

Extended Kalman Filter for Fixed-Wing Aircraft Dynamics

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1 Introduction

The goal of this project is to implement an extended Kalman filter (EKF) to estimate a fixed-wing aircraft state vector from noisy sensor measurements. The first iteration of this project will be focused only on the longitudinal dynamics.

2 Aircraft Model

The nonlinear longitudinal dynamics for conventional fixed-wing aircraft can be written as follows [1, 2]

$$\begin{aligned}\dot{U} &= -QW - g \sin \theta + \frac{X}{m} \\ \dot{W} &= QU + g \cos \theta + \frac{Z}{m} \\ \dot{Q} &= \frac{M}{I_{yy}} \\ \dot{\theta} &= Q\end{aligned}\tag{1}$$

The forces and moments can be broken down as follows

$$\begin{aligned}X &= qS \left(C_X(\alpha) + \frac{\bar{c}}{2V_T} C_{X_Q} Q + C_{X_{\delta_e}} \delta_e \right) + X_{t_0} + X_{\delta_t} \delta_t \\ Z &= qS \left(C_Z(\alpha) + \frac{\bar{c}}{2V_T} C_{Z_Q} Q + C_{Z_{\delta_e}} \delta_e \right) \\ M &= qS\bar{c} \left(C_M(\alpha) + \frac{\bar{c}}{2V_T} C_{M_Q} Q + C_{M_{\delta_e}} \delta_e \right)\end{aligned}\tag{2}$$

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Considering a trim condition in a cruise level flight, the nonlinear system 1 can be further simplified to be on the following from

$$\begin{aligned}
\dot{U} &= -QW - g \cos \theta_0 \Delta \theta + X_U \Delta U + X_W \Delta W + X_{\delta_e} \delta_e + X_{\delta_t} \delta_t \\
\dot{W} &= QU - g \sin \theta_0 \Delta \theta + Z_U \Delta U + Z_W \Delta W + Z_{\delta_e} \delta_e \\
\dot{Q} &= M_U \Delta U + M_W \Delta W + M_Q Q + M_{\delta_e} \delta_e + M_{\delta_t} \delta_t \\
\dot{\theta} &= Q
\end{aligned} \tag{3}$$

The nonlinear system 1 can be linearized and written in a standard linear system form as follows [1, 2]

$$\begin{bmatrix} \dot{U} \\ \dot{W} \\ \dot{Q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \cos \theta_0 \\ Z_u & Z_w & U_0 & -g \sin \theta_0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} U \\ W \\ Q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & M_{\delta_t} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_t \end{bmatrix} \tag{4}$$

In this project we will consider the longitudinal model of the aircraft "DELTA" given in [3, PP. 561–563] whose parameters are given as follows (at $U_0 = 75 \text{ m/s}$ and $\theta_0 = 2.7^\circ$)

$$\begin{aligned}
m &= 300000 \text{ kg} \\
X_U &= -0.02 \\
X_W &= 0.1 \\
Z_U &= -0.23 \\
Z_W &= -0.634 \\
M_U &= -2.55 * 10^{-5} \\
M_W &= -0.005 \\
M_Q &= -0.61 \\
X_{\delta_e} &= 0.14 \\
Z_{\delta_e} &= -2.9 \\
M_{\delta_e} &= -0.64 \\
X_{\delta_t} &= 1.56 \\
M_{\delta_t} &= 0.0054
\end{aligned} \tag{5}$$

where δ_t is considered to be from the trim thrust. As such, δ_t is allowed the between 1 and -0.56 [4].

3 Extended Kalman Filter Equations

In this section, we will summarize the continuous-discrete EKF equations as presented by Crassidis and Junkins [5]. The EKF considers a nonlinear system

with process and measurement noise as follows

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) + G(t)\mathbf{w}(t), \quad \mathbf{w}(t) \sim N(0, Q(t)) \\ \tilde{\mathbf{y}}_k &= \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \quad \mathbf{v}_k \sim N(0, R_k)\end{aligned}\tag{6}$$

where the overscript symbol \sim denotes a measured quantity and the subscript symbol $_k$ denotes the value at time step k . Also, \mathbf{x} is the state vector, \mathbf{u} is the control input vector, \mathbf{w} is the process noise vector, and \mathbf{v} is the measurement noise vector.

The EKF can be initialized as follows

$$\begin{aligned}\hat{\mathbf{x}}(t_0) &= \hat{\mathbf{x}}_0 \\ P_0 &= E\{\tilde{\mathbf{x}}(t_0)\tilde{\mathbf{x}}^T(t_0)\}\end{aligned}\tag{7}$$

where the overscript symbol $\hat{}$ denotes an estimated quantity and P is the state estimate error covariance matrix.

The Kalman gain, updated state, and updated state estimate error covariance can be obtained as follows

$$\begin{aligned}K_k &= P_k^- C_k^T \left(C_k P_k^- C_k^T + R_k \right)^{-1} \\ C_k &\equiv \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}_k^-} \\ \hat{\mathbf{x}}_k^+ &= \hat{\mathbf{x}}_k^- + K_k \left(\tilde{\mathbf{y}}_k - \mathbf{h}(\hat{\mathbf{x}}_k^-) \right) \\ P_k^+ &= \left(I - K_k C_k \right) P_k^-\end{aligned}\tag{8}$$

where the superscript symbol $-$ denotes the state/covariance after the propagation step but before the update step. On the other hand, the superscript symbol $+$ denotes the state/covariance after the update step.

Finally, the state and state estimate error covariance can be propagated as follows

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t) \\ \dot{P}(t) &= A(t)P(t) + P(t)A^T(t) + G(t)Q(t)G^T(t) \\ A(t) &\equiv \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right|_{\hat{\mathbf{x}}(t), \mathbf{u}(t)}\end{aligned}\tag{9}$$

4 Algorithm Structure

Figure 1 shows a simple sketch of the algorithm structure. There are two classes implemented in this project: *LongDynamics* and *EKF*. The first one encapsulates the nonlinear dynamic system equations for the fixed-wing aircraft longitudinal dynamics, whereas the second one encapsulates the EKF algorithm

implementation. The *LongDynamics* class relies on an external package (odeint [6]) for time propagation (numerical integration). In addition, there are some helper functions to load the dataset and compute the root mean square error.

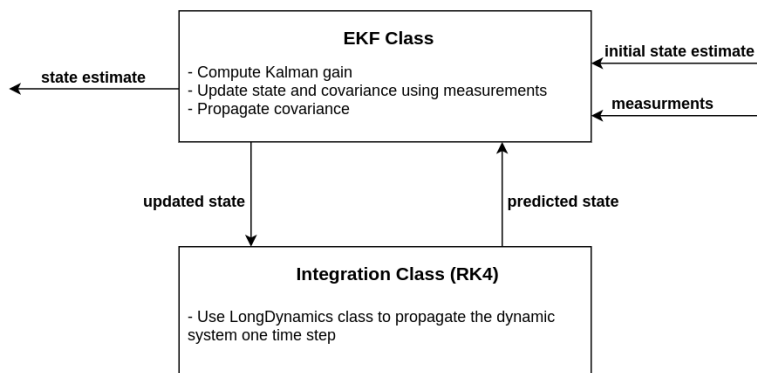


Figure 1: EKF algorithm structure.

5 Results

Figures 2 and 3 show the estimated angle of attack and pitch rate versus the measured and true ones.

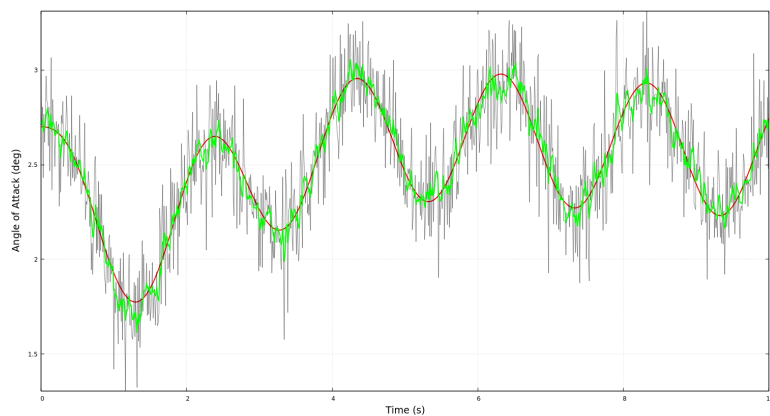


Figure 2: A comparison between the estimated, measured and true angle of attack.

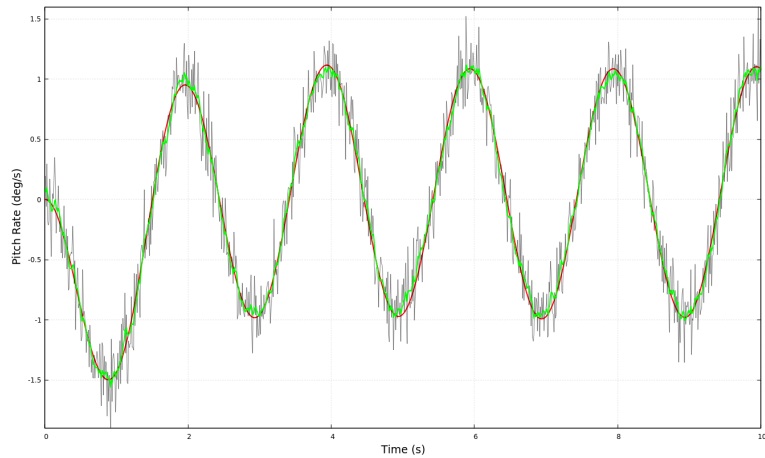


Figure 3: A comparison between the estimated, measured and true pitch rate.

References

- [1] Robert C Nelson. *Flight stability and automatic control*. McGraw-Hill, 2nd edition, 1998.
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- [4] Ahmed M Hassan and Haithem E Taha. Airplane loss of control problem: Linear controllability analysis. *Aerospace Science and Technology*, 55:264–271, 2016.
- [5] John L Crassidis and John L Junkins. *Optimal estimation of dynamic systems*. CRC press, 2011.
- [6] odeint - <http://headmyshoulder.github.io/odeint-v2/>.