NRV example for inelastic scattering: the ^{64}Zn(^{16}O,^{16}O)^{64}Zn*(2^+) reaction

Introduction

This document describes briefly the use of the Nuclear Reaction Video Project (NRV) toolkit for the case of inelastic scattering. We will use as example the ^{16}O+^{64}Zn $\to$ ^{16}O+^{64}Zn(2^+) reaction. This physics case is taken from the reference of Tenreiro *et al*, Physical Review C53, p. 2870 (1996), Ref. I hereafter.

The link to the inelastic scattering interface at the NRV website is:

http://nrv.jinr.ru/nrv/webnrv/inelastic_scattering/default.htm

1 Reminder of the DWBA approximation

To describe an inelastic process of the form $a + A \to a + A^*$ (target excitation) within the DWBA approximation the projectile–target interaction is conveniently written as:

$$V(\mathbf{R}, \xi) = V_0(R) + \Delta V(\mathbf{R}, \xi), \tag{1}$$

where $V_0(R)$ contains in general nuclear and Coulomb parts, and describes the relative motion of the projectile and target, and $\Delta(\mathbf{R}, \xi)$ is the part of the projectile–target interaction which is responsible for the inelastic process. It depends on the internal coordinates of the nucleus being excited ($\{\xi\}$) as well as on the projectile–target relative coordinate ($\mathbf{R}$).

We consider that the target nucleus is initially in its ground state, described by some wavefunction $\phi_i(\xi)$, and is excited to some state $\phi_f(\xi)$. Within the DWBA approximation, the scattering amplitude for this process is given by:

$$\boxed{f_{fi}(\theta) = -\frac{\mu}{2\pi\hbar^2} \int d\mathbf{R}\ \chi_f^{(-)*}(\mathbf{K}_f, \mathbf{R}) \Delta V_{if}(\mathbf{R}) \chi_i^{(+)}(\mathbf{K}_i, \mathbf{R}),} \tag{2}$$

where θ is the scattering angle (in c.m. frame) and $\Delta V_{if}(\mathbf{R})$ is the *transition potential*

$$\boxed{\Delta V_{if}(\mathbf{R}) = \int d\xi \phi_f^*(\xi) \Delta V(\xi, \mathbf{R}) \phi_i(\xi).} \tag{3}$$

In Eq. (2), $\chi_i^{(+)}(\mathbf{K}_i, \mathbf{R})$ is the distorted-wave describing the projectile-target relative motion in the incident channel. This distorted-wave is the solution of the Schrödinger equation with the average potential $V_0(R)$:

$$\left[-\frac{\hbar^2}{2\mu_{aA}} \nabla^2_{\mathbf{R}} + V_0(R) - E_i\right] \chi_i^{(+)}(\mathbf{K}_i, \mathbf{R}) = 0 \tag{4}$$

where E_i is the c.m. kinetic energy in the entrance channel. Typically, the nuclear part of the optical potential $V_0(R)$ is parametrized in terms of some convenient form (e.g. Woods-Saxon shape) and the parameters adjusted to reproduce the elastic angular distribution. Analogously, $\chi_f^{(-)}(\mathbf{K}_i, \mathbf{R})$ is the corresponding distorted wave for the final channel. In practice, we use the same potential as for the incident channel.

2 Radial formfactors in the collective model

In many practical situations, such as in the collective models considered here,

$$\Delta V(\mathbf{R},\xi) = \sum_{\lambda>0} \mathcal{F}_\lambda(R) \sum_{\mu} \mathcal{T}_{\lambda,\mu}(\xi) Y_{\lambda\mu}(\hat{R}) \tag{5}$$

The transition potential is given by:

$$\Delta V_{if}(\mathbf{R}) \equiv \langle I_f M_f | \Delta V(\mathbf{R},\xi) | I_i M_i \rangle = \sum_{\lambda>0} \mathcal{F}_\lambda(R) \langle I_f M_f | \mathcal{T}_{\lambda\mu}(\xi) | I_i M_i \rangle Y_{\lambda\mu}(\hat{R}) \tag{6}$$

where the *radial formfactor* $\mathcal{F}_\lambda(R)$ contains the radial dependence and $\mathcal{T}_{\lambda\mu}$ is a given multipole operator depending on the structure model. By using the Wigner-Eckart theorem:

$$\langle I_f M_f | \mathcal{T}_{\lambda\mu}(\xi) | I_i M_i \rangle = (2I_f + 1)^{-1/2} \langle I_f M_f | I_i M_i \lambda\mu \rangle \langle I_f \| \mathcal{T}_\lambda(\xi) \| I_i \rangle_{\mathrm{BM}} \tag{7}$$

where $\langle I_f \| \mathcal{T}_\lambda(\xi) \| I_i \rangle$ are the so-called reduced matrix elements.

We consider two important and popular cases:

2.1 Coulomb excitations

$$\Delta V_{if}(\mathbf{R}) \equiv \langle f; I_f M_f | \Delta V | i; I_i M_i \rangle = \sum_{\lambda>0,\mu} \frac{4\pi\kappa}{2\lambda+1} \frac{Z_t e}{R^{\lambda+1}} \langle f; I_f M_f | \mathcal{M}(E\lambda,\mu) | i; I_i M_i \rangle Y_{\lambda\mu}(\hat{R}) \tag{8}$$

where we see that $\mathcal{T}_{\lambda,\mu} \to \mathcal{M}(E\lambda,\mu)$ is the electric multipole operator. Its reduced matrix elements are related to the **electric transition probability**

$$B(E\lambda; i \to f) = \frac{1}{2I_i+1} |\langle f; I_f || \mathcal{M}(E\lambda) || i; I_i \rangle|^2. \tag{9}$$

In the collective model, the $B(E\lambda)$ value can be related to the so-called **deformation parameter** (β_λ^C):

$$\sqrt{B(E\lambda; I_i \to I_f)} = \frac{3 Z_t e R_c^{\lambda-1}}{4\pi} \beta_\lambda^C R_c \tag{10}$$

where R_c is an average radius of the charge distribution of the nucleus being excited and Z_t its net charge.

2.2 Nuclear excitations

For small nuclear deformations we have that that the projectile-target nuclear interaction can be expanded as:

$$V(\mathbf{R},\xi) \simeq V(R-R_0) - \sum_{\lambda,\mu} \hat{\delta}_{\lambda\mu} \frac{dV(R-R_0)}{dR} Y_{\lambda\mu}(\theta,\phi) + \ldots \tag{11}$$

where $\hat{\delta}_\lambda$ are deformation length operators. Hence, the transition potentials for nuclear excitations (with $\lambda > 0$) are

$$V_{if}(\mathbf{R}) = -\frac{dV(R-R_0)}{dR}\langle f; I_f M_f|\hat{\delta}_{\lambda\mu}|i; I_i M_i\rangle Y_{\lambda\mu}(\hat{R}) \tag{12}$$

The required structure input are the reduced matrix elements of the deformation operator which, in NRV, are expressed in terms of a nuclear deformation parameter (β_λ) and an average matter radius:

$$\langle f; KI_f\|\hat{\delta}_\lambda\|i; KI_i\rangle = (-1)^\lambda \beta_\lambda^N R_0 \tag{13}$$

So, to summarize, NRV will require the Coulomb (β_λ^C) and nuclear (β_λ^N) deformation parameters.

In general, both the Coulomb and nuclear interactions will contribute to the excitation and so the transition potential will be the sum of the nuclear and Coulomb transition potentials and, consequently, the corresponding scattering amplitude will be given by the coherent sum of the individual nuclear and Coulomb amplitudes, i.e.,

$$f_{if}(\theta) = f_{if}^N(\theta) + f_{if}^C(\theta). \tag{14}$$

Since the corresponding inelastic cross section is proportional to the square of $f_{if}(\theta)$, interference effects will occur between the nuclear and Coulomb parts.

3 NRV calculations

For the NRV calculations, we will use the optical model potentials of Table I of Tenreiro *et al* (see Fig. 1). The reduced radius for both the real and imaginary parts is $r_v = r_w = 1.25$ fm. The considered excitation, $0^+ \to 2^+$ has necessarily $\lambda = 2$ so we will need the corresponding quadrupole Coulomb and nuclear deformation parameters.

The Coulomb deformation parameter (β_C) can be derived from the electric transition probability

$$\boxed{\beta_C = \frac{4\pi\sqrt{B(E2; i \to f)}}{3 Z_t e R_c^2}} \tag{15}$$

where R_c is the charge radius of the nucleus being excited, in our case, ^{64}Zn. For the reduced radius for the charge distribution is $r_C = 1.25$ fm so

$$R_c = r_c A_t^{1/3} = 1.25 \times 64^{1/3} \text{ fm} = 5 \text{ fm}.$$

For $B(E2; gs; 0^+ \to 2^+)$, we will use the experimental value from Att. Nucl. Data Tables 80 p. 35 (2002):

$$B(E2; gs; 0^+ \to 2^+) = 1680 \text{ e}^2\text{fm}^4.$$

For the nuclear part, the strength of the coupling is given in terms of a *nuclear deformation parameter* (β_N), or, equivalently, a *deformation length* ($\delta_N = \beta_N R_0$) such that

$$\delta_N = \beta_N R_0 \equiv \langle f; I_f\|\hat{\delta}_\lambda\|i; I_i\rangle, \tag{16}$$

TABLE I. Summary of the potential parameters obtained from the optical model analysis of the elastic scattering angular distributions and deformation lengths βR obtained from the DWBA analysis, performed on the inelastic scattering angular distributions.

$E_{c.m.}$ (MeV)	V_0 (MeV)	a_v (fm)	W_0 (MeV)	a_w (fm)	$\beta_v R_v$ (fm)	$\beta_w R_w$ (fm)	χ^2
32.0	68.21	0.557	7.05	0.574	0.892	2.823	0.60
32.8	51.09	0.556	9.14	0.551	0.941	1.815	1.09
34.0	44.30	0.562	18.76	0.556	1.106	1.047	0.74
34.8	43.78	0.561	14.76	0.551	1.207	1.106	1.07
35.2	45.69	0.559	12.09	0.563	1.250	0.411	1.26
38.4	44.66	0.560	20.61	0.560	1.117	1.011	1.38
41.6	41.03	0.560	18.56	0.560	0.889	1.013	2.22
43.2	44.40	0.560	18.97	0.559	1.258	0.932	0.62
44.8	43.99	0.560	19.30	0.560	1.258	0.831	1.20
49.6	46.82	0.561	17.43	0.554	1.106	1.359	1.07
51.2	43.52	0.561	19.40	0.552	1.106	1.334	0.82

Figure 1: Optical model parameters from Tenreiro et el, PRC53, 2870 (1996).

where $\langle f; I_f \| \hat{\delta}_\lambda \| i; I_i \rangle$ is the *reduced matrix element* of the deformation length operator.

For a correct implementation of these calculations, the following remarks are in order here:

- For the nuclear deformation, the paper by Tenreiro *et al.* (Table I) provides the product $\delta_N = \beta_N R_0$. In NRV, we need to provide the value β_N, which is then internally multiplied by the radius

  $$R_0 = r_0 \times A_{exc}^{1/3},$$

  where A_{exc} is the mass of the nucleus being excited ($A_{exc} = 64$ in our case).

  Therefore, the values of β_N that we need to introduce in NRV in order to get the desired deformation length, are

  $$\beta_N^{\mathrm{NRV}} = \frac{\delta_N}{R_0} = \frac{\delta_N}{r_0 \times 64^{1/3}}, \tag{17}$$

  where δ_N are the values of $\beta_N R$ listed in Table I.

- Moreover, in Ref. I the authors use different deformations for the real and imaginary parts. NRV assumes the same value for the real and imaginary parts, so we will use the value of the real part (keeping in mind that this will probably yield slightly worse fits of the data).

The results obtained for the elastic and inelastic cross sections are displayed in Fig. 2. The agreement with the inelastic data is not as good as in the paper of Tenreiro *et al.* This is because, in NRV, we are assuming the same deformation parameter for the real and imaginary parts.

It is illustrative to study the separate effect of the Coulomb and nuclear couplings in the inelastic cross sections. For this purpose, we can set to zero the corresponding value of β in NRV. The results are also included in Fig. 2. From these curves, several conclusions can be drawn:

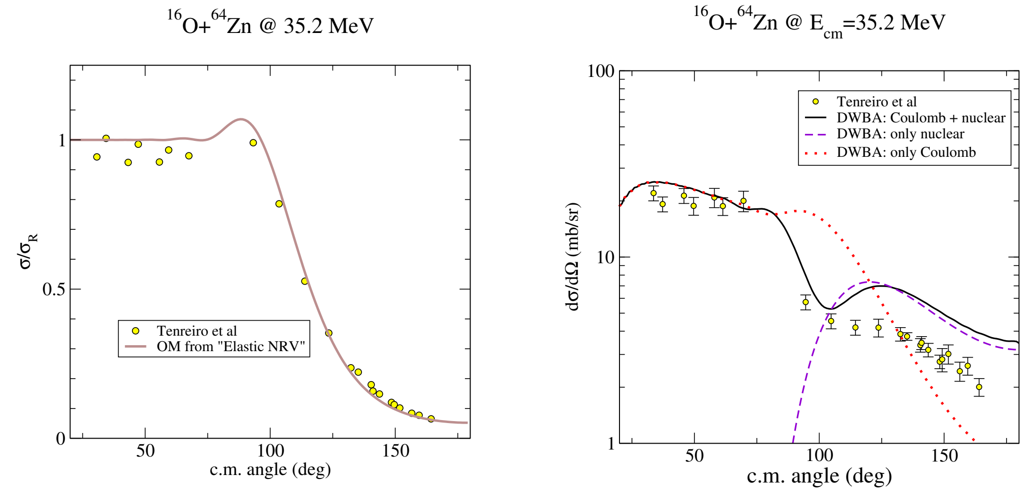

Figure 2: Left: Elastic differential cross section (relative to Rutherford cross section) for $E_{\text{c.m.}} = 35,2$ MeV. Right: inelastic differential cross section.

- The Coulomb excitation mechanism is dominant at the smaller scattering angles, whereas the nuclear excitation dominates the larger scattering angles. This is a direct consequence of the "long-range" versus "short-range" nature of these interactions.
- Around $\theta_{\text{c.m.}} \approx 100°$ the Coulomb and nuclear cross sections are of similar magnitude, and hence the interference effect between both contributions is apparent at these angles. Moreover, we see that this interference is of destructive nature.