The Equivalent Photon Method for Coulomb excitation

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Material available at: `https://github.com/ammoro/R4E`

- Applicable when the de Broglie wavelength is small compared to the collision scale.
- Internal nuclear motion must remain quantum-mechanical.
- Mixed approach: classical projectile motion, quantum internal excitation.
- Tre trajectory must be barely perturbed by the momentum and energy transfer to the projectile:

$$\frac{\Delta\ell}{\ell} \ll 1, \qquad \frac{\Delta\varepsilon_n}{E} \ll 1$$

- Differential cross section:

$$\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{clas}} P_n(\theta)$$

where $(d\sigma/d\Omega)_{\text{clas}}$ is the classical elastic differential cross section.

Application to Coulomb excitation

- Sommerfeld parameter:

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

- Half of distance of closest approach in head-on collision:

$$a_0 = \frac{\kappa Z_p Z_t e^2}{2E} = \frac{\eta}{k}$$

- Distance of closest approach for scattering angle θ:

$$r_{\min}(\theta) = a_0 \left[1 + \frac{1}{\sin(\theta/2)} \right]$$

- Relation between impact parameter(b) and scattering angle (θ)

$$b = a_0 \cot(\theta/2)$$

- Adiabaticity parameter for head-on collision:

$$\xi_{0\to n} = \frac{(E_n - E_0)}{\hbar} \tau_{col} \approx \frac{(E_n - E_0)}{\hbar} \frac{a_0}{v}$$

We can rewrite the Sommerfeld parameter as:

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} = \frac{a_0}{\lambda\!\!\!^{-}}$$

The projectile is described by a wavepacket of dimension $\sim \lambda\!\!\!^{-}$, which must be small compared to the dimensions of the classical trajectory ($\sim a_0$):

$$\boxed{\lambda\!\!\!^{-} \ll a_0 \;\Rightarrow\; \eta \gg 1}$$

- Multipole expansion of Coulomb potential

$$V(\mathbf{r},\xi) = \frac{Z_t Z_p e^2}{4\pi\epsilon_0 r} + \sum_{\lambda>0,\mu} \frac{Z_t e}{\epsilon_0(2\lambda+1)} M(E\lambda,\mu) \frac{Y_{\lambda\mu}(\hat{r})}{r^{\lambda+1}}$$
$$\equiv V_0(r) + V_{\mathrm{coup}}(\mathbf{r},\xi)$$

- $V_0(r)$ determines de trajectory, but does not induce excitations.

- $M(E\lambda,\mu)$ is the electric multipole operator, related to the **electric reduced transition probability**:

$$\boxed{B(E\lambda; i \to f) = \frac{1}{2I_i+1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2}$$

- Internal states are conveniently labeled by their intrinsic spin and its projection:

$$|n\rangle \longrightarrow |I_n M_n\rangle$$

Semiclassical 1st order Eλ excitation (Alder & Winther) (akin EPM method)

- For Eλ excitation to bound states ($0 \to n$):

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{0\to n} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{B(E\lambda, 0 \to n)}{e^2 a_0^{2\lambda-2}} f_\lambda(\theta, \xi)} \qquad \xi_{0\to n} = \frac{(E_n - E_0)}{\hbar}\frac{a_0}{v}$$

- For continuum states (breakup):

$$\boxed{\frac{d\sigma(E\lambda)}{d\Omega dE} = \left(\frac{Z_t e^2}{\hbar v}\right)^2 \frac{1}{e^2 a_0^{2\lambda-2}} \frac{dB(E\lambda)}{dE} \frac{df_\lambda(\theta, \xi)}{d\Omega}}$$

☞ $dB(E\lambda)/dE$ can be extracted from small-angle Coulomb dissociation data.

$$\boxed{\frac{d\sigma}{dE}(\theta < \theta_{\max}) = \int_0^{\theta_{\max}} \frac{d\sigma(E\lambda)}{d\Omega dE} d\Omega \propto \frac{dB(E\lambda)}{dE}}$$

- The pure Coulomb excitation formula is expected to be valid only for trajectories beyond the grazing distance: $r_{\min} > R_g \approx 1.45(A_p^{1/3} + A^{1/3})$ fm.
- We can define a grazing angle (θ_g) such that for angles $\theta < \theta_g$ one has $r_{\min}(\theta) > R_g$ and hence the collision will be mostly Coulomb:
- We can estimate θ_g from the condition $r_{\min}(\theta_g) = R_g$:
  - At high energies, where trajectories are barely affected by Coulomb:

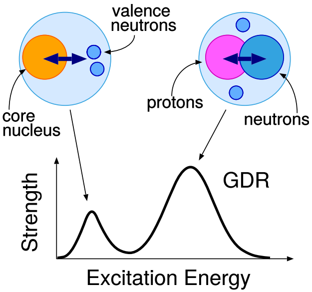
$$R_g \approx b_g = q_0 \cot(\theta_g/2) \quad \Rightarrow \quad \theta_g$$

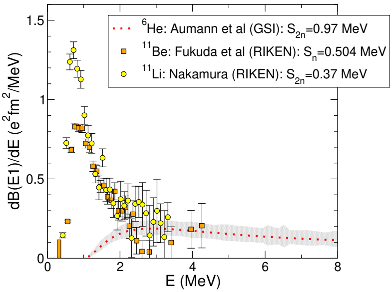
  - Near the Coulomb barrier, where Coulomb deflection is large:

$$R_g = r_{\min}(\theta_g) = a_0 \left(1 + \frac{1}{\sin(\theta_g/2)}\right) \quad \Rightarrow \quad \theta_g$$

Exploring the continuum with breakup reactions

Electric response of weakly-bound nuclei



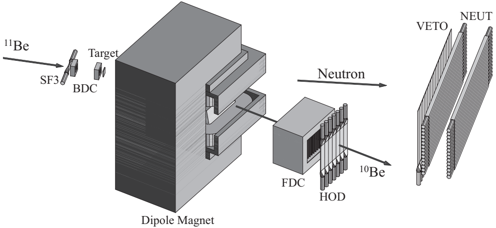


- The $E\lambda$ response can be quantified through the $B(E\lambda)$ probability:

$$\boxed{B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f \| \mathcal{M}(E\lambda) \| \Psi_i \rangle|^2}$$

- Neutron-halo nuclei have large $B(E1)$ strengths near threshold

Example: ^{11}Be+^{208}Pb $\rightarrow$ ^{10}Be+ n+ ^{208}Pb measured at RIKEN (69 MeV/u).
Fukuda et al, PRC70, 054606 (2004))

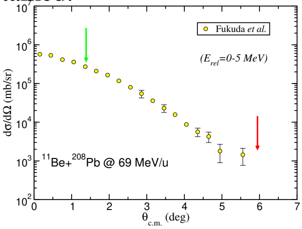


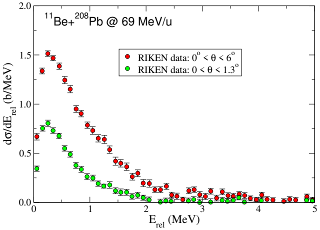
☞ ^{11}Be excitation energy can be reconstructed from core-neutron coincidences (*invariant mass method*)

- Experimentally, one measures angular and relative energy distribution of the ^{11}Be* system:

$$\boxed{\frac{d^2\sigma}{d\Omega\, dE}}$$

- Integrating over the angle or energy, single differential cross sections are obtained:





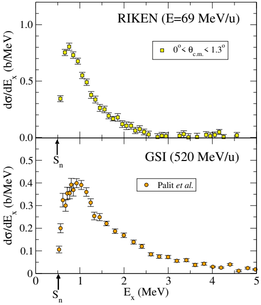
- In the Coulomb dominated region (i.e. small angles), the breakup cross section is expected to be dominated by the $dB(E\lambda)/dE$ distribution, but we need a theory that relates both observables.

Extracting $B(E1)$ of ^{11}Be from ^{11}Be+^{208}Pb Coulomb dissociation

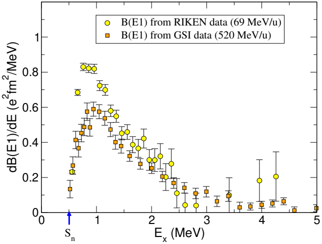
Common assumptions:

- Breakup dominated by Coulomb excitation (mostly E1).
- Nuclear excitation, if present, can be estimated and added incoherently

☞ If the assumptions above are fulfilled, the extracted $dB(E\lambda)dE$ should be independent of the incident energy and target employed, since it reflects a structure property of the projectile.



$$\frac{dB(E\lambda)}{dE} \propto \frac{d\sigma}{dE}$$



RIKEN: Fukuda et al, PRC70 (2004) 054606
GSI: Palit et al, PRC68 (2003) 034318

☞ *The extracted $dB(E\lambda)/dE$ distributions are reasonably compatible, but with apparent differences at the peak*