

ONLINE APPENDIX: Dynamics of Deterrence: A Macroeconomic Perspective on Punitive Justice Policy

1 Additional Details on The Prison Boom 1980s-

Contribution of Demographics- Age, Race, and Employment Status. To be sure the expansion in incarceration and the cohort results are not driven by changes in the racial or employment status composition of these groups, we perform the following experiment on prison admissions data. We divide the population into 12 cells covering the intersections of each of two race groups: black and white, two employment groups: employed and non-employed, and three age groups: 18-24, 25-34, and 35-54. We then calculate the prison admission rate for each group in the first BJS Prison Survey: 1979.¹ Employment in this survey is a self-report of status at the time of arrest. Next, we predict the admission rate for each age group as follows. We first calc λ^y to satisfy:

$$TotAdmitRate^y = \sum_{r,a,e} \lambda^y \phi_{r,a,e}^{1980} \pi_{r,a,e}^y$$

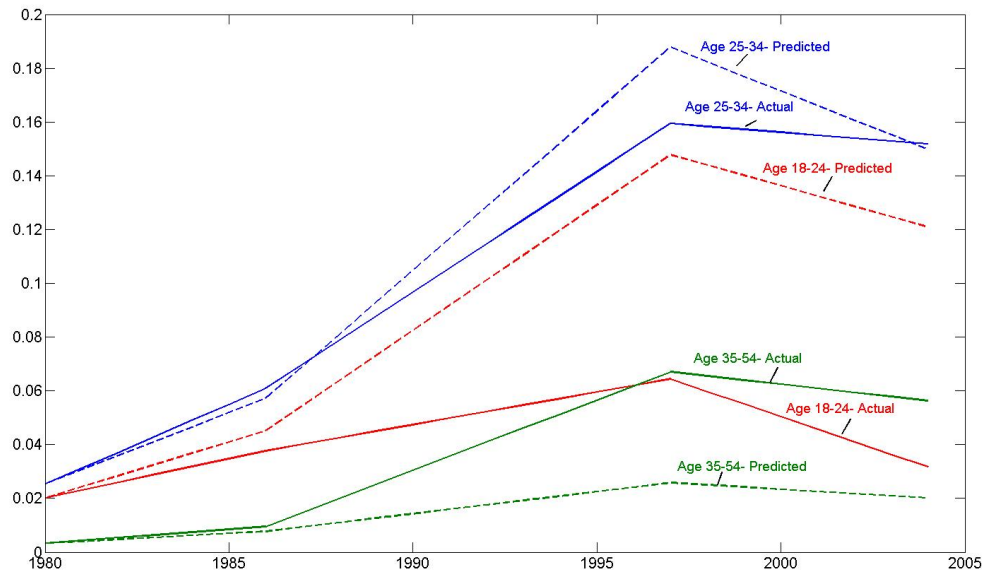
where $\phi_{r,a,e}^{1980}$ is the 1980 admission rate for each demographic cell (r, a, e) where $r \in \{black, white\}$ is race, $e \in \{employed, nonemployed\}$, $a \in \{18 - 24, 25 - 34, 35 - 54\}$. $\pi_{r,a,e}^y$ is the share of each demographic cell in year y . Therefore, λ^y is the percent increase in admissions rate necessary to match the total admission rate in year y , holding year 1980 relative behavior of each demographic cell fixed, but adjusting for changes in each cells share in the total demographics. We then calculate predicted rates for each age group a as:

$$AdmitRate_a^y = \sum_{r,e} \lambda^y \phi_{r,a,e}^{1980} \pi_{r,a,e}^y$$

Figure 1 shows our results. It shows that admissions rates for the youngest group, age 18-24 have been consistently lower than predicted by 1979 behavior. The middle age group, 25-34, is sometimes lower and sometimes higher than predicted. Finally, the oldest age group 35-54, has

¹We limit our analysis to males. We use SEER data to measure total population counts each cell. We adjust this data to be only for those with High School degrees or less using Decennial Census data and interpolating linearly for non-decade years.

rates substantially higher than predicted by the late 1990s.² These patterns are informative about the dynamic role of deterrence and are largely consistent with the theory developed in the paper.



(a) Predicted and Actual Rates by Age

Figure 1: Actual values calculated from BJS Prison Surveys and Census data. Predicted values calculated by holding admission rates fixed to 1979 levels, and raising rates by the same proportion for each age group, adjusting for demographics (race and employment).

The left pane of Figure 2 displays predicted and actual incarceration rates by employment status, controlling for changes in race and age composition across the two over time. Observe that the increase in incarceration rates occurred for both employed and non-employed individuals. Therefore aggregate rates cannot be accounted for by changes in employment status. Finally, the right pane shows that whites account for only slightly less of the increase than would be predicted from pre-1980 outcomes. In other words, the high incarceration rates of blacks relative to whites were not driven by policy changes but accounted for by pre-existing differences prior to 1980.

Stability in Length of Time Served. Prior to the late 1990's, state-level inmate records were not uniform nor well-kept. There was no reporting requirement to federal authorities. Thus, there

²The BJS Prison Survey is only conducted every seven years providing limited data points for this analysis.

is no historical time series of the sentence length served in state and federal prisons. [Neal and Rick \(2014\)](#) and [Pfaff \(2011\)](#) infer sentence lengths from admission, stock, and release data on successive age cohorts of inmates in a sample of state prisons. Both papers concur that the median length of time served in prison for new admits did not change much over the last few decades. This agrees with other papers such as [Raphael and Stoll \(2009\)](#). Therefore, we keep the parameter governing the duration of time served (probability of release) constant through our transition experiment. We set this parameter to provide an expected duration of 2.7 years following [Raphael and Stoll \(2009\)](#) who find an average duration served of 2.64/2.73 in 1984/1998. During this time period, the BJS did report that the time served for Federal prisoners did increase from 15 months to 29 months. However, federal prisoners represent just a small portion of total prison inmates, a consistent 7% during this time.

A Note on the Role of Drug Crime and Enforcement One hypothesis is that criminality associated with drug markets, particularly associated with cocaine, is important to understanding the aforementioned incarceration trends. This hypothesis is met with a great deal of skepticism in the Criminology and Economics literatures. A look at the data reveals why. It is true that prison admissions involving a drug charge have been the category with the largest expansion over the past 30 years. However, the rise in admissions based on drug felonies can only account for 33% of total state and federal admissions at their peak in the 1990's and represent less than 20% of admissions in 2010.³ Sentences for drug felonies are relatively short and so prisoners currently serving for a drug offense comprise an even smaller share of the stock relative to the flow.

The importance of all crime categories to the trend motivates the decision to include all crimes in our analysis. We also choose a parsimonious approach. We do not distinguish in the model nor the main data targets between the four major categories of crime: violent, property, drug and other crime. This is because criminals often operate in more than one category of crime and there is great heterogeneity in these patterns.

³Calculation from Bureau of Justice Statistics Data.

2 Cohort Effects

2.1 Simplified Empirical Model.

In this section we establish a simplified model of crime and incarceration guided by the main mechanisms in our full quantitative model. We use it to derive an empirical strategy to estimate age, time, and cohort effects from semi-aggregated panel data given a set of assumptions that are consistent with our theory. Let $C_{j,t}$ and $I_{j,t}$ be the crime and incarceration rates, respectively, of cohort j at time t . These are our outcomes in the data for which we are interested in measuring cohort effects. The relationship between these variables over time is provided by the following equations.

$$\text{Incarceration Rate } I_{j,t} = \pi_t C_{j,t}$$

$$\text{Initial Crime Choice } X_{j,0} = g^X(\pi)$$

$$\text{Evolution of Crime Rate } C_{j,t} = X_{j,t} A_a + T_t$$

$$X_{j,t} = (\phi + \beta \pi_{t-1}) X_{j,t-1}$$

The interpretation of this model in relation to our research is as follows. The policy variable is π_t : the probability of incarceration conditional on committing a crime. It is exogenous and can change over time. The first line presents the result that, assuming a large population, the incarceration rate for cohort j at time t is equal to that cohort's crime rate $C_{j,t}$ multiplied by the incarceration probability π_t .

The remaining equations explicate an extreme version of the cohort effects found in the full structural model. In the full model, choices made under the policy prevalent during youth persistently affect outcomes even as the policy changes later in life. Here, we model that cohort effect as a permanent component $X_{j,0}$ interpreted as an initial crime choice. The initial crime choice is given by a function $g^X(\pi) \in [0, 1]$. We assume this function is twice continuously differentiable in $(0, 1)$ and that $g'^X(\pi) < 0$, ie: that punitive policy deters.

The final two lines show the evolution of a cohort's crime rate given the initial crime choice and the evolution of the policy. First, the cohort's last period crime rate $X_{j,t-1}$ has a persistent effect on today's crime rate $X_{j,t}$. The coefficient term $(\phi + \beta \pi_{t-1})$ has the following interpretation. The

term $\phi < 1$ captures the direct effect crime today has on crime tomorrow. The term $\beta\pi_{t-1}$ captures the effect that a prison experience yesterday has on crime today. For what follows, we assume that β may be larger than zero in which case a prison experience increases future crime or at least slows its decay. Both ϕ and β can be interpreted as some persistent criminal capital. The age effect is A_a . In the data, crime peaks before age 20 and declines over the life-cycle. Incarceration is hump shaped, peaking between 25-35 before declining.⁴ Therefore A_a will be larger or smaller than one capturing the growth and decay of crime related to the life cycle not otherwise captured by the criminal capital process. Time shows up in two ways. The first is a level effect T_t . The second is through changes in the policy π_t over time.

The first two propositions present steady state comparative statics with respect to π . For these, we suppress the time and cohort subscripts. The first result from this model, summarized in Proposition 2.1, is that the age profile of crime looks different in steady states with different incarceration probabilities π . In particular, as π increases crime is more persistent over the life-cycle resulting in higher incarceration rates for old individuals relative to young.

Proposition 2.1 (A steady state with a higher punitive policy exhibits higher crime and incarceration at older ages relative to young). *Let two policies $\hat{\pi} > \pi$ be given and \hat{X}_a and X_a be the persistent component of crime at age a in the steady-state for each policy, respectively. Then:*

$$\frac{\hat{X}_a}{\hat{X}_{a-s}} > \frac{X_a}{X_{a-s}} \quad \forall \quad s \in (1, a)$$

Proof. We begin by showing $\frac{\hat{X}_a}{\hat{X}_{a-1}} > \frac{X_a}{X_{a-1}}$, then the remainder cases for $s \in (1, a)$ can be completed by induction. Expanding, the result is immediate:

$$\begin{aligned} \frac{\hat{X}_a}{\hat{X}_{a-1}} &= \frac{(\phi + \beta\hat{\pi})\hat{X}_{a-1}}{\hat{X}_{a-1}} \\ &= (\phi + \beta\hat{\pi}) \\ &> (\phi + \beta\pi) \\ &= \frac{(\phi + \beta\pi)X_{a-1}}{X_{a-1}} = \frac{X_a}{X_{a-1}} \end{aligned}$$

⁴This model will not be able to reconcile the monotonic decline of crime with the non-monotone shape of incarceration. This is both because crimes in the data include less serious offenses and because incarceration sentences depend on past criminal records. Instead of including these features, we instead estimate the model twice in the data for each series to capture the two different concepts of crime.

The inequality holds since it is given that $\hat{\pi} > \pi$ □

The change in the life-cycle profile at the steady state when the policy increases occurs regardless of the elasticity of the initial crime choice. Crime becomes more persistent over the life cycle through the prison experience so long as $\beta > 0$. Since incarceration is hump-shaped over the life-cycle, this implies that the peak of life-cycle incarceration will move to older ages for β sufficiently large. This result is particularly important for how we think about time and age effects in the data. It is consistent with shifts towards incarceration at older ages that are salient in the data and suggests the permanent component of this shift can be interpreted as the effect of changes in policy.

The second set of results address how a change in punitive policy (π) affects aggregate crime and incarceration rates.⁵

Proposition 2.2 (Conditions for increased punitive policy to decrease crime.). *Let $\mathbb{C}(\pi)$ be the aggregate crime rate. For a given π , the likelihood of $\frac{\partial \mathbb{C}(\pi)}{\partial \pi} < 0$ is:*

- decreasing in β .
- decreasing in $\sum_{a=0}^M A_a$.
- increasing(decreasing) in π if $g''^X(\pi) >> 0$ ($g''^X(\pi) << 0$).

Proof. The aggregate crime rate given π is the sum of crime across all age groups:

$$\mathbb{C}(\pi) = g^X(\pi) \sum_{a=0}^M (\phi + \beta\pi)^a A_a$$

Then, since we assume $\frac{\partial g^X(\pi)}{\partial \pi} < 0$, it is true that $\frac{\partial \mathbb{C}(\pi)}{\partial \pi} < 0$ iff:

$$-\frac{\partial g^X(\pi)}{\partial \pi} > \frac{\partial \sum_{a=0}^M (\phi + \beta\pi)^a A_a}{\partial \pi}$$

By inspection, the right-hand side is increasing in both β and the sequence A_a , and so the inequality less likely to hold for larger values of these parameters. Also, the left-hand side is increasing in π

⁵It is assumed the maximum age is M , but these proofs will also apply to $\lim_{M \rightarrow \infty}$, so long as parameters are appropriately restricted such that crime is finite.

if $g''^X(\pi) \gg 0$, and so the inequality is more likely to hold for larger values of π if g^X is strictly convex. \square

Corollary 2.3 (Response of incarceration to increased punitiveness.). *Let $\mathbb{I}(\pi)$ be the aggregate incarceration rate. For a given π , is the likelihood of $\frac{\partial \mathbb{I}(\pi)}{\partial \pi} < 0$ is:*

- decreasing in β .
- decreasing in $\sum_{a=0}^M A_a$.
- increasing(decreasing) in π if $g''^X(\pi) \gg 0$ ($g''^X(\pi) \ll 0$).

Proof. Omitted. \square

We now consider the effect of a policy change along the transition. The main result, summarized in Proposition 2.4 explicates the existence of cohort effects.

Proposition 2.4 (The cohort born immediately before an increase in π has higher age-specific crime and incarceration rates at all ages than all cohorts it precedes and follows.). *Let an initial π_0 be given. Denote with hat notation the variables related to the cohort born at $\bar{t} - 1$ where \bar{t} is when the policy is changed to $\pi > \pi_0$. Then:*

$$\begin{aligned} C_{\hat{j},t} &> C_{t-\hat{j}+s,s} \quad \forall \quad t > \bar{t} + 1 \quad \text{and} \quad s \neq \bar{t} + 1 \\ I_{\hat{j},t} &> I_{t-\hat{j}+s,s} \quad \forall \quad t > \bar{t} \quad \text{and} \quad s \neq \bar{t} \end{aligned}$$

Proof. Expanding $C_{\hat{j},\bar{t}}$, we have:

$$\begin{aligned} C_{\hat{j},\bar{t}} &= X_{\hat{j},\bar{t}} A_{\bar{t}-j} + T_{\bar{t}} \\ &= (\phi + \beta \pi_0) X_{\hat{j},\bar{t}-1} A_{\bar{t}-j} + T_{\bar{t}} \\ &= \Pi_{\tau=\hat{j}}^{\bar{t}} [(\phi + \beta \pi_0) * X_{j,0}] A_{\bar{t}-j} + T_{\bar{t}} \\ &= \Pi_{\tau=\hat{j}}^{\bar{t}} [(\phi + \beta \pi_0) * (g^X(\pi_0))] A_{\bar{t}-j} + T_{\bar{t}} \\ &= [(\phi + \beta \pi_0)^{\bar{t}-\hat{j}-1}] * (g^X(\pi_0)) A_{\bar{t}-j} + T_{\bar{t}} \end{aligned}$$

Since time and age effects are invariant to the policy change, we can ignore them. It suffices to

show that $X_{\hat{j},t} > X_{t-\hat{j}+s,s}$ for all $t > \bar{t}$ and all $s \neq \bar{t}$. The evolution of $X_{\hat{j},t}$ for $t > \bar{t}$ is:

$$X_{\hat{j},t} = [(\phi + \beta\pi)^{t-\bar{t}}] * [(\phi + \beta\pi_0)^{\bar{t}-\hat{j}-1}] * (g^X(\pi_0))$$

First, let us consider prior (older) cohorts at the same age in the past: $s < \bar{t} + 1$. Their persistent component is $X_{t-\hat{j}+s,s}$. We want to show the following relationship:

$$\begin{aligned} X_{t-\hat{j}+s,s} &= [(\phi + \beta\pi_0)^{t-\hat{j}+s-1}] * (g^X(\pi_0)) \\ &< [(\phi + \beta\pi)^{t-\bar{t}}] * [(\phi + \beta\pi_0)^{\bar{t}-\hat{j}-1}] * (g^X(\pi_0)) \\ &= X_{\hat{j},t} \quad \forall \quad t > \bar{t} \quad \text{and} \quad s < t \end{aligned}$$

This inequality holds because $\pi > \pi_0$ and $\beta \in (0, \infty)$. Now, for later (younger) cohorts at the same age in the future: $s > \bar{t} + 1$. Their persistent component is $X_{t-\hat{j}+s,s}$. We want to show the following relationship:

$$\begin{aligned} X_{t-\hat{j}+s,s} &= [(\phi + \beta\pi)^{t-\hat{j}+s-1}] * (g^X(\pi)) \\ &< [(\phi + \beta\pi)^{t-\bar{t}}] * [(\phi + \beta\pi_0)^{\bar{t}-\hat{j}-1}] * (g^X(\pi_0)) \\ &= X_{\hat{j},t} \quad \forall \quad t > \bar{t} \quad \text{and} \quad s > \bar{t} \end{aligned}$$

It suffices to show, for any n :

$$[(\phi + \beta\pi)^n] * (g^X(\pi)) < [(\phi + \beta\pi_0)^n] * (g^X(\pi_0))$$

Which holds for $\pi > \pi_0$ and $g'^X(\pi) < 0$, both as assumed. □

Corollary 2.5 establishes an additional restriction required for the cohort effect to translate to a non-monotone transition in crime and incarceration. With respect to crime: it is essentially required that the increase in the incidence and impact of prison on future crime to not be too large relative to the initial crime choice. It does not require that crime fall in the new steady state, but that would suffice. The condition is more stringent with respect to incarceration since the change in incarceration rate is the change in the crime rate times the change in the policy π . Still, it is not required that crime fall in the new steady state in order for the transition to be non-monotone, but again this would be sufficient.

Corollary 2.5 (The transition path of crime and incarceration after an increase in punitiveness are non-monotone if the elasticity of the initial choice is sufficiently large relative to the effect of prison on criminal persistence.). *Let an initial π_0 be given and consider the economy at a steady state for that π_0 . Assume at time-zero the policy switches permanently and unexpectedly to $\pi_1 > \pi_0$. Then:*

- a) *The transition path for crime is non-monotone iff*

$$\frac{g^x(\pi_0)}{g^x(\pi_1)} > \frac{\sum_{a=0}^{M-1} (\phi + \beta\pi_1)^a + 1}{\sum_{a=0}^{M-1} (\phi + \beta\pi_0)^a + 1}$$

- b) *The transition path for crime is non-monotone iff*

$$\frac{\pi_0 g^x(\pi_0)}{\pi_1 g^x(\pi_1)} > \frac{\sum_{a=0}^{M-1} (\phi + \beta\pi_1)^a + 1}{\sum_{a=0}^{M-1} (\phi + \beta\pi_0)^a + 1}$$

Proof. Let $C(\pi_0)$ and $C(\pi_1)$ be the steady state aggregate crime rate at the two policies, π_0 and π_1 , respectively. Let C_0 be the aggregate crime rate the period after the policy change. Then:

$$\begin{aligned} C(\pi_0) &= g^x(\pi_0) \sum_{a=0}^M (\phi + \beta\pi_0)^a A_a \\ C_0 &= g^x(\pi_0)(\phi + \beta\pi_1) \left[\sum_{a=0}^{M-1} (\phi + \beta\pi_0)^a A_a + 1 \right] \\ C(\pi_1) &= g^x(\pi_1) \sum_{a=0}^M (\phi + \beta\pi_1)^a A_a \end{aligned}$$

That $C_0 > C(\pi_0)$, follows directly from Proposition 2.4. Simple algebra comparing C_0 and $C(\pi_1)$ provides the necessary and sufficient condition provided in the statement of this corollary. \square

Corollary 2.6 (If crime falls in the new steady state with increased punitiveness, then the transition path of crime and incarceration to that steady state are non-monotone.). *Let an initial π_0 be given and consider the economy at a steady state for that π_0 . Assume at time-zero the policy switches permanently and unexpectedly to $\pi_1 > \pi_0$. Let $C(\pi)$ and $I(\pi)$ be the aggregate crime and incarceration rates at the steady state of policy π . Then, if $C(\pi_0) > C(\pi_1)$, the transition between steady states is non-monotone for both crime and incarceration.*

Proof. Follows straightforwardly from Proposition 2.4. □

Identification. The manner in which all four effects enter this simple model is crucial for identification. Conceptually, the age effect impacts growth rates while the time and cohort effects impact levels. Finally, we have shown that the age profile can also be changed overtime by changes in the policy π_t . This motivates a two-step procedure to separate these components into growth rate and level effects. We begin by estimating the growth rate effects, generating residuals from predicted values, and then estimating time and cohort effects on these residual levels.

- **Step 1- Growth Effects** The crime rate of cohort j in periods t and $t - 1$ are:

$$\begin{aligned} c_{j,t-1} &= \prod_{s=j}^{t-1} [(\phi + \beta\pi_{s-1})] * g^X(\pi_j) \times A_{t-1-j} \\ c_{j,t} &= \prod_{s=j}^t [(\phi + \beta\pi_{s-1})] * g^X(\pi_j) \times A_{t-j} \end{aligned}$$

Taking logs of the growth rate $\frac{c_{j,t-1}}{c_{j,t}}$, we have:

$$\begin{aligned} \log(c_{j,t-1}) - \log(c_{j,t}) &= \log(\phi + \beta\pi_{t-1}) + \log\left(\frac{A_{t-1}}{A_t}\right) \\ &= \beta_g \mathbf{I}_t + \beta_a \mathbf{I}_a + \epsilon_{j,t} \end{aligned}$$

The last line shows the regression strategy, adding $\epsilon_{j,t}$ as the error term. We will regress the growth rate of cohort-level crime upon dummies for time \mathbf{I}_t and age \mathbf{I}_a . Since we only have two of the three (age, cohort, and time) effects we do not suffer from perfect co-linearity.

- **Step 2- Use predicted crime growth rates to predict cohort's life-cycle crime profiles.** Next, generate predicted values $\hat{c}_{j,t-1}$ by multiplying the crime rate at age 18 by the predicted age-cross-time growth rates estimated in this equation. Then use the residuals $\tilde{c} = c_{j,t-1} - \hat{c}_{j,t-1}$ in the following equation to estimate level effects.
- **Step 3- Level Effects.** The cohort specific level, $g^X(\pi_j)$ did not factor into the growth equation but factors into the level. We now estimate that effect as well as contemporaneous time level effects which affect all individuals equally regardless of their past crime behavior using the following regression:

$$\tilde{c}_{j,t} = \beta_t \mathbf{I}_t + \beta_c \mathbf{I}_c + \epsilon_{j,t}$$

Here we again break the co-linear problem in separating age, cohort, and time effects by only featuring two out of the three: time $\beta_t \mathbf{I}_t$ captured by year dummies and cohort $\beta_c \mathbf{I}_c$ captured with cohort dummies.

A summary of the procedure is to first estimate time-varying growth rate effects, generate residuals from the predicted values, and then estimate time and cohort effects on these residuals. The first stage dealing with growth rates, interprets age as the *time-invariant growth/decay* in crime/incarceration for individuals of all cohorts as they get older. The *time-variant change in growth/decay* in crime/incarceration for all individuals of all cohorts and ages from one year to the next is interpreted as the effect of policy changes. The second stage of our regression deals with levels. We estimate the time and cohort effects to best match the life-cycle profile of cohorts. The cohort component is a constant initial level for each cohort from which the life-cycle profile is created using the first stage regression. In other words, it shifts a single cohort’s age-profile from the first stage up or down. The further time-level effects fill in gaps for years when individuals of all ages increase crime. The critical difference between the time effects in the first and second stage is that in the first stage, individuals are affected in proportion to their prior behavior where as in the second stage it is a common level increase for all individuals.

2.2 Data and Construction of Outcome Variables.

Our two measures of involvement with the criminal justice system are arrest rates and prison admission rates. An important difference between the measures are that arrests span all offenses-misdemeanors and felonies, and need not be accompanied by an actual conviction. In comparison, admissions will be defined as a conviction for a new crime (not only a violation of probation or parole) to a state or federal prison and are therefore usually more serious felonies with a sentence of a year or more.

Arrest Rates Arrests rates are reported directly by the Federal Bureau of Investigation as part of its Uniform Crime Reporting Program. Rates are reported by coarse age groups, gender, and race. They are also categorized by type: Violent crime (murder, forceable rape, robbery, assault, etc.); Property Crime (burglary, larceny, arson, etc.); and Other (fraud, weapons, prostitution, drug violations, etc.). At the time of writing, these data were available from 1980 to 2015.

Admission Rates We calculate admission rates by combining data on admissions from the restricted dataset: National Corrections Reporting Program (NCRP, accessed through ICPSR [U.S. Dept. of Justice \(1983-2000\)](#)) and the US Decennial Census and Current Population Survey (Census, accessed through IPUMS [Ruggles \(2004\)](#)).

The NCRP is a restricted access data set maintained by the Bureau of Justice Statistics. It is a compilation of prison admission, release, parolee, and prison stock data reported to the Department of Justice by individual states. We clean the NCRP data by the following criteria. First, we restrict the sample to states meeting consistency checks from the audit study of [Neal and Rick \(2014\)](#). Next, we include only states that have a consistent time series from 1984 onwards. This leaves us with 12 state prison reports: California, Georgia, Illinois, Michigan, Minnesota, New Jersey, New York, North Dakota, Ohio, South Carolina, Washington, Wisconsin. Within these states, our sample is limited to males entering prison in the calendar year with a new conviction. This means that we exclude persons entering the prison based on a parole violation, returned escapees, or persons without a conviction. We match state-age cells with population data from the Census for the same cells. We linear interpolate population estimates between 1980 and 1990, and between 1990 and 2000. We use CPS annual data following 2000. Finally, we calculate admission rates by crime category according to two metrics: (i) category of offense with the longest sentence; and (ii) category of any offense with a sentence. The latter implies that a single admission can fall under multiple categories of crime if there are multiple offenses spanning more than one category. Note that the NCRP data only lists up to 3 offenses, prioritizing more serious offenses. This means that offenses such as trespassing or possession of drugs are likely to be omitted even if they add to total sentence length.

2.3 Results.

First we check to see that the data are consistent with the assumptions and predictions of the simple model with respect to the age-profile. We assumed that the age profile peaks early on and then decays at a constant rate. We then derived that a change in the policy π should change the shape of the age-profile by increasing mass at older ages. [Figure 3](#) shows that the data are consistent with these features. In particular, the shape of the age profile in the earliest year available, close to the pre-1980’s “steady state” has a shape that exhibits a strikingly constant decay over the life-cycle. The same is true for the last year available only for arrests: 2010. This is the closest we get to the model’s predicted new steady state if we consider the major policy changes to have happened in

the early 1980s. It is also true that the latest year available for each series puts a relatively higher mass on older ages compared to young. Finally, the temporary bumps upwards in the age profile over time departing from a constant decay are hints at cohort effects along the transition. We now estimate these effects more formally following our regression strategy.

In the first step of the regression we consider growth rates. The regression specification is:

$$D1.ln(X_{a,t}) = \beta^p \mathbf{D}_t + \beta^a \mathbf{D}_a + \epsilon_{a,t}$$

The dependent variable $D1.ln(X_{a,t})$ is the first difference of the natural log of the outcome X , either prison admission rate or arrest rate, for age group a in year t . The independent variables include dummies for each year \mathbf{D}_t and age \mathbf{D}_a . The omitted age group is the modal peak age for the series: 21 years old for admissions and 18 years old for arrests.

The second step uses age-time specific growth rates estimated in the first step to predict the shape of the life-cycle profile for a given cohort.⁶ We specifically make this prediction as follows. Let $X_{a,0}$ be the observed admission or arrest rate for age a in the first year of our sample and a_0^c be the first age that we see a cohort c . We construct predicted life-cycle profile using $X_{a_0^c,0}$ as a base and the estimated growth rates to predict values at other ages through an iterative process both forwards for older ages. For example:

$$\begin{aligned}\hat{X}_{a_0^c,t} &= X_{a_0^c,0} \\ \hat{X}_{a_0^c+1,t+1} &= X_{a_0^c,0} * \beta_{a_0^c+1}^a * \beta_{t+1}^p \\ \hat{X}_{a_0^c+2,t+1} &= X_{a_0^c+1,0} * \beta_{a_0^c+2}^a * \beta_{t+2}^p \\ &\dots \quad \text{etc.}\end{aligned}$$

The notation used is β_n^a corresponds to the n^{th} element representing age n in the age vector and so forth.

The dependent variable in the second step is the residual difference between the rates in *levels* using the estimated growth rates from the first step $\hat{X}_{a,t}$ and the actual observed rates $X_{a,t}$: $\tilde{X}_{a,t} = X_{a,t} - \hat{X}_{a,t}$. The independent variables are dummies for cohort and time. This results

⁶The coefficient estimates are transformed into growth rates in accordance to the log-interpretation.

in the simple regression:

$$\tilde{X}_{a,t} = \beta^t \mathbf{D}_t + \beta^c \mathbf{D}_c + \epsilon_{a,t}$$

The essence of our procedure is clear. The first step creates a age profile that changes shape over time. The shape does not exactly match the shape of the actual profiles because it is assumed that the time change in growth rates affects each age group to the same degree. What is left is the clear gap in levels to be filled in by cohort and time effects. The same logic holds here. Conceptually, time effects will capture the variation in levels applicable to all living cohorts at a given time and the cohort effects will take the rest.

The resulting cohort and time coefficients of the second step are presented visually for Incarceration in Figure 4 and for arrests in Figure 5. The peak time effect for prison admissions is the year 1990. This occurs in the first third of the peak of the total crime index. Cohort effects in arrests peak for the cohorts born in the early 1960's while cohort effects for prison admissions peak a couple of years later. These facts together are consistent with our theory of how a more punitive incarceration policy should differentially affect cohorts. The time effect in admissions rises sharply through the late 1980's. The cohort with the highest cohort effect would have been in their early 20's during this escalation, approaching the peak of the age profile of a typical criminal life-cycle profile. Thus they cultivated their criminal careers prior to the time-related increase in punitive admissions and were at the peak of their careers where behavior is less elastic when the policy tightened.

One word of caution is to take into consideration the relative magnitude of cohort and time effects in each series. Time effects are about double the magnitude of cohort effects in prison admissions. Cohorts are relatively more important for arrests.

Our study lumps all crimes together. Still, both cohort and time effects are present when crime is disaggregated into major categories: Drug, Violent and Property. Figure 2.3 and 2.3 show the estimated coefficients for time and cohort effects on levels, following the same regression strategy as the total rates. Many prison admits are convicted of crimes spanning two or more of the categories. For the purpose of this exercise, we classify the crime of incarceration by the most serious offense. The disaggregated arrest series follow patters similar to the aggregate series.

One observation from Figure 2.3 is that drug arrests appear to have a more persistently high time effect. This is also true for prison admissions show in Figure 2.3. These figures also suggest that violent and property crimes could be rising among more recent cohorts (again, relative to trend); whereas drug crime trends are mostly a time phenomenon. Finally, the magnitudes of

cohort effects relative to time effects also differ across crime categories in both series. Cohorts contribute more greatly to violent crime trends and less so to property crime trends where time effects are large in comparison to cohort effects.

3 Calibration

3.1 Probability of Incarceration Conditional on a Crime (π).

Choosing a value of π for the initial steady state and calculating the change in π during the 1980's is not completely straightforward. This is because not all crimes are reported to the police. The closest study to ours, [İmrohoroglu, Merlo, and Rupert \(2004\)](#), considered property crime alone in their calibration and set their probability of apprehension to equal the clearance rate for these crimes. This is not an apt strategy for our paper since we would like to include all crimes to provide a complete view of the total criminal justice system and how it has changed overtime. Specifically, we cannot use the clearance rate for drug crimes. The clearance rate for these crimes are not reported since the incidence of these, and other victimless crimes, are not reported to the police. However, we do have complete information on arrests, convictions, and incarceration on all crimes by category for cases processed in state courts through the Bureau of Justice Statistics "Felony Sentences in State Courts" series published for several years in the time period 1986-2009. These reports collect data from a subset of US counties. For our analysis we use the years 1986 and 2002; the longest time period for which these reports also include national estimates.

To get a clearance rate for drug trafficking crimes, we impute the total crimes by assuming that the clearance rate for drug trafficking crimes is equal to the average clearance rate for crimes in the categories of Murder, Robbery, Aggregated Assault, and Burglary. From there we calculate the number of incarcerations per crime in the usual way: by dividing total admissions in these categories divided by total reported crimes, but we also include total admissions to Federal prisons in these years in our count of admissions.⁷ Using this procedure we arrive at a value of $\pi = 2.4$ for 1986 and a value of $\pi = 7.4$ for 2002. Since, the increase in prison admissions started in the late 1970s, we round the initial π down to 2, which is also consistent with the estimations of [Pettit \(2012\)](#). Since the admission rates to prison peaked prior to 2002, we round the later π up to 8.

⁷While federal prisons do include crimes not in these categories, such as immigration, the majority of Federal prisoners are serving sentences related to drug trafficking and weapons.

The rates at each stage are provided in Table 1 to provide comparison between the imputed and non-imputed results and to understand at what point in the criminal justice process the π is changing. These figures bolster our claim that there has been a large increase in π . The listed average rates of conviction/arrest and incarceration/conviction do not contain imputed values and have almost doubled for every category over this time period. This conversion also strengthens the claim that policy has changed substantially over the period.

Table 1: Source: BJS- Felony Sentences in State Courts Series

1983	Arrest per Reported Crime	Conviction per Arrest	Incarceration per Conviction	Incarceration per Reported Crime
Murder	84.7	56.4	53.7	45.5
Robbery	20.7	37.7	32.8	6.8
Aggregated Assault	36.7	12.5	8.9	3.3
Burglary	8.9	35.6	26.2	2.3
Drug Trafficking	15.6*	41.2	26.2	10.8*
Total	15.6*	29.6	21.7	2.4*†
2002	Arrest per Reported Crime	Conviction per Arrest	Incarceration per Conviction	Incarceration per Reported Crime
Murder	79.0	70.2	66.7	52.7
Robbery	19.3	47.2	40.6	7.9
Aggregated Assault	45.9	23.3	16.5	7.6
Burglary	9.4	49.9	35.9	3.4
Drug Trafficking	20.3*	79.9	54.3	11.0*
Total	20.3*	46.9	33.6	7.8*†

¹ * imputed value;

² † also include Federal Prison Admissions;

³ Arrests are adults only and Convictions are felony convictions only.

4 Targeted and Non-Targeted Statistics

4.1 Employment and Wages: National Longitudinal Study of Youth 1979 (NLSY79)

We use data from the July 18, 2013 release of the NLSY79. The NLSY79 includes a nationally representative panel of respondents 1422 years old in 1979. Respondents were surveyed annually from 1979 to 1994 and biannually thereafter. The sample is restricted to Black or White males

that do not graduate high school by age 25.⁸

The NLSY data includes variables on both labor market outcomes and incarceration. Labor market variables, including labor force participation, employment and unemployment status, hourly wages, and job characteristics are available on a weekly frequency. Incarceration status is observed once per year after 1980 and asked retrospectively in 1980.

In our model, all jobs are found through search and there is no intensive margin. Accordingly, we define employment in the NLSY sample as any non-self employed job worked a median of 35-100 hrs per week over the employment relationship. We match each job to its characteristics using the Employer History Roster. Hourly wage for each job in each week is also taken from the Employer History Roster.⁹ We use CPI to calculate wages in 1987 dollars and exclude wages less than \$2.00 or greater than \$200.00 per hour as missing.

4.2 Wage Regression

Our theory of wage dynamics is a Ben-Porath type of progression following [Ljungqvist and Sargent \(1998\)](#). Wages increase probabilistically following a period of employment and decrease probabilistically following a period of non-employment along a pre-determined grid. To calibrate the grid points and the transition probabilities, we follow [Kitao, Ljungqvist, and Sargent \(2017\)](#). This is a quantitative paper that introduces a quadratic life-cycle wage profile into the [Ljungqvist and Sargent \(1998\)](#) framework. Figure 8 below confirms our sample of interest also exhibits a quadratic life-cycle wage profile and so this approach is well-suited for our needs.

We construct a regression where the dependent variable is the natural-log of the hourly wage ($\ln(w_{it})$).¹⁰ The regression includes a quadratic term to capture the typical life-cycle wage profile (A_{it}) which will be used to set transition probabilities and the grid shape for the employed. The regression also includes a quadratic transformation of the length of total non-employment over the past two years (N_{it}). This is motivated by the wage scarring literature showing persistent wage effects from periods of non-employment ([Michaud \(2018\)](#)). Finally, the regression also includes individual fixed (γ_i) effects to control for level differences across individuals as we are concerned with growth rates, not levels.

⁸GED holders are included in the sample. This is especially relevant since many incarcerated individuals earn a GED in prison or are mandated to do so as part of their parole release.

⁹If a worker is employed in two jobs in the same week, we consider the longest held job.

¹⁰The construction of all variables reported in the previous sub-section

$$\ln(w_{it}) = \alpha + \beta^A A_{it} + \beta^{A2} (A_{it})^2 + \beta^N N_{it} + \beta^{N2} (N_{it})^2 + \gamma_i + \epsilon_{it}$$

We consider two variations on measures for the life-cycle: (1) age and (2) measured experience (months of employment).¹¹ We also provide robustness as to the type of non-employment: (a) all non-employment spells aggregated; (b) non-employment and prison spells separated; (c) non-participation, unemployment, and prison spells separated.

We also use the weekly data from the NLSY to calculate labor market flows to compare with the model’s calibration. The states and flows are identified as follows. Employment is defined in the same way as described above for the Mincer regression specification with experience. Non-employment is categorized according to a question asking the respondent’s labor market status into “unemployed” or “non-participant”. If the respondent has a job, but that job does not meet our requirement to be classified as “employed”, we categorize the individual as “unemployed”. We use the tenure variable to clean for spurious flows including transitory changes in hours that would move a respondent across states. We do this as follows. If we see a switch from “employed” to any of our non-employment categories at time “t”, we then check the tenure variable reported for the next 4 weeks. If we see the respondent becomes “employed” in the next four weeks and the tenure is greater than one month, then we count the individual as having had been continuously employed. In other words, if they regain employment at tenure greater than one month, we conclude the transition is spurious and drop it as a true transition.

5 Characteristics at Arrest: Survey of Inmates of State Correctional Facilities

The Survey of Inmates of State Correctional Facilities is a representative survey of inmates in adult correctional facilities. We use the 1979 survey consisting of approximately 12,000 inmates in 300 institutions for the initial calibration of the model.

The specific sample selection is: Black or White Males, ages 18-64 at survey date, less than high school degree¹² Further, the sample is restricted to inmates entering the prison in the 1970’s.¹³

¹¹The data are censored with a maximum age of 50 on account of the single-cohort panel structure of the NLSY.

¹²Specifically, coded as not having completely attended the twelfth year in high school.

¹³All observations are weighted with frequency weights provided in the survey construction. Employed includes both part and full time.

Table 2: Wage Regressions table

	(1) Wagem_ b/se	(2) Wagem_ b/se	(3) Wagem_ b/se	(4) Wagem_ b/se	(5) Wagem_ b/se
Age (yrs)	0.02377*** (0.00033)			0.02375*** (0.00033)	0.02219*** (0.00034)
Age ²	-0.00045*** (0.00001)			-0.00045*** (0.00001)	-0.00041*** (0.00001)
Experience (mo)		0.00160*** (0.00003)			
Experience ²		-0.00000*** (0.00000)			
Age 25-34			0.12937*** (0.00178)		
Age 35-50			0.21097*** (0.00206)		
Non-Employed (mo)	-0.00436*** (0.00011)	-0.00433*** (0.00011)	-0.00468*** (0.00011)	-0.00437*** (0.00011)	
Non-Employed ²	-0.00001* (0.00000)	-0.00001 (0.00000)	-0.00001 (0.00000)	-0.00001* (0.00000)	
Jail Last Yr				-0.39459*** (0.05093)	-0.40735*** (0.05091)
Non-Participant (mo)					-0.00318*** (0.00033)
Non-Participant ²					0.00005** (0.00001)
Unemployed (mo)					-0.00642*** (0.00034)
Unemployed ²					0.00013*** (0.00001)
Constant	1.58557*** (0.00264)	1.59744*** (0.00267)	1.67146*** (0.00198)	1.58605*** (0.00264)	1.59659*** (0.00275)
Observations	182071	182071	182071	182071	182071

Standard errors in parentheses

Experience is total months over lifetime.

Non-employment, Non-participation, and Unemployment are total months in past two years.

Table 3: Weekly Employment Transition Rates

Status at $t - 1$ Status at t	Employed			Non-Employed	Unemployed	Non-Participant
	NE	U	N	E	E	E
By Age						
18-24	1.91	1.01	0.89	3.20	4.12	2.43
25-34	1.04	0.51	0.53	2.56	3.87	1.83
35-50	0.53	0.23	0.31	1.09	1.89	0.75
Total (18-50)						
Never Incarcerated [†]	1.05	0.54	0.51	2.81	3.82	1.94
Incarcerated w/in last year [‡]	3.06	1.18	1.88	1.21	2.33	0.97
Total	1.19	0.59	0.60	2.44	3.54	1.67

[†] Never observed as incarcerated in entire sample: age 14-19 to age 50.

[‡] We have 36,002 observations of employment status for 257 individuals incarcerated within a last year.

Percentage of the demographic group in Prison uses total population in the demographic group estimated from the 1980 Decennial Census.

Targeted & Non-Targeted Statistics from the Prison Survey 1979

Population in State Prison	
Age 18-24	0.026
25-34	0.043
35-64	0.005
Estimated Pop. State & Federal Prison	
Age 18-24	0.028
25-34	0.047
35-64	0.005
Employed Month of Crime	
Age 18-64	0.699
Unemployed Month of Crime	
Age 18-64	0.153
NiLF Month of Crime	
Age 18-64	0.144

Total prison populations, including state and federal facilities were estimated by scaling populations to adjust for the share of males with sentences greater than one year in State Prisons in 1979 out of Federal and State facilities combined (92.7%) (National Prison Statistics accessed using the Corrections Statistical Analysis Tool (bjs.gov)).

6 Recidivism of Prisoners Released Series, 1983, 1994, and 2005

The Bureau of Justice Statistics organized the compilation of demographic and criminal history data for prisoners released in 1983, 1994, and 2005. The data cover a representative sample of 16,000/38,624 released prisoners in 1983/2005 from California, Florida, Illinois, Michigan, Minnesota, New Jersey, New York, North Carolina, Ohio, Oregon, and Texas.¹⁴ Prisoners in these states comprise approximately two-thirds of the prison population. The files have two layers of data. The first layer includes socio-demographic data and corrections records data at the time of inmate release. The second layer contains information on subsequent events over the three years after release including arrest, imprisonment, and non-criminal data. The 2005 survey significantly increased the number of states involved to 30 and followed individuals over 5 years instead of 3.

We have obtained access to the restricted micro-data for the first two surveys, but not the 2005 iteration. Our statistics for recidivism in the 2000's instead come from restricted micro-data we have obtained from the study: "Criminal Recidivism in a Large Cohort of Offenders Released from Prison in Florida, 2004-2008. The study provides similar variables to the Recidivism of Prisoners Released Series for over 156,000 offenders released from the Florida Department of Corrections between 1996-2004. Outcomes for each released individual are available from state criminal records for 3 years following their release. We restrict our analysis to individuals released from prisons after 2000 for comparability.

There are obvious hazards in comparing the Florida survey to the 1983 and 1994 surveys. Clearly, Florida on its own is less representative than the 11 states used in those survey, but it is consistently a top-3 state in number of state prison inmates accounting for 7-10% of the prison population. The greater concern is that the survey covers only recidivism activities taking place within Florida. For this reason we would expect to under-estimate recidivism activities relative to the 1983 and 1994 surveys. However, a comparison of re-imprisonment rates over a three year time horizon with those reported for the 2005 iteration of the Recidivism of Prisoners Released Survey provides some confidence in the comparability of the Florida Survey. We calculated a total 3-year reimprison rate of 36% from our sub-sample of the Florida data which is remarkably close to the same statistic of 36.1% in the BJS report from the 2005 Recidivism of Prisoners Released Survey.¹⁵

¹⁴ Arizona, Delaware, and Virginia were added in the 1994 survey, but we exclude them for consistent comparison across surveys.

¹⁵ That report cautions against comparisons across years because of the stark demographic changes of the prisoner population towards older individuals. However, this is exactly what our theory predicts! It is not a problem for us to compare recidivism rates across these surveys with recidivism rates from our model data because the (endogenous!)

We are interested in a single dimension of recidivism most consistent with our model and measurements in other datasets: re-imprisonment for a new felony charge. The table below presents trends in this statistic by the age partition used in our model.¹⁶

3-yr Re-imprisonment Rate on a Felony Charge			
Age	1983 (<i>a</i>)	1994 (<i>a</i>)	2000-2003 (<i>b</i>)
18-24	22.1	29.8	42.6
25-34	18.3	26.3	36.4
35-64	9.6	20.1	30.4
Total (18-64)	17.8	25.0	35.0

Re-imprisonment rate for inmates released from state prison during the stated year.

a authors' calculations from the Recidivism of Prisoners Released Series (BJS) micro-data

b authors' calculations from Criminal Recidivism in a Large Cohort of Offenders Released from Prison in Florida 2004-2008 (FDOC) micro-data

7 Recidivism of Felons on Probation, 1986-1989

Recidivism of Felons on Probation, 1986-1989 is a data release from the Bureau of Justice statistics. These data include information from sentencing records, probation files, and criminal histories collected in 1989, pertaining to individuals under felony probation in 1986. The sample includes 32 urban and suburban areas. There are 12,369 observations from a representative sample of the 81,927 total individuals on probation in 1986. Although the time period is slightly later than our initial calibration, we use these statistics to calibrate parameters of the model related to skill shocks by employment/jail status which we assume are time and policy invariant.

The specific sample selection is: Black or White Males, ages 18-64 at survey date, less than high school degree¹⁷ Observations weighted using provided "Probation case weight" variable which

age demographics in our model are changing in a similar way as the data.

¹⁶Caution must be used when comparing these data with the BJS summary papers on the surveys. Our analysis of the micro-data exactly replicates these reports when using the "Received" records from prisons/jails to identify re-incarceration. Using this measure we match their 40% 3-year recidivism rate for 1983 which breaks down to 51%/38%/29% for young, middle, and old respectively. However, this measure includes both jails which we are not considering in other datasets and includes re-confinement for violation of conditions of release, probation, or parole which we also do not model and do not include in the admission data from the NCRP data.

¹⁷Here we cannot distinguish between GED and non-GED high school graduates, so GED's are excluded.

considers demographics, offense, location, and sentence.¹⁸

Targeted Statistics from Recidivism of Felons 1986-1989		
Statistic	Proportion	3 Year Recidivism
Total Pop	1	17.1
Wks Employed		
>60%	31.4 (29.0-33.6)	9.7 (7.4-12.0)
40-60%	18.5 (16.75-20.25)	12.9 (9.1-16.7)
<40%	50.1 (49.0-52.2)	23.3 (20.8-25.8)
Hourly Wage		
< \$5	22.7 (17.7-27.6)	12.8 (4.5-21.1)
\$5-\$10	47.7 (41.9-53.6)	14.1 (7.8-20.5)
>\$10	29.6 (24.4-34.8)	10.1 (3.7-39.0)
Sample: Emp >60%	Proportion	3 Yr Recidivism
< \$5	21.9 (16.5-27.2)	6.3 (1.2-11.4)
\$5-\$10	51.7 (45.1-58.2)	10.5 (4.8-16.3)
>\$10	26.5 (20.9-32.0)	6.9 (0.1-13.6)

Weeks employed are for the first year after release. Workers earning less than the minimum wage (\$3.1) are treated as unemployed. Wages are truncated above at \$10 with 24% of employed individuals earning more than \$10.

¹⁸Statistics by wage for those with more than 60% of weeks employed are pooled due to small sample by race. Pool includes 446 individuals.

8 Decennial Census and Current Population Survey

We use data from the Decennial Census and Current Population Surveys to calculate labor market statistics for our focus population: white and black males without high school degrees.

The specific sample selection is: Males, ages 18-64 at survey date, less than high school degree¹⁹, civilian non-institutionalized. We consider both employment defined as employed at survey and as defined by working 50-52 weeks in the last year. Similarly, we consider unemployment as unemployed at survey and unemployed greater than three weeks in the last year.

Employment Rate and Employment to Population Ratio		
Statistic	Employment Rate	Employment to Pop
1970	95.4	81.9
Young (18-24)	90.3	66.9
Middle (25-34)	95.2	88.6
Old (35-64)	96.3	83.5
1980	90.1	71.1
Young (18-24)	82.0	61.5
Middle (25-34)	87.9	77.9
Old (35-64)	93.3	72.7
1990	87.7	64.4
Young (18-24)	79.0	56.4
Middle (25-34)	85.8	72.5
Old (35-64)	91.5	64.3
2000	88.5	59.1
Young (18-24)	82.0	55.7
Middle (25-34)	88.8	67.6
Old (35-64)	91.3	57.8
2010	79.9	54.5
Young (18-24)	66.4	38.1
Middle (25-34)	79.9	65.8
Old (35-64)	84.2	56.8

¹⁹Specifically, less than high school is coded as (*educd* < 50 & *educd* > 1) and we also consider adding (*educd* == 63 || *educd* == 62). We are trying to include GEDs, but omit those completing twelfth grade with a diploma.

For the estimation of the model, we target statistics averaged across the 1970-1980 Current Population Survey for our sample. First the employment rate is 70%. As a robustness check on wages, we compare growth rates implied by our NLSY regression to life-cycle wage growth (cross-section) in the 1980 Census. Wages are constructed as total labor income divided by the product of weeks worked in the year times average hours per week.²⁰

Median Hourly Wage by Age Group (1980-dollars)			
Statistic	Value	Growth	Standard Dev.
Young (18-24)			
All	5.41		6.38
Middle (25-34)		Growth Young to Middle	
All	6.95	22.1%	6.69
Old (35-64)		Growth Middle to Old	
All	8.45	21.5%	7.67
Total (18-64)		Growth Young to Old	
All	7.58	56.2%	7.37

9 A Simple Theoretical Model

Here we provide a simpler version of the model presented in the paper. We assume that the only source of ex-post heterogeneity across individuals is the employment status. That is, we assume all individuals are infinitely-lived, have identical human capital and criminal capital (low), and prison has no explicit effect on job finding probability.

Let V_p , V_u , V_e represent the value functions for an incarcerated, unemployed and employed individual, respectively, we can formulate these value functions as:

Incarcerated Individual:

$$rV_p = \tau (V_u - V_p) \quad (9.1)$$

Unemployed Individual:

$$rV_u = b + \lambda_w \max \{V_e - V_u, 0\} + \eta \int \max \{\pi (V_p - V_u) + \kappa, 0\} dH(\kappa) \quad (9.2)$$

²⁰We set hourly wages below \$2 or above \$200 (1980) dollars equal to missing

Employed Individual:

$$rV_e = w + \delta(V_u - V_e) + \eta \int \max\{\pi(V_p - V_e) + \kappa, 0\} dH(\kappa) \quad (9.3)$$

As long as $w > b$, we have a cut-off rule for the crime decision.

Lemma 9.1. *There exists κ_u^* (κ_e^*) such that unemployed (employed) individual commits every crime if the reward is higher than or equal to κ_u^* (κ_e^*), and κ_u^* is given by*

$$\kappa_u^* = \pi(V_u - V_p) \quad (9.4)$$

$$\kappa_e^* = \pi(V_e - V_p) \quad (9.5)$$

We can also prove that $V_e - V_u > 0$. This gives us the following corollary:

Corollary 9.2. *If $w > b$, then the threshold of crime reward for employed is higher than for unemployed: $\kappa_e^* > \kappa_u^*$.*

The above corollary implies each employed individual commits fewer crimes than an unemployed individual. Thus, employment is an individual characteristic that “deters” crime.

We can further characterize the values for employment, unemployment and incarceration as functions of the cut-off values κ_e^* and κ_u^* .

Lemma 9.3. *The values for incarceration, V_p , unemployment, V_u , and employment, V_e , can be expressed as:*

$$\begin{aligned} V_p &= \frac{\tau \kappa_u^*}{r\pi} \\ V_u &= \frac{(r + \tau) \kappa_u^*}{r\pi} \\ V_e &= \frac{r\kappa_e^* + \tau \kappa_u^*}{r\pi} \end{aligned}$$

Using these values, we can show the following two important propositions:

Proposition 1. *If $\delta > \tau$, an increase in the probability of getting caught, π , increases the crime threshold for both unemployed and employed, i.e. decreases the crime rate.*

Proposition 2. *If $w > b$, an increase in the job offer arrival rate increases the crime threshold for the unemployed (decreases the crime rate). Furthermore if $\delta > \tau$ ($\delta < \tau$) an increase in the job offer arrival rate increases (decreases) the crime threshold for the employed and decreases (increases) the crime rate.*

These propositions show that as the criminal policy becomes more punitive (an increase in π), unemployed and employed individuals respond by committing less crimes. However, the net effect on incarceration probability is ambiguous. Notice that incarceration probability in the model is $\pi\eta(1 - H(\kappa_i^*))$ for $i \in \{u, e\}$. An increase in π directly increases this probability. However, as individuals respond, κ_i^* increases, which decreases this probability. The net effect depends on the magnitude of these two forces. The second important message of the above propositions is the effect of a change in job arrival rate on crime propensities. The second proposition shows that as labor market opportunities for individuals improve (an increase in job arrival rate), individuals decrease their crime propensities.

9.1 Firm's Problem:

Let J be the value of a match between a firm and individual and V_f be the value of a vacancy. We have the following flow equations for the firm:

$$\begin{aligned} rV_f &= -k + \lambda_f (J - V) \\ rJ &= p - w + \delta (V_f - J) + \eta (1 - H(\kappa_e^*)) \pi (V_f - J) \end{aligned}$$

Lastly, the free-entry condition pins down the market tightness:

$$V_f = 0$$

Combining the free-entry condition with the above value functions, we get

$$\lambda_f = \frac{k(r + \delta + \eta(1 - H(\kappa_e^*))\pi)}{p - w} \quad (9.6)$$

Lemma 1. *An increase in the probability of incarceration, $\pi\eta(1 - H(\kappa_e^*))$, increases λ_f , the worker arrival rate for the firms, which means job offer arrival rate for the workers, λ_w , decreases.*

This is an immediate consequence of equation (9.6). The expected duration of a new match decreases as the probability a worker ends the match by going to prison increases. Shorter match duration implies lower profits for the firm. To maintain the equilibrium zero expected profits condition, fewer firms post vacancies and the job arrival rate for workers decreases. Referring back to a worker's problem, observe a decrease in the job arrival rate results in lower crime thresholds. Thus, if the deterrence effect of an increase in incarceration policy (the probability of prison) is small enough such that total flows into prison increase, then the equilibrium response of firms to post fewer vacancies can further reduce the deterrence effect of the policy. ²¹

9.2 Steady-State Flows:

The equations characterizing the steady-state are as follows:

$$\begin{aligned} p\tau &= u\eta\pi f_u + (1 - u - p)\eta\pi f_e \\ u(\lambda_w + \eta\pi f_u) &= p\tau + (1 - u - p)\delta \\ (1 - u - p)(\delta + \eta\pi f_e) &= u\lambda_w \end{aligned}$$

where p and u are the measure of incarcerated and unemployed individuals, respectively, and f_u and f_e are the probability of committing crime conditional on receiving an opportunity. That is, $f_u = (1 - H(\kappa_u^*))$ and $f_e = (1 - H(\kappa_e^*))$. Then, we have

$$\begin{aligned} u &= \frac{\tau(\delta + \eta\pi f_e)}{(\tau + \eta\pi f_e)(\lambda_w + \eta\pi f_u + \delta) + \eta\pi(\delta - \tau)(f_u - f_e)} \\ p &= \frac{\eta\pi(f_e + u(f_u - f_e))}{\tau + \eta\pi f_e} \end{aligned}$$

If we assume that $f_u = f_e = 1$, which is the case when the crime reward distribution is degenerate and the reward is sufficiently large, then we can show that the unemployment rate, which is defined as $\frac{u}{1-p}$ becomes

$$\frac{u}{1-p} = \frac{\delta + \eta\pi}{\lambda_w + \delta + \eta\pi} \quad (9.7)$$

Proposition 3. *An increase in the probability of getting caught, π , increases the unemployment*

²¹This requires that crime increases when the job arrival rate falls. By Proposition 2, this requirement is true with certainty if the job duration is shorter than the prison duration, but may or may not hold otherwise (Proposition 2).

rate.

10 Proofs

Proof. From equation (9.1) we have

$$V_u = \frac{r + \tau}{\tau} V_p \quad (10.1)$$

We can express the difference between the value of employment and incarceration by combining equations (9.1) and (9.3) :

$$(r + \eta(1 - H(\kappa_e^*))\pi + \delta)(V_e - V_p) = w + (\delta - \tau)(V_u - V_p) + \eta\bar{\kappa}_e$$

where $\bar{\kappa}_e = \int_{\kappa_e^*} \kappa dH(\kappa)$. Substituting (9.4) and (9.5) into the above equation we get

$$(r + \delta) \frac{\kappa_e^*}{\pi} + \eta(1 - H(\kappa_e^*))\kappa_e^* = w + (\delta - \tau) \frac{\kappa_u^*}{\pi} + \eta\bar{\kappa}_e \quad (10.2)$$

Similarly, using equations (9.1) and (9.2), we can express the difference between the value of unemployment and incarceration as:

$$(r + \eta(1 - H(\kappa_u^*))\pi + \lambda_w + \tau)(V_u - V_p) = b + \lambda_w(V_e - V_p) + \eta\bar{\kappa}_u$$

Again, substituting equations (9.4) and (9.5) into the above equation we get

$$(r + \lambda_w + \tau) \frac{\kappa_u^*}{\pi} + \eta(1 - H(\kappa_u^*))\kappa_u^* = b + \lambda_w \frac{\kappa_e^*}{\pi} + \eta\bar{\kappa}_u \quad (10.3)$$

Then, equations (10.2) and (10.3) give us κ_u^* and κ_e^* . Given these thresholds, we can express the value functions as

$$\begin{aligned} V_p &= \frac{\tau\kappa_u^*}{r\pi} \\ V_u &= \frac{(r + \tau)\kappa_u^*}{r\pi} \\ V_e &= \frac{r\kappa_e^* + \tau\kappa_u^*}{r\pi} \end{aligned}$$

□

Proof. Using implicit function theorem on equation (10.2) and (10.3) we have

$$\frac{d\kappa_e^*}{d\pi} = \frac{(\delta - \tau) \frac{d\kappa_u^*}{d\pi} + w + \eta \int_{\kappa_e^*} (1 - H(\kappa)) d\kappa}{r + \delta + \eta\pi (1 - H(\kappa_e^*))}$$

If $\delta > \tau$, it is immediate to see that $\frac{d\kappa_e^*}{d\pi} > 0$. Using implicit function theorem on equation (10.3)

$$\frac{d\kappa_u^*}{d\pi} = \frac{\lambda_w \frac{d\kappa_e^*}{d\pi} + b + \eta \int_{\kappa_u^*} (1 - H(\kappa)) d\kappa}{r + \lambda_w + \tau + \eta\pi (1 - H(\kappa_u^*))}$$

Since $\frac{d\kappa_e^*}{d\pi} > 0$, then we also have $\frac{d\kappa_u^*}{d\pi} > 0$. □

Proof. Using integration by parts, we can express equation (10.2) as

$$(r + \delta) \frac{\kappa_e^*}{\pi} = w + (\delta - \tau) \frac{\kappa_u^*}{\pi} + \eta \int_{\kappa_e^*} (1 - H(\kappa)) d\kappa$$

Using the implicit function theorem, we have

$$\frac{d\kappa_e^*}{d\kappa_u^*} = \frac{\delta - \tau}{r + \delta + \eta\pi (1 - H(\kappa_e^*))}$$

Then, if $\delta < \tau$, we have $\frac{d\kappa_e^*}{d\kappa_u^*} < 0$ and if $\delta > \tau$, we have $1 > \frac{d\kappa_e^*}{d\kappa_u^*} > 0$. Similarly, using equation (10.3) and the implicit function theorem, we have

$$\frac{d\kappa_u^*}{d\lambda_w} = \frac{\kappa_e^* - \kappa_u^*}{r + \lambda_w + \tau + \eta\pi (1 - H(\kappa_u^*)) - \lambda_w \frac{d\kappa_e^*}{d\kappa_u^*}}$$

We know that $w > b$ implies $\kappa_e^* - \kappa_u^* > 0$. Since we also know that $\frac{d\kappa_e^*}{d\kappa_u^*} < 1$, then we have $\frac{d\kappa_u^*}{d\lambda_w} > 0$. This result is independent of the relation between δ and τ . But, depending on the relation between δ and τ we have two opposite results. If $\delta < \tau$, since we have $\frac{d\kappa_e^*}{d\kappa_u^*} < 0$, then we get $\frac{d\kappa_e^*}{d\lambda_w} < 0$. Otherwise, if $\delta > \tau$, we have $\frac{d\kappa_e^*}{d\lambda_w} > 0$. □

Proof. Equation (9.7) shows the relation between the unemployment rate and π . As π increases

unemployment increases. Moreover, an increase in π decreases the offer arrival rate, which further increases the unemployment rate. \square

11 Value Functions

11.1 Individuals

Given the five-dimensional heterogeneity of the individuals, the value function for the individuals depend on five state variables: age, criminal type, addiction type, labor market status and skill. We denote $V_i^{x,m,k}(h)$ as the value of an individual with labor market status $i \in \{e, u, p\}$, addiction type $x \in \{a, na\}$, criminal type $k \in \{0, 1\}$, skill level h and age m . Employed individuals receive wage, wh , which potentially depends on the skill level and market wage rate.

For notational convenience we denote $I_f^{x,m,k}(h)$ as the indicator function for the continuation of the match between the firm and the individual of type x, k , skill level h , and age m . $I_f^{x,m,k}(h) = 1$ if the match continues and $I_f^{x,m,k}(h) = 0$ if the match dissolves:

$$I_f^{x,m,k}(h) = \begin{cases} 1 & \text{if } V_e^{x,m,k}(h) \geq V_u^{x,m,k}(h) \\ 0 & \text{o.w.} \end{cases} \quad (11.1)$$

For notational convenience we define the following value functions:

$$\begin{aligned} V_{eu}^{x,m,k}(h) &= I_f^{x,m,k}(h) V_e^{x,m,k}(h) + (1 - I_f^{x,m,k}(h)) V_u^{x,m,k}(h) \\ V_{px}^{x,m,k}(h) &= \nu_x V_p^{-x,m,k}(h) + (1 - \nu_x) V_p^{x,m,k}(h) \\ V_{ijx}(h) &= \nu_x V_{ij}^{-x,m,k}(h) + (1 - \nu_x) V_{ij}^{x,m,k}(h) \end{aligned}$$

The value of an employed worker with skill level h , age m , and type x, k becomes the following equation:

$$\begin{aligned} wh + \eta_m \int \max \{ V_{ec}^{x,m,k}(h, \kappa) - V_e^{x,m,k}(h), 0 \} dF(\kappa) + \eta_m^x (V_{eca}^{x,m,k}(h) - V_e^{x,m,k}(h)) + \\ rV_e^{x,m,k}(h) = \psi_e (V_{eu}^{x,m,k}(f_e(h)) - V_e^{x,m,k}(h)) + \delta (V_u^{x,m,k}(h) - V_e^{x,m,k}(h)) + \\ \vartheta_m (V_{eu}^{x,m+1,k}(h) - V_e^{x,m,k}(h)) + \xi^x (V_{eu}^{-x,m,k}(h) - V_e^{x,m,k}(h)) \end{aligned} \quad (11.2)$$

s.to

$$\begin{aligned} V_{eca}^{x,m,k}(h) &= \pi V_{px}^{x,m,-k}(h) + (1 - \pi) V_{eux}^{x,m,k}(h) \\ V_{ec}^{x,m,k}(h, \kappa) &= \pi V_{px}^{x,m,-k}(h) + (1 - \pi) V_{eux}^{x,m,k}(h) + \kappa \end{aligned}$$

Unemployed individuals receive unemployment benefit. Denoting b as the replacement ratio, the unemployment benefit for an individual with human capital level h becomes bh . The value of an unemployed individual with skill level h , age m , and type x, k can be written as follows:

$$\begin{aligned} &bh + \eta_m \int \max \{ V_{uc}^{x,m,k}(h, \kappa) - V_u^{x,m,k}(h), 0 \} dF(\kappa) + \eta_m^x (V_{uca}^{x,m,k}(h) - V_u^{x,m,k}(h)) + \\ rV_u^{x,m,k}(h) &= \lambda_w^{k,m} s^\phi (V_{eu}^{x,m,k}(h) - V_u^{x,m,k}(h)) + \xi^x (V_u^{-x,m,k}(h) - V_u^{x,m,k}(h)) + \\ &\vartheta_m (V_{eu}^{x,m+1,k}(h)) + \psi_u (V_u^{x,m,k}(f_u(h)) - V_u^{x,m,k}(h)) \end{aligned} \quad (11.3)$$

s.to.

$$\begin{aligned} V_{uca}^{x,m,k}(h) &= \pi V_{px}^{x,m,-k}(h) + (1 - \pi) V_{ux}^{x,m,k}(h) \\ V_{uc}^{x,m,k}(h, \kappa) &= \pi V_{px}^{x,m,-k}(h) + (1 - \pi) V_{ux}^{x,m,k}(h) + \kappa \end{aligned}$$

Lastly, incarcerated individual receives no benefit. The value of an incarcerated individual becomes the following:

$$rV_p^{x,m,k}(h) = \tau (V_u^{x,m,k}(h) - V_p^{x,m,k}(h)) + \vartheta_m (V_p^{x,m+1,k}(h) - V_p^{x,m,k}(h)) + \quad (11.4)$$

$$\xi^x (V_p^{-x,m,k}(h) - V_p^{x,m,k}(h)) + \psi_p (V_p^{x,m,k}(f_p(h)) - V_p^{x,m,k}(h)) \quad (11.5)$$

11.2 Firms

$$rV_f^{k,m} = -c_{k,m} + \lambda_f^{k,m} \int (s_u^{x,k,m})^\psi (J^{x,m,k}(h) - V_f) d\mu_u(h, x|m, k) \quad (11.6)$$

$$rJ^{x,m,k}(h) = \left\{ \begin{aligned} &(1 - w)h + \delta (V_f^{k,m} - J^{x,m,k}(h)) + \psi_e (J^{x,m,k}(f_e(h)) - J^{x,m,k}(h)) + \\ &(\eta_m (1 - F(\kappa_e^{x,k,m}(h))) + \eta_m^x) \pi (V_f^{k,m} - J^{x,m,k}(h)) + \vartheta_m (J^{x,m+1,k}(h) - J^{x,m,k}(h)) \\ &((\eta_m (1 - F(\kappa_e^{x,k,m}(h))) + \eta_m^x) (1 - \pi) \nu^x + \xi^x) (J^{-x,m,k}(h) - J^{x,m,k}(h)) \end{aligned} \right\} \quad (11.7)$$

where $\kappa_e^{x,k,m}(h)$ denotes the criminal reward threshold for the employed individual with characteristics (x, k, m, h) .

11.3 Equilibrium

Definition 11.1. A stationary equilibrium consists value functions for individuals V, J decision rules I_f , policy functions κ_u and κ_e for the unemployed and employed, market tightness $\theta^{k,m}$, job offer arrival rates $\lambda_w^{k,m}$, worker arrival rates $\lambda_f^{k,m}$, and stationary measure of individuals μ , such that

1. Given market tightness and job and worker arrival rates, policy functions s_u and s_e and value functions V, V_f and J solve (11.2) – (11.6), decision rule I_f solves (11.1).
2. Stationary measure μ is consistent with the decision rules.
3. Perfect competition among firms should result $V_f^{k,m} = 0$.
4. Job arrival rates and worker arrival rates satisfy (9.6).

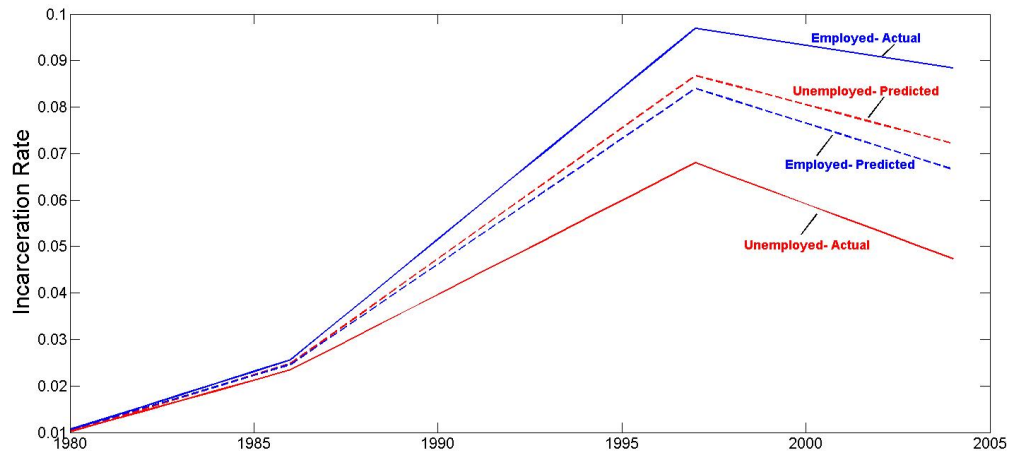
12 Additional Results

12.1 Incapacitation versus Deterrence Effect - v2

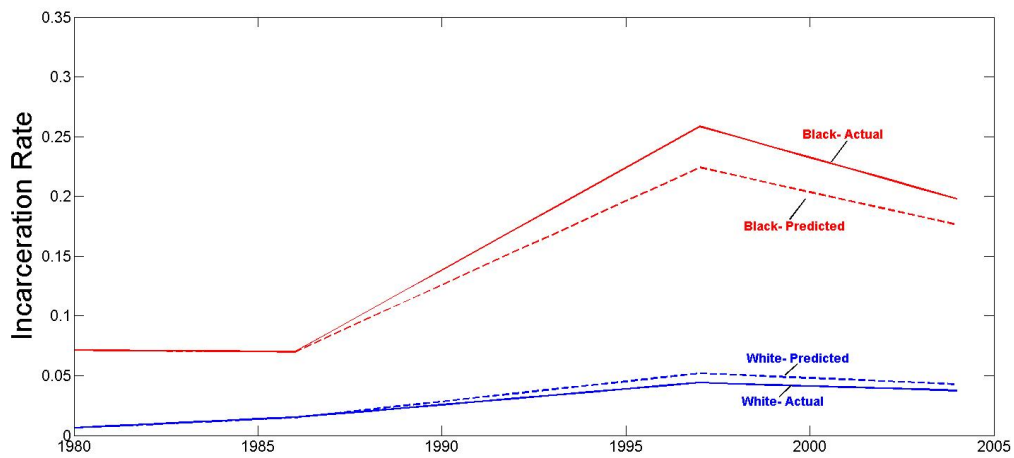
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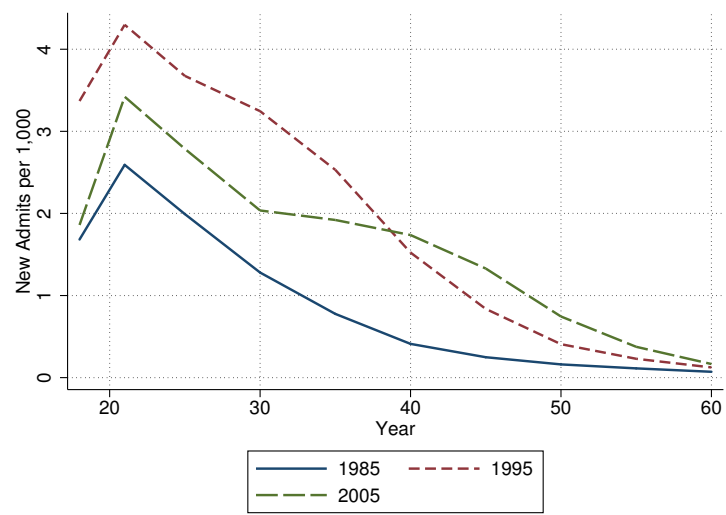


(a) Predicted and Actual Rates by Employment Status

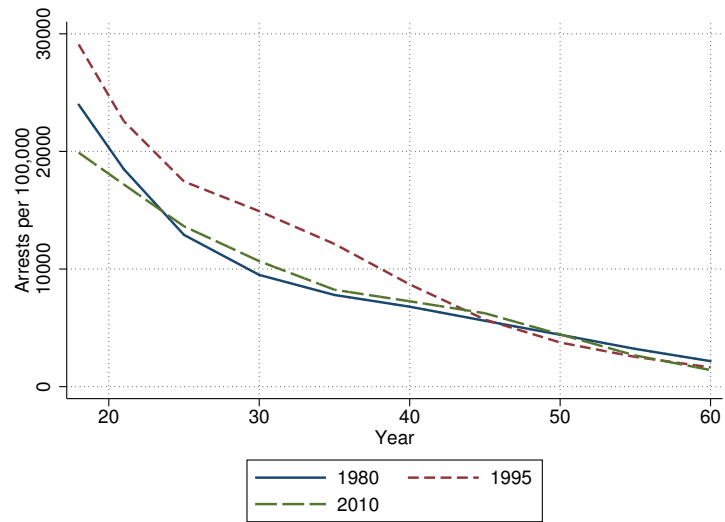


(b) Predicted and Actual Rates by Race

Figure 2: Actual values calculated from BJS Prison Surveys and Census data. Predicted values calculated by holding admission rates fixed to 1979 levels, and raising rates by the same proportion for each group, adjusting for demographics (race/or employment status and age).



(a) New Prison Admission Rate by Age



(b) Arrest Rate by Age

Figure 3: Age Profiles of Crime and Incarceration

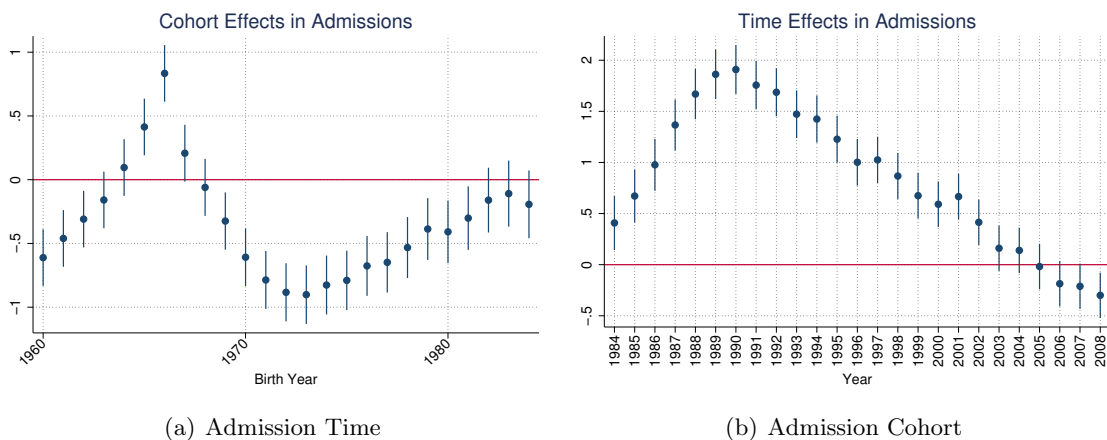


Figure 4: Admissions for new offenses from National Corrections Reporting data.

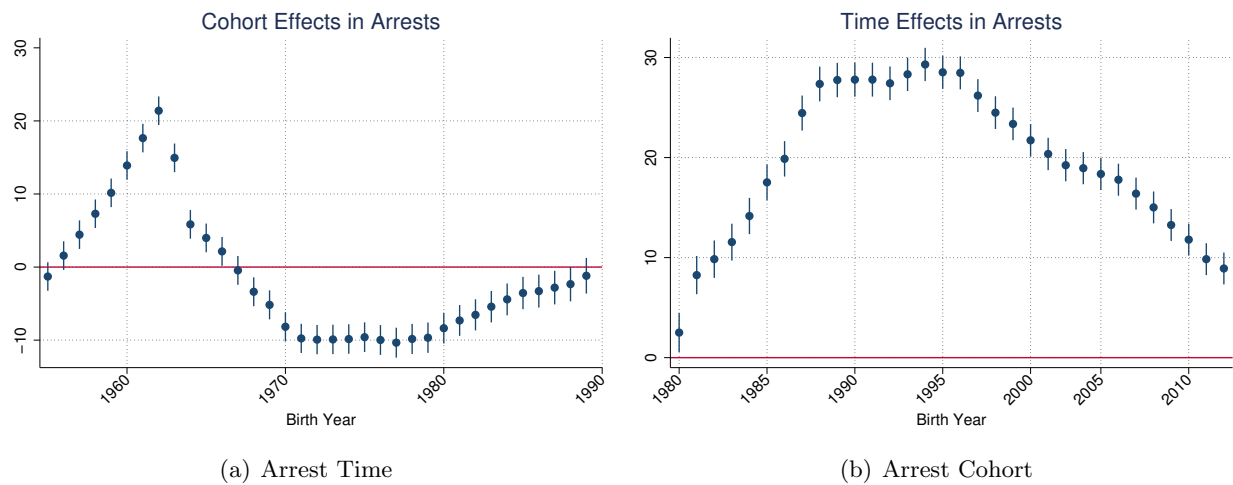


Figure 5: Calculated using FBI Uniform crime reports arrest rates.

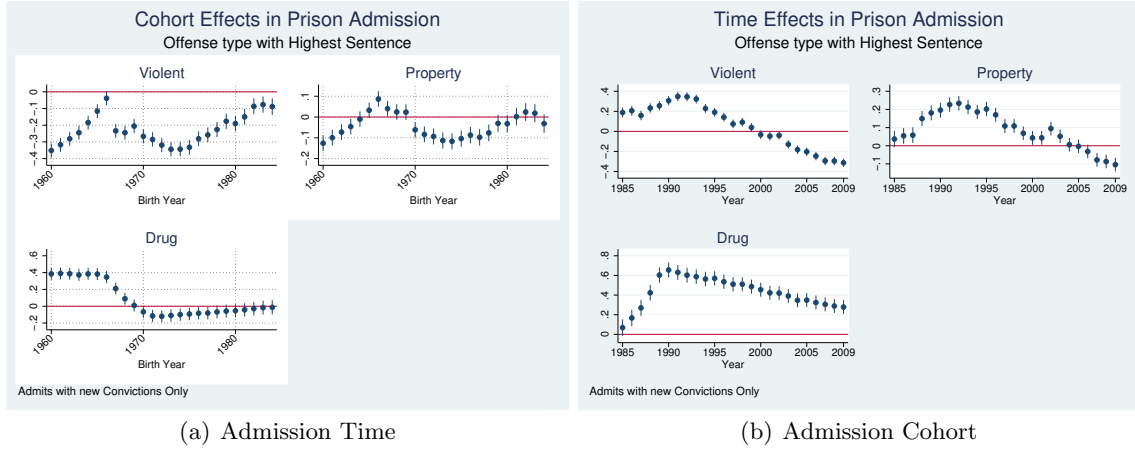


Figure 6: Admissions for new offenses from National Corrections Reporting data.

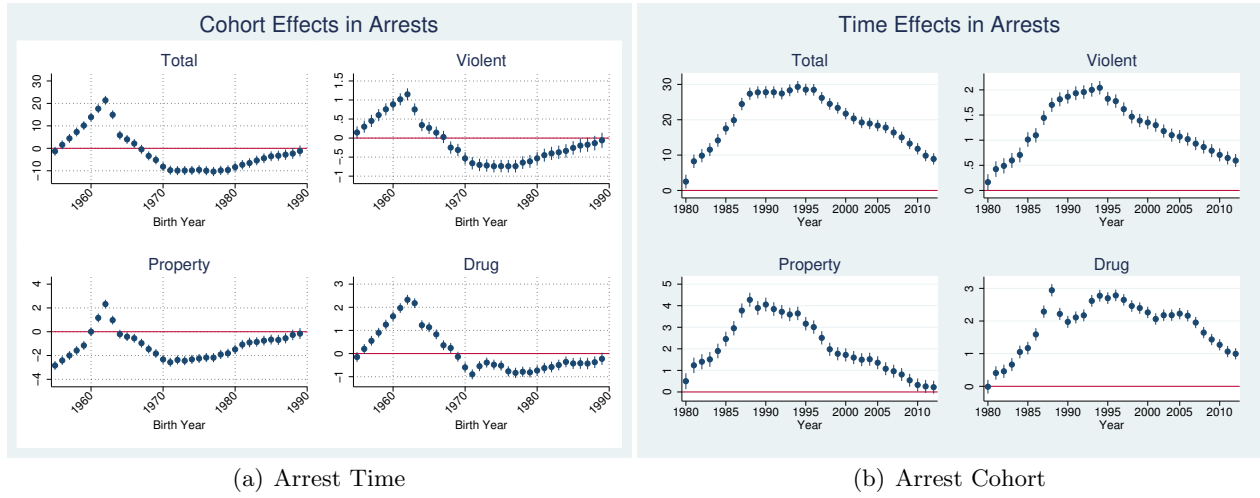


Figure 7: Calculated using FBI Uniform crime reports arrest rates.

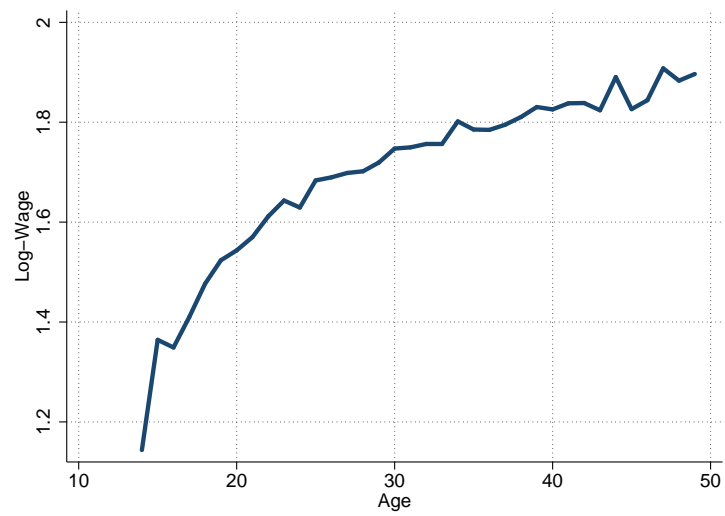
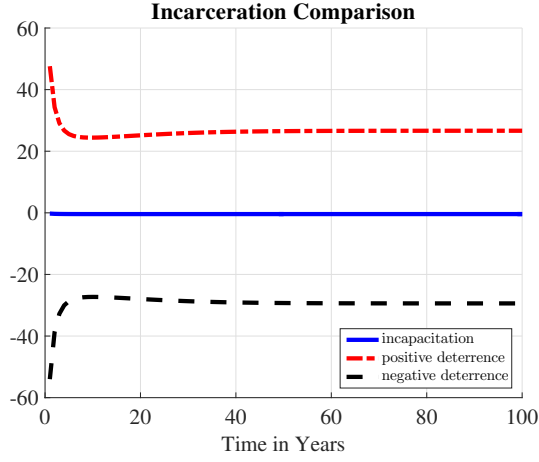
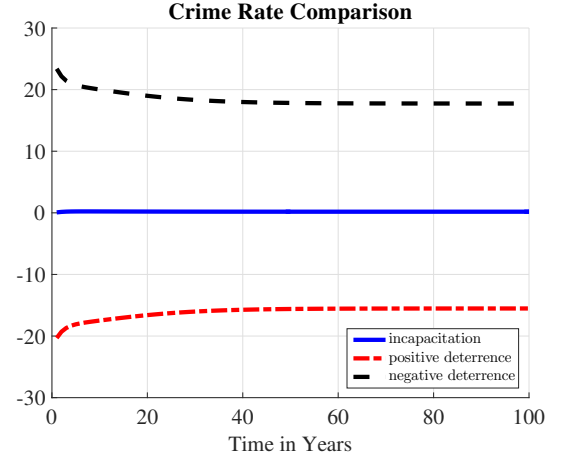


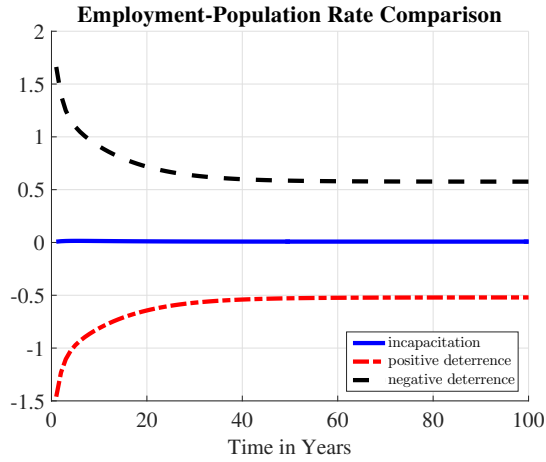
Figure 8: Mean Log-Wage by Age, NLSY 79



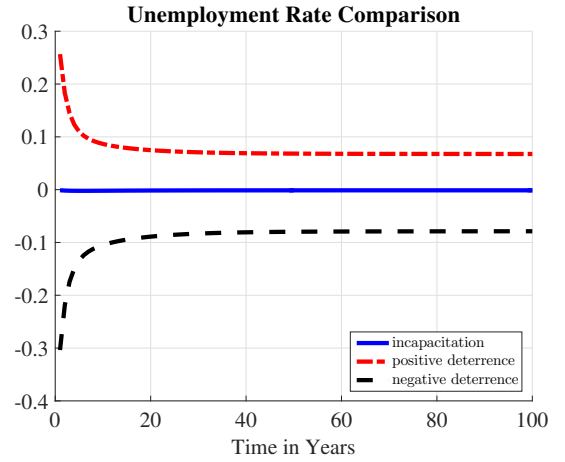
(a) Incarceration



(b) Crime Rate



(c) Employment-Population Rate



(d) Unemployment Rate

Figure 9: Various Elasticities: The figures compare the evolution of elasticities of various statistics. The solid line shows the elasticities corresponding to a 50% increase in the prison release rate. The long dashed line plots the elasticities corresponding to a 50% increase in the probability of getting caught, and lastly the dashed line plots the elasticities corresponding to a 50% decrease in the probability of getting caught. Elasticities are computed as the percentage difference of the variable of interest one year after the shock with respect to the corresponding value of these variables in the benchmark economy.

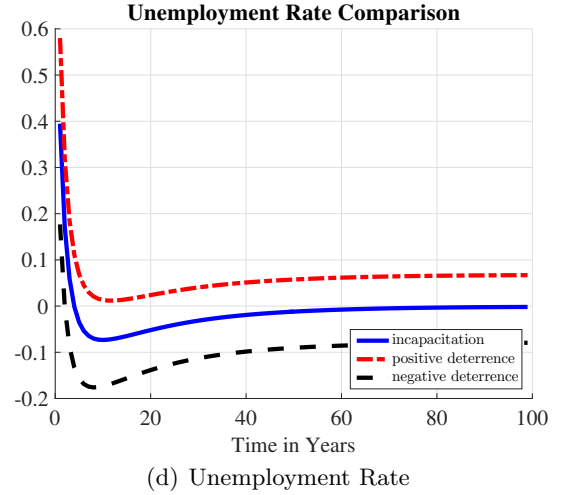
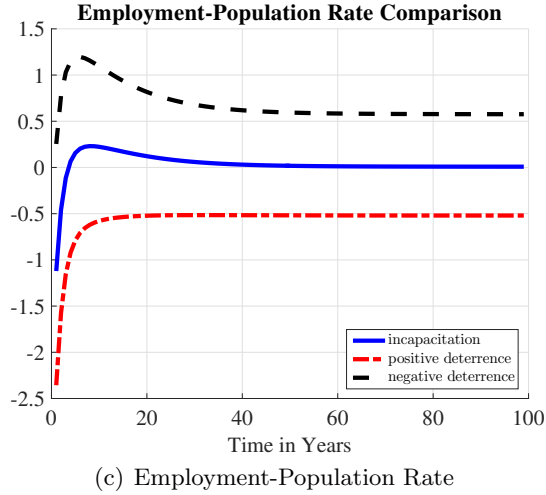
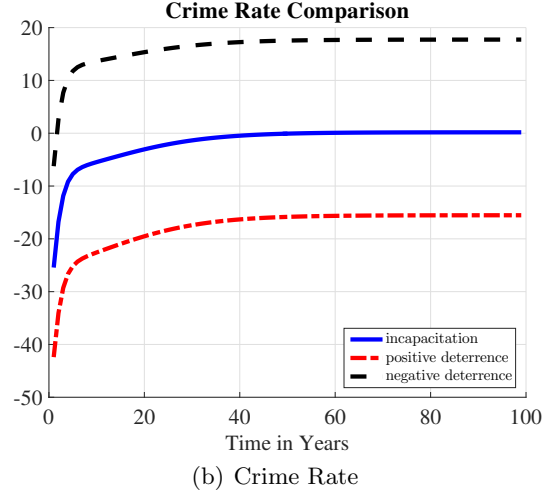
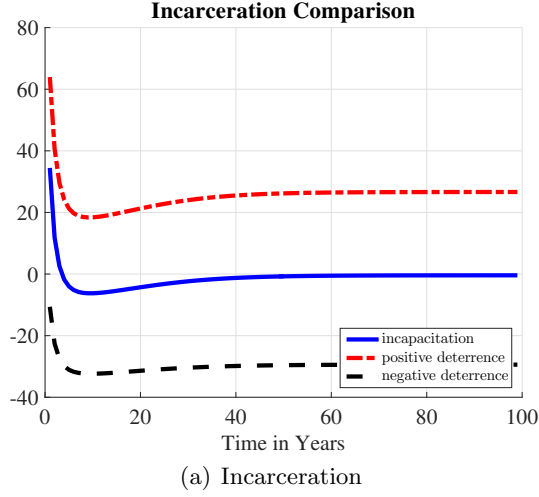


Figure 10: Various Elasticities: The figures compare the evolution of elasticities of various statistics. The solid line shows the elasticities corresponding to a 50% increase in the prison release rate. The long dashed line plots the elasticities corresponding to a 50% increase in the probability of getting caught, and lastly the dashed line plots the elasticities corresponding to a 50% decrease in the probability of getting caught. Elasticities show the percentage change in the variable of interest one year after the shock with respect to its value at the time of the shock.