Extended Kalman Filter for Fixed-Wing Aircraft Dynamics

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1 Introduction

The goal of this project is to implement an extended Kalman filter (EKF) to estimate a fixed-wing aicraft state vector from noisy sensor measurments. The first iteration of this project will be focused only on the longitudinal dynamics.

2 Aircraft Model

The nonlinear longitudinal dynamics for conventional fixed-wing aircraft can be written as follows [3, 4]

$$
\begin{aligned}
\dot{U} &= -QW - g\sin\theta + \frac{X}{m} \\
\dot{W} &= QU + g\cos\theta + \frac{Z}{m} \\
\dot{Q} &= \frac{M}{I_{yy}} \\
\dot{\theta} &= Q
\end{aligned}
\tag{1}
$$

The forces and moments can be broken down as follows

$$
\begin{aligned}
X &= qS\left(C_X(\alpha) + \frac{\bar{c}}{2V_T}C_{X_Q}Q + C_{X_{\delta_e}}\delta_e\right) + X_{t_0} + X_{\delta_t}\delta_t \\
Z &= qS\left(C_Z(\alpha) + \frac{\bar{c}}{2V_T}C_{Z_Q}Q + C_{Z_{\delta_e}}\delta_e\right) \\
M &= qS\bar{c}\left(C_M(\alpha) + \frac{\bar{c}}{2V_T}C_{M_Q}Q + C_{M_{\delta_e}}\delta_e\right)
\end{aligned}
\tag{2}
$$

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Considering a trim condition in a cruise level flight, the nonlinear system 1 can be further simplified to be on the following from

$$\begin{aligned}
\dot{U} &= -QW - g\cos\theta_0\Delta\theta + X_U\Delta U + X_W\Delta W + X_{\delta_e}\delta_e + X_{\delta_t}\delta_t \\
\dot{W} &= QU - g\sin\theta_0\Delta\theta + Z_U\Delta U + Z_W\Delta W + Z_{\delta_e}\delta_e \\
\dot{Q} &= M_U\Delta U + M_W\Delta W + M_Q Q + M_{\delta_e}\delta_e + M_{\delta_t}\delta_t \\
\dot{\theta} &= Q
\end{aligned} \tag{3}$$

The nonlinear system 1 can be linearized and written in a standard linear system form as follows [3, 4]

$$\begin{bmatrix} \dot{U} \\ \dot{W} \\ \dot{Q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta_0 \\ Z_u & Z_w & U_0 & -g\sin\theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \\ Q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & M_{\delta_t} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix} \tag{4}$$

In this project we will consider the longitudinal model of the aircraft "DELTA" given in [2, PP. 561–563] whose parameters are given as follows (at $U_0 = 75\ m/s$ and $\theta_0 = 2.7°$)

$$\begin{aligned}
m &= 300000kg \\
X_U &= -0.02 \\
X_W &= 0.1 \\
Z_U &= -0.23 \\
Z_W &= -0.634 \\
M_U &= -2.55 * 10^{-5} \\
M_W &= -0.005 \\
M_Q &= -0.61 \\
X_{\delta_e} &= 0.14 \\
Z_{\delta_e} &= -2.9 \\
M_{\delta_e} &= -0.64 \\
X_{\delta_t} &= 1.56 \\
M_{\delta_t} &= 0.0054
\end{aligned} \tag{5}$$

where δ_t is considered to be from the trim thrust. As such, δ_t is allowed the between 1 and -0.56 [1].

3 Extended Kalman Filter Equations

4 Algorithm Structure

References

References

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