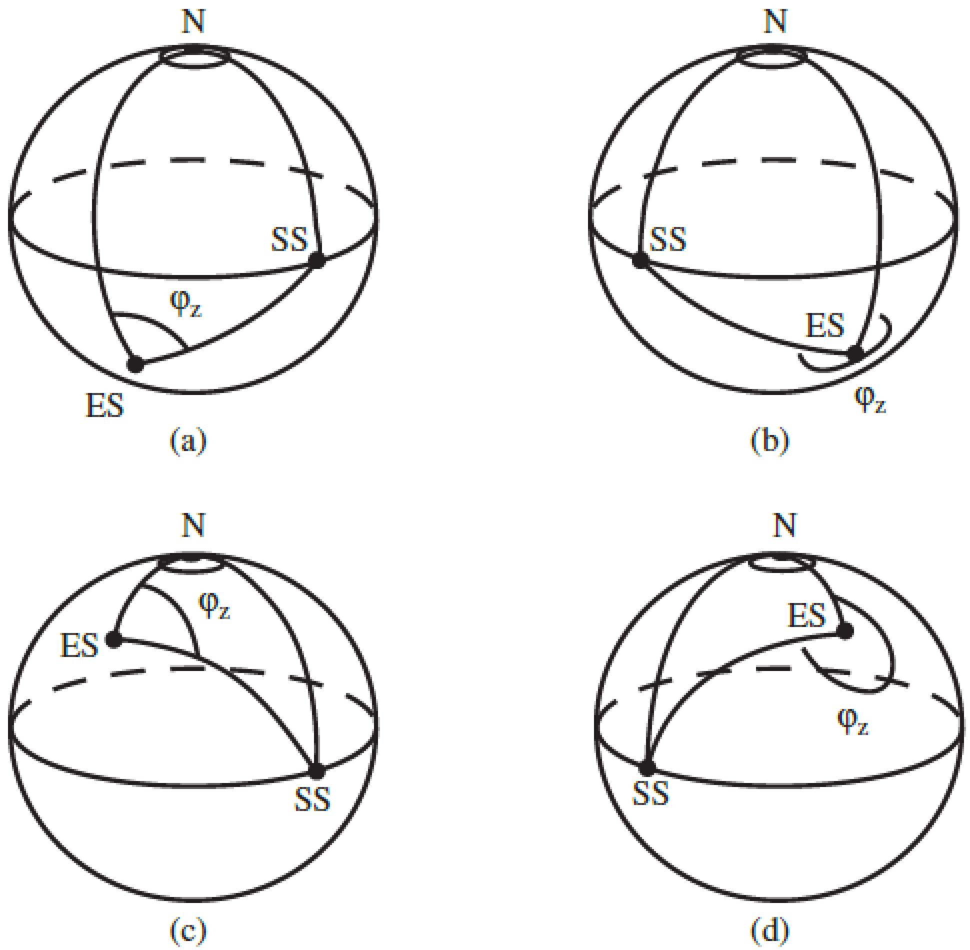


Figure 2.14 Determination of azimuth angle condition.

SAMPLE CALCULATION

Example

- Consider an earth station located in Washington, DC, and a GSO satellite located at 97°W
- The input parameters are:

Earth Station: Washington, DC

Latitude: $L_E = 39^o N = +39$

Longitude: $l_E = 77^o W = -77$

Altitude: $H = 0$ km

Satellite:

Latitude: $L_S = 0^o$ (inclination angle = 0)

Longitude: $l_S = 97^o W = -97$

Find the range, d, the elevation angle, θ, and the azimuth angle φ_Z to the satellite.

Step 1) Determine the differential longitude, B,

$$B = l_E - l_S = (-77) - (-97) = +20$$

Step 2) Determine the earth radius at the earth station, R,

$$\ell = \left(\frac{r_e}{\sqrt{1 - e_e^2 \sin^2(L_E)}} + H \right) \cos(L_E)$$

$$= \left(\frac{6,378.14}{\sqrt{1 - (0.08182)^2 \sin^2(39^o)}} + 0 \right) \cos(39^o) = 4,963.33 \text{ km}$$

$$z = \left(\frac{r_e(1 - e_e^2)}{\sqrt{1 - e_e^2 \sin^2(L_E)}} + H \right) \sin(L_E)$$

$$= \left(\frac{6,378.14(1 - 0.08182^2)}{\sqrt{1 - 0.08182^2 \sin^2(39^o)}} + 0 \right) \sin(39^o) = 3,992.32 \text{ km}$$

$$\Psi_E = \tan^{-1}\left(\frac{z}{\ell}\right) = \tan^{-1}\left(\frac{3,992.32}{4,963.33}\right) = 38.81^o$$

$$R = \sqrt{\ell^2 + z^2} = \sqrt{4,963.33^2 + 3,992.32^2} = 6,369.7 \text{ km}$$

Step 3) Determine the range d,

$$d = \sqrt{R^2 + r_s{}^2 - 2\,R\,r_s\,\cos(\Psi_E)\,\cos(B)}$$
$$= \sqrt{6369.7^2 + 42,164^2 - 2 * 6369.7 * 42164 * \cos(38.81^o) * \cos(20^o)}$$

$$d = 37{,}750\ \text{km}$$

Step 4) Determine the elevation angle θ,

$$\theta = \cos^{-1}\left(\frac{r_e + h_{GSO}}{d}\sqrt{1 - \cos^2(B)\cos^2(L_e)}\right)$$
$$= \cos^{-1}\left(\frac{6,378.14 + 35{,}786}{37{,}750}\sqrt{1 - \cos^2(20^o)\cos^2(39^o)}\right)$$
$$\theta = 40.27^o$$

Step 5) Determine the intermediate angle A_i,

$$\beta = \cos^{-1}[\cos(B)\cos(L_E)]$$
$$= \cos^{-1}[\cos(20)\cos(39)]$$
$$= 43.09^o$$

$$A_i = \sin^{-1}\left(\frac{\sin(|B|)}{\sin(\beta)}\right)$$
$$= \sin^{-1}\left(\frac{\sin(20)}{\sin(43.09)}\right)$$
$$= 30.04^o$$

Table 2.2 Determination of azimuth angle from intermediate angle.

Condition[a)]	$\varphi_Z =$	Figure (2.14)
SS point is NE of ES	A_i	(a)
SS point is NW of ES	$360 - A_i$	(b)
SS point is SE of ES	$180 - A_i$	(c)
SS point is SW of ES	$180 + A_i$	(d)

Step 6) Determine the azimuth angle φ_Z from the intermediate angle A_i,

Since the subsatellite point SS is southwest of the Earth station ES, condition (d) holds and,

$$\varphi_Z = 180 + A_i$$
$$= 180 + 30.04$$
$$= 210.04^o$$

Summary: The orbital parameters for the Washington DC ground station are:

$$d = 37{,}750 \text{ km}$$
$$\theta = 40.27^\circ$$
$$\varphi_Z = 210.04^\circ$$