IUAC School on Nuclear Reactions

Antonio M. Moro

Universidad de Sevilla, Spain

November 15th-20th, 2021

Material available at: `https://github.com/ammoro/IUAC`

About myself

Myself:

- PhD in Seville (2001)
- Postdoc in Lisbon (Portugal): 2001-04
- Researcher at U. Seville: 2004-2010
- Associate Professor (since 2010)
- Full Professor (since 2020)

Topics:

- Few-body nuclear structures
- Continuum of weakly-bound nuclei
- Reactions with exotic nuclei

⇐ **Real Alcazar of Seville (XIV century A.C.)**

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⇐ "Plaza de España" in "Star Wars: The Attack of the Clones"

Course outline I

1. The CDCC method
   - Reminder of the coupled-channels method
   - Single-particle and cluster excitations
   - Partial wave analysis: radial equations
   - Some examples of applications of the CDCC method
   - Exploring the continuum with breakup reactions
   - Radiative capture from Coulomb dissociation data
2. Advanced CDCC and extensions
   - Extension to 3-body projectiles
   - Core excitations
3. Application of CDCC to transfer reactions
   - Transfer reactions with weakly bound nuclei
   - Transfer populating unbound states
4. The Coupled-Reaction Channels (CRC) formalism
5. Non-elastic breakup

Motivation of the CDCC method

How does the halo structure affect the elastic scattering?

- For $E = 19$ MeV (below the barrier, $V_b \approx 21$ MeV) ^{4}He follows Rutherford formula.
- ^{6}He drastically departs from Rutherford formula at both energies!

Motivation of the CDCC method

How does the halo structure affect the elastic scattering?

- ⇨ ^{4}He+^{208}Pb shows typical Fresnel pattern and "standard" optical model parameters
- ⇨ ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to nonelastic channels (e.g. breakup, transfer...)
- ☞ Understanding and disentangling these nonelastic channels requires going beyond the optical model (eg. coupled-channels method)

Reminder of the coupled-channels method

Inelastic scattering

- Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.
- Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.

Models for inelastic excitations

1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.

2. FEW-BODY/SINGLE-PARTICLE: Involve the excitation of a nucleon or cluster.

The coupled-channels method for inelastic scattering

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (e.g. projectile).

$$\boxed{H = T_R + h(\xi) + V(\mathbf{R}, \xi)}$$

- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the project (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the projectile.

$$\boxed{h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)}$$

- $\phi_n(\xi)$: internal states of the projectile.
- $V(\mathbf{R}, \xi)$: Projectile-target interaction.

Modelscape and scattering wavefunction: d+^{10}Be → d+^{10}Be* example

☞ The modelspace is composed by ground states (elastic channel) and some excited states (inelastic scattering)

Boundary conditions for scattering wavefunction:

$$\Psi^{(+)}_{\mathbf{K}_0}(\mathbf{R},\xi) \xrightarrow{R\gg} \underbrace{e^{i\mathbf{K}_0\cdot\mathbf{R}}\phi_0(\xi)}_{\text{incident}} + \underbrace{f_{0,0}(\theta)\frac{e^{iK_0R}}{R}\phi_0(\xi)}_{\text{elastic}} + \underbrace{\sum_{n>0} f_{n,0}(\theta)\frac{e^{iK_nR}}{R}\phi_n(\xi)}_{\text{inelastic}}$$

Cross sections:

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{0\to n} = \frac{K_n}{K_0}|f_{n,0}(\theta)|^2 \qquad f_{n,0}(\theta) = \text{scattering amplitude}$$

CC model wavefunction (target excitation)

We expand the total wave function in a subset of internal states representing the adopted modelspace:

$$\boxed{\Psi_{\text{model}}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K}_0,\mathbf{R}) + \sum_{n>0}\phi_n(\xi)\chi_n(\mathbf{K}_n,\mathbf{R})}$$

and impose the boundary conditions for the (unknown) $\chi_n(\mathbf{R})$:

$$\chi_0^{(+)}(\mathbf{K}_0,\mathbf{R}) \to e^{i\mathbf{K}_0\cdot\mathbf{R}} + f_{0,0}(\theta)\frac{e^{iK_0R}}{R} \qquad \text{for n=0 (elastic)}$$

$$\chi_n^{(+)}(\mathbf{K}_n,\mathbf{R}) \to f_{n,0}(\theta)\frac{e^{iK_nR}}{R} \qquad \text{for n>0 (non-elastic)}$$

Calculation of $\chi_n^{(+)}(\mathbf{R})$: the coupled equations

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi^{(+)}_{\text{model}}(\mathbf{R}, \xi) = 0$$

- Multiply on the left by each $\phi_n^*(\xi)$, and integrate over $\xi \Rightarrow$ coupled channels equations for $\{\chi_n(\mathbf{R})\}$:

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- The structure information is embedded in the coupling potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\xi \phi^*_{n'}(\xi) V(\mathbf{R}, \xi)\phi_n(\xi)}$$

☞ *$\phi_n(\xi)$ will depend on the assumed structure model (collective, few-body, etc).*

Optical Model vs. Coupled-Channels method

Optical Model

- The Hamiltonian:
  $H = T_R + V(\mathbf{R})$
- Internal states: Just $\phi_0(\xi)$
- Model wavefunction:
  $\Psi_{\text{mod}}(\mathbf{R}, \xi) \equiv \chi_0(\mathbf{K}, \mathbf{R})\phi_0(\xi)$
- Schrödinger equation:
  $[H - E]\chi_0(\mathbf{K}, \mathbf{R}) = 0$

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Coupled-channels method

- The Hamiltonian:
  $H = T_R + h(\xi) + V(\mathbf{R}, \xi)$
- Internal states:
  $h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)$
- Model wavefunction:
  $\Psi_{\text{model}}(\mathbf{R}, \xi) = \phi_0(\xi)\chi_0(\mathbf{K}, \mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{K}, \mathbf{R})$
- Schrödinger equation:

$$[H - E]\Psi_{\text{model}}(\mathbf{R}, \xi) = 0$$

$$\Downarrow$$

$$[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{K}, \mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{K}, \mathbf{R})$$

DWBA approximation as 1st order CC

- Two-states model $n = 0, 1$:

$$\Psi(\mathbf{R}, \xi) = \underbrace{\phi_0(\xi)\chi_0(\mathbf{R})}_{\text{elastic}} + \underbrace{\phi_1(\xi)\chi_1(\mathbf{R})}_{\text{inelastic}}$$

- Coupled-channels equations:

$$[E - \varepsilon_0 - T_0 - V_{00}(\mathbf{R})]\chi_0(\mathbf{R}) = V_{01}(\mathbf{R})\chi_1(\mathbf{R})$$
$$[E - \varepsilon_1 - T_1 - V_{11}(\mathbf{R})]\chi_1(\mathbf{R}) = V_{10}(\mathbf{R})\chi_0(\mathbf{R})$$

- Iterative solution of the CC equations (DWBA):

$$[E - \varepsilon_0 - T_0 - V_{00}(\mathbf{R})]\chi_0(\mathbf{R}) \approx 0$$
$$[E - \varepsilon_1 - T_1 - V_{11}(\mathbf{R})]\chi_1(\mathbf{R}) \approx V_{10}(\mathbf{R})\chi_0(\mathbf{R})$$

DWBA approximation as 1st order CC

- Asymptotically:

$$\chi_1^{(+)}(\mathbf{R}) \to f_{10}(\theta)\frac{e^{iK_1R}}{R}$$

with (not proven here!)

$$\boxed{f_{10}(\theta) = -\frac{2\mu}{4\pi\hbar^2}\int d\mathbf{R}\widetilde{\chi}_1^{(-)*}(\mathbf{K}_1,\mathbf{R})V_{10}(\mathbf{R})\widetilde{\chi}_0^{(+)}(\mathbf{K}_0,\mathbf{R})}$$

where $\widetilde{\chi}_0(\mathbf{K}_0,\mathbf{R}), \widetilde{\chi}_1(\mathbf{K}_1,\mathbf{R})$ are solutions of:

$$[E-\varepsilon_0-T_0-V_{00}(\mathbf{R})]\widetilde{\chi}_0(\mathbf{K}_0,\mathbf{R}) = 0$$
$$[E-\varepsilon_1-T_1-V_{11}(\mathbf{R})]\widetilde{\chi}_1(\mathbf{K}_1,\mathbf{R}) = 0$$

- *The DWBA approximation amounts at solving the CC equations to 1st order (Born approximation)*

Single-particle and cluster excitations

Many-body to few-body reduction

$$\mathcal{V}_{pt} = \sum_{ij} V_{ij}(\mathbf{r}_{ij})$$

$$\mathcal{V}_{pt} = U_{ct}(\mathbf{r}_{ct}) + U_{nt}(\mathbf{r}_{nt})$$

- Effective three-body Hamiltonian:

$$\boxed{H = T_{\mathbf{R}} + h_r(\mathbf{r}) + U_{ct}(\mathbf{r}_{ct}) + U_{nt}(\mathbf{r}_{nt})}$$

- $U_{ct}(\mathbf{r}_{ct})$, $U_{nt}(\mathbf{r}_{nt})$ are optical potentials describing fragment-target elastic scattering (eg. target excitation is treated effectively, through absorption)

Inelastic scattering in a few-body model

- Some nuclei allow a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
- Projectile-target interaction:

$$V(\mathbf{R},\xi) \equiv V(\mathbf{R},\mathbf{r}) = U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)$$

- Transition potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})\left[U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)\right]\phi_{n'}(\mathbf{r})}$$

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Example: ^{7}Li=α+t

$$\mathbf{r}_\alpha = \mathbf{R} - \frac{m_t}{m_\alpha + m_t}\mathbf{r}; \quad \mathbf{r}_t = \mathbf{R} + \frac{m_\alpha}{m_\alpha + m_t}\mathbf{r}$$

Internal states: (two-body cluster model)

$$[T_\mathbf{r} + V_{\alpha-t}(\mathbf{r}) - \varepsilon_n]\phi_n(\mathbf{r}) = 0$$

Example: ^{7}Li(α+t) +^{208}Pb at 68 MeV

➪ CC calculation with 2 channels ($3/2^-$, $1/2^-$):

Data from Davinson et al, Phys. Lett. 139B (1984) 150)

☞ Fresco input available at `https://github.com/ammoro/IUAC`

Application of the CC method to weakly-bound systems

Example: Three-body calculation (p+n+^{58}Ni) with Watanabe potential:

$$V_{dt}(\mathbf{R}) = \int d\mathbf{r}\phi^*_{\rm gs}(\mathbf{r})\left\{V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})\right\}\phi_{\rm gs}(\mathbf{r})$$

☞*Three-body calculations omitting breakup channels fail to describe the experimental data.*

Extension of the CC to unbound states

☞*We want to include explicitly in the modelspace the breakup channels of the projectile or target.*

Bound versus scattering states

Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Bound versus scattering states

Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r) | u_{k',\ell sj}(r) \rangle \propto \delta(k - k')$

SOLUTION $\Rightarrow$ continuum discretization

The origins of CDCC

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)] to describe deuteron scattering as an effective three-body problem $p + n + A$.

PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

George H. Rawitscher*
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139,†
and Department of Physics, University of Surrey, Guildford, Surrey, England

(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transition matrix element for relative neutron-proton angular momenta $l=2$ are found to be comparable in magnitude to the ones for $l=0$ for values of ϵ_a larger than about 3 MeV, and both decrease only slowly with ϵ_a, suggesting that a large breakup spectrum is involved in deuteron-nucleus collisions. The effect of the various breakup transitions on the elastic phase shifts is estimated by numerically solving a set of coupled equations. These equations couple the functions $\chi_a(\vec{R})$ which are the coefficients of the expansion of the neutron-proton-nucleus wave function in a set of the $\phi_a(\vec{r})$'s. The equations are rendered manageable by performing a (rather crude) discretization in the neutron-proton relative-momentum variable k_a. Numer-

George Rawitscher (1928-2018)

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Continuum discretization for deuteron scattering

- Select a number of angular momenta ($\ell = 0, \ldots, \ell_{\max}$).
- For each ℓ, set a maximum excitation energy $\varepsilon_{\max}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\max}$ in a set of sub-intervals (*bins*).
- For each bin, calculate a representative wavefunction.

CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\phi^{[k_1,k_2]}_{\ell jm}(\mathbf{r}) = \frac{u^{[k_1,k_2]}_{\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \qquad [k_1, k_2] = \text{bin interval}$$

$$\boxed{u^{[k_1,k_2]}_{\ell sjm}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell sj}(r) dk}$$

- k: linear momentum
- $u_{k,\ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function

CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:

$$\Psi^{(+)}(\mathbf{R},\mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r}) = 0$

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Transition potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})\left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2})\right]\phi_{n'}(\mathbf{r})$$

Partial-wave decomposition of CDCC wavefunction

- In practical calculations, the CDCC wf is expanded in the so-called channel basis $\langle \hat{R}, \mathbf{r}, |\beta; J_T\rangle = \left[Y_L(\hat{R}) \otimes \phi_{n,J_p}(\mathbf{r})\right]_{J_T}$:

$$\Psi_{\beta_0,J_T,M_T}(\vec{R},\vec{r},\xi) = \sum_{\beta} \frac{\chi^{J_T}_{\beta,\beta_0}(R)}{R} |\beta; J_T\rangle \quad \beta \equiv \{L, J_p, n\}$$

- The radial coefficients verify

$$\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + \varepsilon_n - E\right)\chi^{J_T}_{\beta,\beta_0}(R) + \sum_{\beta} V^{J_T}_{\beta,\beta'}(R)\chi^{J_T}_{\beta'}(R) = 0$$

with the coupling potentials:

$$V^{J_T}_{\beta,\beta'}(R) = \langle \beta; J_T | V_1(\vec{R},\vec{r}) + V_2(\vec{R},\vec{r}) | \beta'; J_T\rangle$$

- Boundary conditions:

$$\chi^{J_T}_{\beta,\beta_0}(R) \to e^{i\sigma_L}\frac{i}{2}\left[H_L^{(-)}(K_\beta R)\delta_{\beta_0,\beta} - S^{J_T}_{\beta,\beta_0} H_L^{(+)}(K_\beta R)\right]$$

Applications of the CDCC formalism: d+ ^{58}Ni

Coupling to continuum states produce:

- Polarization of the projectile (modification of real part)
- Flux removal (absorption) from the elastic channel (imaginary part)

Two- and three-body breakup observables

- From the breakup scattering amplitudes we can obtain **two-body breakup** observables:

$$\boxed{\frac{d\sigma_n}{d\Omega_{\text{c.m.}}} = |f_{0,n}(\theta)|^2 \Rightarrow \frac{d^2\sigma}{d\Omega_{\text{c.m.}} d\epsilon_{pn}} \simeq \frac{1}{\Delta_n} \frac{d\sigma_n}{d\Omega_{\text{c.m.}}}}$$

with:

  - Δ_n=width of the bin containing the relative energy ϵ_{pn}
  - $\Omega_{\text{c.m.}}$=C.M. scattering angle of the projectile c.m. (not easy to measure!)

- **Three-body observables** can be also calculated using a suitable combination of the scattering amplitudes (Tostevin, PRC63, 024617 (2001)):

$$\boxed{\frac{d^3\sigma}{d\Omega_n d\Omega_p dE_p}}$$

Breakup observables with CDCC: exclusive breakup of d+ ^{12}C → p+n+^{12}C

Data: Matsuoka *et al.*, NPA391, 357 (1986).

A.Deltuva, A.M.M., E.Cravo, F.M.Nunes, A.C.Fonseca, PRC 76, 064602 (2007)

Application of the CDCC method to other weakly bound two-body nuclei

- The CDCC has been also applied to nuclei with a cluster structure:
  - ^{6}Li=α + d ($S_{\alpha,d}$=1.47 MeV)
  - ^{11}Be=^{10}Be + n (S_n=0.504 MeV)

Exploring the continuum with breakup reactions

Electric response of weakly-bound nuclei

- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

$$B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

How to probe/extract the $B(E1)$ of halo nuclei?

Example: ^{11}Be+^{208}Pb $\rightarrow$ ^{10}Be+ n+ ^{208}Pb measured at RIKEN (69 MeV/u).
Fukuda et al, PRC70, 054606 (2004))

☞ ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

What observables are measured in Coulomb dissociation experiments?

- In a fully exclusive experiment, one can in principle measure the angular and relative energy distribution of the ^{11}Be* system:

$$\frac{d^2\sigma}{d\Omega\, dE_{\rm rel}}$$

- Integrating over the angle or energy, single differential cross sections are obtained:

- In the Coulomb dominated region (i.e. small angles), the breakup cross section is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

Semiclassical 1st order Eλ excitation (Alder & Winther) (akin EPM method)

- For Eλ excitation to bound states ($0 \to n$):

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0\to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta,\xi)} \qquad \xi_{0\to n} = \frac{(E_n - E_0)}{\hbar}\frac{a_0}{v}$$

- For continuum states (breakup):

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE}\frac{df_\lambda(\theta,\xi)}{d\Omega}}$$

☞ $dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

$$\boxed{\frac{d\sigma}{dE}(\theta < \theta_{\max}) = \int_0^{\theta_{\max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}}$$

Extracting $B(E1)$ of ^{11}Be from ^{11}Be+^{208}Pb Coulomb dissociation

Assumptions:

- Breakup dominated by Coulomb excitation
- Nuclear excitation, if present, can be estimated and added incoherently
- If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the weakly-bound projectile

$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$

RIKEN: Fukuda et al, PRC70 (2004) 054606
GSI: Palit et al, PRC68 (2003) 034318

The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak

CDCC analysis of Coulomb dissociation data

- Nuclear excitation not negligible, even for small θ
- Nuclear contribution interferes with Coulomb
- Higher-order couplings can affect the cross sections
- These ingredients can be naturally incorporated within the CDCC method (at the expense of more complexity!)

E.g.: CDCC analysis based on two-body ^{10}Be+n model:

PLB 811 (2020) 135959

- Different structure models yield different $B(E1)$ strengths and hence different breakup cross sections
- Comparison with the angular distribution evicences the defficiencies of the semiclassical EPM model

Application to radiative capture reactions

Radiative capture: $b + c \rightarrow a + \gamma$

Photodissociation: $a + \gamma \rightarrow b + c$

➪ Related by detailed balance:

$$\sigma_{E\lambda}^{(rc)} = \frac{2(2J_a+1)}{(2J_b+1)(2J_c+1)} \frac{k_\gamma^2}{k^2} \sigma_{E\lambda}^{(phot)} \qquad (\hbar k_\gamma = E_\gamma / c)$$

➪ Astrophysical S-factor:

$$S(E_{\text{c.m.}}) = E_{\text{c.m.}} \sigma_{E\lambda}^{(rc)} \exp[2\pi\eta(E_{\text{c.m.}})]$$

➪ Capture cross sections are difficult to measure because they are very small at relevant astrophysical energies.

Example: p+^{16}O → ^{17}F + γ

Morlock, PRL79, 3837 (1997)

Indirect determination of radiative capture from Coulomb breakup

➪ The photodissociation ($\gamma + a \to b + c$) cross section is related to the $B(E\lambda)$

$$\sigma_{E\lambda}^{\text{photo}} = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda)}{dE}$$

➪ Then, in 1st order semiclassical limit, the Coulomb breakup x-section is proportional to photodissociation x-section:

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE_\gamma} = \frac{1}{E_\gamma}\frac{dn_{E\lambda}}{d\Omega}\sigma_{E\lambda}^{\text{photo}}}$$

(Equivalent Photon Method)

with the virtual photon number

$$\frac{dn_{E\lambda}}{d\Omega} = Z_t^2\alpha\frac{\lambda[(2\lambda+1)!!]^2}{(2\pi)^3(\lambda+1)}\xi^{2(1-\lambda)}\left(\frac{c}{v}\right)^{2\lambda}\frac{df_{E\lambda}}{d\Omega}$$

Radiative capture from Coulomb dissociation experiments

- Capture reactions have typically small cross sections
- Use breakup (Coulomb dissociation) reactions:

$$\frac{d\sigma}{d\Omega dE_{c.m.}} \rightarrow \sigma_{E\lambda}^{(\text{phot})} \rightarrow \sigma_{E\lambda}^{(rc)} \rightarrow S(E_{\text{c.m.}})$$

$^{15}C+^{208}Pb \rightarrow {}^{14}C+n+^{208}Pb$

$^{14}C(n,\gamma)^{15}C$

(dots: direct; shaded region: from Coulomb breakup)
Summers and Nunes, PRC 78, 011601(R), 2008

Exploring structures in the continuum

The continuum spectrum is not "homogeneous"; it contains in general energy regions with special structures, such as resonances and virtual states

What is a resonance?

- It is a pole of the S-matrix in the complex energy plane.
- It is a structure on the continuum which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, continuum wavefunctions have a large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. For practical reasons, a normalized wave-packet (or "bin") can be constructed to represent the resonance.

Distinctive features of a resonance

In the energy range of the resonance, the continuum wavefunctions have a large probability of being within the range of the potential.

Cuts and areas ordered by size

(Courtesy of C. Dasso)

Resonances and phase-shifts

Resonances and phase-shifts

Resonances in nuclear breakup experiments

RIKEN data

Fukuda *et al*, PRC70 (2004) 054606)

CDCC analysis

Howell *et al.*, JPG31 (2005) S1881

Suggested exercise

Consider a simple two-body model for a one-neutron halo nucleus. For E1 transitions, the electric transition probability is given by

$$\frac{dB(E1)}{dE_c} = \left(\frac{4\pi}{k}\right)^2 \frac{\mu k}{(2\pi)^3\hbar^2} \frac{3}{4\pi} \left(\frac{Ze}{A}\right)^2 \left|\langle \ell_i 010|\ell_f 0\rangle \int dr\, r^2 \phi_f(k,r)\phi_i(r)\right|^2$$

where μ is the reduced mass o the two-body system, $\phi_i(r)$ is the radial part of the ground-state wavefunction and $\phi_f(k,r)$ is the radial part of the positive-energy state with energy $E_c = \hbar^2 k^2/2\mu$ and asymptotic behaviour $\phi_f(k,r) \xrightarrow{r\gg} (kr)j_\ell(kr)$

- Show that, in the special case in which the final states are approximated by plane-waves, the $B(E1)$ distribution is related to the Fourier transform of the ground state wavefunction. Give arguments to justify that, in the case of weakly bound systems, the $B(E1)$ distribution is concentrated at low excitation energies.

Exercises

- In the situation above, the ground state wavefunction can be approximated by its asymptotic form which, for a s-wave configuration, can be written as

$$\phi_i(r) \simeq \sqrt{2k_0}e^{-k_0 r}/r.$$

  Give the expression for k_0 in terms of the neutron separation energy (S_n) and the reduced mass of the neutron-core system. Compute it numerically for the ^{11}Be (^{10}Be+n) case.

- Show that, under these approximations, the $B(E1)$ distribution is given by

$$\frac{dB(E1)}{dE_c} = \frac{3\hbar^2}{\pi^2\mu}\left(\frac{Ze}{A}\right)^2 \frac{\sqrt{S_n}E_c^{3/2}}{(E_c+S_n)^4}.$$

  Hint: $\int_0^\infty dr\, r^2 j_1(br)e^{-ar} = 2b/(a^2+b^2)^2$

- Show that the maximum of this $B(E1)$ distribution is located at $E_c = \frac{3}{5}S_n$.

Advanced CDCC applications

Extension to 3-body projectiles

To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{x},\mathbf{y})\, \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{\alpha t}(\mathbf{r}_3)\}\, \phi_{n'}(\mathbf{x},\mathbf{y})$$

☞ $\phi_n(\mathbf{x},\mathbf{y})$ three-body WFs for bound and continuum states: hyperspherical coordinates, Faddeev, etc (difficult to calculate!)

Calculation of three-body states in the HH formalism

$$\Psi^{j\mu}(\rho,\Omega) = \rho^{-5/2}\sum_{\beta}\chi^{j}_{\beta}(\rho)\mathcal{Y}^{j\mu}_{\beta}(\Omega) \qquad \beta \equiv \{K, l_x, l_y, l, S_x, J; I\}$$

Hyperspherical Harmonics (HH) expansion

hypermomentum $\widehat{K}$

$$\mathcal{Y}^{j\mu}_{\beta}(\Omega) = \left[\left(\Upsilon^{l_x l_y}_{Klm_l}(\Omega)\otimes\kappa_{S_x}\right)_J \otimes \phi_I\right]_{j\mu}$$

$$\Upsilon^{l_x l_y}_{Klm_l}(\Omega) = \varphi^{l_x l_y}_{K}(\alpha)\left[Y_{l_x}(\widehat{x})\otimes Y_{l_y}(\widehat{y})\right]_{lm_l}$$

$$\varphi^{l_x l_y}_{K}(\alpha) = N^{l_x l_y}_{K}\,(\sin\alpha)^{l_x}\,(\cos\alpha)^{l_y}\,P_n^{l_x+\frac{1}{2},l_y+\frac{1}{2}}(\cos 2\alpha)$$

Jacobi coordinates $\{x, y, \widehat{x}, \widehat{y}\}$

Hyperspherical coordinates $\{\rho, \alpha, \widehat{x}, \widehat{y}\}$ $\quad \rho = \sqrt{x^2+y^2} \quad \alpha = \arctan\dfrac{x}{y}$

Calculation of three-body states in the HH formalism

$$\Psi^{j\mu}(\rho,\Omega) = \rho^{-5/2}\sum_{\beta}\chi^{j}_{\beta}(\rho)\mathcal{Y}^{j\mu}_{\beta}(\Omega) \qquad \beta \equiv \{K, l_x, l_y, l, S_x, J; I\}$$

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$$\Upsilon^{l_x l_y}_{Klm_l}(\Omega) = \varphi^{l_x l_y}_{K}(\alpha)\left[Y_{l_x}(\widehat{x})\otimes Y_{l_y}(\widehat{y})\right]_{lm_l}$$

$$\varphi^{l_x l_y}_{K}(\alpha) = N^{l_x l_y}_{K}(\sin\alpha)^{l_x}(\cos\alpha)^{l_y}P_n^{l_x+\frac{1}{2},l_y+\frac{1}{2}}(\cos 2\alpha)$$

Hyperradial functions are the solution of the coupled equations:

$$\left[-\frac{\hbar^2}{2m}\left(\frac{d^2}{d\rho^2}-\frac{15/4+K(K+4)}{\rho^2}\right)-\varepsilon\right]\chi^{j}_{\beta}(\rho)+\sum_{\beta'}V^{j\mu}_{\beta'\beta}(\rho)\chi^{j}_{\beta'}(\rho)=0$$

with coupling potentials $V^{j\mu}_{\beta'\beta}(\rho)$. Model space defined by **a given K_{max}**

Calculation of three-body states in the HH formalism

$$V^{j\mu}_{\beta'\beta}(\rho) = \left\langle \mathcal{Y}^{j\mu}_{\beta}(\Omega) \middle| V_{12} + V_{13} + V_{23} \middle| \mathcal{Y}^{j\mu}_{\beta'}(\Omega) \right\rangle + \delta_{\beta\beta'} V_{3\mathrm{b}}(\rho)$$

- V_{ij} interaction between pairs
  central, spin-orbit, spin-spin, tensor. To reproduce binary subsystem
- $V_{3\mathrm{b}}$ phenomenological three-body force
  diagonal term. Fixed to fine-tune the three-body energies

Pseudo-State (PS) method

$$\chi^{j}_{\beta}(\rho) = \sum_{i=0}^{N} C^{j}_{i\beta} U_{i\beta}(\rho)$$

expanded in $\mathcal{L}^2$ basis

N: number of hyperradial excitations included

$$\mathcal{H}\Psi^{j\mu}_{n} = \varepsilon_n \Psi^{j\mu}_{n}$$

- $\varepsilon_n < 0$ **bound states**
- $\varepsilon_n > 0$ **discretized continuum**

Calculation of three-body states in the HH formalism

Analytical **T**ransformed **H**armonic **O**scillator **(THO)** basis

$$U_{i\beta}^{\mathsf{THO}}(\rho) = \sqrt{\frac{ds}{d\rho}}\, U_{iK}^{\mathsf{HO}}[s(\rho)]$$

$$s(\rho) = \frac{1}{\sqrt{2}\boldsymbol{b}}\left[\frac{1}{\left(\frac{1}{\rho}\right)^4 + \left(\frac{1}{\gamma\sqrt{\rho}}\right)^4}\right]^{\frac{1}{4}}$$

Example:

$\Psi_n^{j\mu}(\rho,\Omega)$ PS spectra, ε_n
$b = 0.7$ fm

The ratio γ/b controls the density of PS as a function of the energy.

[PRC**88**(2013)014327]

Four-body CDCC calculations for ^{6}He scattering

N.b.: 1-channel potential considers only g.s. → g.s. coupling potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_{\text{g.s.}}(\mathbf{x},\mathbf{y})\, \{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3)\}\, \phi_{\text{g.s.}}(\mathbf{x},\mathbf{y})$$

Data (LLN): Sánchez-Benítez et al, NPA 803, 30 (2008) L. Acosta et al, PRC 84, 044604 (2011)
Calculations: Rodríguez-Gallardo et al, PRC 80, 051601 (2009)

Polarization potential from CDCC calculations

$$\left[E - \varepsilon_0 - \hat{T}_{\mathbf{R}} - V_{0,0}(\mathbf{R})\right]\chi_0(\mathbf{R}) = \sum_{i\neq 0} V_{i,0}(\mathbf{R})\chi_i(\mathbf{R}) \equiv U_{\mathrm{TELP}}(R)\chi_0(\mathbf{R}).$$

Example: ^{6}He+^{208}Pb at 22 MeV

- Polarization potentials are long-ranged.
- Both nuclear and Coulomb couplings contribute to the polarization effect.

Beyond the strict few-body picture: the effect of core excitation

To what extent can one ignore the dynamics of the core?

Few-body versus Microscopic

Microscopic models

- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

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Many-body

Few-body

Inert cluster models

- ✖ Ignores cluster excitations (only few-body d.o.f).
- ✖ Phenomenological inter-cluster interactions (aprox. Pauli).
- ✔ Exactly solvable (in some cases).
- ✔ Achieved for 3-body and 4-body (eg. coupled-channels, Faddeev).

Few-body versus Microscopic

Microscopic models

- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

Non-inert-core few-body models

- ✔ Few-body + some relevant collective d.o.f.
- ✔ Pauli approximately accounted for.
- ✔ Achieved for 3-body problems (coupled-channels, Faddeev).

Inert cluster models

- ✖ Ignores cluster excitations (only few-body d.o.f).
- ✖ Phenomenological inter-cluster interactions (aprox. Pauli).
- ✔ Exactly solvable (in some cases).
- ✔ Achieved for 3-body and 4-body (eg. coupled-channels, Faddeev).

Many-body

Few-body

Effect of core excitation in scattering observables

- Elastic scattering (adiabatic recoil model): K. Horii *et al*, PRC81 (2010) 061602

  ☞ *Some effects found in* ^{8}B +^{12} C.

- Transfer (DWBA, CCBA, Faddeev): Winfield *et al*, NPA 683 (2001) 48, Fortier *et al*, PLB 461 (1999) 22, Deltuva, Phys.Rev. C 88, 011601 (2013)

- Knock-out: Batham et al, PRC71, 064608 (2005)

  ☞ Small effect on stripping; large effect on diffraction

- Breakup
  - DWBA: Crespo *et al*, PRC83, 044622 (2011), A.M.M. *et al* PRC85, 054613 (2012), A.M.M. and J.A. Lay, PRL109, 232502 (2012)
  - CDCC: Summers *et al*, PRC74, 014606(2006), PRC76,014611 (2007), De Diego *al*, PRC89, 064609 (2014), PRC 95, 044611 (2017)

How do core excitations affect the breakup of weakly-bound nuclei?

Core excitations will affect:

1. the structure of the projectile ➪ core-excited admixtures

$$\Psi_{JM}(\vec{r},\xi) = \sum_{\ell,j,I} \left[\varphi^J_{\ell,j,I}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$

2. the dynamics ➪ collective excitations of the ^{10}Be during the collision compete with halo (single-particle) excitations.

➪ Both effects have been recently implemented in an extended version of of the CDCC formalism (CDCC): Summers *et al*, PRC74 (2006) 014606, R. de Diego *et al*, PRC 89, 064609 (2014)

Core excitation in reactions: *frozen-halo* picture

$$\Psi_{JM}(\vec{r},\xi) = \left[\varphi^J_{\ell,j}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$

- $\varphi^J_{\ell,j}(\vec{r})$= valence particle wavefunction
- $\Phi_I(\xi)$= core wavefunction (*frozen*)

Core excitation in reactions: *frozen-halo* picture

$$\Psi_{JM}(\vec{r},\xi) = \left[\varphi^J_{\ell,j}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$

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- $\Phi_I(\xi)$= core wavefunction (*frozen*)

Core excitation mechanism in breakup

$$\Psi_{JM}(\vec{r},\xi) = \sum_{\ell,j,I} \left[\varphi^J_{\ell,j,I}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$

$e[\,^{10}Be(0^+)\ x\ 1d_{3/2}\,] + f\,[\,^{10}Be(2^+)\ x\ 2s_{1/2}\,]$

$c[\,^{10}Be(0^+)\ x\ 1d_{5/2}\,] + d\,[\,^{10}Be(2^+)\ x\ 1d_{5/2}]$

$A[\,^{10}Be(0^+)\ x\ 1p_{1/2}\,] + B\,[\,^{10}Be(2^+)\ x\ 1p_{3/2}\,]$

$a[\,^{10}Be(0^+)\ x\ 2s_{1/2}\,] + b\,[\,^{10}Be(2^+)\ x\ 1d_{5/2}]$

Core excitation mechanism in breakup

$$\Psi_{JM}(\vec{r},\xi) = \sum_{\ell,j,I} \left[\varphi^J_{\ell,j,I}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$

☞ *Dynamic core excitation contributes to the inelastic/breakup probabilities*

Extending CDCC to include core excitations

- Standard CDCC ⇒ use coupling potentials:

$$V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi^{\alpha'}_{J'M'}(\vec{r}) | V_{vt}(r_{vt}) + V_{ct}(r_{ct}) | \Psi^{\alpha}_{JM}(\vec{r}) \rangle$$

- Extended CDCC (XCDCC) ⇒ use generalized coupling potentials

$$V_{\alpha;\alpha'}(\mathbf{R}) = \langle \Psi^{\alpha'}_{J'M'}(\vec{r},\xi) | V_{vt}(r_{vt}) + V_{ct}(r_{ct},\xi) | \Psi^{\alpha}_{JM}(\vec{r},\xi) \rangle$$

  - $\Psi^{\alpha}_{JM}(\vec{r},\xi)$: projectile WFs involving core-excited admixtures (structure).
  - $V_{ct}(r_{ct},\xi)$: non-central potential allowing for core excitations/de-excitations (dynamic core excitation).

- Summers *et al*, PRC74 (2006) 014606 (bins)
- R. de Diego *et al*, PRC 89, 064609 (2014) (THO pseudo-states)

Evidences of core excitation in nuclear breakup

Structure part: particle-core model

- Particle-plus-core Hamiltonian:

$$H_{\text{proj}} = T_r + h_{\text{core}}(\xi) + V_{vc}(\vec{r}, \xi)$$

- Projectile states expanded in $|\alpha;\ JM\rangle \equiv |(\ell s)j, I; JM\rangle$ basis:

$$\boxed{\Psi_{JM}(\vec{r}, \xi) = \sum_{\ell,j,I} R^J_{\ell,j,I}(r) \left[\left[Y_\ell(\hat{r}) \otimes \chi_s \right]_j \otimes \Phi_I(\xi) \right]_{JM}}$$

- The unknowns $R^J_{\ell,j,I}(r)$ can be obtained by direct integration of the Schrödinger equation or by diagonalization in a suitable discrete basis (pseudo-state method).

Valence-core and core-target interactions in a simple collective model

➪ Valence-core:

$$V_{vc}(\vec{r},\xi) \simeq V_{vc}^{(0)}(r) - \delta_2 \frac{dV_{vc}^{(0)}}{dr} Y_{20}(\hat{r})$$

➪ Core-target:

$$V_{ct}(\vec{r}_{ct},\xi) \simeq \underbrace{V_{ct}^{(0)}(r_{ct})}_{\text{Valence excitation}} \underbrace{- \delta_2 \frac{dV_{ct}^{(0)}}{dr} Y_{20}(\hat{r}_{ct})}_{\text{Core excitation}}$$

$\delta_2 = \beta_2 R_0$=deformation length

➪ *More sophisticated models for the projectile structure and core-target interaction are possible!*

Exclusive breakup measurements of halo nuclei

RIKEN experiments:

- ^{19}C+p: Satou et al., PLB660 (2008) 320
- ^{11}Be+^{12}C: Fukuda et al, PRC70, 054606 (2004)

- Excitation energy and angular distribution of the projectile can be reconstructed from core-neutron coincidences (*invariant mass method*)

Application to ^{11}Be: spectroscopic factors

State	Model	$\|0^+ \otimes (\ell s)j\rangle$	$\|2^+ \otimes s_{1/2}\rangle$	$\|2^+ \otimes d_{5/2}\rangle$
$1/2^+$ (g.s.)	PRM	0.857	–	0.121
	SM (WBT)	0.76	–	0.184
$5/2^+$ (1.78 MeV)	PRM	0.702	0.177	0.112
	SM(WBT)	0.682	0.177	0.095
$3/2^+$ (3.41 MeV)	PRM	0.165	0.737	0.081
	SM(WBT)	0.068	0.534	0.167

- $1/2^+_1, 5/2^+_1 \Rightarrow$ dominant ^{10}Be(gs) $\otimes$ nlj configuration
- $3/2^+_1 \Rightarrow$ dominant ^{10}Be(2^+) $\otimes$ $2s_{1/2}$ configuration

Evidence of *dynamical* core excitations in p(^{11}Be,p') at 64 MeV/u (MSU)

Data: Shrivastava et al, PLB596 (2004) 54 (MSU)

R.de Diego et al, PRC85, 054613 (2014)

- E_{rel}=0–2.5 MeV contains $5/2^+$ resonance (expected single-particle mechanism)
- E_{rel}=2.5–5 MeV contains $3/2^+$ resonance (expected core excitation mechanism)

⇨ Dynamical core excitations give additional (and significant!) contributions to breakup

Dominance of *dynamical* core excitations in ^{19}C resonant breakup

^{19}C + p → ^{18}C+n + p @ 70 MeV/u (RIKEN)

Satou et al, PLB660 (2008) 320

- No general consensus about the ^{19}C spectrum.
- Resonant peak consistent with $5/2^+_1$, $5/2^+_2$ or even a combination of both.

Dominance of *dynamical* core excitations in ^{19}C resonant breakup

^{19}C + p → ^{18}C+n + p @ 70 MeV/u (RIKEN)

Satou et al, PLB660 (2008) 320

J.A. Lay et al, PRC 94, 021602 (2016)

➪ The core-excitation mechanism gives the dominant contribution to the cross section

Transfer reactions

Outline

Example: d+^{208}Pb → p + ^{209}Pb

1. What do we measure in a transfer reaction?
   1. For a typical transfer reaction (e.g. d+^{208}Pb → p + ^{209}Pb), one measures the angular and energy distribution of outgoing fragments (e.g. protons).
   2. Additionally, one may collect information of decay products of ^{209}Pb (e.g. γ-rays, n, p, etc)
2. What information can we infer from a transfer reaction?
   1. Excitation energies of the residual nucleus (^{209}Pb).
   2. Angular momentum assignment.
   3. Single-particle content of populated states (i.e. spectroscopic factors).

What do we measure in a transfer reaction?

Example: d+^{208}Pb → p + ^{209}Pb

Phys. Rev. 159 (1967) 1039

- The proton energy spectrum shows some peaks which reflect the energy spectrum of the residual nucleus (^{209}Pb).
- Each peak has a characteristic angular distribution, which depends on the structure of the associated state.
- The population probability will depend on the reaction dynamics and on the structure properties of these states.

DWBA method for transfer reactions

- Transfer process: $\underbrace{(b+x)}_{a} + A \rightarrow b + \underbrace{(A+x)}_{B}$

- Complications arise with respect to inelastic scattering because now we have two different mass partitions involved

$$\underbrace{a+A}_{\alpha} \rightarrow \underbrace{b+B}_{\beta}$$

The important (d,p) case

- Introduce auxiliary potentials in entrance ($U_\alpha(\mathbf{R})$) and exit ($U_\beta(\mathbf{R}')$) channels.
- Projectile-target interaction: $V_\beta = V_{pn} + U_{pA} = U_{pB}(\mathbf{R}') + \underbrace{V_{pn} + U_{pA} - U_{pB}}_{\Delta V_\beta} \equiv U_\beta(\mathbf{R}') + \Delta V_\beta$
- Post-form DWBA transition amplitude:

$$\boxed{\mathcal{T}_{d,p}^{\mathrm{DWBA}} = \int\int \underbrace{\chi_p^{(-)*}(\mathbf{K}_p,\mathbf{R}')\Phi_B(\xi',\mathbf{r}')}_{\text{Final state}} \underbrace{(V_{pn} + U_{pA} - U_{pB})}_{\Delta V_\beta} \underbrace{\chi_d^{(+)}(\mathbf{K}_d,\mathbf{R})\varphi_d(\mathbf{r})\phi_A(\xi')}_{\text{Initial state}} d\xi' d\mathbf{r}' d\mathbf{R}'}$$

- $\chi_{\alpha,\beta}^{(\pm)}$ are distorted waves for entrance and exit channels, obtained with the optical potentials $U_\alpha(\mathbf{R})$, $U_\beta(\mathbf{R}')$
- For medium-mass/heavy targets: $U_{pA} \approx U_{pB} \Rightarrow V_{pn} + U_{pA} - U_{pB} \approx V_{pn}(\mathbf{r})$

(d,p) case: parentage decomposition of target nucleus

➪ Approximation of the overlap integral

$$\int d\xi' \; \phi_B^*(\xi', \mathbf{r}')\phi_A(\xi') \approx C_{BA}^{\ell j} \varphi_{nA}^{\ell j}(\mathbf{r}')$$

- $C_{BA}^{\ell j}$ = spectroscopic amplitude
- $|C_{BA}^{\ell j}|^2 = S_{BA}^{\ell j}$ = spectroscopic factor
- $\varphi_{nA}^{\ell j}(\mathbf{r}')$ = single-particle wavefunction describing motion of n with respect to A.

$$\boxed{\mathcal{T}_{d,p}^{\rm DWBA} = C_{BA}^{\ell j} \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j,*}(\mathbf{r}') \; V_{pn}(\mathbf{r}) \; \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r}) d\mathbf{r}' d\mathbf{R}'}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{\beta,\alpha} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} S_{BA}^{\ell j} \left| \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j,*}(\mathbf{r}') \; V_{pn}(\mathbf{r}) \; \chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})\varphi_d(\mathbf{r}) d\mathbf{r}' d\mathbf{R}' \right|^2}$$

☞ *In DWBA, the (d,p) cross section is proportional to the spectroscopic factor. Comparing the data with DWBA calculations, one can extract the values of $S_{BA}^{\ell j}$*

Application of CDCC to transfer reactions

Transfer reactions with weakly bound nuclei

- DWBA approximates the total WF by the elastic channel and assumes that the transfer occurs in one step (Born approximation).
- For weakly bound projectiles (eg. deuterons), breakup is an important channel and can influence the transfer process.

- $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R},\mathbf{r})$ includes breakup components, but these are lost when we make the DWBA approximation ($\Psi^{(+)} \approx \chi^{(+)}_d(\mathbf{K}_d,\mathbf{R})\varphi_d(\mathbf{r})$) $\Rightarrow$ need to go beyond DWBA

Adiabatic distorted wave approximation (ADWA)

- $\chi_d^{(+)}(\mathbf{K}_d, \mathbf{R})$ describes deuteron elastic scattering but, for the (d,p) transfer matrix element, we need only $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R}, \mathbf{r})$ for small $|\mathbf{r}|$
- R.C. Johnson and col. have derived an approximation of $\Psi^{(+)}_{\mathbf{K}_d}(\mathbf{R}, \mathbf{r})$ valid for $r \approx 0$, which includes the effect of deuteron breakup effectively (adiabatic approx.):
  1. Zero-range approximation (Johnson-Soper):

$$\boxed{U^{JS}(R) = U_{pb}(R) + U_{nb}(R)} \qquad \Rightarrow \chi_d^{JS}(\mathbf{R})$$

  2. Finite-range version (Johnson–Tandy):

$$\boxed{U^{JT}(R) = \frac{\langle \varphi_{pn}(\mathbf{r}) | V_{pn}(U_{nb} + U_{pb}) | \varphi_{pn}(\mathbf{r}) \rangle}{\langle \phi_{pn}(\mathbf{r}) | V_{pn} | \phi_{pn}(\mathbf{r}) \rangle}}$$

DWBA vs ADWA

From Timofeyuk and, Progress in Particle and Nuclear Physics 111 (2020) 103738

CDCC-BA approximation

- Exact transition amplitude for a general $\alpha \to \beta$ process:

$$\boxed{\mathcal{T}_{d,p}^{\rm CDCC} = C_{BA}^{\ell j} \int\int \chi_p^{(-)*}(\mathbf{K}_p, \mathbf{R}')\varphi_{nA}^{\ell j,*}(\mathbf{r}')\, V_{pn}(\mathbf{r})\, \underbrace{\Psi_{\mathbf{K}_\alpha}^{(+)}(\mathbf{R}_\alpha, \xi_\alpha)}\, d\mathbf{r}'\, d\mathbf{R}'}$$

- Use CDCC approximation for $\Psi_{\mathbf{K}_\alpha}^{(+)}$:

$$\Psi_{\mathbf{K}_\alpha}^{(+)} \approx \Psi^{\rm CDCC} = \underbrace{\chi_0(\mathbf{K}_\alpha, \mathbf{R})\phi_0(\mathbf{r})}_{\text{elastic}} + \sum_{n',j,\pi} \underbrace{\phi_{n'}^{j\pi}(k_{n'}, \mathbf{r})\chi_{n',j,\pi}(\mathbf{K}_{n'}, \mathbf{R})}_{\text{breakup}}$$

- Coupling to deuteron breakup states included explicitly.

DWBA, ADWA and CDCC-BA compared

DWBA vs ADWA vs CDCC

Example: ^{58}Ni(d,p)^{59}Ni

☞*CDCC and ADWBA provide better description of the data and lead also to more realistic spectroscopic information (e.g. spectroscopic factors)*

Pang *et al*, PRC 90, 044611 (2014)

Exploring resonances from transfer reactions

- Inclusion of continuum states in DWBA poses numerical problems due to oscillatory behaviour of unbound wavefunctions
- Some regularization method must be applied:
  - Representing the resonance by a weakly bound state with the same quantum numbers
  - Vincent & Fortune contour integration in the complex radius plane (PRC2 (1970) 782)
  - Representing the resonance by a continuum *bin*

E.g.: $d_{3/2}$ resonance in ^{17}O resonance at $E_r = 0.95$ MeV

E.g.: ^{11}Be resonance at E_x = 1.78 MeV from ^{10}Be(d,p)^{11}Be

Schmitt et al, PRC88, 064612 (2013)

Virtual states

- Neutrons in s-wave cannot produce resonant states (no barrier)
- Still, they can exhibit distinctive structures, characterized by a rapid increase of the phase-shift near zero energy (virtual states). This behaviour is commonly characterized in terms of the scattering length:

$$a_s = -\lim_{k\to 0} \tan \frac{\delta(k)}{k}$$

Levinson theorem:

$$\delta_\ell(E_n = 0) \to n\pi$$

(n=number of bound states for partial wave ℓ)

Resonances and virtual states in the complex energy/momentum plane

- A bound state is a normalizable solution of the Schrödinger equation for a imaginary momentum $+i|k|$ and (negative) energy: $E_s = \hbar^2(+i|k|)^2/2\mu = -\hbar^2|k|^2/2\mu$.
- A virtual state is a non-normalizable solution of the Schrödinger equation for a imaginary momentum $-i|k|$ and (negative) energy: $E_s = \hbar^2(-i|k|)^2/2\mu = -\hbar^2|k|^2/2\mu$.

Fig. 3.4. The correspondences between the energy (left) and momentum (right) complex planes. The arrows show the trajectory of a bound state caused by a progressively weaker potential: it becomes a resonance for $L > 0$ or when there is a Coulomb barrier, otherwise it becomes a virtual state. Because $E \propto k^2$, bound states on the positive imaginary k axis and virtual states on the negative imaginary axis both map onto the negative energy axis.

(*figure borrowed from Thompson & Nunes' book*)

Bound vs virtual (anti-bound) states in the N-N system

- $p-n$: $\delta(E) \xrightarrow{E\to 0} \pi \Rightarrow$ one-bound state (deuteron)
- $p-p$, $n-n$: no bound states, but they do have an anti-bound (virtual) state.
- $p-p$: δ becomes negative for large $E \Rightarrow$ evidence of repulsive core!

Spectroscopy in the continuum: shell evolution with neutron/proton asymmetry

➪Systematic studies of shell evolution requires extensions to the continuum

Shell-evolution for N=7 isotones

From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

SM explanation: Otsuka et al, PRL95,232502(2005)

Spectroscopy in the continuum: the ^{10}Li case

☞ Detected protons carry information on the ^{9}Li+n excitation spectrum.

H.P. Jeppesen et al, PLB642 (2006) 449

Populating resonances via transfer reactions $^{9}Li(d,p)^{10}Li^{*}$

From: P.G. Hansen and J.A. Tostevin, Ann Rev Nucl Part Sci **53** (2003) 219

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Coupled-Reaction Channels (CRC) method

Motivation of the CRC method

- The DWBA method includes transfer couplings to first order in the residual interaction (ΔV).
- Transfer couplings should affect the elastic channel, but this effect is not considered in DWBA
- When couplings are strong, higher orders may be important.

Post and prior Hamiltonians

- Transfer process: $\underbrace{(b+x)}_{a} + A \rightarrow b + \underbrace{(b+x)}_{B}$

- Full Hamiltonian:
  - Prior form: $\xi_\alpha = \{\xi, \xi', \mathbf{r}\}$

$$H = \hat{T}_{\mathbf{R}} + H_\alpha(\xi_\alpha) + V_\alpha(\mathbf{R}, \mathbf{r}) = \hat{T}_{\mathbf{R}} + \underbrace{H_a(\xi, \mathbf{r}) + H_A(\xi')}_{H_\alpha(\xi_\alpha)} + \underbrace{V_{xA} + U_{bA}}_{V_\alpha(\mathbf{R},\mathbf{r})}$$

  - Post form: $\xi_\beta = \{\xi, \xi', \mathbf{r}'\}$

$$H = \hat{T}_{\mathbf{R}'} + H_\beta(\xi_\beta) + V_\beta(\mathbf{R}', \mathbf{r}') = \hat{T}_{\mathbf{R}'} + \underbrace{H_b(\xi) + H_B(\xi', \mathbf{r}')}_{H_\beta(\xi_\beta)} + \underbrace{V_{xb} + U_{bA}}_{V_\beta(\mathbf{R}',\mathbf{r}')}$$

Coupled Reaction Channels

- Trial model wavefunction:

$$\Psi = \phi_a(\xi, \mathbf{r})\phi_A(\xi')\chi_\alpha(\mathbf{R}_\alpha) + \phi_b(\xi)\phi_B(\xi', \mathbf{r}')\chi_\beta(\mathbf{R}_\beta)$$

- Coupled-reaction channels (CRC) equations: $[H - E]\,\Psi = 0$

$$\begin{aligned}
[E - \varepsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\,\chi_\alpha(\mathbf{R}_\alpha) &= \int d\mathbf{R}_\beta K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\beta(\mathbf{R}_\beta) \\
\left[E - \varepsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)\right]\chi_\beta(\mathbf{R}_\beta) &= \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\alpha(\mathbf{R}_\alpha)
\end{aligned}$$

- Non-local kernels:

$$\begin{aligned}
K_{\alpha,\beta}(\mathbf{R}_\beta, \mathbf{R}_\alpha) &= \int d\xi d\xi' d\mathbf{r}\; \phi_a^*(\xi, \mathbf{r})\phi_A^*(\xi')(H - E)\phi_b(\xi)\phi_B(\xi', \mathbf{r}') \\
&\equiv \int d\xi_\alpha \Phi_\beta^*(\xi_\beta)(H - E)\Phi_\alpha(\xi_\alpha)
\end{aligned}$$

☞ *CRC equations have to be solved iteratively due to NL kernels.*

Evaluation of the non-local kernels

- In the NL kernels, H can be replaced by either H^{prior} or H^{post}
- E.g., for $H = H^{\text{prior}}$:

$$\boxed{K_{\alpha,\beta} = I^{\text{prior}}_{\alpha,\beta} + N^{\text{post}}_{\alpha,\beta}}$$

  - Interaction kernel:

$$\boxed{I^{\text{prior}}_{\alpha,\beta} = J_{\alpha,\beta} \int d\xi_\alpha \Phi^*_\beta(\xi_\beta)(V_\alpha - U_\alpha)\Phi_\alpha(\xi_\alpha)}$$

  - Non-orthogonality kernel:

$$\boxed{N^{\text{post}}_{\alpha,\beta} = J_{\alpha,\beta}[T_{\mathbf{R}} + U_\alpha - (E - \varepsilon_\alpha)] \int d\xi_\alpha \Phi^*_\beta(\xi_\beta)\Phi_\alpha(\xi_\alpha)}$$

Coupled radial equations

- As in other CC applications, the total WF is expanded in partial waves, leading to a set of coupled radial differential equations:

$$\left[E_{\kappa pt} - T_{\kappa L}(R_\kappa) - U_\kappa(R_\kappa)\right] f_\alpha(R_\kappa) = \sum_{\alpha',\Gamma>0} i^{L'-L}\, V^{\Gamma}_{\alpha:\alpha'}(R_{\kappa'}) f_{\alpha'}(R_{\kappa'}) + \sum_{\alpha',\kappa'\neq\kappa} i^{L'-L} \int_0^{R_m} V_{\alpha:\alpha'}(R_\kappa, R_{\kappa'}) f_{\alpha'}(R_{\kappa'}) dR_{\kappa'}$$

  - Asymptotic kinetic energies: $E_{\kappa pt} = E + Q_\kappa - \epsilon_p - \epsilon_t$
  - R_κ, $R_{\kappa'}$=projectile-target radial coordinates in partitions k,k'.
  - $V^{\Gamma}_{\alpha:\alpha'}(R_{\kappa'})$= coupling potential of multipolarity Γ within partition k' (local)
  - $V_{\alpha:\alpha'}(R_\kappa, R_{\kappa'}$=transfer coupling potentials (nonlocal)
- Boundary conditions:

$$f_\alpha(R_{\kappa'}) \to \frac{i}{2}\left[\delta_{\alpha\alpha_0} H^{(-)}_{L\eta_\alpha}(K_\alpha R_\kappa) - S_{\alpha_0\alpha} H^{(+)}_{L\eta_\alpha}(K_\alpha R_\kappa)\right]$$

Some remarks on CRC calcultions

- For each transfer coupling, we need to provide the appropriate spectroscopic amplitude. Results will depend on its phase!
- The diagonal potential describing each channel will not coincide with the optical potential describing elastic scattering in that channel, since the effect of some channels are already taken into account explicitly.

Comparison with data

Example: d+^{40}Ca E_d=58 MeV

- Full CRC calculation includes coupling to deuteron breakup, (d,t) transfer and $(d,3He)$ transfer
- The final potential employed for the d+^{40}Ca differs from that used in the one-channel optical model case.

Keeley & Mackintosh, PRC77, 054603 (2008)

DWBA approximation from CRC

- Iterative solution of the CRC equations:

$$
\begin{aligned}
[E-\varepsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\chi_\alpha(\mathbf{R}_\alpha) &\approx 0 \\
\left[E-\varepsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)\right]\chi_\beta(\mathbf{R}_\beta) &\approx \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha,\mathbf{R}_\beta)\chi_\alpha(\mathbf{R}_\alpha)
\end{aligned}
$$

- DWBA scattering amplitude (prior):

$$
\boxed{f^{\rm DWBA}_{\beta,\alpha} = -\frac{\mu_\beta}{2\pi\hbar^2}\int\int \widetilde{\chi}^{(-)*}_\beta(\mathbf{R}_\beta)(\phi_b\phi_B|V_{\rm prior}|\phi_a\phi_A)\widetilde{\chi}^{(+)}_\alpha(\mathbf{R}_\alpha)d\mathbf{R}_\alpha d\mathbf{r}}
$$

- Distorted waves:

$$
\begin{aligned}
[E-\varepsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\widetilde{\chi}_\alpha(\mathbf{R}_\alpha) &= 0 \\
\left[E-\varepsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)\right]\widetilde{\chi}_\beta(\mathbf{R}_\beta) &= 0
\end{aligned}
$$

- Structure form-factor:

$$
(\phi_b\phi_B|V_{\rm prior}|\phi_a\phi_A) \equiv \int d\xi d\xi'\ \phi_b(\xi)\phi_B(\xi',\mathbf{r}')V_{\rm prior}\phi_a(\xi,\mathbf{r})\phi_A(\xi')
$$

The problem of inclusive breakup

α production in ^{6}He scattering

$$^6\text{He} + ^{208}\text{Pb} \rightarrow \alpha + \text{X}$$

☞CDCC reproduces elastic scattering, but not inclusive α's.

Evidence of NEB contributions in inclusive (^{6}Li,αX)

^{6}Li+^{209}Bi @ 32 MeV

Santra *et al*, PRC85,014612(2008)

If elastic breakup were the dominant mechanism, we would expect $N_\alpha \approx N_d$ but, experimentally, one finds $N_\alpha \gg N_d$

"Breakup" modes of few-body projectiles

⇨ For a reaction of the form $\underbrace{(b+x)}_{a} + A \rightarrow b + \text{anything}$

$$\boxed{\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}}$$

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Explicit evaluation of inclusive breakup in Ichimura-Austern-Vincent model

- Inclusive breakup:

$$a(= b + x) + A \rightarrow b + (x + A)^*$$

Inclusion of all relevant $x + A$ channels is not feasible in general $\Rightarrow$ use closed-form models

- Inclusive differential cross section: $\sigma_b^{\rm BU} = \sigma_b^{\rm EBU} + \sigma_b^{\rm NEB}$:
  - $\sigma_b^{\rm EBU}$ is breakup leaving A in g.s. (e.g. CDCC)
  - $\sigma_b^{\rm NEB}$ corresponds to non-elastic x+A processes and can be calculated as the absorption in the $x + A_{gs}$ channel:

$$\frac{d\sigma^{\rm NEB}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x^{(0)}|W_{xA}|\varphi_x^{(0)}\rangle \quad \text{(optical theorem)}$$

where $\varphi_x^{(0)}$ describes the $x - A$ relative motion and is computed from:

$$[K_x + U_{xA} - E_x]\varphi_x^{(0)}(\mathbf{r}_x) = (\chi_b^{(-)}|V_{bx}|\chi_{aA}\phi_a\rangle$$

Application to ^{209}Bi (^{6}Li+,α X)

- Large α yield ($\sigma_\alpha \gg \sigma_d$) $\Rightarrow$ evidence of NEB channels

Application to ^{209}Bi (^{6}Li+,α X)

Assume: $\sigma_\alpha \simeq \sigma^{\rm EBU} + \sigma^{\rm NEB}$:

- EBU ⇨ CDCC
- NEB ⇨ IAV with DWBA approximation: $\Psi^{(+)}_{3b} \approx \chi_a(\mathbf{R})\phi_a(\mathbf{r})$

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- EBU ⇨ CDCC
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J. Lei and AMM, PRC92, 044616 (2015)

- Inclusive data well accounted for by EBU+NEB
- Inclusive α's dominated by NEB
- EBU (^{6}Li $\rightarrow \alpha + d$) only relevant for small scattering angles.