

12.05 Virtual Lecture Notes (Part 1)

Calculating the Greatest Common Divisor Recursively

The greatest common divisor (GCD) of two integer values, x and y , is the largest number that evenly divides both x and y . Euclid discovered that the greatest common divisor of x and y must always be the same as that of x and r , where r is the remainder of x divided by y .

Euclid's algorithm for finding the Greatest Common Divisor:

1. Divide the larger number by the smaller number.
2. If there is no remainder, the smaller number is the GCD.
3. Otherwise, the smaller number becomes the larger and the remainder becomes the smaller, and go back to Step 1.

Example

$x = 126, y = 90$	$x \text{ or } y \neq 0$	\therefore	$126 / 90 = 1 \text{ remainder } 36$
$x = 90, y = 36$	$x \text{ or } y \neq 0$	\therefore	$90 / 36 = 2 \text{ remainder } 18$
$x = 36, y = 18$	$x \text{ or } y \neq 0$	\therefore	$36 / 18 = 2 \text{ remainder } 0$
$x = 18, y = 0$	$x \text{ or } y = 0$	\therefore	$\text{GCD} = 18$

Base Case: $\text{GCD}(x, y) = y$ if y is a divisor of x
Recursive Cases: $\text{GCD}(x, y) = \text{GCD}(y, x)$ if $x < y$,
 $\text{GCD}(x, y) = \text{GCD}(y, x \% y)$ otherwise.

Implementation

```
//precondition x, y non-negative
int gcd(int x, int y)
{
    if(x < y)
        return gcd(y, x);

    Else if(y == 0)
        return x;
    else
        return gcd(y, x%y);
}
```