Extended Kalman Filter for Fixed-Wing Aircraft Dynamics

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1 Introduction

The goal of this project is to implement an extended Kalman filter (EKF) to estimate a fixed-wing aicraft state vector from noisy sensor measurments. The first iteration of this project will be focused only on the longitudinal dynamics.

2 Aircraft Model

The nonlinear longitudinal dynamics for conventional fixed-wing aircraft can be written as follows [1, 2]

$$\begin{aligned}
\dot{U} &= -QW - g\sin\theta + \frac{X}{m} \\
\dot{W} &= QU + g\cos\theta + \frac{Z}{m} \\
\dot{Q} &= \frac{M}{I_{yy}} \\
\dot{\theta} &= Q
\end{aligned} \tag{1}$$

The forces and moments can be broken down as follows

$$\begin{aligned}
X &= qS\left(C_X(\alpha) + \frac{\bar{c}}{2V_T}C_{X_Q}Q + C_{X_{\delta_e}}\delta_e\right) + X_{t_0} + X_{\delta_t}\delta_t \\
Z &= qS\left(C_Z(\alpha) + \frac{\bar{c}}{2V_T}C_{Z_Q}Q + C_{Z_{\delta_e}}\delta_e\right) \\
M &= qS\bar{c}\left(C_M(\alpha) + \frac{\bar{c}}{2V_T}C_{M_Q}Q + C_{M_{\delta_e}}\delta_e\right)
\end{aligned} \tag{2}$$

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Considering a trim condition in a cruise level flight, the nonlinear system 1 can be further simplified to be on the following from

$$\begin{aligned}
\dot{U} &= -QW - g\cos\theta_0\Delta\theta + X_U\Delta U + X_W\Delta W + X_{\delta_e}\delta_e + X_{\delta_t}\delta_t \\
\dot{W} &= QU - g\sin\theta_0\Delta\theta + Z_U\Delta U + Z_W\Delta W + Z_{\delta_e}\delta_e \\
\dot{Q} &= M_U\Delta U + M_W\Delta W + M_Q Q + M_{\delta_e}\delta_e + M_{\delta_t}\delta_t \\
\dot{\theta} &= Q
\end{aligned} \tag{3}$$

The nonlinear system 1 can be linearized and written in a standard linear system form as follows [1, 2]

$$\begin{bmatrix} \dot{U} \\ \dot{W} \\ \dot{Q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\theta_0 \\ Z_u & Z_w & U_0 & -g\sin\theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \\ Q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & M_{\delta_t} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix} \tag{4}$$

In this project we will consider the longitudinal model of the aircraft "DELTA" given in [3, PP. 561–563] whose parameters are given as follows (at $U_0 = 75\ m/s$ and $\theta_0 = 2.7°$)

$$\begin{aligned}
m &= 300000kg \\
X_U &= -0.02 \\
X_W &= 0.1 \\
Z_U &= -0.23 \\
Z_W &= -0.634 \\
M_U &= -2.55 * 10^{-5} \\
M_W &= -0.005 \\
M_Q &= -0.61 \\
X_{\delta_e} &= 0.14 \\
Z_{\delta_e} &= -2.9 \\
M_{\delta_e} &= -0.64 \\
X_{\delta_t} &= 1.56 \\
M_{\delta_t} &= 0.0054
\end{aligned} \tag{5}$$

where δ_t is considered to be from the trim thrust. As such, δ_t is allowed the between 1 and -0.56 [4].

3 Extended Kalman Filter Equations

In this section, we will summarize the continous-discrete EKF equations as presented by Crassidis and Junkins [5]. The EKF considers a nonlinear system with process and measurment noise as follows

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) + G(t)\boldsymbol{w}(t),\ \ \boldsymbol{w}(t) \sim N(0, Q(t)) \\
\widetilde{\boldsymbol{y}}_k &= \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{v}_k,\ \ \boldsymbol{v}_k \sim N(0, R_k)
\end{aligned}
\tag{6}
$$

where the overscript symbol $\widetilde{}$ denotes a measured quantity and the subscript symbol $_k$ denotes the value at time step k. Also, $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the control input vector, $\boldsymbol{w}$ is the process noise vector, and $\boldsymbol{v}$ is the measurment noise vector.

The EKF can be initialized as follows

$$
\begin{aligned}
\hat{\boldsymbol{x}}(t_0) &= \hat{\boldsymbol{x}}_0 \\
P_0 &= E\{\widetilde{\boldsymbol{x}}(t_0)\widetilde{\boldsymbol{x}}^T(t_0)\}
\end{aligned}
\tag{7}
$$

where the overscript symbol $\hat{}$ denotes an estimated quantity and P is the state estimate error covariance matrix.

The Kalman gain, updated state, and updated state estimate error covariance can be obtained as follows

$$
\begin{aligned}
K_k &= P_k^- C_k^T \left(C_k P_k^- C_k^T + R_k \right)^{-1} \\
C_k &\equiv \left. \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}} \right|_{\hat{\boldsymbol{x}}_k^-} \\
\hat{\boldsymbol{x}}_k^+ &= \hat{\boldsymbol{x}}_k^- + K_k \left(\widetilde{\boldsymbol{y}}_k - \boldsymbol{h}(\hat{\boldsymbol{x}}_k^-) \right) \\
P_k^+ &= \left(I - K_k C_k \right) P_k^-
\end{aligned}
\tag{8}
$$

where the superscript symbol $^-$ denotes the state/covariance after the propagation step but before the update step. On the other hand, the superscript symbol $^+$ denotes the state/covariance after the update step.

Finally, the state and state estimate error covariance can be propagated as follows

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \\
\dot{P}(t) &= A(t)P(t) + P(t)A^T(t) + G(t)Q(t)G^T(t) \\
A(t) &\equiv \left. \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} \right|_{\hat{\boldsymbol{x}}(t), \boldsymbol{u}(t)}
\end{aligned}
\tag{9}
$$

4 Algorithm Structure

Figure 1 shows a simple sketch of the algorithm structure. There are two classes implemented in this project: *LongDynamics* and *EKF*. The first one encapsulates the nonlinear dynamic system equations for the fixed-wing aircraft longitudinal dynamics, whereas the second one encapsulates the EKF algorithm implementation. The *LongDynamics* class relies on an external package (odeint [6]) for time propagation (numerical integration). In addition, there are some helper functions to load the dataset and compute the root mean square error.

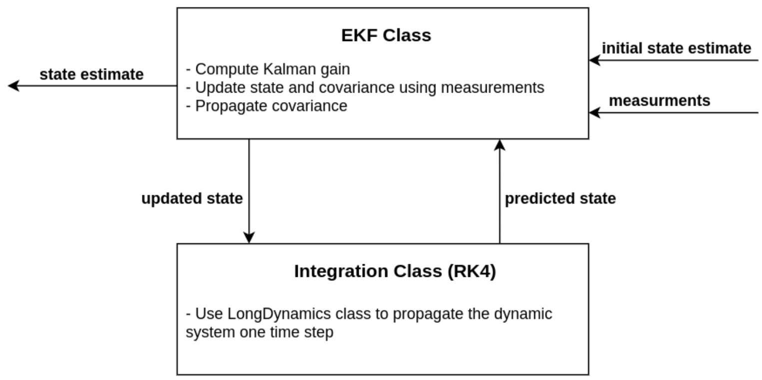

Figure 1: EKF algorithm structure.

References

[1] Robert C Nelson. *Flight stability and automatic control.* McGraw-Hill, 2nd edition, 1998.

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[3] D. McLean. *Automatic flight control systems.* Prentice Hall, 1990.

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[5] John L Crassidis and John L Junkins. *Optimal estimation of dynamic systems.* CRC press, 2011.

[6] odeint - http://headmyshoulder.github.io/odeint-v2/.