MATH 351 Homework

Analysis Due at 2pm Monday 9/25

Read Chapter 2 of the book. For this homework, using LaTeX gives one point of extra credit for each problem.

1. Prove that if A and B are countably infinite sets, then so is $A \cup B$.
2. Given any set A, consider the dodgeball set $D = \{X, O\}$. Show that set F consisting of all functions $f : A \to D$ has the same cardinality as the power set $\mathcal{P}(A)$ consisting of all subsets of A.

Definition 1. Given a countable collection of sets A_n, $n \in \mathbb{N}$, we define the intersection and union of these sets as

$$\bigcap_{n=1}^{\infty} A_n = \{x \,|\, \forall n \in \mathbb{N}, x \in A_n\} \qquad \bigcup_{n=1}^{\infty} A_n = \{x \,|\, \exists n \in \mathbb{N} \text{ s.t. } x \in A_n\}$$

1. Nested intervals
   (a) Use one of the definition above to restate the Nested Interval Property.
   (b) Find with proof a sequence of nonempty nested intervals of the form $I_n = (a_n, b_n)$ such that $\bigcap_{n=1}^{\infty} I_n = \emptyset$.
   (c) What does this say about the Nested Interval Property?
2. Find with proof a sequence of (not necessarily nested) intervals of the form $I_n = [a_n, b_n]$ such that $\bigcup_{n=1}^{\infty} I_n = (0, 1)$.
3. Consider the intervals $A_n = \{x \in \mathbb{R} \,|\, x \geq n\}$.
   (a) Are the sets A_n nested? Why or why not?
   (b) Find with proof the intersection $\displaystyle\bigcap_{n=1}^{\infty} A_n$.
   (c) What does this say about the Nested Interval Property?
4. Suppose A is a nonempty bounded set of real numbers and z is an upper bound for A.
   (a) Prove or disprove: if $z \in A$, then $z = \sup(A)$.
   (b) Prove or disprove: if $z = \sup(A)$, then for every $\varepsilon > 0$, there is some $x \in A$ such that $x > z - \varepsilon/2$.