Reaction4Exp commented example: Coulomb excitation and dissociation of ^{11}Be+^{197}Au,^{208}Pb

Introduction

This document describes briefly the use of the Reaction4Exp website for the case of inelastic scattering and coulomb dissociation within the semiclassical equivalent photon method (EPM). We will use as example the ^{11}Be+^{197}Au reaction. The link to the elastic scattering website is:

https://reaction4exp.us.es/elastic/index.php

1 Reminder of the EPM method

In its simplest form, the theory of Alder and Winther (AW) [1] assumes that the projectile moves along a classical trajectory, which is weakly affected by the internal excitations of the colliding nuclei. This means that:

$$\frac{\Delta\ell}{\ell} \ll 1 \quad \text{and} \quad \frac{\Delta\varepsilon_n}{E} \ll 1.$$

where ε_n are the excitation energies of the excited states under consideration.

In a semiclassical treatment of inelastic scattering process, the differential cross section for a given $0 \to n$ transition can be expressed as:

$$\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{clas}} P_n(\theta) \tag{1}$$

where $(d\sigma/d\Omega)_{\text{clas}}$ is the classical differential elastic cross section which, for a pure Coulomb case, coincides with the Rutherford cross section, and P_n is the excitation probability for the scattering angle θ.

In the important case of pure Coulomb scattering, which was the case studied in detail by Alder and Winther [1], and treating the excitation in first-order of the perturbation expansion, the differential cross section for a $0 \to n$ transition induced by a Eλ transition is given by the analytical formula,

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0\to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta,\xi)\,, \qquad (2)}$$

which is valid only for angles smaller than the grazing[1] one ($\theta < \theta_{\text{gr}}$) and where a_0 is half the distance of closest approach in a classical head-on collision, $\xi_{0\to n} = \frac{(\varepsilon_n - \varepsilon_0)}{\hbar}\frac{a_0}{v}$

[1] The grazing angle refers to the angle at which the projectile interacts with the surface of the target in such a way that the projectile barely "grazes" the surface of the target, rather than fully penetrating or colliding head-on.

is the adiabaticity parameter and $f_\lambda(\theta,\xi)$ is an analytical function, depending on the kinematical conditions, but independent of the structure of the projectile.

For weakly bound nuclei, the excitation will typically populate unbound (continuum) states. The previous formula can be generalized to:

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega d\varepsilon} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{d\varepsilon} \frac{df_\lambda(\theta,\xi)}{d\Omega} \qquad (\theta < \theta_{\rm gr})} \tag{3}$$

where $df_\lambda(\theta,\xi)/d\Omega$ is also a well-defined analytic function and $dB(E\lambda)/d\varepsilon$ is the electric reduced probability to the continuum states.

It is common to express (3) in terms of the so-called number of equivalent photons,

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega d\varepsilon} = \frac{16\pi^3}{9\hbar c} \frac{dN_{E\lambda}(E_x\theta)}{d\Omega} \frac{dB(E\lambda)}{d\varepsilon}} \tag{4}$$

where $dN_{E\lambda}(E_x\theta)/d\Omega$ is the number of equivalent photons per solid angle. This is typically referred to as **Equivalent Photon Method (EPM)**.

2 Application to ^{11}Be+^{197}Au, ^{208}Pb reactions

2.1 Inelastic scattering

For the inelastic scattering calculation between discrete states we will consider the reaction ^{11}Be+^{197}Au, for which elastic, inelastic (excitation of the bound excited state at E_x = 320 keV) and inclusive breakup (^{10}Be production) has been reported [5] at two incident energies: $E_{\rm lab}$ = 31.9 MeV and $E_{\rm lab}$ = 39.6 MeV.

According to Eq. (2), the semiclassical calculation requires as input the incident energy, the target nucleus charge and the reduced electric transition probability connecting the initial and final states. In this case, it corresponds to the E1 transition between the ground state ($1/2^+$) and the first excited state ($1/2^+$) in ^{11}Be. Using the value determined experimentally by Millener *et al*,

$$B(E1; g.s. \to 1/2^-) = 0.116 \text{ e}^2\text{fm}^2 \tag{5}$$

in the R4E platform, one gets the angular distributions shown in Fig. 1 for the 31.9 MeV (left) and 39.6 MeV (right) incident energies.

2.2 Coulomb dissociation

For the Coulomb dissociation data we will consider the reaction ^{11}Be+^{208}Pb at 69 MeV/u [2]. In this case, the key structure input is the reduced electric transition probability connecting the initial state with the unbound (continuum) states. For the present example, we will consider the set III of Table I from Ref. [6] which will be hereafter

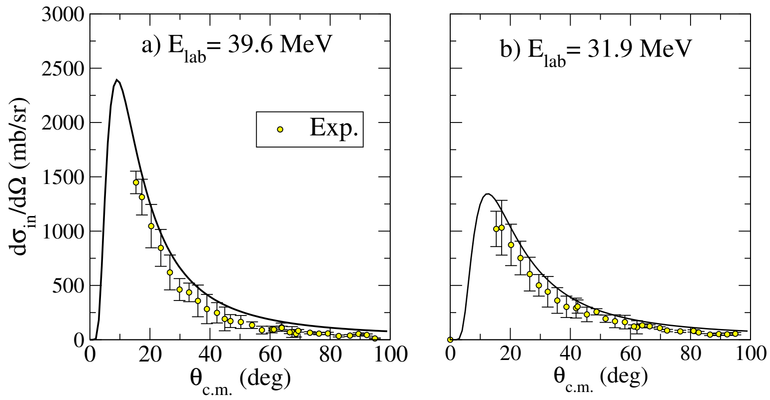

Figure 1: Differential cross sections angular distributions for the inelastic scattering of ^{11}Be on ^{197}Au at $E_{\text{lab}} = 39.6$ MeV (left) and $E_{\text{lab}} = 31.9$ MeV (right), respectively. Experimental data are from Ref. [5].

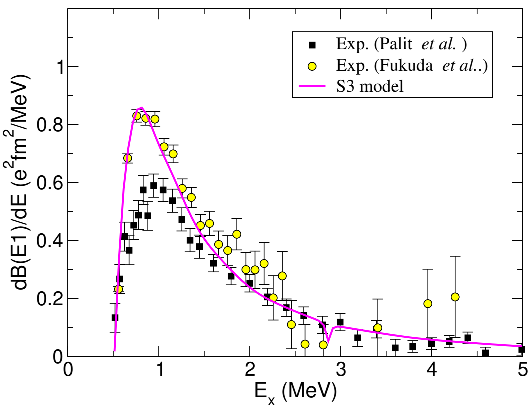

Figure 2: Electric transition probability for ^{11}Be calculated with the S3 model of Ref. [6]. For comparison, the $B(E1)$ distributions extracted from Coulomb dissociation data in Refs. [4] and [2] are also shown. A critical analysis of the differences between these distributions can be found in Ref. [3].

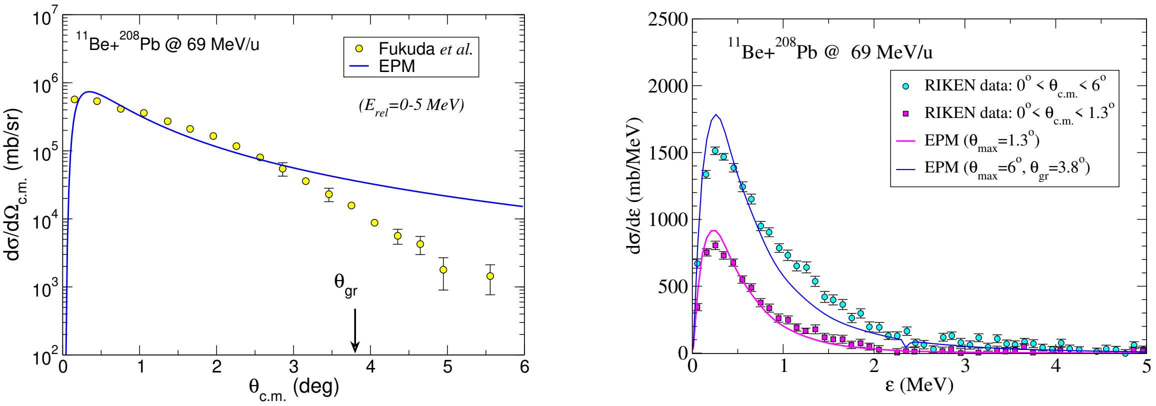

Figure 3: Experimental [2] and calculated (EPM) differential cross section for the Coulomb breakup of ^{11}Be on ^{208}Pb at 69 MeV/u. Left panel: energy-integrated angular distribution. Right panel: angular integrated energy distribution as a function of the n-^{10}Be relative energy in the continuum.

denoted “S3 model” . This is shown in Fig. 2 in comparison with the $B(E1)$ “experimental” distributions obtained from two experiments performed at RIKEN [2] and GSI [4].

In this case, the EPM calculation provides a double-differential cross section with respect to the angle and relative energy of the fragments. Integrating over the energy or the angle, one can obtain the corresponding single-differential cross sections. This is illustrated in Fig. 3. The left panel displays the energy-integrated angular distribution. Note that the EPM calculation is only justified for angles below the grazing one. In this reaction, this was estimated in [2] as $\theta_{\rm gr} = 3.8^{\circ}$[2] , and it is indicated with a vertical arrow in Fig. 3. Beyond this angle, nuclear effects are expected to be important and assumption of pure Coulomb excitation breaks down.

The right panel of Fig. 3 shows the relative-energy distribution resulting from the integration of the double differential cross section with respect to the scattering angle. Two angular cuts are considered in the data and in the calculations: $\theta_{\rm c.m.} < 1.3^{\circ}$ and $\theta_{\rm c.m.} < 6^{\circ}$. For the former, which is well below the grazing angle, the reaction is expected to be largely dominated by Coulomb excitation mechanism. For the wider angular cut, the maximum angle is greater than the grazing angle. For $\theta_{\rm gr} < \theta < 6^{\circ}$, absorptive effects will tend to suppress the Coulomb dissociation probability. To account for this effect in a simple way, these contributions have been omitted from EPM calculation.

Some additional remarks are in order:

- In these examples, we have not taken into account the experimental resolutions. For a meaningful comparison between the calculations and the data, the former should be convoluted with these resolutions.

[2]This value can be obtained from the relation between the impact parameter b and the scattering angle θ for a classical Coulomb trajectory, $b = a_0 \cot(\theta/2)$. Approximating $b_{\rm gr} \approx 1.2(11^{1/3} + 208^{1/3}) = 9.8$ fm one gets $\theta_{\rm gr} \approx 2\cot^{-1}(b_{\rm gr}/a_0) = 3.8^{\circ}$.

- In some applications [4], nuclear breakup contributions are computed separately and added incoherently to the pure Coulomb excitation cross section. Note that this procedure ignores possible interference effects [3].

References

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