Optimal Borrowing Constraints and Growth in a Small Open Economy

Amanda Michaud Jacek Rothert*

Indiana University U.S. Naval Academy

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Abstract

Chinese high growth has been accompanied by government restrictions on international borrowing (capital controls). In this paper, we ask: are such restrictions a useful policy tool to facilitate growth? We provide a theory of borrowing constraints on households as a tool to correct a learning-by-doing externality. Borrowing constraints operate as a policy tool through two channels: (i) increasing labor supply and (ii) reallocating labor towards traded goods. We find welfare gains are closest to that of the First-Best Planner allocation when the externality is not too large. We compute the sequence of optimal constraints along the growth path and show how use of this policy tool contributes to repressed wages, current account balance, and slow real exchange rate appreciation.

Keywords: learning-by-doing, borrowing constraints, Chinese economy, capital controls

*Corresponding author; E-mail: jacek.rothert@gmail.com; Phone: (1)-410-293-5379;

Mailing address:

Department of Economics, United States Naval Academy; 589 McNair Rd, Annapolis, MD 21402

1 Introduction

Financial liberalizations of the late 1980's and 1990's came on the heels of economic literature linking underdeveloped financial markets to poor economic outcomes.[1] China has followed a different path. For several decades Chinese policy actively limited households' access to financial markets while the country experienced extraordinary economic growth. As the world seeks to take a lesson from the Chinese experience, important questions remain unanswered. Did China grow in spite of these policies, or can financial repression of households promote growth? If the latter is true and financial repression can increase growth, is it at the cost or the benefit to the welfare of households? Our goal is to address these questions by analyzing financially repressive polices, directed at households, as a tool to facilitate growth.

We provide a theory of borrowing constraints on households as a means to facilitate growth through a "learning-by-doing" (LBD) externality (Arrow (1962), Romer (1986)). LBD is the idea that increased production accelerates productivity growth through institutional learning. The classic channel through which borrowing constraints correct such externalities is through reallocation of resources from the non-traded sector to the traded goods sector where empirical evidence suggest LBD externalities are largest[2]. We introduce a new labor supply channel through elastic labor supply that complements the reallocation channel in increasing labor in the traded goods sector.

The mechanism of the new labor supply channel is straightforward. Borrowing constraints reduce current consumption of households.[3] If leisure is a normal good and/or if leisure and consumption are complements, households choose less leisure and work more when current consumption is reduced. The LBD externality translates higher labor today into higher

[1] Financial reforms were one part of a general package of market-based reforms prescribed to developing countries by the IMF, World Bank, and US Treasury during this time. This broad-based unanimity of this view in these international institutions is referred to as the Washington Consensus.

[2] Deaton and Laroque (1999) provide a theory of a "virtuous" cycle where borrowing constraints lead households to save for land purchases, thus raising capital allocated to the production sector. Fernandes and Isgut (2005) provide evidence of learning by exporting for Colombia; Ma and Zhang (2008) for China.

[3] It is not simply a wealth effect since total wealth is unchanged, but the availability of wealth in a given period is constrained.

growth by increasing future productivity. Thus, borrowing constraints facilitate growth.

What is not obvious is how borrowing constraints affect welfare. Our main finding is optimal borrowing constraints produce welfare improvements closest to the First-Best Planner for values of the learning-by-doing externality that are not too large or too small. This result is explained as follows. Correcting a large learning by doing externality implies a large increase in permanent income. However, if borrowing constraints are used to correct the externality, households will be limited in smoothing consumption. Therefore, households enjoy a *smaller share* of the welfare gains from correcting the externality when LBD is large. For smaller values of LBD, the increase in permanent income is smaller, and the contemporaneous increase in output from higher labor supply today better smooths consumption in absence of borrowing. Therefore households enjoy a *larger share* of the welfare gains from correcting the externality when LBD is moderate-sized.

We provide a quantitative example to explore our theory in the context of the Chinese growth experience. We consider a two-sector model of traded and non-traded goods, where LBD occurs in the traded sector. We calibrate the model to the Chinese economy from 1990-2009, considering several LBD elasticities and choosing a sequence of borrowing constraints to match the time series of China's trade balance. When calibrated to standard preference parameters, we find a welfare loss equivalent to a 13% decline in annual consumption relative to laissez-faire despite a moderate LBD elasticity of 5%. We conclude that Chinese policy constrained households beyond what could be justified by correcting this LBD externality alone. However, the mechanics at work in the model are consistent with the data. Including borrowing constraints provides annual real wage growth and real exchange rate appreciation of magnitudes that are significantly closer to the data than the laissez faire allocation. We also quantify the importance of our new channel of elastic labor supply for this calibration; it accounts for about 25% of the increase in labor in the traded sector labor relative to laissez-faire. All of these results are highly sensitive to preference parameters. We will discuss how increasing complementarity between traded and non-traded goods, or the intertemporal elasticity of substitution for consumption or for leisure all reduce welfare losses, and can bring welfare gains.

Borrowing constraints on households are equivalent to capital controls in our environment.[4] Indeed, other second best macroeconomic policies (ex: real exchange rate manipulation) would reallocate labor to the traded goods sector as well. However, we choose to analyze the borrowing constraint tool in acknowledgement that it would function similarly in a richer environment while these alternative macroeconomic tools would not. For instance, borrowing constraints on households may be distinct from capital controls when firms have financial considerations or when capital controls are ineffective because heterogenous agents provide a domestic credit market with both borrowers and savers as in Bacchetta et al. (2013) or Itskhoki and Moll (2014).[5] We also briefly discuss how Pigovian taxes can achieve the first-best, but do not consider them implementable in a practical sense. This is because: (i) difficulties implementing subsidies specifically targeted at sources of growth externalities; (ii) WTO regulations may preclude uses in tradeable sectors; and (iii) non-distortionary (lump-sum) taxes/transfers necessary to achieve the first best are unrealistic. As we do observe repression households access to international financial markets in fast growing countries like China, we aim to isolate how these policies affect labor supply and sectoral production (traded vs nontraded).

Related Literature The use of welfare improving capital controls to correct externalities is an area of active research.[6] Recent literature mainly focuses on prudential controls to regulate pecuniary externalities from over-borrowing.[7] Another application is regulating the interaction between private credit markets and sovereign debt markets.[8] We focus instead on growth externalities, similar to the ones studied in Korinek and Serven (2010) or Aizenman and Lee (2010). Our contribution is (i) an analytic and quantitative study of the effects of capital controls on labor supply and allocation across sectors; and (ii) we compute the entire

[4]By the representative agent result.

[5]Incidentally, Itskhoki and Moll (2014) show the externality created by credit market imperfections in their model are isomorphic to a learning-by-doing externality in labor akin to that which we study in this paper.

[6]See Farhi and Werning (2012) and Korinek (2011) for overviews of applications.

[7]Including Schmitt-Grohé and Uribe (2012), Benigno et al. (2013) and Bianchi (2011) in two-sector small open economies similar to our environment.

[8]Wright (2006), Jeske (2006), Dovis (2012)

time path of the optimal policy, in the constrained Ramsey sense, both on and off equilibrium path. Therefore, we assume no commitment to future policies on the government's behalf.

Our work belongs to the literature in which financial repression fosters growth. The mechanism typically analyzed is how financial repression increases savings rates providing higher capital investment to firms (Jappelli and Pagano (1994), Castro et al. (2004))[9]. In contrast, we consider that financial repression reduces current consumption of households and can increase labor supply to firms.

We integrate our work with the previous literature by incorporating the channel of substitution studied in Deaton and Laroque (1999). They show the laissez-faire allocation is inefficient if assets in one sector (manufacturing) contribute to growth while assets in other sectors (agriculture or construction) do not. Policies reallocating resources to sectors with the learning-by-doing externality can then improve welfare. We model this channel using a two sector model where the growth externality is higher in traded than non-traded sectors. We then decompose the effect of the borrowing constraint into an increase in overall labor supply and a reallocation of labor across sectors.

A difference between our work and Deaton and Laroque (1999) and Jappelli and Pagano (1994) is that we consider an open economy. This connects us to the literature on learning-by-doing and the current account. Korinek and Serven (2010) show real exchange rate (RER) undervaluation can correct a learning-by-doing externality. Aizenman and Lee (2010) show undervaluation of RER will improve welfare only if learning-by-doing occurs through increased employment (rather than higher capital stock). These papers also focus on the classical reallocation channel. We add to this literature by developing the labor supply channel and considering a non-monetary policy tool directed at households. This is a critical distinction in calculating welfare gains. Also, our policy tool targets households directly and is applicable to broader frameworks beyond representative household where exchange rate manipulation or capital controls are either ineffective or have perverse ancillary effects.

Several papers on current account surpluses in East Asia,[10] focus on credit market imperfections. Our distinction is we consider government imposed restrictions on household

[9] See Pagano (1993) for an early discussion of the two competing effects of financial repression.

[10] See Gourinchas and Jeanne (2013) for the empirical analysis.

borrowing. This is different from Buera and Shin (2009) or Song et al. (2011) in that we focus on households rather than firms. It is also different from Mendoza et al. (2009) or Carroll and Jeanne (2009) in that we consider government imposed restrictions, rather than exogenous "credit market imperfections" or "underdeveloped banking sectors".

The empirical literature provides evidence the learning-by-doing externality we consider may be significant.[11] These estimates depend on where one is looking for the externality. Estimates for industrial sectors and learning-by-exporting are generally significant(Harrison and Rodriguez-Clare (2009), Rodrik (2008)). Empirical studies of China, the country motivating our work, also find strong evidence for these externalities, (Jarreau and Poncet (2009) and Du et al. (2012)). The inconclusiveness of this literature precisely motivates our theoretical study to provide additional testable implications to guide empirical efforts.

2 Theory

We break the model apart to clarify the economics behind each mechanism. We use a one-sector model with elastic labor supply to isolate how borrowing constraints increase labor supply. We then use a two-sector model with inelastic labor supply to isolate the reallocation effect borrowing constraints have in moving labor from non-tradeables to tradeables.

2.1 Borrowing Constraints: The Labor Supply Channel (One-Sector)

We first present a two-period example. A stand-in household has preferences over consumption and supply of labor of the following form:

$$U(C_1, C_2, L_1, L_2) = \log(C_1) + \log(C_2) - \psi L_1 - \psi L_2 \tag{HH}$$

[11] Giles and Williams (2001a) and Giles and Williams (2001b) provide a literature review.

The resource constraints in the two periods are:

$$C_1 + B_2 = F(L_1) + B_1 \tag{2.1}$$

$$C_2 = A_2 F(L_2) + B_2 \tag{2.2}$$

and we assume production function of the form: $F(L) = L^{\alpha}$. C_t and L_t denote consumption and labor supply in period $t = 1, 2$, B_2 is asset holdings at the beginning of period 2 (i.e. debt issued in the first period $= -B_2$), and B_1 is initial asset holdings (taken as given at the beginning of period 1). The world gross interest rate rate is exogenous and equals 1. We assume $\alpha < 1$. Productivity in the second period (A_2) depends on the aggregate labor supplied in the first period:

$$A_2 = L_1^{\phi}, \tag{2.3}$$

where $\phi > 0$ is the elasticity of future productivity with respect to current labor supply. The household does not internalize the effect of L_1 on A_2.

2.1.1 Laissez-faire Equilibrium

In a laissez-faire equilibrium, the stand-in household maximizes (HH), subject to the following budget constraints:

$$C_1 + B_2 = w_1 \cdot L_1 + \pi_1 + B_1 \tag{2.4}$$

$$C_2 = w_2 \cdot L_2 + \pi_2 + B_2 \tag{2.5}$$

where w_t and π_t are wages earned by the household and profits made by competitive firms at $t = 1, 2$. The profit maximization problem of a representative firm in each period is:

$$\max_{L_1} L_1^{\alpha} - w_1 L_1$$
$$\max_{L_2} A_2 L_2^{\alpha} - w_2 L_2$$

The laissez-faire equilibrium allocation is characterized by feasibility (2.1)-(2.3) and three

Euler conditions derived from households' and firms' optimization:

$$\psi = \frac{1}{C_1} \cdot \alpha L_1^{\alpha-1} \tag{2.6}$$

$$\psi = \frac{1}{C_2} \cdot A_2 \alpha L_2^{\alpha-1} \tag{2.7}$$

$$\frac{C_2}{C_1} = 1 \tag{2.8}$$

Inefficiency of the laissez-faire equilibrium allocation Because the effect of labor supply on future productivity is not internalized (A_2 is taken as given), the allocation in the laissez-faire equilibrium is not optimal. To see this consider the following function that defines the household's welfare from supplying L_1 amount of labor in period one and choosing B_2 of assets for the next period:

$$W(L_1, B_2) = \log(L_1^\alpha + B_1 - B_2) - \psi L_1 + V(L_1^\phi, B_2)$$

where

$$V(A, B_2) := \max_{L_2} \log(A L_2^\alpha + B_2) - \psi L_2 \tag{2.9}$$

is the continuation value if productivity at $t = 2$ is A and asset holdings are B_2. Let L_1^{LF}, B_2^{LF} be the labor supply at $t = 1$ and asset holdings in the laissez-faire equilibrium. Starting from a laissez-faire equilibrium allocation, a marginal increase in labor supply in the first period will improve welfare:

$$\left.\frac{\partial W}{\partial L_1}\right|_{(L_1,B_2)=(L_1^{LF},B_2^{LF})} = \underbrace{\frac{\alpha {L_1^{LF}}^{\alpha-1}}{{L_1^{LF}}^{\alpha} + B_1^{LF} - B_2^{LF}} - \psi}_{\text{short term loss} = 0} + \underbrace{\phi \cdot \left(L_1^{LF}\right)^{\phi-1} V_A\left(\left(L_1^{LF}\right)^{\phi}, B_2^{LF}\right)}_{\text{dynamic gain} > 0} > 0$$

The first term on the RHS is zero, because the laissez-faire allocation satisfies (2.6). The second term is positive, because Envelope Theorem implies V is differentiable with respect to A and $V_A > 0$. This same logic will hold in the proof for the many period model. A marginal increase in labor supply from the laissez faire has zero short-term cost but a positive dynamic gain through the externality.

2.1.2 First-Best Planner's Allocation

The First-Best Planner directly chooses an allocation, internalizing the effect of labor supply in the first period on the productivity in the second period. Thus, the First-Best Planner's allocation is defined as follows.

Definition 2.1 (**First-Best Planner's Allocation**). The First Best Planner's allocation is $(C_1^{FB}, C_2^{FB}, L_1^{FB}, L_2^{FB}, B_2^{FB})$ that maximizes household's utility (HH) and (ii) is feasible, i.e. it satisfies (2.1)-(2.3).

First-best Policy Intervention The First-Best Planner's allocation can be implemented with (i) a labor subsidy in period 1, combined with (ii) a capital income tax in period 2, and a lump sam tax/subsidy in each period. With these tools, the household's budget constraints would become:

$$
\begin{aligned}
C_1 + B_2 &= w_1(1+\tau_\ell)L_1 + B_1 - T_1 + \pi_1 \\
C_2 &= w_2 L_2 + B_2 \cdot (1-\tau_B) + T_2 + \pi_2
\end{aligned}
$$

where π_1, π_2 are firms' profits in each period, w_1, w_2 are wages, τ_ℓ is the labor subsidy, τ_B is the capital income tax, $T_1, T_2 > 0$ are the lump tax / subsidies given by $T_1 = L_1 w_1 \tau_\ell$, and $T_2 = B_2 \cdot \tau_B$. Using the consumption-leisure trade-off from the first period, and the inter-temporal Euler condition, we can pin down $(\tau_\ell, \tau_B, T_1, T_2)$ that implements the First-Best Planner's allocation. In a general case, the policy takes the following form:

$$
\begin{aligned}
\tau_\ell &= -\frac{U_{L_1}(C_1^{FB}, C_2^{FB}, L_1^{FB}, L_2^{FB})}{U_{C_1}(C_1^{FB}, C_2^{FB}, L_1^{FB}, L_2^{FB})} \cdot \frac{1}{F'(L_1^{FB})} - 1 \\
\tau_B &= 1 - \frac{U_{C_1}(C_1^{FB}, C_2^{FB}, L_1^{FB}, L_2^{FB})}{U_{C_2}(C_1^{FB}, C_2^{FB}, L_1^{FB}, L_2^{FB})} \\
T_1 &= F'(L_1^{FB}) \cdot L_1^{FB} \cdot \tau_\ell \\
T_2 &= B_2^{FB} \cdot \tau_B
\end{aligned}
$$

With the functional forms considered in this section, the optimal policy takes a particularly simple form[12]:

$$\tau_\ell = \frac{\psi C_1^{FB}}{\alpha L_1^{FB^{\alpha-1}}} - 1$$
$$T_1 = \alpha (L_1^{FB})^\alpha \tau_\ell$$

2.1.3 Constrained Ramsey Equilibrium

We now consider a problem of a benevolent policy maker, who is limited to choosing a borrowing constraint taking the corresponding competitive equilibrium allocation as given. The borrowing constraint ($\underline{b}$) limits household debt in the first period[13]:

$$B_2 \geq \underline{b} \tag{2.10}$$

We define a borrowing-constrained allocation as follows.

Definition 2.2. A borrowing-constrained allocation is $(C_t(\underline{b}), L_t(\underline{b}))_{t=1,2}$ that maximizes (HH) subject to the resource constraints (2.1)-(2.2), the intra-temporal Euler Equation (2.6), and the borrowing constraint (2.10).

In words, a borrowing-constrained allocation is the allocation that arises in a competitive equilibrium, in which household faces the borrowing constraint (2.10).

Formally, the policy maker solves a constrained Ramsey problem of the following form:

$$\max_{\underline{b}} \log(C_1(\underline{b})) + \log(C_2(\underline{b})) - \psi L_1((\underline{b})) - \psi L_2(\underline{b})$$

where $(C_t(\underline{b}), L_t(\underline{b}))_{t=1,2}$ is the borrowing-constrained allocation defined above. For a given size of the LBD externality, the value of the borrowing-constrained allocation is given by:

$$W(\underline{b}) = \max_{C_1, L_1, B_2} \log(C_1) - \psi L_1 + V\left(L_1^\phi, B_2\right) \tag{2.11}$$

[12]With the separable log utility of consumption, the optimal capital income tax is $\tau_B = 0$

[13]Note that the planner may choose a positive value of $\underline{b}$, in which case should be interpreted as forced savings instead of a borrowing-constraint.

subject to the resource constraint (2.1), the borrowing constraint (2.10) and household's endogenous reaction to the policy[14]. This is summarized by the Euler Equation (2.6), referred to in this context as the implementability condition.

Definition 2.3 (Constrained Ramsey Equilibrium Allocation). The Constrained Ramsey Equilibrium Allocation is $(C_t(\underline{b}^*), L_t(\underline{b}^*))_{t=1,2}$ with the borrowing constraint set to $\underline{b}^* = \arg\max_{\underline{b}} W(\underline{b})$.

Proposition 2.1. ***The Constrained Ramsey problem is solved by an unique optimal borrowing constraint that binds and improves welfare.***
Let B_2^{LF} be the asset holdings chosen in the laissez-faire allocation. There exists a unique $\underline{b}^* > B_2^{LF}$ such that $\underline{b}^* = \arg\max_{\underline{b}} W(\underline{b})$, with W defined in (2.11).

Proof. See Appendix B. □

In any equilibrium the intra-temporal Euler equation must hold. If leisure is a normal good, then tighter borrowing constraint will raise labor supply:

$$\frac{\alpha L_1^{\alpha-1}}{L_1^\alpha + B_1 - \underline{b}} - \psi = 0 \Rightarrow \frac{dL_1}{d\underline{b}} > 0$$

As a result, productivity and the continuation value in the second period increase. When $\underline{b} = B_2^{LF}$, a marginal increase in $\underline{b}$ will have a first-order effect only on the continuation value. The proof of uniqueness exploits the non-monotone effect of the optimal constraint on welfare. Very tight constraint (high $\underline{b}$) will drive consumption down too much[15]. This is illustrated in Figure 1.

2.1.4 "Usefulness" of optimal borrowing constraints

We now turn to our next question: how useful are borrowing constraints relative to other policy tools in correcting this learning-by-doing externality? We address the qualitative

[14] Households take the policy as given and do not internalize the effect of their behavior on the choice of the policy maker

[15] Formally, existence of an optimal constraint follows from the fact we are choosing $\underline{b}$ from $[B_2^{LF}, \infty)$ and that $\lim_{\underline{b}\to\infty} \frac{\partial W}{\partial \underline{b}} = -\infty$, which implies we can consider a truncation of $[B_2^{LF}, \infty)$ and have W (a continuous function) defined over a compact set.

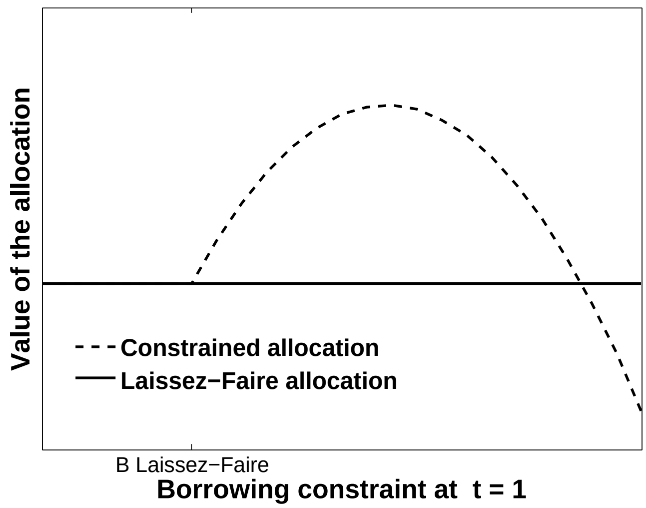

Figure 1: Welfare in a borrowing-constrained allocation

Parameters: $\alpha = \psi = 0.67; B_1 = 0$.

aspect of this question by comparing (i) the welfare gains from imposing optimal constraints to (ii) welfare gains from moving to the First-Best Planner's allocation. Specifically, we explore how the size of the externality affects the fraction of the First-Best Planner's welfare gains captured by a Ramsey Planner limited to using borrowing constraints as the only policy tool.

When the size of the externality is ϕ, the utility achieved in the First-Best Planner's allocation is:

$$V^{FB}(\phi) = \max_{L_1, B_2} \log\left(L_1^{\alpha} + B_1 - B_2\right) - \psi L_1 + V\left(L_1^{\phi}, B_2\right)$$

where V is the continuation value defined in (2.9). Notice that no additional constraints are imposed in the above maximization problem.

The value of the equilibrium allocation with optimal constraints is given by:

$$V^{RP}(\phi) = \max_{\underline{b}} W(\underline{b}; \phi)$$

where $W(\underline{b}; \phi)$ is defined in (2.11) on page 10. We measure welfare gains as the percent increase in annual consumption over the laissez-faire equilibrium allocation necessary to achieve the specified level of utility.

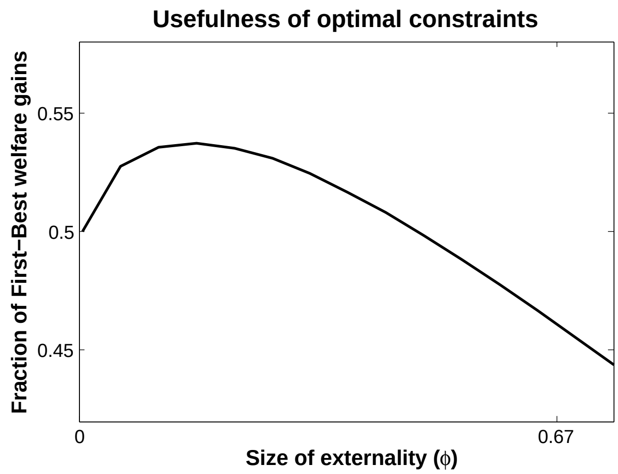

Figure 2: Size of externality and fraction of First-Best Planner's welfare gains
Parameters: $\alpha = \psi = 0.67; B_1 = 0$.

The fraction of the welfare gain in First-Best Planner's allocation (relative to laissez-faire) captured by the constrained Ramsey Planner is depicted in Figure 2 for different values of the externality. Two intuitive results emerge that hold in the many-period model: (1) the First-Best Planner's allocation always out-performs the constrained Ramsey allocation; and (2) the constrained Ramsey equilibrium achieves the largest fraction of the First-Best Planner's welfare gain if the LBD externality is neither too large, nor too small (Figure 2). The intuition for this is as follows. There is larger labor supply in both the First-Best Planner's allocation and in the constrained Ramsey allocation than in the laissez-faire allocation. This has two effects: first, current income rises and second, permanent income rises. The First-Best Planner's allocation takes advantage of both effects, but the optimal constraints only take advantage of the first. Initially (small LBD), an increase in the size of externality has a larger impact on current income relative to permanent income, so the constrained Ramsey planner captures a larger and larger fraction of the First-Best Planner's welfare gains. For large values of the externality, the permanent income effect starts to dominate. It then becomes optimal for the households to borrow to take advantage of the increase in permanent income. The optimal borrowing constraint, by construction, does not allow households to smooth consumption. An increasing fraction of the welfare gain cannot be captured as the effect of the externality on permanent income increases.

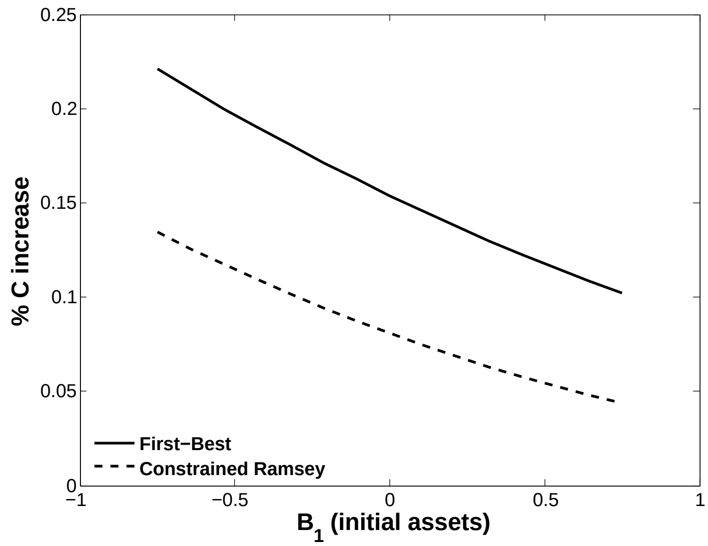

Figure 3: Size of initial assets and welfare gain
Parameters: $\alpha = \psi = 0.67; \phi = 0.05$.

The initial asset position of the economy also determines the "usefulness" of the borrowing constraint as a policy tool[16]. Welfare gains of both the First-Best Planner and the constrained Ramsey Planner allocations are decreasing in the size of initial asset holdings (Figure 3). This is intuitive, because the benefit of the LBD externality is an increase wealth. Welfare gains fall for wealthy agents if the marginal value of wealth is small relative to the cost of lost leisure. As the welfare gain from the First-Best Planner and the constrained Ramsey allocations each decrease, the constrained Ramsey allocation also captures less of the welfare gain of the First-Best Planner's allocation.

Implications for the Current Account The qualitative effect of the optimal borrowing constraint on the current account in the one-sector model is immediate. By construction it will reduce debt (or increase savings) and increase the current account. Comparison relative

[16]For positive initial asset position, it is also likely the optimal policy will proscribe a positive "borrowing constraint", ie: forced savings.

to the First-Best Planner's allocation is more nuanced. Figure 4 plots the level of first-period borrowing in each allocation against the size of the externality ϕ. The solid line corresponds to the First-Best Planner's allocation. The dot-dashed line corresponds to the Laissez-Faire allocation. The dashed line corresponds to the constrained Ramsey allocation.[17]

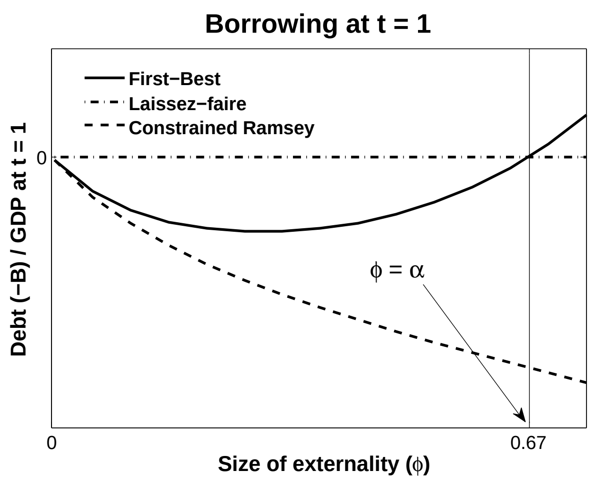

Figure 4: Size of externality and debt in the 2-period model
Parameters: $\alpha = \psi = 0.67; B_1 = 0$.

Figure 4 shows the debt in the First-Best Planner's allocation is smaller than in the laissez-fair allocation if and only if $\phi < \alpha$. This is driven by the same current income versus permanent income dynamic that drives the implications for welfare gains discussed in the prior section. When the learning-by-doing externality is small, increasing labor in the first period has a larger effect on current income than it has on permanent income. As a result, less borrowing, and perhaps even savings, is necessary for consumption smoothing. When the externality is large, increasing labor in the first period has larger effects on *permanent* income and more borrowing is necessary for consumption smoothing. The turning point is

[17] For the clarity of the exposition the figure is presented for a special case when $\psi = \alpha$, so that in the laissez-faire allocation $L_1^{LF} = 1$ and $Debt_1^{LF} = -B_2^{LF} = 0$, regardless of the size of the externality (the result, however, is general and extends beyond this special case).

at $\alpha = \phi$, where the elasticities of current output and future productivity with respect to current labor supply are equal and the two effects exactly offset each other.

2.2 Infinite Horizon Quantitative Model

We now present the infinite-horizon model used for quantitative analysis of the one-sector model. We continue to simplify the analysis using labor as the only factor of production, but our results carry to a model with capital as shown in Section A.1 of the Appendix.

Households A stand-in household has preferences over consumption of a single tradable good C and labor L and maximizes the lifetime utility given by:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{2.12}$$

In each period t it faces the following sequence of budget constraints and debt limits:

$$C_t + B_{t+1} \leq w_t L_t + R^* B_t + \pi_t \tag{2.13}$$

$$B_{t+1} \geq \underline{b}_t \tag{2.14}$$

In the constraints above w denotes wage, b are bond holdings and π are firms' profits. Our assumption of a small open economy implies R^* is exogenous (in our model it is constant: $R^* = 1/\beta$). We assume total labor is bounded $L \in [0, 1]$. We allow for the debt limit in (2.14) to be time varying, since we focus on the borrowing constraint as a policy tool that may vary over time.

Firms The single consumption good is produced by competitive firms. A representative firm chooses labor L to maximize profits (π) subject to production technology $F(\cdot)$:

$$\max_{L_t} \pi_t = A_t F(L_t) - w_t L_t$$

A_t is total factor productivity productivity, relative to the world frontier (which is normalized to 1). Productivity next period depends on learning-by-doing technology $H(\cdot)$:

$$A_{t+1} = \phi(A_t, L_t)$$

The effect of today's labor L_t on tomorrow's productivity A_{t+1} is not internalized by households. In addition to standard regularity assumptions on utility and production, we assume the following properties on the learning-by-doing technology:

Assumption 2.1. *$H(A_t, L_t) \in [A_t, 1]$ for all (A_t, L_t); and $H_L(A_t, L_t) > 0$ for every $A_t \in (0, 1)$ and every $L_t \in (0, 1)$.*

Definition 2.4 (Equilibrium)**.** Given initial productivity A_0, bond holdings B_0, and the sequence of debt limits $(\underline{b}_t)_{t=0}^{\infty}$, an equilibrium consists of sequences of allocations $(C_t^*, L_t^*, B_{t+1}^*)_{t=0}^{\infty}$, wages $(w_t^*)_{t=0}^{\infty}$, profits $(\pi_t^*)_{t=0}^{\infty}$ and the sequence of productivity $(A_{t+1}^*)_{t=0}^{\infty}$ such that, given the wages and profits, (i) allocations solve the household's and firms' maximization problems and (ii) markets clear.

As in the 2-period model, we focus on three allocations: (i) the Laissez-faire allocation, which we treat as a benchmark, (ii) the First-Best Planner's allocation, and (iii) the constrained Ramsey Planner equilibrium allocation, which arises in a borrowing-constrained equilibrium where a benevolent policy-maker chooses a sequence of borrowing constraints that maximizes household's welfare. We will start by defining and characterizing each of these allocations in this ∞-horizon model.

2.2.1 Laissez-faire Equilibrium

Definition 2.5. A laissez-faire equilibrium is an equilibrium such that (2.14) never binds (the Lagrange multiplier associated with (2.14) is 0 for every t).

Characterization The Laissez-faire Equilibrium Allocation solves the following dynamic program:

$$V^{LF}(A, B) = \max\{U(C, L) + \beta V^{LF}(A', B')\}$$

subject to:

$$A' = H(C, L) \tag{2.15}$$

$$C + B' = AF(L) + R^* B \tag{2.16}$$

$$-U_L(c, L) = U_C(C, L) \cdot AF'(L) \tag{2.17}$$

$$U_C(C, L) = \beta R^* U_C(C', L') \tag{2.18}$$

$$B' \geq \underline{B} \tag{2.19}$$

where $\underline{B}$ is the natural borrowing limit. The first two constraints ensure the allocation is feasible. The implementability conditions (2.17)-(2.18) ensure the household's first order conditions are satisfied. The last condition is the no-Ponzi constraint. As was the case in the two-period model, the laissez-faire allocation is inefficient, because the household does not internalize the effect of labor supply on future productivity.

2.2.2 First-Best Planner's Allocation

Definition 2.6 (First-Best Planner's Allocation)**.** The First-Best Planner's allocation is a sequence $(A_{t+1}, C_t, L_t, b_{t+1})_{t=0}^{\infty}$ that maximizes (3.1) subject to

$$A_{t+1} = H(A_t, L_t), \quad t = 0, 1, 2, ...$$
$$\sum_{t=0}^{\infty} \frac{C_t}{{R^*}^t} \leq \sum_{t=0}^{\infty} \frac{A_t F(L_t)}{{R^*}^t} + R^* B_0,$$

Characterization From the above definition it follows immediately the First-Best Planner's allocation solves the following program.

$$V^{FB}(A, B) = \max\{U(C, L) + \beta V^{FB}(A', B')\}$$

subject to (2.15), (2.16) and (2.19). The difference, compared to the Laissez-faire allocation, is that the First-Best Planner is not limited to choosing allocation that would satisfy the implementability conditions (2.17)-(2.18).

2.2.3 Constrained Ramsey Equilibrium

We again consider a constrained Ramsey Planner who is limited to choosing an optimal borrowing limit (or forced savings if positive) to improve upon the Laissez-Faire allocation. The policy maker realizes the resulting *competitive equilibrium* allocation must satisfy the intra-temporal Euler condition:

$$0 = U_L(C, L) + U_C(C, L) \cdot AF'(L)$$

The effect of a *binding* borrowing constraint on current labor supply can be seen by differentiating the above condition:

$$\frac{dL}{d\underline{b}} = \frac{AF'U_{C,C} + U_{C,L}}{U_{C,C}(AF')^2 + AF''U_C + AF'U_{L,L} + 2AF'U_{C,L}} > 0 \tag{2.20}$$

This condition implies that a binding borrowing constraint increases labor supply if leisure is a normal good. This effect is further strengthened if consumption and leisure are complements. In the infinite horizon model the optimal borrowing constraints are an entire optimal policy profile of constraints set in each period. They can be found by setting up a simple constrained Ramsey problem—finding a sequence of borrowing constraints $(\underline{b}_t)_{t=0}^{\infty}$, that yields maximum level of utility of the competitive equilibrium allocation given $(\underline{b}_t)_{t=0}^{\infty}$.

Definition 2.7. Let $W\left((\underline{b}_t)_{t=0}^{\infty}\right) := \sum_{t=0}^{\infty} \beta^t U\left(C_t\left((\underline{b}_t)_{t=0}^{\infty}\right), L_t\left((\underline{b}_t)_{t=0}^{\infty}\right)\right)$, where $\left(C_t\left((\underline{b}_t)_{t=0}^{\infty}\right)\right)$, $\left(L_t\left((\underline{b}_t)_{t=0}^{\infty}\right)\right)$ are competitive equilibrium allocations of consumption and labor, given the sequence of borrowing constraints $(\underline{b}_t)_{t=0}^{\infty}$. A constrained Ramsey problem is to find $(\underline{b}_t^*)_{t=0}^{\infty} \in \arg\max_{(\underline{b}_t)_{t=0}^{\infty}} W\left((\underline{b}_t)_{t=0}^{\infty}\right)$.

Characterization A time-consistent profile of optimal constraints can be characterized recursively as follows:

$$V^{RP}(A, B) = \max_{\underline{b}} \{U(C, L) + \beta V^{OC}(A', B')\} \tag{2.21}$$

s.t.

$$
\begin{aligned}
A' &= H(A, L) \\
C + B' &\leq AF(L) + R^* B \\
-U_L(C, L) &= U_C(C, L) \cdot AF'(L) \\
B' &\geq \underline{b}
\end{aligned}
$$

The dynamic program (2.21) yields policy functions for consumption, labor and the net foreign asset position: $C^{RP}(A, B), L^{RP}(A, B)$ and ${B'}^{RP}(A, B)$. These policy functions constitute equilibrium allocations in an economy in which a borrowing constraints are set to $B'(A, B)$. Notice these borrowing constraints take into account (i) the optimal policy that would be set in the state next period, i.e. $B''(A', B')$, as well as the optimal policy that would be set if the economy reaches a state off-the equilibrium path. In other words, under this characterization the optimal policy is time-consistent[18].

2.2.4 Quantitative Analysis

For our quantitative welfare analysis we consider a period utility function of the form: $U(C, L) = \log(C) + \log(1 - L)$. The strength of the effect of the borrowing constraint on labor depends on the cross-elasticity between consumption and leisure, here set at unity where the constraint does not bind[19]. Where the constraint does bind, the cross-elasticity between consumption and leisure will be higher.[20]

The production function is $F(L) = L^\alpha$. The learning-by-doing technology assumes con-

[18]This is distinct from the constrained Ramsey problem in sequence form which does not require time-consistency or specification of off-path policies.

[19]Castaneda et al. (2003)find the ratio cross-sectional coefficient of variation of consumption relative leisure in the US is 3, but Pijoan-Mas (2006) finds a coefficient between 1 and 1.5 is needed to match statistics for households the first quartile of income. Motivated by China, we choose the statistic relevant for low income households. In Section 4, we provide analysis and discussion for a range of values.

[20]Similarly, Domeij and Floden (2006) show a general downwards bias in intertemporal elasticity of leisure when agents are liquidity constrained and cannot smooth consumption.

stant productivity catch-up:

$$A' = \min\{1, A \cdot e^{\phi L}\} \tag{2.22}$$

where ϕ measures the effect of L on future productivity. We consider a range of values for the learning-by-doing parameter: $\phi \in [0, 0.20]$. We also consider how the results depend on the initial level of productivity (relative to the world frontier). The remaining parameters are set to standard values: $\beta = 0.96$, $\alpha = 0.67$, $R^* = \frac{1}{\beta}$. Initial assets are set to $B_0 = 0$.

The fraction of the welfare gain of the First-Best Planner's allocation (relative to Laissez-Faire) captured by the constrained Ramsey policy is increasing in the size of the externality (Figure 5). This is for the range we consider in our exercise. It declines in the distance from the frontier. For the economy whose productivity is initially at 50% of the world frontier, optimal constraints achieve as much as 25% - 35% of maximum possible welfare gains for empirically plausible values of the learning-by-doing parameter.

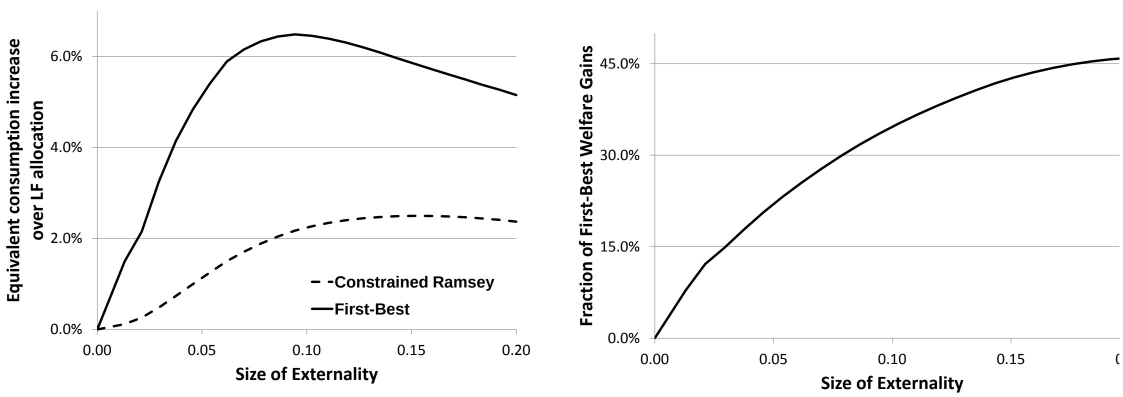

Figure 5: Welfare Gains - role of the size of externality

Initial TFP: $A_0 = 0.5$.

A larger externality or a larger distance from the frontier each increase welfare gains (Figure 6). For an economy whose productivity is initially at 50% of the world frontier, the gain from optimal constraints is equivalent to 0.5% - 2.5% increase in life-time consumption, depending on the size of the externality. For a given size of externality, the welfare gain is increasing in the distance from the frontier.

Figure 7 depicts the time paths of each allocation. The tradeoff between the long-term benefit and short-term costs of the constrained Ramsey policy relative to Laissez-faire are

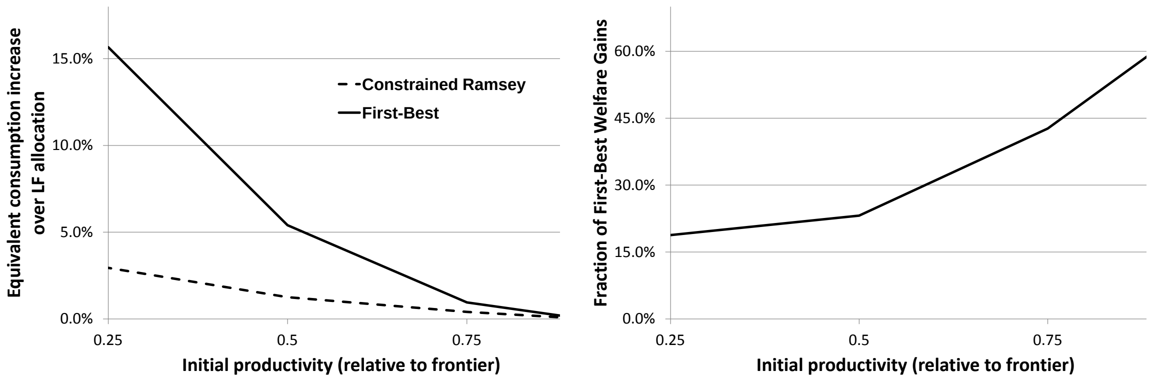

Figure 6: Welfare Gains - role of initial productivity

LBD parameter: $\phi = 0.05$.

apparent. In the short-term, the constrained Ramsey allocation has higher labor and lower consumption. However, productivity growth is accelerated such that in later periods the constrained Ramsey allocation enjoys more leisure and consumption than the Laissez-faire. Figure 7 also shows the optimal borrowing constraint has a lower welfare improvement than the First-Best Planner's solution both because: (i) agents cannot consumption smooth; but also (ii) the constraints increase labor supply less than the First-Best.

3 Borrowing Constraints: The Reallocation Channel (Two-Sector)

We analyze a two sector model with a tradeable and non-tradeable sector to highlight the ability of capital controls to *reallocate resources* to the tradeable sector. This is motivated by the empirical literature on "export-led-growth" or "learning by exporting" claiming the strongest growth externalities are in tradeable goods sectors.[21]

[21] Loecker (2010) provides an overview of modern techniques to identify learning by exporting. Ma and Zhang (2008) provide evidence this externality was especially strong in China.

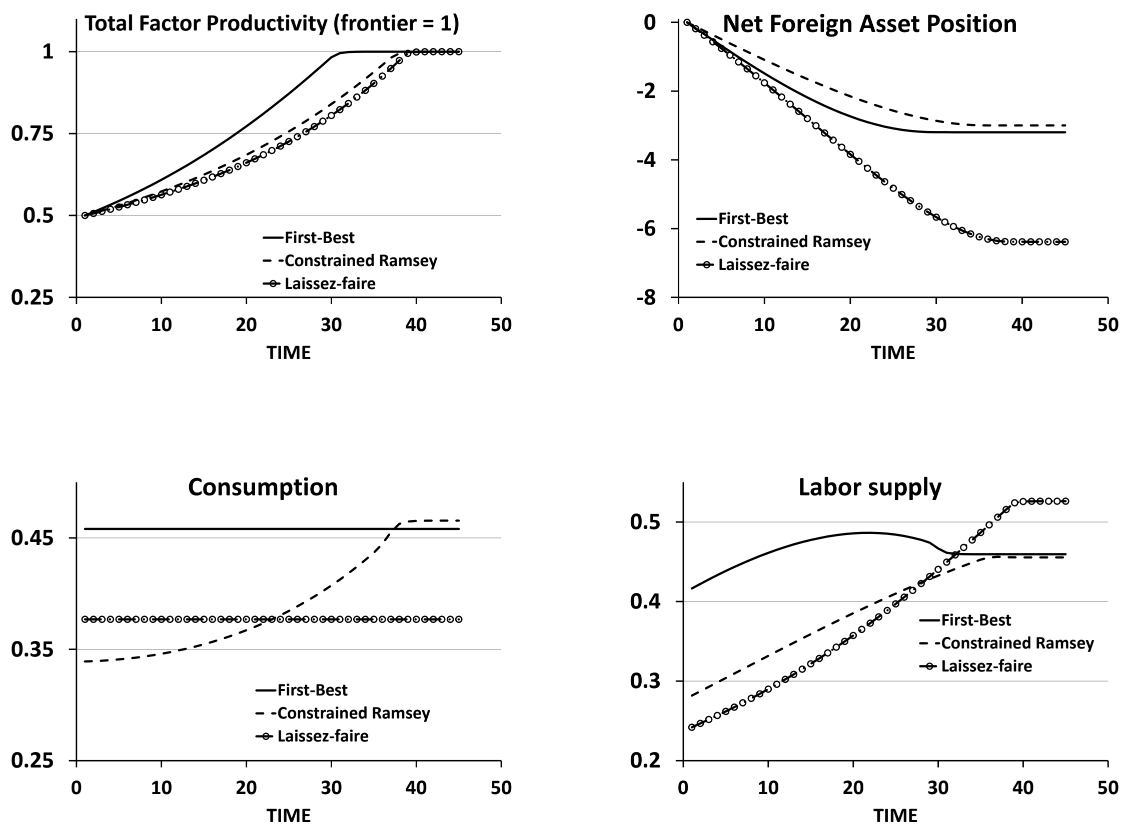

Figure 7: Time paths

LBD parameter: $\phi = 0.05$.

Environment A stand-in household has preferences over consumption of tradable good C^T and a non-tradable good C^N. To simplify the analysis and focus on the reallocation, we assume inelastic labor supply: $L = 1$.

$$\sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N) \tag{3.1}$$

In each period t it faces the following constraints:

$$C_t^T + p_t C_t^N + B_{t+1} \leq w_t + R^* B_t + \pi_t \tag{3.2}$$

$$B_{t+1} \geq \underline{b} \tag{3.3}$$

The first constraint is the budget constraint, the second is the no-Ponzi condition. In the constraints above, w denotes wage, p denotes relative price of new construction goods, R^* is the world (fixed) interest rate, B are bond holdings and π are firms' profits that are rebated in a lump-sum fashion to households that own the firms.

Both goods are produced by competitive firms. The production functions are as follows:

$$
\begin{aligned}
Y_t^T &= A_t F(L_t^T) \\
Y_t^N &= G(L_t^N),
\end{aligned}
$$

where A_t is the productivity in the manufacturing (tradable) sector, relative to the world frontier (which is normalized to 1). We assume initial $A_0 < 1$. As long as the country is below the frontier, there is scope for productivity catch-up. The catch-up arises endogenously through learning-by-doing in the tradable sector:

$$
A_{t+1} = H(A_t, L_t^T)
$$

The remaining market clearing conditions are as follows:

$$
\begin{aligned}
L_t^T + L_t^N &= 1 \\
C_t^T + B_{t+1} &= Y_t^T + R^* B_t
\end{aligned}
$$

The first condition equates the total labor supplied inelastically by the households with total labor demanded by firms. The second condition is the resource constraint in the tradable sector.

Definition 3.1. Given initial productivity A_0 and bond holdings B_0, an equilibrium consists of sequences of allocations $(C_t^T, C_t^N, L_t^T, L_t^N, L_t, B_{t+1})_{t=0}^{\infty}$, prices $(w_t, p_t)_{t=0}^{\infty}$ and sequence of productivity $(A_{t+1})_{t=0}^{\infty}$ such that, given the prices, (i) allocations solve the household's and firms' maximization problems and (ii) markets clear.

Instead of the of labor versus leisure in the 1-sector model, the key tradeoff in the 2-sector model is tradeable consumption versus non-tradeable. The Euler equating the relative price of non-tradeable goods with marginal rate of substitution between tradables and non-tradables is:

$$0 = U_{C^T}(C^T, C^N)p - U_{C^N}(C^T, C^N) \tag{3.4}$$

where, in equilibrium,

$$\begin{aligned} C^T &= AF(L^T) + R^*B - B', \qquad B' \geq \underline{b} \\ C^N &= G(1 - L^T) \\ p &= \frac{AF'(L^T)}{G'(1 - L^T)} \end{aligned}$$

For a general borrowing constraint (capital controls) to improve welfare, it must reallocate resources to the tradeable sector where the learning-by-doing externality occurs. Our main result is that this requires only mild assumptions: (1) households' indifference curves in the (C^N, C^T) space are convex and (2) production function be strictly concave. Applying the implicit function theorem when the borrowing constraint binds, i.e. $B' = \underline{b}$, and basic algebra to the key equation (3.4) yields:

$$\frac{\partial L^T}{\partial \underline{b}} = -\frac{A_{\underline{b}}}{A_{L^T}} > 0 \qquad \text{and} \qquad \frac{\partial L^N}{\partial \underline{b}} < 0. \tag{3.5}$$

where[22]

$$\begin{aligned} A_{\underline{b}} &= U_{C^T C^T} \cdot AF' - U_{C^T C^N} G' < 0 \\ A_{L^T} &= G' \cdot (U_{C^T C^N} AF' - U_{C^N C^N} G') - U_{C^N} G'' - [U_{C^T} AF'' + U_{C^T C^T} AF' - U_{C^T C^N} G'] > 0 \end{aligned}$$

The first equation of (3.5) shows it is assured that the increase of labor in the tradeable sector when the borrowing constraint binds if tradeables and non-tradeables are complements (in the sense that the cross-derivative is positive). Since the borrowing constraint forces lower consumption of tradeables, this complementarity induces the choice of lower consumption of non-tradeables as well. As assumed for illustrative purposes of this reallocation, labor supply is inelastic and excess labor beyond what is produced for domestic consumption must go to

[22] The signs follow from the fact that U is strictly concave in both arguments and $U_{c^T, c^N} \geq 0$ (convex indifference curves).

production of tradeable goods. An immediate result of (3.5) is the borrowing constraint generates undervalued real exchange rate (i.e. a fall in relative price of non-tradables). From the equilibrium condition equating the value of marginal product of labor in the two sectors, we have $p = \frac{AF'(L^T)}{G'(1-L^T)}$. Hence, we have:

$$\frac{\partial p}{\partial \underline{b}} = \frac{\partial p}{\partial L^T}\frac{\partial L^T}{\partial \underline{b}} < 0$$

because $\frac{\partial p}{\partial L^T} < 0$. In short, a tighter constraint reduces supply of the tradable good, which raises its relative price and induces reallocation of labor into that sector. The welfare result is now straight-forward.

Proposition 3.1. ***There always exists a binding borrowing constraint that improves welfare***

Proof. See Appendix B. □

3.1 Quantitative Analysis

We consider the following functional forms. Utility: $U(C^T, C^N) = \log((C^T)^\eta (C^N)^{1-\eta})$. Production: $y^T = A(L^T)^\alpha$ and $y^N = (L^N)^\alpha$, with $L^T + L^N = 1$. Learning by doing technology: $A' = \min\{1, A \cdot e^{\phi L^T}\}$. The parametrization is identical to the one-sector analysis, with the addition of preference parameter $\eta = 0.5$. As in the one-sector model, the welfare gain from the optimal borrowing constraint is increasing in the size of the externality, but is always less than the welfare gain from moving to the Planner's allocation. The magnitudes are also similar. Figure 8 shows how the welfare gains depend on the size of the externality ϕ.

4 The Chinese Experience

In this section, we perform a positive quantitative analysis of China's growth experience since opening to trade in 1991. We use our model to infer a sequence of borrowing constraints consistent with Chinese economic growth and to provide a counterfactual experience under no borrowing constraints (laissez-faire). This allows us to derive implications for welfare and

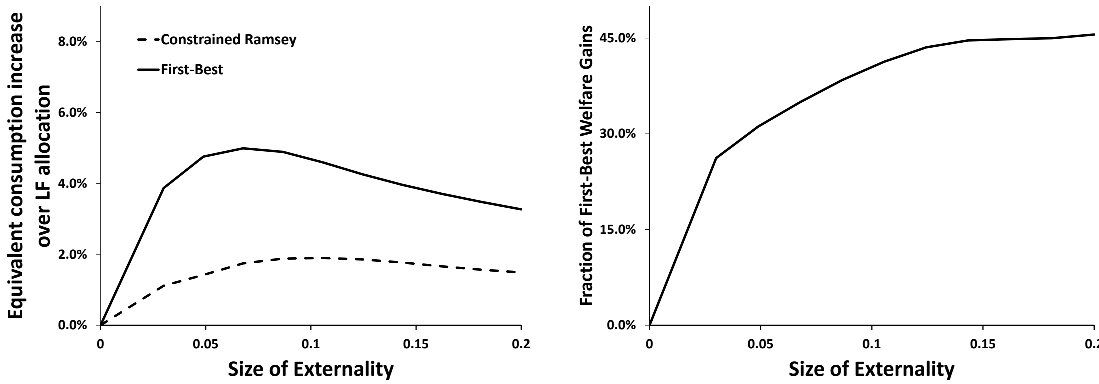

Figure 8: Welfare gains in the 2-sector model - inelastic labor

generate non-targeted statistics under each scenario. We compare these statistics to the data as evidence supporting the channels operational in our theory.

We choose to analyze the Chinese economy for three reasons. First, the Chinese economy is large and has experienced one of the fastest, persistent growth episodes in recent time. Second, there is evidence aggressive restrictions on borrowing have been placed on households by the government (or state run banks)[23]. Third, Chinese growth has been distinguished by three anomalies that are consistent with our theory: (i) depressed real wage growth, (ii) undervalued real exchange rate, and (iii) low debt accumulation.

4.1 Quantitative Model

In our quantitative exercise, we consider a 2-sector model with elastic labor.

Preferences The period utility function for the household is assumed to take the following form:

$$U(C^T, C^N, L) = \frac{G(C^T, C^N)^{1-\sigma}}{1-\sigma} + \psi\frac{(1-L)^{1-\nu}}{1-\nu}$$

[23]Interest rate ceilings are well documented. Further, prudential funds mandate savings of 11-30% of income for some workers, varying by province and time period Sin (2005)

where

$$G(C^T, C^N) = \left[\omega_T {C^T}^{\frac{\eta-1}{\eta}} + (1-\omega_T){C^N}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

Under these preferences, the intertemporal elasticity of consumption and the cross-elasticity between consumption and leisure, two potentially important margins for our analysis, can be separately analyzed.

The objective is to maximize total life-time utility of households given by:

$$\sum_t \beta^t U(C_t^T, C_t^N, L_t).$$

Household's constraints Households' constraints are standard:

$$C_t^T + p_t C_t^N + B_{t+1} \leq w_t L_t + \pi_t + R^* B_t \tag{4.1}$$

$$B_{t+1} \geq \underline{b}_t \tag{4.2}$$

The last constraint is the borrowing constraint. For this exercise it is calibrated exogenously and not the result of an optimal policy problem.

Technology Technological and resource constraints are identical to the ones in Section 3. We consider the following functional forms for production functions and law of motion of productivity:

$$\begin{aligned}
Y_t^T &= A_t^T {L_t^T}^{\alpha_T} && \text{tradable good} \\
Y_t^N &= A_t^N {L_t^N}^{\alpha_N} && \text{non-tradable good} \\
A_{t+1}^T &= \min\{1, A_t^T \cdot e^{\gamma+\phi\cdot L_t^T}\} && \text{law of motion (learning by doing)} \\
A_{t+1}^N &= \min\{1, A_t^N \cdot e^{\gamma+\phi\cdot L_t^T}\} && \text{law of motion (learning by doing)}
\end{aligned}$$

where γ is the exogenous productivity catchup and ϕ is the elasticity of future productivity with respect to current employment in the tradable sector (the productivity at the world frontier is normalized to 1). We assume that the learning occurs only in the tradable sector, but its effect is spilled over to the non-tradable sector[24]. Such spillover across sectors increases the potential welfare gains from the borrowing constraints.

Resource constraints and market clearing Total labor supply is divided into employment in tradeable and non-tradeable sectors: $L_t = L_t^T + L_t^N$. The output of the non-tradeable sector must be consumed within the same period: $C_t^N = Y_t^N$. The resource constraint in the tradable sector is: $C_t^T + NX_t = Y_t^T$. Combining these two conditions with the household's budget constraint, yields a relationship between trade balance and the household's borrowing/savings decisions:

$$NX_t = B_{t+1} - R^* B_t$$

4.2 Parameters

The model is yearly and we choose a discount factor of $\beta = 0.96$. Preference parameters are $\nu = 1.69$ and $\sigma = 1.5$. These provide an inter-temporal elasticity of substitution of consumption $1/\sigma = 0.67$ which is standard in macroeconomic models. The inter-temporal elasticity of substitution of leisure $1/\nu = 0.59$, may be larger than often used as motivated by Domeij and Floden (2006) finding for larger elasticities when controlling for borrowing constraints as are present in our model.

The elasticity of substitution between traded and non-traded goods is set to a fairly standard value of $\eta = 0.5$, implying the two are complements[25]. Finally, labor shares in tradeables and non-tradeables are set to $\alpha^T = 0.6$, and $\alpha^N = 0.4$, respectively, following the multi-country analysis of Echevarria (1997).

Table 2 presents parameters that have been calibrated. The consumption share parameter ω_T is calibrated using US data. We assume the United States is at the world frontier

[24] The extent and nature of learning by doing is contentious. Still, several empirical studies support learning in the traded sector, specifically manufacturing (recently, De Loecker (2007)). The mode of transmission to the non-tradeable sector can come through development of intermediate goods technology used in each sector or human capital embodied in workers (Dasgupta (2012)).

[25] See e.g. Kehoe and Ruhl (2009).

Table 1: Imposed parameter values

Discount factor	β	0.96
Elast. of subst. C^T vs. C^N	η	0.5
Inter-temp. elast. of subst.	$1/\sigma$	0.67
Inter-temp elast. of leisure	$1/\nu$	0.59
Labor shares	α_T	0.6
	α_N	0.4

$(A^T = A^N = 1)$ and has a steady-state net foreign asset position of 0. Then, ω_T is calibrated so that, in the steady-state, the share of tradable output in GDP equals the average share of manufacturing output in GDP in the United States in years 1980 - 1991, which is 0.17. This yields $\omega_T = 0.077$. While this parameter seems small, it implies a reasonable non-tradeable share of GDP of 70% in the calibrated model.

Table 2: Calibrated parameter values

Parameter		**Value**
Consumption shares	ω_T	0.077
	ω_N	$1-\omega_T$
Initial TFP	A_0^T	0.034
	A_0^N	0.132

Having calibrated ω_T, using the US data, we jointly calibrate A_0^T and A_0^N using Chinese data. We match two moments: (i) the ratio of Chinese to American GDP per capita in 1991 (assuming $A_0^T = A_0^N = 1$ in the United States), which equals 0.03; and (ii) the share of tradable output in Chinese GDP in 1991, which equals 0.31. The calibrated values for A_0^T and A_0^N are 0.034 and 0.132, respectively.

For our quantitative analysis, we must choose a value for the elasticity of "learning-by-doing" and a sequence of borrowing constraints. Both are difficult to quantify directly from empirical observations. Instead, we consider different values for the learning-by-doing elasticity ϕ ranging between 0 and 0.08. For a given ϕ we jointly calibrate the exogenous growth parameter γ, and the sequence of constraints $(\underline{b}_t)$. The exogenous growth parameter is calibrated to match the difference between average growth of Chinese and US real GDP per capita (0.0752) between 1991 and 2007; the sequence of constraints $(\underline{b}_t)$ is calibrated period-by-period, to match the ratio of trade balance to GDP.

4.3 Results

For our baseline, we consider an elasticity of 5%, within the typical estimates found in the literature[26]. We find that the calibrated sequence of borrowing constraints imply a welfare loss equal to a 13% fall in annual consumption relative to laissez-faire. We conclude that Chinese policy constrained households' borrowing beyond what could be justified by this LBD externality. However, we do find that the presence of the LBD externality reduces welfare losses by 41% compared to a 22% fall in annual cosumption relative to laissez-faire when all growth is exogenous.[27]

Further, the presence of the LBD externality and operation of ancillary channels in our model bring the frictionless baseline closer to two unique (un-targeted) aspects of the Chinese growth experience: (i) depressed real wage growth and (ii) slow real exchange rate appreciation. Table 3 summarizes these results. Displayed are relevant statistics from the data, the model with the calibrated constraints, and the Laissez-faire equilibrium with no binding borrowing constraint.

Our model is successful in accounting for slow real exchange rate (RER) appreciation, although we overshoot slightly. In the model, the RER is simply the price of the aggregate consumption good, which is computed as the solution to the following problem:

$$RER_t = \min_{C_t^T, C_t^N} C_t^T + p_t \cdot C_t^N$$

s.t.

$$\left[\omega_T {C^T}^{\frac{\eta-1}{\eta}} + (1-\omega_T) {C^N}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} = 1$$

[26] Fernandes and Isgut (2005) suggest an estimate of 5%. Badinger and Egger (2008) suggest provide estimates that range between 3% and 8%.

[27] For all changes in parameter values, including this comparative static with respect to LBD, the exogenous catch up parameter is recalibrated to match the observed difference in Chinese vs US real GDP/capita growth.

Table 3: Contribution to Chinese puzzles; LBD elasticity = 0.05

Moment	**Data**	**Laissez-faire**	**Constraints**	**% gap closed**
$\Delta W - \Delta GDP$	-0.04	0.09	0.00	66.9%
RER appreciation	0.01	0.09	0.00	118.4%

Empirical estimate of $\Delta W - \Delta GDP$ from Song et al. (2011). RER appreciation is average annual appreciation of real effective exchange rate over the period 1991-2007, using the World Bank data.

Thus, the RER movements are proportional to the movements in the price of non-tradable good p. In a laissez-faire allocation, the average annual appreciation of the RER is about 9%, which is many orders of magnitude larger than 1% reported by the World Bank. In the allocation with calibrated borrowing constraints, the average RER appreciation is 0%. This is because the reallocation of labor towards the tradable sector lowers the marginal product of labor in that sector and increases the marginal product of labor in the non-tradable sector (all relative to laissez-faire). Thus, the price of non-tradables has to remain low for the wages in both sectors to be equal.

We also consider the effect of the borrowing constraints on labor market outcomes. Song et al. (2011) report the Chinese experience in the 1990s and early 2000s has been accompanied by the real wage growth substantially slower than the growth of the real GDP per capita. In our model, borrowing constraints raise labor supply which reduces marginal product of labor. As a result, real wage is growing slower than GDP. In a laissez-faire allocation, real wage grows on average 9.7% points faster than real GDP. In the allocation with calibrated borrowing constraints, that difference reduces to .06% points, which is about 66.9% closer to the empirical difference of -4% points.

We decompose the increase of employment in the tradeable sector into two channels: (i) the classic channel of reallocating the labor away from the non-tradable sector, and (ii) the new channel of elastic labor supply. We calculate the % contribution of elastic labor as $\frac{\Delta L}{\Delta L^T}$, and the % contribution of the reallocation channel as $-\frac{\Delta L^N}{\Delta L^T}$ (note that $\Delta L^N < 0$). We find that, while the constraints are in place, on average 25% of the increase in labor in the tradeable sector, relative to Laissez-Faire, comes from the increase in over-all labor supply (time paths are shown in figure 9). The quantitative importance of this channel is highly sensitive to preference parameters discussed in the next section.[28]

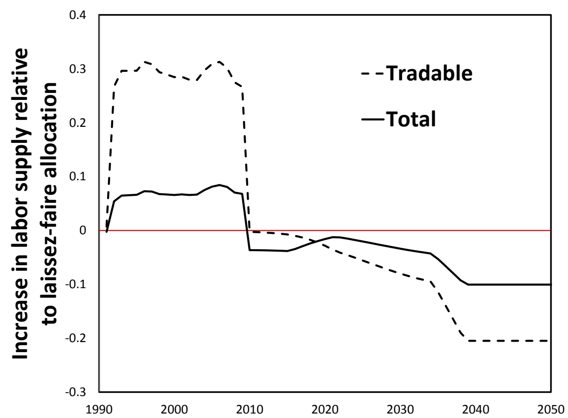

Figure 9: Change in sectorial labor at observed constraints versus Laissez-Faire

4.3.1 Sensitivity Analysis: Preference Parameters

All of the results discussed for the baseline calibration are highly sensitive to choices of the three preference parameters: the elasticity of substitution between tradeable and non-tradeable goods; the intertemporal elasticity of substitution of consumption; and the intertemporal elasticity of substitution of leisure.[29] Table 4 displays welfare and TFP growth comparative statics, changing one parameter only and recalibrating the model to the same targets as the baseline. Figure 10 displays the welfare comparisons for a range of LBD parameters and selected preference parameters.

The first column of Table 4 considers alternative elasticities of substitution for tradeable and non tradeable goods (η). The higher the elasticity of substitution, the lower the welfare

[28]It should also be recalled these statistics are for the inferred borrowing constraints, not the constrained Ramsey Planner for which this magnitude may be different.

[29]We omit analysis of the cross-elasticity of consumption and leisure, opting instead to isolate the intertemporal elasticities of consumption and leisure independently by use of separable utility.

Table 4: The role of preference parameters in welfare and growth effects

		EoS C^T v C^N (η)		IES of L ($1/\nu$)		IES of C ($1/\sigma$)	
	Benchmark	$\eta = 0.05$	$= 0.25$	$1/\nu = 1$	$= 6$	$1/\sigma = 1$	$= 6$
Welfare	-0.131	0.034	-0.039	-0.110	-0.068	-0.078	-0.002
TFP Growth	0.018	0.020	0.018	0.021	0.025	0.014	0.006

LBD parameter is set to $\phi = 0.05$. In each case, the model has been re-calibrated following the procedure described in Section 4.2.
Benchmark values: $\eta = 0.5; 1/\nu = 0.59; 1/\sigma = 0.67$.

gains (higher the welfare losses). This result depends on two competing effects. The borrowing constraint disproportionably lowers consumption of the tradeable good since the price is fixed at the international price. When the elasticity of substitution is high, households substitute towards the cheaper non-tradeable good. This improves welfare by reducing the distortion in households' consumption of tradeable vs non-tradeable good, but at the cost of lower TFP growth as less labor is supplied to the traded good sector through the reallocation channel. For the range of parameter values considered, the latter effect dominates the former and welfare gain is reduced as the elasticity of substitution increases. This also highlights the importance of this parameter in determining the contribution of the reallocation channel in increasing labor supplied to the traded sector. As the elasticity increases, the reallocation channel becomes less important. Indeed, when traded and non-traded goods are perfect substitutes, the reallocation channel shuts down completely.

Increasing the intertemporal elasticity of substitution (IES) for leisure ($1/\nu$) unambiguously increases welfare gains. As this elasticity increases, households are more willing to substitute leisure across time. This implies both that households face less disutility as the borrowing constraint distorts leisure smoothing and that TFP grows faster as the current labor supply increases more. These two ways in which this parameter affects welfare are similar to the elasticity of substitution between tradeable and non-tradeable goods, but in this case they work in the same direction. Increasing the IES for leisure also strengthens the labor supply channel.

Increasing the intertemporal elasticity of substitution (IES) for consumption ($1/\sigma$) also increases welfare gains, but at the cost of lower increase in TFP growth. This highlights the key tradeoff in using borrowing constraints to correct this LBD externality. When the IES of consumption is high, households suffer less disutility as the borrowing constraint distorts consumption smoothing. On the other hand, when the IES is high, households have less incentive to work harder to smooth consumption and the increase in TFP growth is lowered as the labor supply channel increasing labor in the traded sector is reduced. For the values we consider, the former effect outweighs the latter. It should also be noted, higher IES of consumption reduces the strength of the labor supply channel.

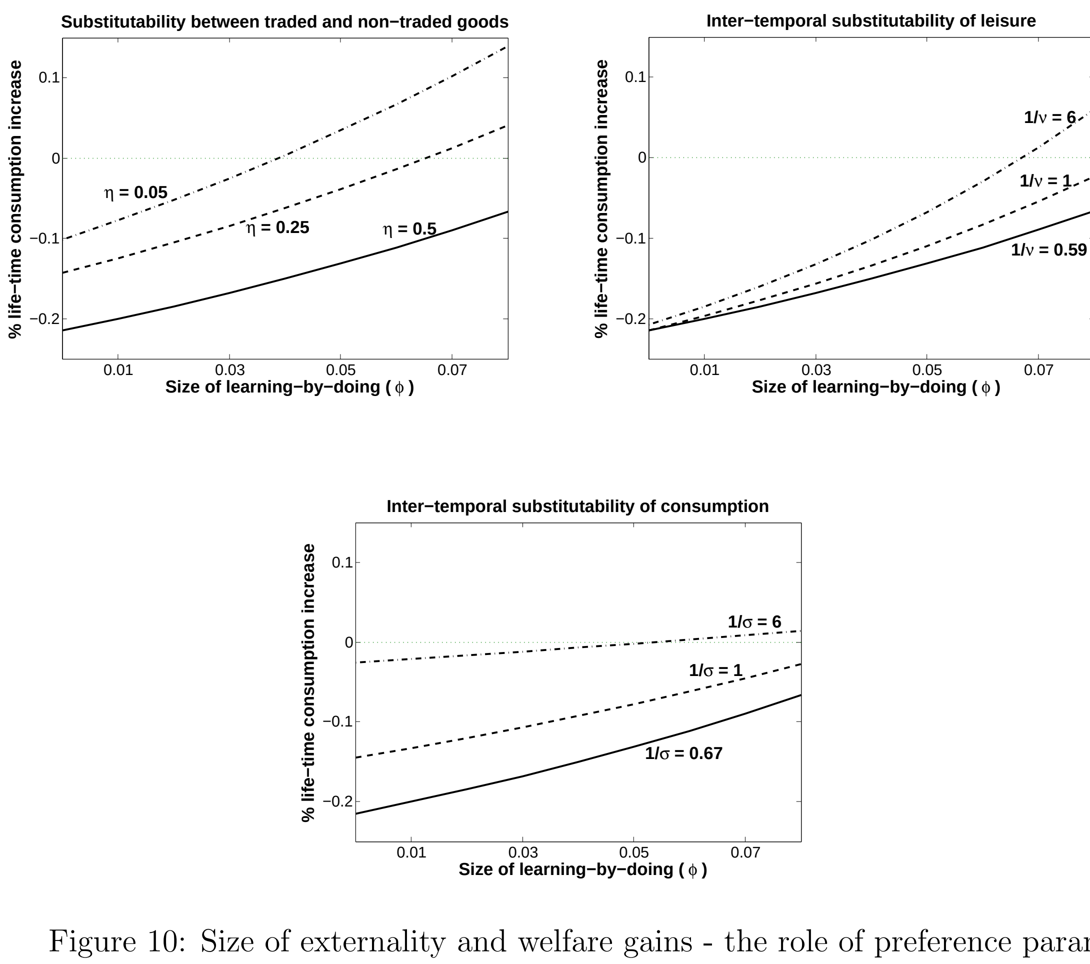

Figure 10: Size of externality and welfare gains - the role of preference parameters

We conclude that, beyond the China exercise, preference parameters drive the key trade-offs in evaluating the welfare implications of borrowing constraints as a policy tool to correct the LBD externality considered in this paper. They determine the efficacy of tightening the borrowing constraint as a means to increase aggregate labor supply and reallocate labor towards the traded sector. However, they also determine the disutility households suffer from the intra- and inter-temporal distortions in consumption and leisure that precisely drive the efficacy of the constraint in correcting the LBD externality.

5 Conclusions

We asked if borrowing constraints are a useful policy tool to facilitate growth. We specifically considered how they can correct a learning by doing externality by (i) raising labor supply and (ii) reallocating factors of production across sectors. Our answer is yes, sometimes. Borrowing constraints can correct learning-by-doing externalities and they come closest to achieving the first best when the externality is not too large. Further, welfare gains are increasing in the intertemporal elasticity of substitution of consumption and of leisure, but are decreasing in the elasticity of substitution between tradeable and non-tradeable goods.

Our analysis has provided understanding of an alternative policy tool to inform practical policy matters. Yet, our analysis is useful beyond normative aims. We have illuminated how household borrowing constraints affect aggregate labor supply and allocation across sectors, as well as welfare, in the presence of an empirically relevant growth externality. The theoretical foundations we have built can be used to guide empirical efforts. We showed borrowing constraints distort economies in two ways: by raising labor supply and reallocating resources across sectors. Further empirical exploration of these channels could help determine how financial repression facilitates or hinders growth.

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A Extensions

A.1 One-Sector SOE with Capital

Our results carry to the model with capital, if labor supply is the major contributor to future productivity.

A.1.1 Environment

The environment is very similar, except we now have additional factor of production - capital. The constraints a household faces are:

$$C_t + X_t + B_{t+1} \leq w_t L_t + R^* B_t + r_t K_t + \Pi_t \tag{A.1}$$

$$K_{t+1} \leq (1-\delta) K_t + X_t \tag{A.2}$$

$$B_{t+1} \geq \underline{b} \tag{A.3}$$

The consumption and investment goods are produced by competitive firms using capital and labor:

$$Y_t = A_t F(K_t, L_t)$$

where A_t is the total factor productivity productivity, relative to the world frontier. The production function F is strictly increasing and concave in each of its arguments, and homogenous of degree one. Productivity next period depends on today's productivity A_t and the aggregate supply of labor L_t:

$$A_{t+1} = H(A_t, L_t), \qquad H(A_t, L_t) \in [A_t, 1], \quad H_L(A, L) > 0 \text{ if } H(A, L) < 1.$$

A.1.2 An optimal borrowing constraint exists, is unique and binding

Since the effect of labor supply on future productivity is not internalized it is immediate that the laissez-faire allocation is inefficient. We also show that a binding borrowing constraint will improve welfare and that the effect of the borrowing constraint on welfare is non-monotone.

First, we consider an infinitesimal increase in $\underline{b}$ above the laissez-faire choice of assets in one period (with constraints in all future periods not binding): $\underline{b} = B' + \epsilon$. It is enough to show that such increase will result in higher labor supply today. Suppose not, i.e. $L(\underline{b}) \leq L^{LF}$. We will show this must imply lower consumption. The intra-temporal Euler condition will then imply $L(\underline{b}) > L^{LF}$ which will be a contradiction.

If $L(\underline{b}) \leq L^{LF}$, then $A'(\underline{b}) \leq A'^{LF}$. Suppose that $C(\underline{b}) \geq C_{LF}$. Then $K'(\underline{b}) < K'^{LF}$. Household's first order condition w.r.t. next period asset holdings imply that $\frac{U_C(C(\underline{b}),L_{CE})}{\beta U_C(C'^{LF},L'^{LF})} > R^*$, hence $1-\delta+A'F_K(K',L'^{LF}) > R^*$. Notice that if $C(\underline{b}) \geq C_{LF}$, the resource constraint then implies $K'^{LF} - K'(\underline{b}) > \underline{b}(S) - B'^{LF}$, i.e. decrease in the next period's capital stock is greater than the increase in net foreign asset position (both relative to laissez-faire policy). Given that $1-\delta+A'F_K(K',L'_{LF}) > R^*$ - gross return on K' is greater than on B', and given that $A'(\underline{b}) \leq A'^{LF}$, household's permanent income falls. That, combined with the decline in current disposable income (due to forced higher choice of B') results in $C(\underline{b}) < C^{LF}$. In a competitive equilibrium the following condition must hold:

$$U_C(C(\underline{b}), L(\underline{b}))AF_L(K, L(\underline{b})) + U_L(C(\underline{b}), L(\underline{b})) = 0$$

Since $C(\underline{b}) < C^{LF}$ we have $U_C(C, L^{LF}) > U_C(C^{LF}, L^{LF})$ then for the above condition to be satisfied we must have $L > L^{LF}(S)$, which contradicts our starting assumption. Since we have shown that $L(\underline{b}) > L^{LF}$, the result that $\frac{A'(\underline{b})}{A} > \frac{A'^{LF}}{A}$ is obvious.

Uniqueness again follows from the fact the constraint has non-monotone effect on welfare (increasing $\underline{b}$ too much will drive down consumption to zero).

B Proofs

Proof of Proposition 2.1

We exploit differentiability of V and the fact that $\frac{dL_1}{d\underline{b}} > 0$ to conclude that:

$$\left.\frac{\partial W}{\partial \underline{b}}\right|_{\underline{b}=B^*} = \underbrace{\frac{1}{C_1^*} \cdot \alpha L_1^{*\alpha-1} - \psi}_{=0} + \underbrace{V_B - \frac{1}{C_1^*}}_{=0} + \frac{dL_1}{d\underline{b}} \phi L_1^{*\phi-1} V_A > 0$$

where * denote the laissez-faire allocations. The first two terms are Euler conditions from household's optimization problem and the LF allocations satisfy them. Hence, the only non-zero term is $\frac{dL_1}{d\underline{b}} \phi L_1^{*\phi-1} V_A > 0$. Which yields the existence of a binding constraint that improves welfare. Uniqueness follows from the fact that W is strictly concave in $\underline{b}$, because V is strictly concave in both A and B. $\square$

B.1 Proof of Proposition 3.1

First, notice the value of a laissez-faire allocation is given by:

$$V^{LF}(A_0, B_0) = \max \sum_t \beta^t U(C_t^T, C_t^N)$$

subject to:

$$\begin{aligned}
A_{t+1} &= H(A_t, L_t^T) \\
C_t^N &= G(L_t^N) \\
C_t^T + B_{t+1} &= A_t F(L_t^T) + R^* B_t \\
U_{C^N}(C_t^T, C_t^N) &= \frac{A_t F'(L_t^T)}{G'(L_t^N)} U_{C^T}(C_t^T, C_t^N) \\
U_{C^T}(C_t^T, C_t^N) &= \beta R^* U_{C^T}(C_{t+1}^T, C_{t+1}^N) \\
B_{t+1} &\geq \underline{b} \\
A_0, B_0 & \quad \text{given.}
\end{aligned}$$

where $\underline{b}$ is the natural debt limit. Envelope theorem implies that $V_A > 0$.

Let $(C_t^{T*}, C_t^{N*}, L_t^{T*}, L_t^{N*}, B_{t+1}^*, A_{t+1}^*)_{t=0}^{\infty}$ be the laissez-faire allocation. Fix t such that $A_{t+1}^* < 1$ and let $(A, B) = (A_t^*, B_t^*)$. Define a function:

$$W(B'; A, B) = \max_{L^T, L^N} U(AF(L^T) + R^*B - B', G(L^N)) + \beta V^{LF}(H(A, L^T), B')$$

subject to (3.4). It is enough to show that:

$$\left.\frac{\partial W}{\partial B'}\right|_{(L^T, L^T, B') = (L_t^{T*}, L_t^{N*}, B_{t+1}^*)} > 0$$

We know that $\frac{\partial L^M}{\partial B'} > 0$. Then, we have:

$$\left.\frac{\partial W}{\partial B'}\right|_{(L^T, L^N, B') = (L_t^{T*}, L_t^{N*}, B_{t+1}^*)} = \frac{\partial L^T}{\partial B'} \beta H_L(A_t^*, L_t^{T*}) V_A^{LF}(A_{t+1}^*, B_{t+1}^*) > 0$$

□

C Numerical Algorithm

General outline Define grids on

- TFP: $Agrid = \{0.10, ..., 1.0\}$, length $= AA$
- Assets: $Bgrid = \{B_{min}, ..., 0.50\}$, length $= BB$

General outline:

1. For each initial $(a, b) \in Agrid \times Bgrid$ compute the laissez-faire equilibrium path and associated $V_{LSF}(A, B)$.
2. Compute V_{RP} (optimal constraints) and policy functions $C_{RP}(A, B)$, $L_{RP}(A, B)$, $B'_{RP}(A, B)$, $A'_{RP}(A, B)$ with VFI and PFI.
3. Compute V_{FB} (First-Best Planner) and policy functions $C_{FB}(A, B)$, $L_{FB}(A, B)$, $B'_{FB}(A, B)$, $A'_{FB}(A, B)$ with VFI and PFI.

Computing V_{OC} Set $V_{RP}(AA,:) = V_{LF}(AA,:)$. Set $A = AA - 1$. Set $\epsilon > 0$ small.

1. Make a guess on $V_{RP}(A,:)$. For every B use golden search to find $L(A, B)$ that maximizes
$$U(C, L) + \beta V_{RP}(A'(A, L), B'(A, B, L)) =: V1_{RP}(A, B)$$
Find C using intra-temporal Euler condition. Evaluate $V_{RP}(A', B')$ using 2D piece-wise linear interpolation for (A', B') between grid-points.

2. Compute $d = norm(V_{RP}(A,:) - V1_{RP}(A,:))$

3. Set $V_{RP}(A,:) = V1_{RP}(A,:)$, iterate until convergence.

4. Set $A = A - 1$. Go back to point 1.

Computing V_{PO} Set $V_{FB}(AA,:) = V_{LF}(AA,:)$. Set $A = AA - 1$. Set $\epsilon > 0$ small.

1. Make a guess on $V_{FB}(A,:)$ and $C_{FB}(A,:)$. For every B use golden search to find $L(A, B)$ that maximizes
$$U(C^*, L) + \beta V_{FB}(A'(A, L), B'(A, B, L)) =: V1_{FB}(A, B)$$
where C^* must satisfy the inter-temporal Euler condition
$$C^{*-\sigma} = \beta R^* C'(A', B')^{-\sigma}$$
Evaluate $V(A', B')$ and $C'(A', B')$ using 2D piece-wise linear interpolation for (A', B') between grid-points. Set $C1(A, B) = C^*$

2. Compute $d1 = norm(V_{FB}(A,:) - V1_{FB}(A,:))$ and $d2 = norm(C1(A,:) - C_{FB}(A,:))$.

3. If $\max\{d1, d2\} < \epsilon$, set $V_{FB}(A,:) = V1_{FB}(A,:)$ and $C_{FB}(A,:) = C1(A,:)$, iterate until convergence.

4. Set $A = A - 1$. Go back to point 1.