

The EPM method

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July 6, 2025

Material available at: <https://github.com/ammoro/R4E>

- Applicable when the de Broglie wavelength is small compared to the collision scale.
- Internal nucleon motion must remain quantum-mechanical.
- Mixed approach: classical projectile motion, quantum internal excitation.
- The trajectory must be barely perturbed by the momentum and energy transfer to the projectile:

$$\frac{\Delta \ell}{\ell} \ll 1, \quad \frac{\Delta \varepsilon_n}{E} \ll 1$$

- Differential cross section:

$$\left(\frac{d\sigma}{d\Omega} \right)_{0 \rightarrow n} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{clas}} P_n(\theta)$$

- Half of distance of closest approach in head-on collision:

$$a_0 = \frac{\kappa Z_p Z_t e^2}{2E}$$

- Distance of closest approach for scattering angle θ :

$$r_{\min}(\theta) = a_0 \left[1 + \frac{1}{\sin(\theta/2)} \right]$$

- **Adiabaticity parameter** for head-on collision:

$$\xi_{0 \rightarrow n} \equiv \xi_{0 \rightarrow n}(\theta = \pi) = \frac{(E_n - E_0)}{\hbar} \tau_{col} \approx \frac{(E_n - E_0)}{\hbar} \frac{a_0}{v}$$

- **Adiabaticity parameter** for general collision

$$\xi_{0 \rightarrow n}(\theta) = \frac{(E_n - E_0)}{\hbar} \frac{r_{\min}(\theta)}{2v} = \xi_{0 \rightarrow n} \left[1 + \frac{1}{\sin(\theta/2)} \right]$$

Sommerfeld parameter:

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} = \frac{a_0}{\lambda}$$

The projectile is described by a wavepacket of dimension $\sim \lambda$, which must be small compared to the dimensions of the classical trajectory ($\sim a_0$):

$$\lambda \ll a_0 \Rightarrow \eta \gg 1$$

- Multipole expansion of Coulomb potential

$$V(\mathbf{r}, \xi) = \frac{Z_t Z_p e^2}{4\pi\epsilon_0 r} + \sum_{\lambda > 0, \mu} \frac{Z_t e}{\epsilon_0 (2\lambda + 1)} M(E\lambda, \mu) \frac{Y_{\lambda\mu}(\hat{r})}{r^{\lambda+1}}$$
$$\equiv V_0(r) + V_{\text{coup}}(\mathbf{r}, \xi)$$

☞ $V_0(r)$ determines de trajectory, but does not induce excitations

- $M(E\lambda, \mu)$ is the electric multipole operator
- Internal states are conveniently labeled by their intrinsic spin and its projection:

$$|n\rangle \rightarrow |I_n M_n\rangle$$

- For $E\lambda$ excitation to bound states ($0 \rightarrow n$):

$$\left(\frac{d\sigma}{d\Omega}\right)_{0 \rightarrow n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \rightarrow n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi) \quad \xi_{0 \rightarrow n} = \frac{(E_n - E_0)}{\hbar} \frac{a_0}{v}$$

- For continuum states (breakup):

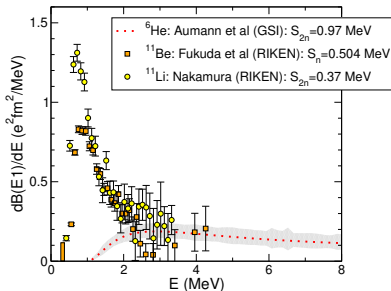
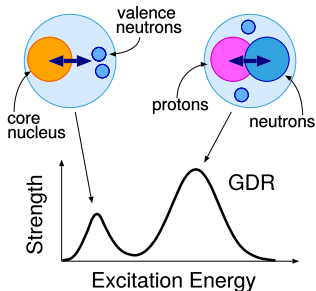
$$\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}$$

👉 $dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

$$\frac{d\sigma}{dE}(\theta < \theta_{\max}) = \int_0^{\theta_{\max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}$$

Exploring the continuum with breakup reactions

Electric response of weakly-bound nuclei



- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

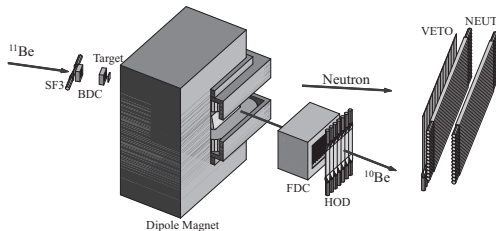
$$B(E\lambda; i \rightarrow f) = \frac{1}{2I_i + 1} |\langle \Psi_f | \mathcal{M}(E\lambda) | \Psi_i \rangle|^2$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

How to probe/extract the $B(E1)$ of halo nuclei?

Example: $^{11}\text{Be} + ^{208}\text{Pb} \rightarrow ^{10}\text{Be} + n + ^{208}\text{Pb}$ measured at RIKEN (69 MeV/u).

Fukuda et al, PRC70, 054606 (2004)



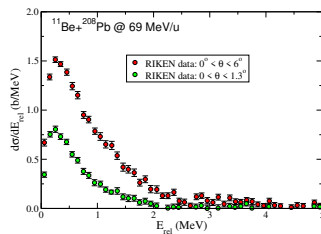
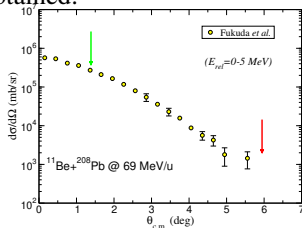
☞ ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

What observables are measured in Coulomb dissociation experiments?

- Experimentally, one measures angular and relative energy distribution of the $^{11}\text{Be}^*$ system:

$$\frac{d^2\sigma}{d\Omega dE}$$

- Integrating over the angle or energy, single differential cross sections are obtained:

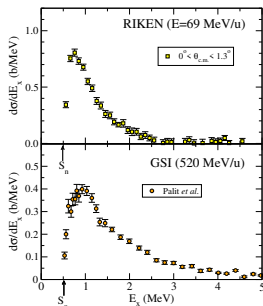


- In the Coulomb dominated region (i.e. small angles), the **breakup cross section** is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

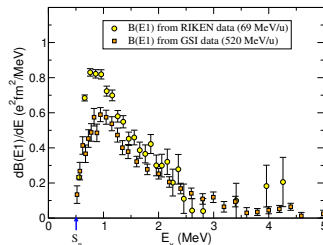
Extracting $B(E1)$ of ^{11}Be from $^{11}\text{Be} + ^{208}\text{Pb}$ Coulomb dissociation

Common assumptions:

- Breakup dominated by Coulomb excitation (mostly E1).
- Nuclear excitation, if present, can be estimated and added incoherently
- ☞ If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the projectile.



$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$



RIKEN: Fukuda et al, PRC70 (2004) 054606

GSI: Palit et al, PRC68 (2003) 034318

☞ The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak