Models for nuclear reactions with weakly-bound systems

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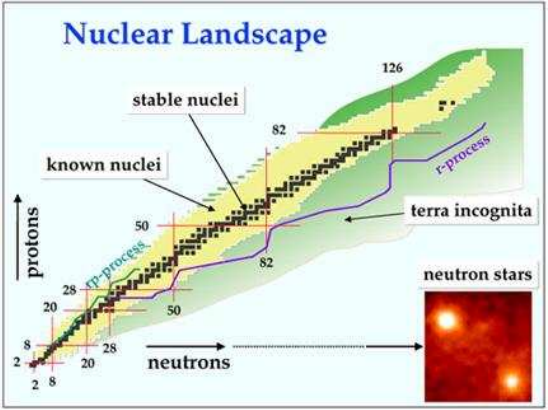
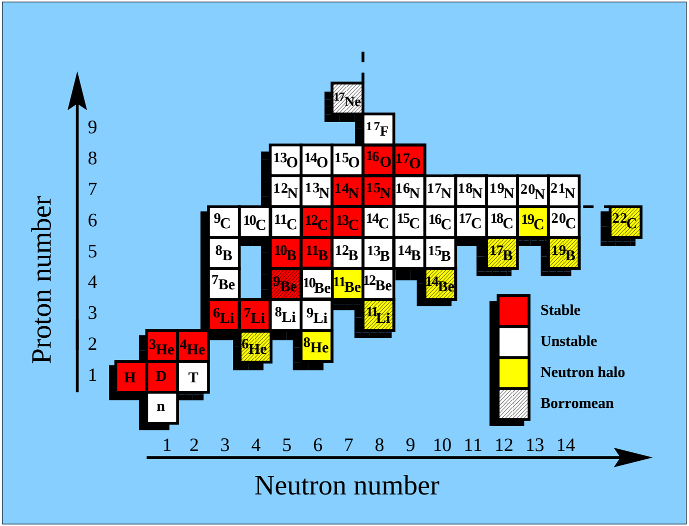
Unstable nuclei and the limits of stability



Note that:

- Not all unstable nuclei are weakly-bound.
- There are weakly-bound nuclei which are not unstable (eg. deuteron).

Light exotic nuclei: halo nuclei and Borromean systems



Light exotic nuclei: halo nuclei and Borromean systems

- Radioactive nuclei: they typically decay by β emission.

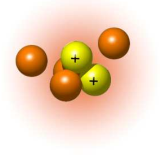
  E.g.: $^6\text{He} \xrightarrow{\beta^-} {^6\text{Li}} \quad (\tau_{1/2} \simeq 807 \text{ ms})$

- Weakly bound: typical separation energies are around 1 MeV or less.

- Spatially extended

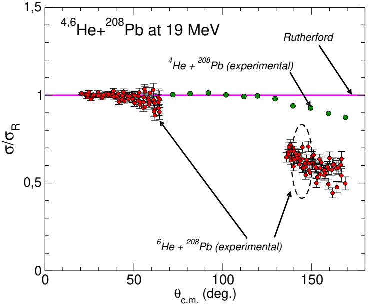
- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.

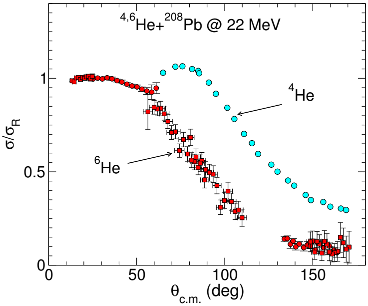
- Borromean nuclei: Three-body systems with no bound binary sub-systems.



Signatures of weakly-bound nuclei in reaction observables

Elastic scattering: Rutherford experiment...100 years later

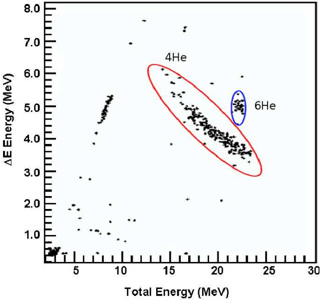


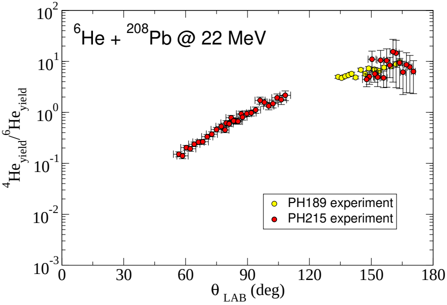


- ^{4}He follows Rutherford formula at 19 MeV but not at 22 MeV.
- ^{6}He drastically departs from Rutherford formula at both energies!

Inclusive breakup cross sections

$$\boxed{{}^{6}\mathrm{He} + {}^{208}\mathrm{Pb} \rightarrow \alpha + \mathrm{X}}$$





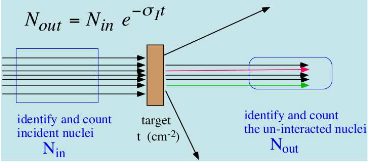
⇨At large angles, there are more α's than ^{6}He (elastic) !

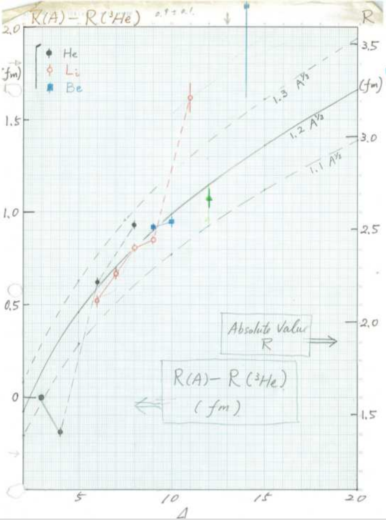
⇨What are the mechanisms behind the α producion and how can we compute it?

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies are proportional to the size of the colliding nuclei.

$$\boxed{\sigma_I \simeq \pi(R_p + R_t)^2}$$



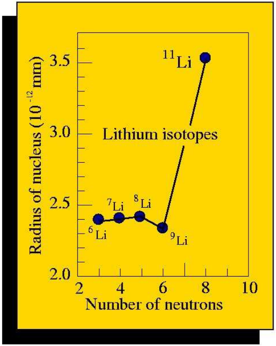


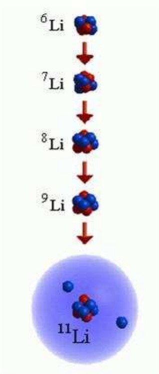
From I. Tanihata

High-energy interaction cross sections with light targets

Interaction cross sections of nuclei on light targets and high energies (hundreds MeV/nucleon) are proportional to the size of the colliding nuclei.

$$\boxed{\sigma_I \simeq \pi(R_p + R_t)^2}$$

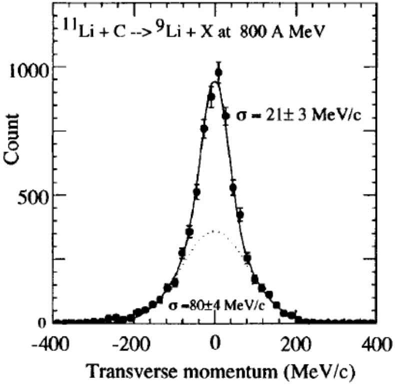




Tanihata *et al*, PRL55, 2676 (1985)

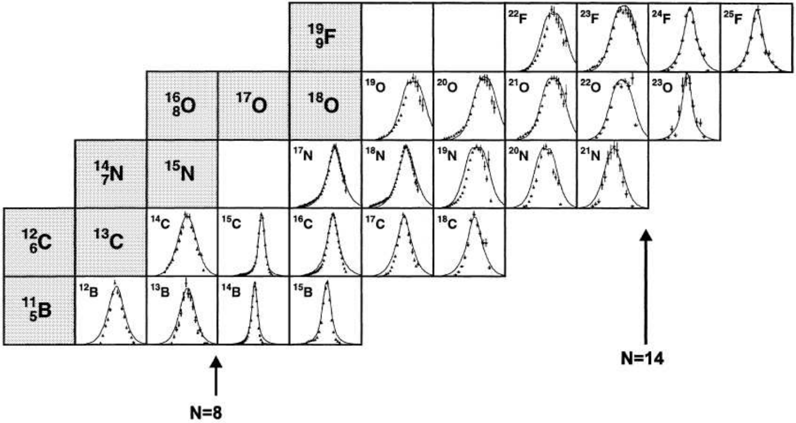
High-energy interaction cross sections with light targets

Momentum distributions in breakup reactions



☞ *A narrow momentum distribution is a signature of an extended spatial distribuion*

High-energy interaction cross sections with light targets



Modelling nuclear reactions

Why reaction theory is important?

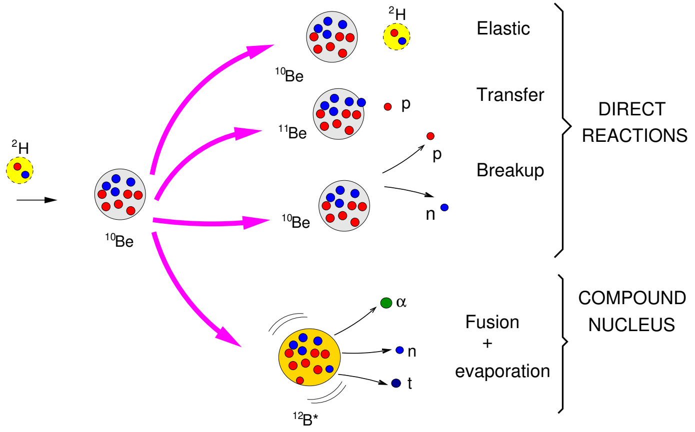
- Reaction theory provides the necessary framework to extract meaningful structure information from measured cross sections and also permits the understanding of the dynamics of nuclear collisions.

- The many-body scattering problem is not solvable in general, so specific models tailored to specific types of reactions are used (elastic, breakup, transfer, knockout...) each of them emphasizing some particular degrees of freedom.

- In particular, exotic nuclei close to driplines are usually weakly-bound and breakup (coupling to the continuum) is important and must be taken into account in the reaction model.

- Few-body models provide an appealing simplification of this complicated problem.

Direct and compound nucleus processes



Direct versus compound reactions

DIRECT: elastic, inelastic, transfer,...

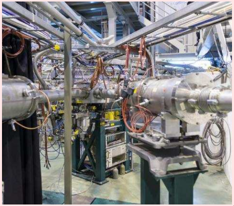
- "fast" collisions (10^{-21} s).
- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

COMPOUND: complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- "loss of memory" $\Rightarrow$ almost symmetric distributions forward/backward

Linking theory with experiments: the cross section

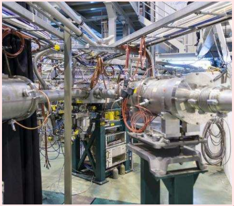
EXPERIMENT



THEORY
$(H\Psi = E\Psi)$

Linking theory with experiments: the cross section

EXPERIMENT

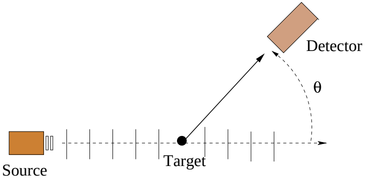


THEORY
$(H\Psi = E\Psi)$

CROSS SECTIONS

$$\frac{d\sigma}{d\Omega}, \frac{d\sigma}{dE}, etc$$

Experimental cross section



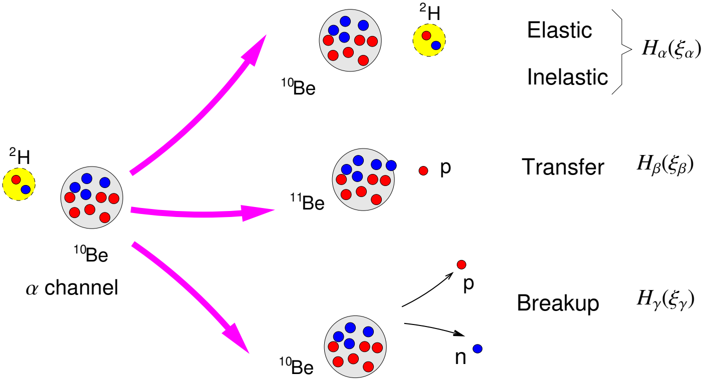
$$\Delta I = I_0 \, n_t \, \frac{d\sigma}{d\Omega} \Delta\Omega$$

- ΔI: detected particles per unit time in $\Delta\Omega$
- I_0: incident particles per unit time
- n_t: number of target nuclei per unit surface
- $\Delta\Omega$: solid angle of detector
- $d\sigma/d\Omega$: differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{\text{flux of scattered particles through } dA = r^2 d\Omega}{\text{incident flux}}$$

Projectile and target internal Hamiltonians

- Mass partitions: $\alpha, \beta, \ldots$
- Internal proj. + target Hamiltonians: $H_\alpha(\xi_\alpha) \equiv H_p(\xi_p) + H_t(\xi_t)$
- Internal states: $[H_\alpha(\xi_\alpha) - \varepsilon_\alpha]\Phi_\alpha(\xi_\alpha) = 0 \quad \{\varepsilon_\alpha\}$ = excitation energies
- Different mass partitions have different Hamiltonians: $H_\alpha(\xi_\alpha)$, $H_\beta(\xi_\beta)$, etc



Model Hamiltonian and model wavefunction

Full Hamiltonian

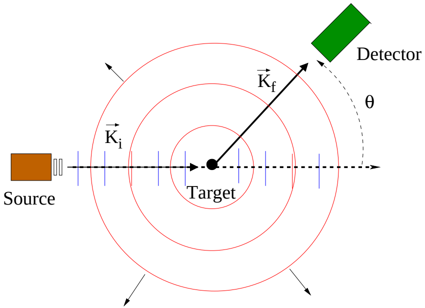
$$\boxed{H = \hat{T}_{\mathbf{R}} + H_p(\xi_p) + H_t(\xi_t) + V(\mathbf{R}, \xi_p, \xi_t)}$$

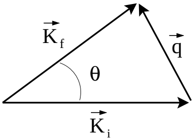
- $\hat{T}_{\mathbf{R}}$: proj.–target kinetic energy
- $H_p(\xi_p)$: projectile Hamiltonian
- $H_t(\xi_t)$: target Hamiltonian
- $V(\mathbf{R}, \xi_p, \xi_t)$: projectile–target interaction

Time-independent Schrodinger equation:

$$\boxed{[H - E]\Psi(\mathbf{R}, \xi_p, \xi_t) = 0}$$

Scattering wavefunction





Among the many mathematical solutions of $[H - E]\Psi = 0$ we are interested in those behaving asymptotically as:

$$\boxed{\Psi^{(+)}_{\mathbf{K}_\alpha} \to \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha \cdot \mathbf{R}_\alpha} + (\text{outgoing spherical waves in } \alpha, \beta, \ldots)}$$

Scattering amplitude and cross sections

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \xrightarrow{R_\alpha \gg} \Phi_\alpha(\xi_\alpha) e^{i\mathbf{K}_\alpha \cdot \mathbf{R}_\alpha} + \Phi_\alpha(\xi_\alpha) f_{\alpha,\alpha}(\theta) \frac{e^{iK_\alpha R_\alpha}}{R_\alpha} \qquad \text{(elastic)}$$

$$+ \sum_{\alpha' \neq \alpha} \Phi_{\alpha'}(\xi_\alpha) f_{\alpha',\alpha}(\theta) \frac{e^{iK_{\alpha'} R_\alpha}}{R_\alpha} \qquad \text{(inelastic)}$$

$$\Psi^{(+)}_{\mathbf{K}_\alpha} \xrightarrow{R_\beta \gg} \sum_\beta \Phi_\beta(\xi_\beta) f_{\beta,\alpha}(\theta) \frac{e^{iK_\beta R_\beta}}{R_\beta} \qquad \text{(transfer)}$$

Cross sections:

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{\alpha\to\beta} = \frac{K_\beta}{K_\alpha} \left| f_{\beta,\alpha}(\theta) \right|^2} \qquad E = \frac{\hbar^2 K_\alpha^2}{2\mu_\alpha} + \varepsilon_\alpha = \frac{\hbar^2 K_\beta^2}{2\mu_\beta} + \varepsilon_\beta$$

$f_{\beta,\alpha}$ is called scattering amplitude

Ideally, the strategy would be:

1. Choose structure model for $H_\alpha(\xi)$
2. Compute $\Psi^{(+)}$ by solving $[H - E]\Psi^{(+)} = 0$
3. Consider the limit $R \gg$ of $\Psi^{(+)}$
4. Project it on the desired final state to extract the scattering amplitude:

$$(\Phi_{\alpha'}(\xi_\alpha)|\Psi^{(+)}\rangle = f_{\alpha',\alpha}(\theta)\frac{e^{iK_{\alpha'}R_\alpha}}{R_\alpha}$$

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But...

- Ψ is a solution of a complicated many-body problem, not solvable in most cases.
- The number of accesible channels and states can be huge.

⇨*So, in practice, we will be happy with an approximation of Ψ (or $f(\theta)$) in a restricted modelspace*

Defining our model space: Feshbach formalism

- Divide the full space into two groups: P and Q
  - ⇨ P: channels of interest
  - ⇨ Q: remaining channels
- Write $\Psi = \Psi_P + \Psi_Q$

$$\boxed{\begin{aligned}(E - H_{PP})\Psi_P &= H_{PQ}\Psi_Q \\ (E - H_{QQ})\Psi_Q &= H_{QP}\Psi_P\end{aligned}} \qquad (H_{PP} = PHP, H_{PQ} = PHQ, \text{etc})$$

- Eliminate (formally) Ψ_Q:

$$\boxed{\underbrace{\left[H_{PP} + H_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}H_{QP}\right]}_{H_{\rm eff}}\Psi_P = E\Psi_P}$$

- $H_{\rm eff}$ too complicated (complex, energy dependent, non-local) ⇒ needs to be replaced by a simpler Hamiltonian:

$$H_{\rm eff} \longrightarrow H_{\rm model} \quad \text{(complex, energy dependent)}$$

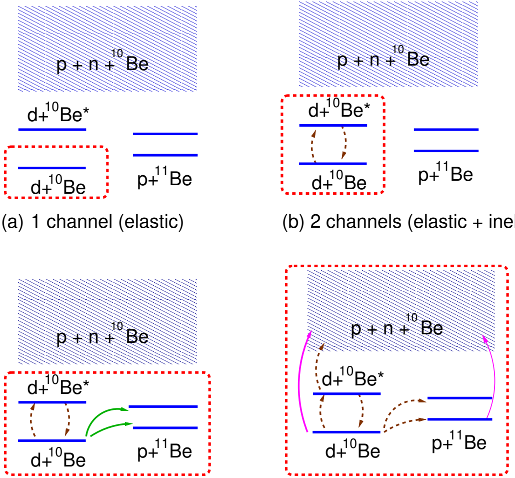
Strategy for reaction calculaions

We need to make a choice for:

1. Modelspace: what channels are to be included?
2. Structure model: for projectile and target

   (Microscopic, collective, cluster...)
3. Reaction formalism

Choice of the modespace: the d+^{10}Be example



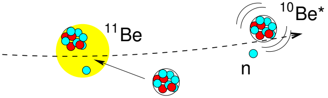
(a) 1 channel (elastic)

(b) 2 channels (elastic + inelastic)

(c) elastic + inelastic + transfer

(d) elastic + inelastic + transfer + breakup

Choice of structure model: from the many-body problem to the few-body picture

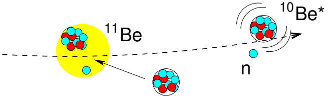


Microscopic models

- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

Choice of structure model: from the many-body problem to the few-body picture

Microscopic models



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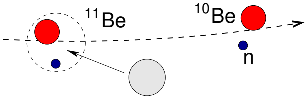
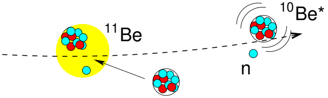
Inert cluster models



- ✖ Ignores cluster excitations (only few-body d.o.f).
- ✖ Phenomenological inter-cluster interactions (aprox. Pauli).
- ✔ Exactly solvable (in some cases).
- ✔ Achieved for 3-body and 4-body (eg. coupled-channels, Faddeev).

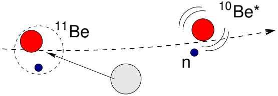
Choice of structure model: from the many-body problem to the few-body picture

Microscopic models



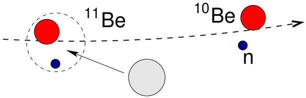
- ✔ Fragments described microscopically
- ✔ Realistic NN interactions (Pauli properly accounted for)
- ✖ Numerically demanding / not simple interpretation.

Non-inert-core few-body models



- ✔ Few-body + some relevant collective d.o.f.
- ✔ Pauli approximately accounted for.
- ✔ Achieved for 3-body problems (coupled-channels, Faddeev).

Inert cluster models



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- ✔ Exactly solvable (in some cases).
- ✔ Achieved for 3-body and 4-body (eg. coupled-channels, Faddeev).

Single-channel scattering: optical model potential

- P space represents just the ground state of projectile and target
- Wavefunction:

$$\Psi = \underbrace{\Psi_P}_{\text{elastic}} + \underbrace{\Psi_Q}_{\text{non-elastic}}$$

- Schrodinger equation in modelspace:

$$\boxed{\left[T + H_\alpha(\xi_\alpha) + \mathcal{V}\right]\Psi_P = E\Psi_P}$$

$$\boxed{\mathcal{V} = \underbrace{V_{PP}}_{\text{Bare interaction}} + \underbrace{V_{PQ}\frac{1}{E - H_{QQ} + i\epsilon}V_{QP}}_{\text{“Polarization” potential}} \equiv V_{\text{bare}} + V_{\text{pol}}}$$

- $\mathcal{V}$ too complicated $\Rightarrow$ usually replaced by some phenomenological (complex) potential $U(\mathbf{R})$

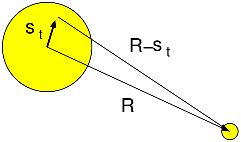
Microscopic folding model for $\mathcal{V}$

Start from some (effective) nucleon-nucleon potential v_{NN} (JLM, M3Y, etc):

1. Single-folding potential:

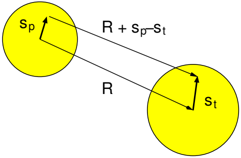
$$V(\mathbf{R}) = \int \rho_t(\mathbf{s}_t) v_{NN}(|\mathbf{R} - \mathbf{s}|) d\mathbf{s}$$

☞ $\rho_t(\mathbf{s}_t)$=target g.s. density.



2. Double-folding potential:

$$V(\mathbf{R}) = \int \rho_p(\mathbf{s}_p)\rho_t(\mathbf{s}_t) v_{NN}(|\mathbf{R} + \mathbf{s}_p - \mathbf{s}_t|) d\mathbf{s}_p d\mathbf{s}_t$$



- ➩ If ρ_p and ρ_t are g.s. densities, $V(\mathbf{R})$ accounts only for the bare potential (V_{PP}) (P-space part) and ignores the effect of non-elastic channels.
- ➩ A model for V_{pol} must be supplied.
- ➩ If v_{NN} is real, $V(\mathbf{R})$ is also real.

Phenomenological optical model

Effective potential: $\mathcal{V} \approx U(R) = U_{\rm nuc}(R) + U_{\rm coul}(R)$

- Coulomb potential: charge sphere distribution

$$\boxed{U_{\rm coul}(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c}\left(3 - \frac{R^2}{R_c^2}\right) & \text{if } R \le R_c \\ \frac{Z_1 Z_2 e^2}{R} & \text{if } R \ge R_c \end{cases}}$$

- Nuclear potential (complex): Eg. Woods-Saxon parametrization

$$\boxed{U_{\rm nuc}(R) = V(r) + iW(r) = -\frac{V_0}{1+\exp\left(\frac{R-R_0}{a_0}\right)} - i\,\frac{W_0}{1+\exp\left(\frac{R-R_i}{a_i}\right)}}$$

⇨ Popular parametrization: $R_0 = r_0(A_p^{1/3} + A_t^{1/3})$ (r_0=reduced radius)

⇨ For "normal" nuclei:

- $r_0 \approx r_0 \sim 1.1 - 1.4$ fm
- $a_0 \approx a_i \sim 0.5 - 0.7$ fm

Elastic scattering within the optical model

- Effective Hamiltonian:

$$H = T_{\mathbf{R}} + H_\alpha(\xi_\alpha) + U(\mathbf{R}) \qquad (U(\mathbf{R}) \text{ complex!})$$

- $U(\mathbf{R})$ independent of $\{\xi_\alpha\}$

$$\Psi^{(+)}_{\mathbf{K}}(\xi_\alpha, \mathbf{R}) = \Phi_0(\xi_\alpha)\chi^{(+)}_0(\mathbf{K}, \mathbf{R})$$

- Schrödinger equation:

$$\boxed{[T_{\mathbf{R}} + U(\mathbf{R}) - E_\alpha]\chi^{(+)}_0(\mathbf{K}, \mathbf{R}) = 0} \qquad \left(E_\alpha = E - \varepsilon_\alpha = \frac{\hbar^2 K^2}{2\mu}\right)$$

- Boundary condition:

$$\chi^{(+)}_0(\mathbf{K}, \mathbf{R}) \to e^{i\mathbf{K}\cdot\mathbf{R}} + f(\theta)\frac{e^{iKR}}{R}$$

Partial wave decomposition

- For a central potential ($U(\mathbf{R}) = U(R)$):

$$\boxed{\chi_0^{(+)}(\mathbf{K},\mathbf{R}) = \frac{1}{KR}\sum_{\ell m} i^\ell (2\ell+1)\chi_\ell(K,R)P_\ell(\cos\theta)} \qquad (\theta = \hat{R}\cdot\hat{K})$$

- $\chi_\ell(K,R)$ obtained from:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{R^2} + U(R) - E_0\right]\chi_\ell(K,R) = 0.$$

- For $U(R) = 0$, $\chi_0^{(+)}(\mathbf{K},\mathbf{R})$ must reduce to the plane wave:

$$\boxed{e^{i\mathbf{K}\cdot\mathbf{R}} = \frac{1}{KR}\sum_{\ell} i^\ell (2\ell+1)F_\ell(KR)P_\ell(\cos\theta)}$$

➯ So, for $U = 0 \Rightarrow \chi_\ell(K,R) = F_\ell(KR) = (KR)j_\ell(KR) \to \sin(KR - \ell\pi/2)$

Asymptotic solution for the case $U(R) \neq 0$

- For $R \gg \Rightarrow U(R) = 0 \Rightarrow \chi_\ell(K,R)$ will be a combination of F_ℓ and G_ℓ

$$\boxed{F_\ell(KR) \to \sin(KR - \ell\pi/2); \qquad G_\ell(KR) \to \cos(KR - \ell\pi/2)}$$

or their *outgoing/ingoing* combinations:

$$\boxed{H^{(\pm)}(KR) \equiv G_\ell(KR) \pm iF_\ell(KR) \to e^{\pm i(KR - \ell\pi/2)}}$$

- The physical solution is determined by the known boundary conditions:

$$\begin{array}{lllll}
 & \chi_0^{(+)}(\mathbf{KR}) & \to & e^{i\mathbf{K}\cdot\mathbf{R}} & + \; f(\theta)\dfrac{e^{iKR}}{R} \\
 & \Downarrow & & \Downarrow & \quad \Downarrow \\
U = 0 & \chi_\ell(KR) & \to & F_\ell(KR) & + \; 0 \\
U \neq 0 & \chi_\ell(KR) & \to & F_\ell(KR) & + \; T_\ell H^{(+)}(KR)
\end{array}$$

☞ The coefficients T_ℓ are called transition matrix elements.

Numerical procedure

1. Fix a *matching radius*, R_m, such that $U(R_m) \approx 0$
2. Integrate $\chi_\ell(R)$ from $R = 0$ up to R_m, starting with the condition:

$$\lim_{R\to 0} \chi_\ell(K,R) = 0$$

3. At $R = R_m$ impose the boundary condition:

$$\chi_\ell(K,R) \to F_\ell(KR) + T_\ell H_\ell^{(+)}(KR)$$
$$= \frac{i}{2}[H_\ell^{(-)}(KR) - S_\ell H_\ell^{(+)}(KR)]$$

☞ S_ℓ=reflection coefficient (S-matrix)

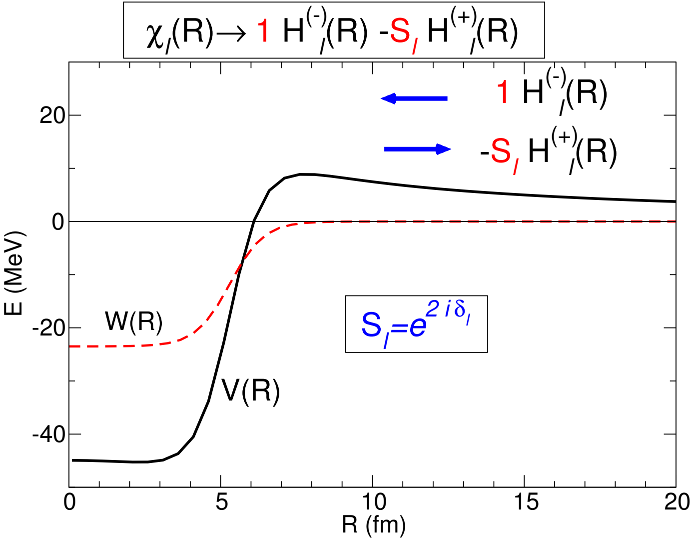
4. Phase-shifts:

$$\boxed{S_\ell = 1 + 2iT_\ell \equiv e^{i2\delta_\ell}} \qquad \boxed{T_\ell = e^{i\delta_\ell}\sin(\delta_\ell)}$$

The S-matrix and phase-shifts

- S_ℓ =coefficient of the outgoing wave for partial wave ℓ.
- $|S_\ell|^2$ is the *survival* probability for the partial wave ℓ:
  - U real $\Rightarrow |S_\ell| = 1 \Rightarrow \delta_\ell$ real
  - U complex $\Rightarrow |S_\ell| < 1 \Rightarrow \delta_\ell$ complex
- Sign of $Re[\delta]$:
  - $\delta > 0 \Rightarrow$ attractive potential
  - $\delta < 0 \Rightarrow$ repulsive potential
  - $\delta = 0$ $(S_\ell = 1) \Rightarrow$ no potential $(U(R) = 0)$
- For $\ell \gg \Rightarrow S_\ell \to 1$

The S-matrix and phase-shifts



The scattering amplitude

- Replace the asymptotic $\chi_\ell(K,R)$ in the general expansion:

$$\boxed{\begin{aligned}\chi_0^{(+)}(\mathbf{K},\mathbf{R}) &\to \frac{1}{KR}\sum_\ell i^\ell(2\ell+1)\left\{F_\ell(KR)+T_\ell H_\ell^{(+)}(KR)\right\}P_\ell(\cos\theta)\\ &= e^{i\mathbf{K}\cdot\mathbf{R}}+\frac{1}{K}\sum_\ell(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\frac{e^{iKR}}{R}\end{aligned}}$$

- The scattering amplitude is the coefficient of e^{iKR}/R:

$$\boxed{\begin{aligned}f(\theta) &= \frac{1}{K}\sum_\ell(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\\ &= \frac{1}{2iK}\sum_\ell(2\ell+1)(S_\ell-1)P_\ell(\cos\theta).\end{aligned}}$$

- Elastic cross section:

$$\frac{d\sigma}{d\Omega}=|f(\theta)|^2.$$

Coulomb plus nuclear case

Radial equation:

$$\boxed{\left[\frac{d^2}{dR^2} + K^2 - \frac{2\eta K}{R} + \frac{2\mu}{\hbar^2}U(R) + \frac{\ell(\ell+1)}{R^2}\right]\chi_\ell(K,R) = 0}$$

$$\boxed{\eta = \frac{Z_p Z_t e^2}{\hbar v} = \frac{Z_p Z_t e^2 \mu}{\hbar^2 K}}$$

(Sommerfeld parameter)

Asymptotic condition:

$$\chi^{(+)}(\mathbf{K},\mathbf{R}) \to e^{i[\mathbf{K}\cdot\mathbf{R}+\eta\log(kR-\mathbf{K}\cdot\mathbf{R})]} + f(\theta)\frac{e^{i(KR-\eta\log 2KR)}}{R}$$

$$\boxed{\begin{aligned}\chi_\ell(K,R) &\to e^{i\sigma_\ell}\left[F_\ell(\eta,KR) + T_\ell H_\ell^{(+)}(\eta,KR)\right] \\ &= (i/2)e^{i\sigma_\ell}\left[H_\ell^{(-)}(\eta,KR) - S_\ell H_\ell^{(+)}(\eta,KR)\right]\end{aligned}}$$

- ☞ $\sigma_\ell(\eta)$=Coulomb phase shift
- ☞ $F_\ell(\eta,KR)$=regular Coulomb wave
- ☞ $H_\ell^{(\pm)}(\eta,KR)$=outgoing/ingoing Coulomb wave

Coulomb plus nuclear case: scattering amplitude

Total scattering amplitude:

$$\boxed{f(\theta) = f_C(\theta) + \frac{1}{2iK}\sum_\ell (2\ell+1) e^{2i\sigma_\ell}(S_\ell - 1) P_\ell(\cos\theta)}$$

☞ $f_C(\theta)$ is the amplitude for pure Coulomb:

$$\boxed{\frac{d\sigma_R}{d\Omega} = |f_C(\theta)|^2 = \frac{\eta^2}{4K^2 \sin^4(\frac{1}{2}\theta)} = \left(\frac{Z_p Z_t e^2}{4E}\right)^2 \frac{1}{\sin^4(\frac{1}{2}\theta)}}$$

Integrated cross sections

- Total elastic cross section (uncharged particles!)

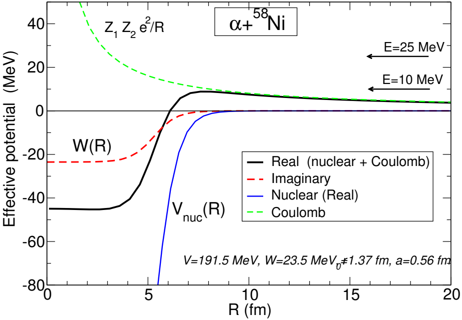
$$\boxed{\sigma_{el} = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)|1 - S_\ell|^2}$$

- Total reaction cross section (loss of flux from elastic channel)

$$\boxed{\sigma_{reac} = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)(1 - |S_\ell|^2) = \frac{\pi}{K^2} \sum_{\ell} (2\ell + 1)|T_\ell|^2}$$

^{4}He+^{58}Ni example

Effective potential: $U(R) = U_{nuc}(R) + U_{coul}(R)$



☞ The maximum of $V_{\rm nuc}(R) + V_C(R)$ defines the Coulomb barrier. Approximately:

$$\boxed{R_b \simeq 1.44(A_p^{1/3} + A_t^{1/3})\ \text{fm}} \qquad \boxed{E_b \simeq \frac{Z_p Z_t e^2}{R_b} = \frac{Z_p Z_t}{(A_p^{1/3} + A_t^{1/3})}\ \text{MeV}}$$

Effect of indicent energy

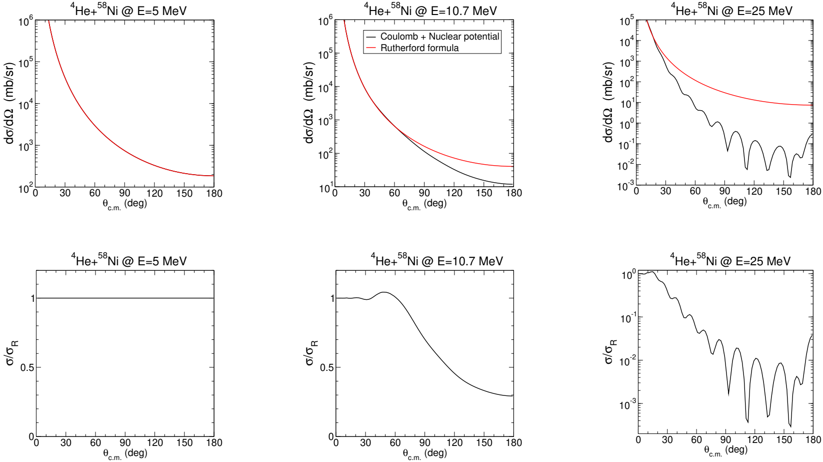
☞ Depending on the bombarding energy E and the charges of the interacting nuclei, we observe different patterns of elastic scattering.

☞ For medium/heavy systems, this can be characterized in terms of the Coulomb (or Sommerfeld) parameter:

$$\eta = \frac{Z_p Z_t e^2}{4\pi\epsilon_0 \hbar v}$$

- E well above the Coulomb barrier ($\eta \lesssim 1$) $\Rightarrow$ Fraunhofer scattering
- E around the Coulomb barrier ($\eta \gg 1$) $\Rightarrow$ Fresnel scattering
- E well below the Coulomb barrier ($\eta \ggg 1$) $\Rightarrow$ Rutherford scattering

Elastic scattering: energy dependence



Rutherford scattering **Fresnel** **Fraunhöfer**

Effect of indicent energy (cont.)

Example: ^{4}He+^{58}Ni at E=5, 10.7, 25 and 50 MeV

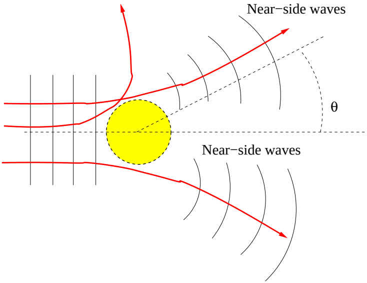
☞ **Coulomb barrier:** $R_b \simeq 7.8$ fm ; $V_b \simeq 10.2$ MeV

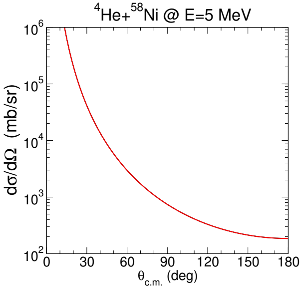
$E_{\rm lab}$ (MeV)	η	K (fm^{-1})	$\lambda\!\!\!^{-} = 1/K$ (fm)	$2a_0$(*) (fm)
5	7.95	0.920	1.087	17.2
10.7	5.62	1.34	0.746	8.06
25	3.55	2.06	0.485	3.44
50	2.51	2.91	0.343	1.69

(*) classical distance of closest approach in head-on collision.

- $\eta \ggg 1$: Rutherford scattering: $\sigma(\theta) \propto 1/\sin^4(\theta/2)$
- $\eta \gg 1$: Fresnel scattering (rainbow)
- $\eta \leq 1$: Fraunhofer scattering (oscillatory behaviour):

Rutherford scattering



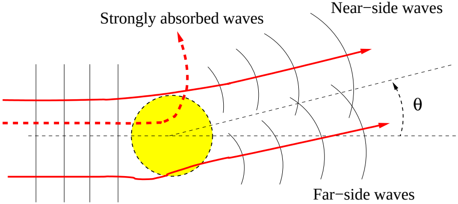


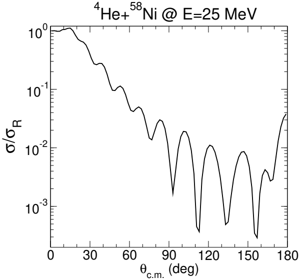
- Bombarding energy well below the Coulomb barrier
- Purely Coulomb potential ($\eta \ggg 1$)
- Obeys Rutherford law:

$$\frac{d\sigma}{d\Omega} = \frac{Z_p Z_t e^2}{4E} \frac{1}{\sin^4(\theta/2)}$$

.

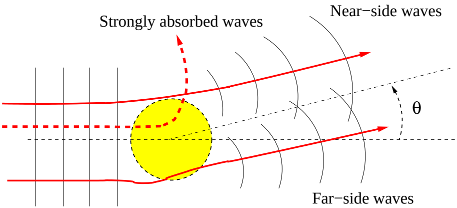
Fraunhofer scattering

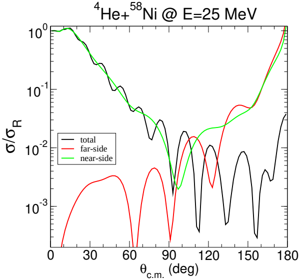




- Bombarding energy well above Coulomb barrier
- Coulomb weak ($\eta \lesssim 1$)
- Nearside/farside interference pattern (difraccion)

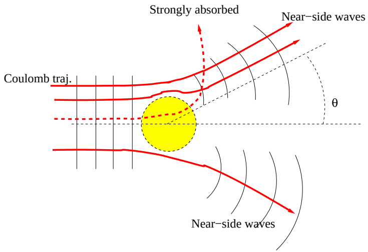
Fraunhofer scattering

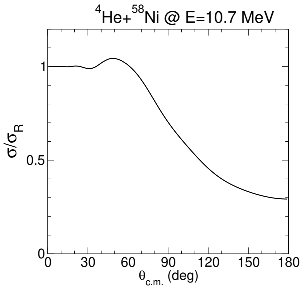




- Bombarding energy well above Coulomb barrier
- Coulomb weak ($\eta \lesssim 1$)
- Nearside/farside interference pattern (difracction)

Fresnel scattering

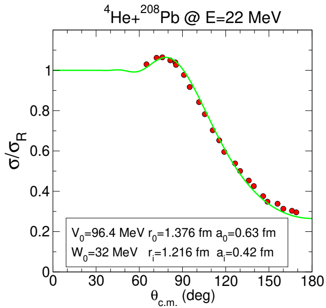


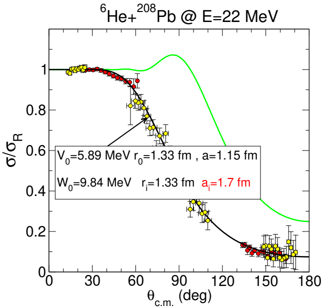


- Bombarding energy around or near the Coulomb barrier
- Coulomb strong ($\eta \gg 1$)
- 'Illuminated' region $\Rightarrow$ Coulomb + nuclear trajectories
  'Shadow' region $\Rightarrow$ strong absorption

Normal versus halo nuclei

How does the halo structure affect the elastic scatterig?

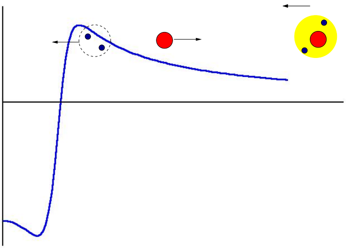


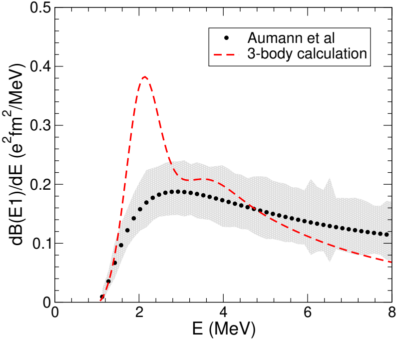


- ^{4}He+^{208}Pb shows typical Fresnel pattern and "standard" optical model parameters
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section, suggesting that part of the incident flux goes to non-elastic channels (eg. breakup)

  Understanding and disentangling these non-elastic channels requires going beyond the optical model (eg. coupled-channels method ⇒ next lectures)

Origin of the long-range absorption in ^{6}He

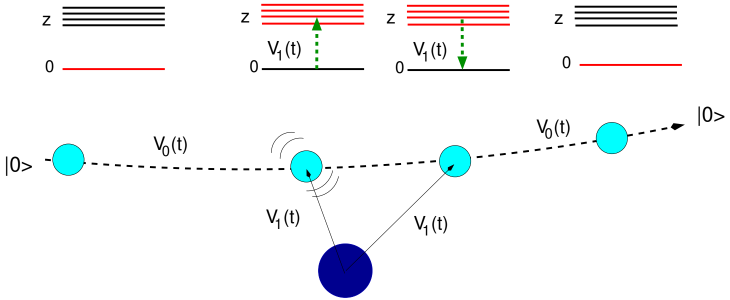




- The Coulomb force on the core induces a tidal force which may eventually break ^{6}He.
- From the structure point of view, this translates into a large $B(E1)$ strength near breakup threshold.

Second order perturbative amplitude

$$c_n^{(2)} = \sum_z \left(\frac{-i}{\hbar}\right)^2 \int_{-\infty}^{+\infty} dt \langle n|V_1(t)|z\rangle \exp\left\{\frac{i}{\hbar}(E_n - E_z)t\right\}$$
$$\times \int_{-\infty}^{t} dt' \langle z|V_1(t')|0\rangle \exp\left\{\frac{i}{\hbar}(E_z - E_0)t'\right\}$$



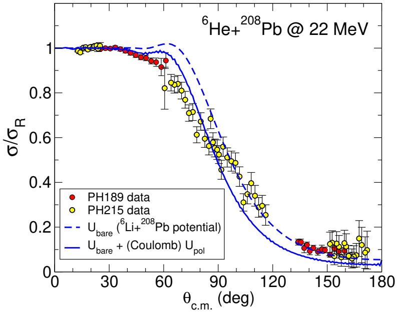
How does a weakly bound nucleus behaves in the field of a heavy target?

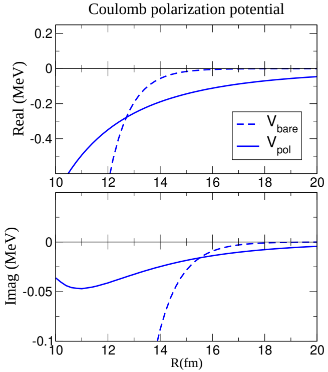
1. The strong Coulomb field will produce a polarization ("stretching") of the projectile, giving rise to a dipole contribution on the real potential:

$$V(R) \approx \frac{Z_1 Z_2 e^2}{R} - \alpha \frac{Z_1 Z_2 e^2}{2R^4}$$

2. The weakly bound nucleus can eventually break up, leading to a loss of flux of the elastic channel $\Rightarrow$ imaginary polarization potential.

The effect of E1 on elastic scattering of weakly-bound nuclei

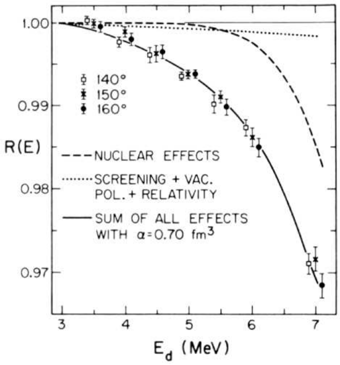


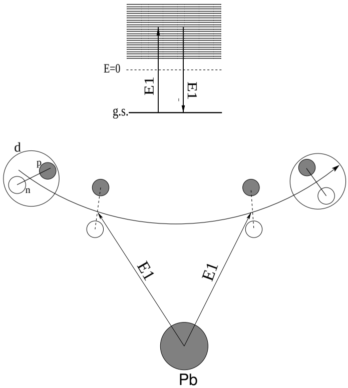


- E1 Coulomb couplings produces a sizable effect on the elastic cross section of neutron-halo nuclei (we have learnt something!) but...some additional physics is still missing.

Eg: deuteron polarizability from d+^{208}Pb:

☞ Adiabatic limit ($E_x \gg$): $V_{\rm pol}^{\rm dip} = -\alpha \frac{Z_1 Z_2 e^2}{2R^4}$



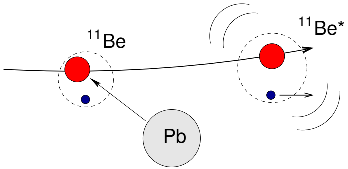


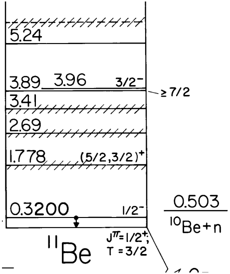
Rodning et al, PRL49, 909 (1982) ⇒ $\alpha = 0.70 \pm 0.05$ fm^3

Inelastic scattering

Inelastic scattering

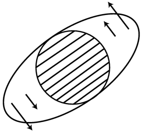
- Nuclei are not inert or *frozen* objects; they do have an internal structure of protons and neutrons that can be modified (excited) during the collision.
- Quantum systems exhibit, in general, an energy spectrum with bound and unbound levels.



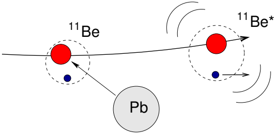


Models for inelastic excitations

1. COLLECTIVE: Involve a collective motion of several nucleons which can be interpreted macroscopically as rotations or surface vibrations of the nucleus.



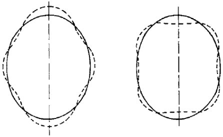
2. FEW-BODY/SIGLE-PARTICLE: Involve the excitation of a nucleon or cluster.



Types of collective excitations

The nucleons can move inside the nucleus in a coherent (collective) way.

1. Vibrations (spherical nuclei): small surface oscillations in shape.

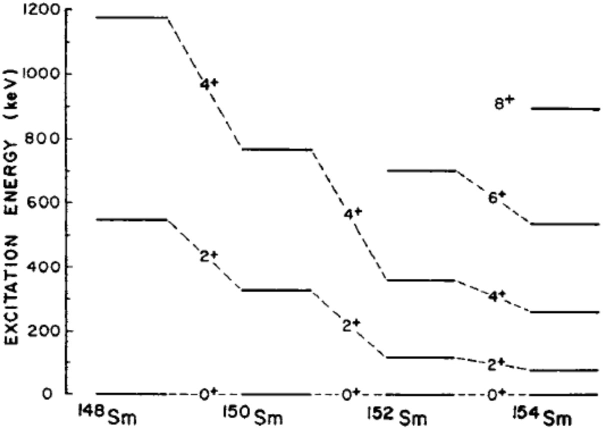


2. Rotations (non-spherical nuclei): permanent deformation.
3. Monopole (*breathing*) mode: oscillations in the size (radius).
4. Isovector excitations (protons and neutrons move out of phase) (eg. giant dipole resonance)

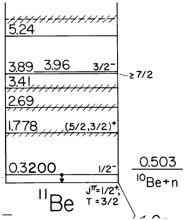
Types of collective excitations

☞ The type of collective motion is closely related to the kind of energy spectrum.

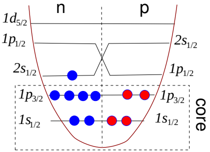
- Rotor: $E_J \propto J(J+1)$
- Vibrator: $E_J \approx n\hbar\omega$



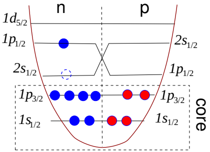
Microscopic description in the IPM: the ^{11}Be case



Ground state $(1/2^{+})$

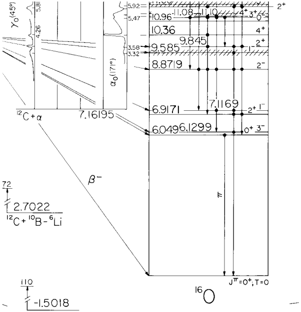


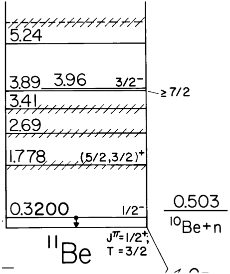
First excited state $(1/2^{-})$



Models for inelastic excitations

Microscopically, what we describe in both cases are quantum transitions between discrete or continuum states:





☞*Collective excitations can be regarded as a coherent superposition of many single-particle excitations.*

- By doing inelastic scattering experiments we *measure* the *response* of the nucleus to an external field (Coulomb, nuclear). This response is related to some structure property of the nucleus.

  Example: for a Coulomb field:

$$\boxed{B(E\lambda; i \to f) = \frac{1}{2I_i + 1} |\langle \Psi_f | \mathcal{M}(E\lambda) | \Psi_i \rangle|^2}$$

  where $\mathcal{M}(E\lambda, \mu)$ is the electric multipole operator:

$$\boxed{\mathcal{M}(E\lambda, \mu) \equiv e \sum_{i}^{Z_p} r_i^{\lambda} Y_{\lambda\mu}^{*}(\hat{r}_i)}$$

- The structure $\Psi_{i,f}$ can be described in a collective, few-body or microscopic model.

Multi-channel case: the coupled-channels method

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (eg. target).

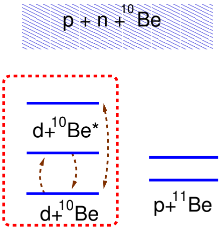
$$\boxed{H = T_R + h(\xi) + V(\mathbf{R}, \xi)}$$

- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the target (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the target.

$$\boxed{h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)}$$

- $V(\mathbf{R}, \xi)$: Projectile-target interaction.

Defining the modelspace: d+^{10}Be → d+^{10}Be* example



☞ P space composed by ground states (elastic channel) and some excited states (inelastic scattering)

Boundary conditions:

$$\Psi^{(+)}_{\mathbf{K}_0}(\mathbf{R},\xi) \xrightarrow{R\gg} \underbrace{e^{i\mathbf{K}_0\cdot\mathbf{R}}\phi_0(\xi)}_{\text{incident}} + \underbrace{f_{0,0}(\theta)\frac{e^{iK_0R}}{R}\phi_0(\xi)}_{\text{elastic}} + \underbrace{\sum_{n>0} f_{n,0}(\theta)\frac{e^{iK_nR}}{R}\phi_n(\xi)}_{\text{inelastic}}$$

Cross sections:

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{0\to n} = \frac{K_n}{K_0}|f_{n,0}(\theta)|^2$$

CC model wavefunction (target excitation)

We expand the total wave function in a subset of internal states (the P space):

$$\boxed{\Psi_{\rm model}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K}_0,\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{K}_n,\mathbf{R})}$$

Boundary conditions for the $\chi_n(\mathbf{R})$ (unknowns):

$$\chi_0^{(+)}(\mathbf{K}_0,\mathbf{R}) \to e^{i\mathbf{K}_0\cdot\mathbf{R}} + f_{0,0}(\theta)\frac{e^{iK_0R}}{R} \qquad \text{for n=0 (elastic)}$$

$$\chi_n^{(+)}(\mathbf{K}_n,\mathbf{R}) \to f_{n,0}(\theta)\frac{e^{iK_nR}}{R} \qquad \text{for n>0 (non-elastic)}$$

Calculation of $\chi_n^{(+)}(\mathbf{R})$: the coupled equations

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi^{(+)}_{\text{model}}(\mathbf{R}, \xi) = 0$$

- Multiply on the left by each $\phi_n(\xi)^*$, and integrate over $\xi \Rightarrow$ coupled channels equations for $\{\chi_n(\mathbf{R})\}$:

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Coupling potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\xi \phi_{n'}(\xi)^* V(\mathbf{R}, \xi)\phi_n(\xi)}$$

☞ *$\phi_n(\xi)$ will depend on the assumed structure model (collective, few-body, etc).*

Optical Model vs. Coupled-Channels method

Optical Model

- The Hamiltonian:
  $H = T_R + V(\mathbf{R})$
- Internal states: Just $\phi_0(\xi)$
- Model wavefunction:
  $\Psi_{\rm mod}(\mathbf{R}, \xi) \equiv \chi_0(\mathbf{K}, \mathbf{R})\phi_0(\xi)$
- Schrödinger equation:
  $[H - E]\chi_0(\mathbf{K}, \mathbf{R}) = 0$

Optical Model vs. Coupled-Channels method

Optical Model

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- Schrödinger equation:
  $[H-E]\chi_0(\mathbf{K},\mathbf{R}) = 0$

Coupled-channels method

- The Hamiltonian:
  $H = T_R + h(\xi) + V(\mathbf{R},\xi)$
- Internal states:
  $h(\xi)\phi_n(\xi) = \varepsilon_n\phi_n(\xi)$
- Model wavefunction:
  $\Psi_{\rm model}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{K},\mathbf{R}) + \sum_{n>0}\phi_n(\xi)\chi_n(\mathbf{K},\mathbf{R})$
- Schrödinger equation:

$$[H-E]\Psi_{\rm model}(\mathbf{R},\xi) = 0$$

$$\Downarrow$$

$$[E-\varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{K},\mathbf{R}) = \sum_{n'\neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{K},\mathbf{R})$$

A first-order formula for $f(\theta)$: the DWBA approximation

- Assume that we can write the p-t interaction as: $V(\mathbf{R},\xi) = V_0(R) + \Delta V(\mathbf{R},\xi)$
- Use central $V_0(R)$ part to calculate the (distorted) waves for p-t relative motion:

$$\left[\hat{T}_{\mathbf{R}} + V_0(R) - E_i\right]\chi_i^{(+)}(\mathbf{K}_i,\mathbf{R}) = 0 \qquad (E_i = E - \varepsilon_i)$$

$$\left[\hat{T}_{\mathbf{R}} + V_0(R) - E_f\right]\chi_f^{(+)}(\mathbf{K}_f,\mathbf{R}) = 0 \qquad (E_f = E - \varepsilon_f)$$

- In first order of $\Delta V(\mathbf{R},\xi)$ (DWBA) :

$$\boxed{f_{i\to f}^{\mathrm{DWBA}}(\theta) = -\frac{\mu}{2\pi\hbar^2}\int \chi_f^{(-)*}(\mathbf{K}_f,\mathbf{R})\,\Delta V_{if}(\mathbf{R})\,\chi_i^{(+)}(\mathbf{K}_i,\mathbf{R})\,d\mathbf{R}}$$

with the transition potential:

$$\boxed{\Delta V_{if}(\mathbf{R}) \equiv \int \phi_f^*(\xi)\,\Delta V(\mathbf{R},\xi)\,\phi_i(\xi)\,d\xi}$$

Physical interpretation of the DWBA method

- DWBA can be interpreted as a first-order approximation of a full coupled-channels calculation:

- The auxiliary potential U_β generating the entrance and exit distorted waves is usually chosen in order to reproduce the elastic scattering at the corresponding c.m. energy.

Models for inelastic scattering

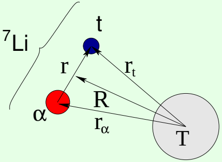
Inelastic scattering in a few-body model

- Some nuclei allow a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
- Projectile-target interaction:

$$V(\mathbf{R},\xi) \equiv V(\mathbf{R},\mathbf{r}) = U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)$$

Example: ^{7}Li=α+t

$$\mathbf{r}_\alpha = \mathbf{R} - \frac{m_t}{m_\alpha + m_t}\mathbf{r}; \quad \mathbf{r}_t = \mathbf{R} + \frac{m_\alpha}{m_\alpha + m_t}\mathbf{r}$$



Internal states:

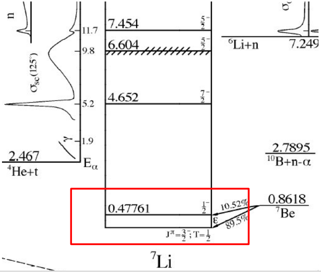
$$[T_\mathbf{r} + V_{\alpha-t}(\mathbf{r}) - \varepsilon_n]\phi_n(\mathbf{r}) = 0$$

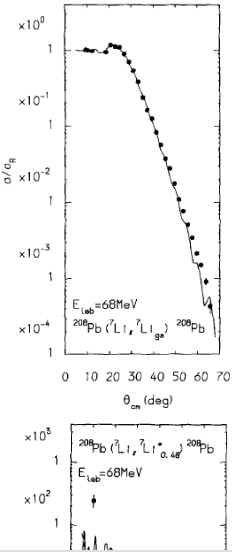
- Transition potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi^*_n(\mathbf{r})\left[U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)\right]\phi_{n'}(\mathbf{r})}$$

Example: ^{7}Li(α+t) +^{208}Pb at 68 MeV

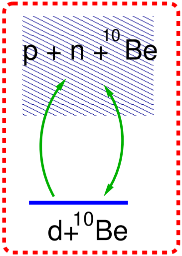
➩ CC calculation with 2 channels (3/2$^-$, 1/2$^-$) (Phys. Lett. 139B (1984) 150)





Inclusion of breakup channels: the CDCC method

Breakup modelspace



☞*We want to include explicitly in the modelspace the breakup channels of the projectile or target.*

The CC method for bound states

We need to incorporate explicitly in the Hamiltonian the internal structure of the nucleus being excited (eg. target).

$$\boxed{H = T_R + h(\xi) + V(\mathbf{R}, \xi)}$$

- T_R: Kinetic energy for projectile-target relative motion.
- $\{\xi\}$: Internal degrees of freedom of the target (depend on the model).
- $h(\xi)$: Internal Hamiltonian of the target.

$$\boxed{h(\xi)\phi_n(\xi) = \varepsilon_n \phi_n(\xi)}$$

- $V(\mathbf{R}, \xi)$: Projectile-target interaction

The CC method (continued): CC model wavefunction

We expand the total wave function in a subset of internal states (the P space):

$$\boxed{\Psi^{(+)}_{\rm model}(\mathbf{R},\xi) = \phi_0(\xi)\chi_0(\mathbf{R}) + \sum_{n>0}\phi_n(\xi)\chi_n(\mathbf{R})}$$

Boundary conditions for the $\chi_n(\mathbf{R})$ (unknowns):

$$\chi_0^{(+)}(\mathbf{R}) \to e^{i\mathbf{K}_0\cdot\mathbf{R}} + f_{0,0}(\theta)\frac{e^{iK_0R}}{R} \qquad \text{for n=0 (elastic)}$$

$$\chi_n^{(+)}(\mathbf{R}) \to f_{n,0}(\theta)\frac{e^{iK_nR}}{R} \qquad \text{for n>0 (non-elastic)}$$

The CC method (continued): calculation of $\chi_n^{(+)}(\mathbf{R})$; the coupled equations

- The model wavefunction must satisfy the Schrödinger equation:

$$[H - E]\Psi^{(+)}_{\rm model}(\mathbf{R}, \xi) = 0$$

- Projecting onto the internal states one gets a system of coupled-equations for the functions $\{\chi_n(\mathbf{R})\}$:

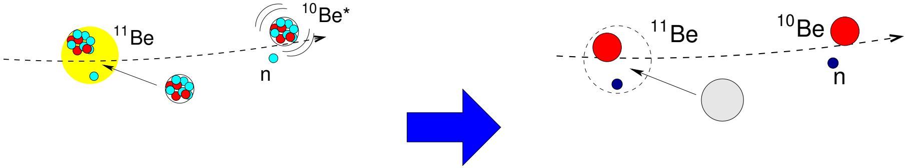
$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- The structure information is embedded in the coupling potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\xi \phi_{n'}(\xi)^* V(\mathbf{R}, \xi)\phi_n(\xi)}$$

☞ *$\phi_n(\xi)$ will depend on the structure model (collective, single-particle,etc).*

Choice of structure model: the few-body (cluster) case



$$\mathcal{V}_{pt} = \sum_{ij} V_{ij}(\mathbf{r}_{ij})$$

$$\mathcal{V}_{pt} = U_{ct}(\mathbf{r}_{ct}) + U_{nt}(\mathbf{r}_{nt})$$

- Effective three-body Hamiltonian:

$$\boxed{H = T_{\mathbf{R}} + h_r(\mathbf{r}) + U_{ct}(\mathbf{r}_{ct}) + U_{nt}(\mathbf{r}_{nt})}$$

- $U_{ct}(\mathbf{r}_{ct})$, $U_{nt}(\mathbf{r}_{nt})$ are optical potentials describing fragment-target elastic scattering (eg. target excitation is treated effectively, through absorption)

Inelastic scattering in a few-body model

- Some nuclei allow a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
- Projectile-target interaction:

$$V(\mathbf{R},\xi) \equiv V(\mathbf{R},\mathbf{r}) = U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)$$

- Transition potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi^*_n(\mathbf{r})\,[U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2)]\,\phi_{n'}(\mathbf{r})}$$

Inelastic scattering in a few-body model

- Some nuclei allow a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
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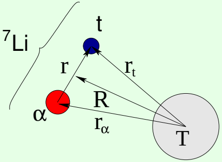
Example: ^{7}Li=α+t

$$\mathbf{r}_\alpha = \mathbf{R} - \frac{m_t}{m_\alpha + m_t}\mathbf{r}; \quad \mathbf{r}_t = \mathbf{R} + \frac{m_\alpha}{m_\alpha + m_t}\mathbf{r}$$

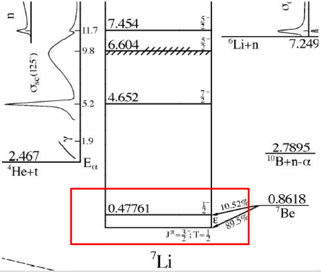


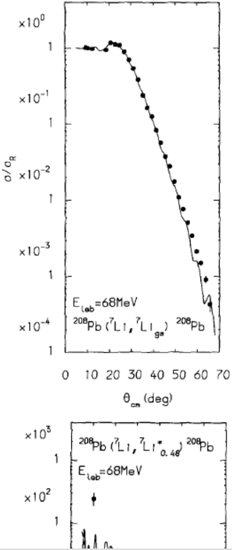
Internal states: (two-body cluster model)

$$[T_\mathbf{r} + V_{\alpha-t}(\mathbf{r}) - \varepsilon_n]\phi_n(\mathbf{r}) = 0$$

Example: ^{7}Li(α+t) +^{208}Pb at 68 MeV

➯ CC calculation with 2 channels ($3/2^-$, $1/2^-$) (Phys. Lett. 139B (1984) 150)

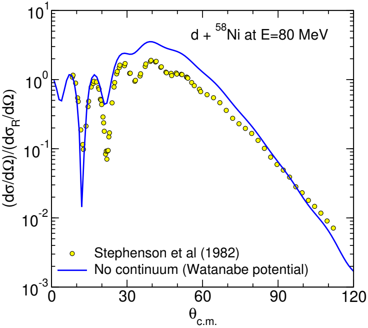


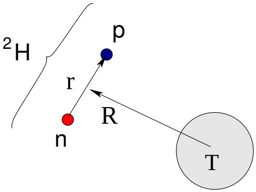


Application of the CC method to weakly-bound systems

Example: Three-body calculation (p+n+^{58}Ni) with Watanabe potential:

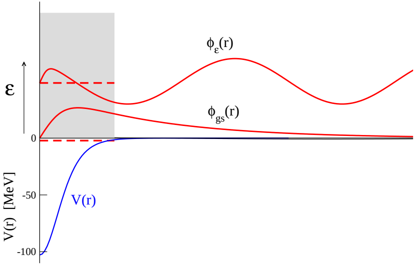
$$V_{dt}(\mathbf{R}) = \int d\mathbf{r}\phi^*_{gs}(\mathbf{r})\left\{V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})\right\}\phi_{gs}(\mathbf{r})$$





☞*Three-body calculations omitting breakup channels fail to describe the experimental data.*

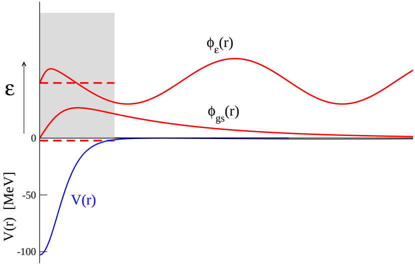
Bound versus scattering states



Continuum wavefunctions:

$$\boxed{\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \qquad \varepsilon = \frac{\hbar^2 k^2}{2\mu}}$$

Bound versus scattering states



Continuum wavefunctions:

$$\varphi_{k,\ell jm}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

$$\varepsilon = \frac{\hbar^2 k^2}{2\mu}$$

Unbound states are not suitable for CC calculations:

- They have a continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r) | u_{k',\ell sj}(r) \rangle \propto \delta(k - k')$

SOLUTION $\Rightarrow$ continuum discretization

The role of the continuum in the scattering of weakly bound nuclei

- Continuum discretization method proposed by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)].

PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

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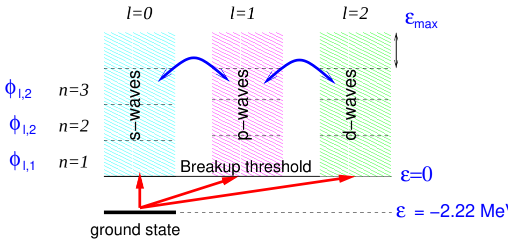
and Department of Physics, University of Surrey, Guildford, Surrey, England
(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R}-\frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R}+\frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transi-

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Continuum discretization for deuteron scattering



- Select a number of angular momenta ($\ell = 0, \ldots, \ell_{\max}$).
- For each ℓ, set a maximum excitation energy $\varepsilon_{\max}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\max}$ in a set of sub-intervals (*bins*).
- For each bin, calculate a representative wavefunction.

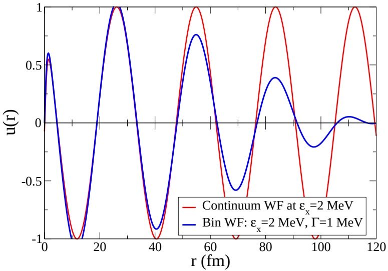
CDCC formalism: construction of the bin wavefunctions

Bin wavefunction:

$$\varphi^{[k_1,k_2]}_{\ell jm}(\mathbf{r}) = \frac{u^{[k_1,k_2]}_{\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm} \qquad [k_1, k_2] = \text{bin interval}$$

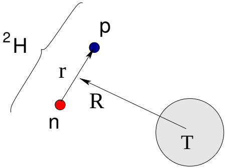
$$\boxed{u^{[k_1,k_2]}_{\ell sjm}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell sj}(r) dk}$$

- k: linear momentum
- $u_{k,\ell sj}(r)$: scattering states (radial part)
- $w(k)$: weight function



CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_{\mathbf{R}} + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:



$$\Psi^{(+)}(\mathbf{R},\mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

  - Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r}) = 0$

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\chi_n(\mathbf{R}) = \sum_{n'\neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

  - Transition potentials:

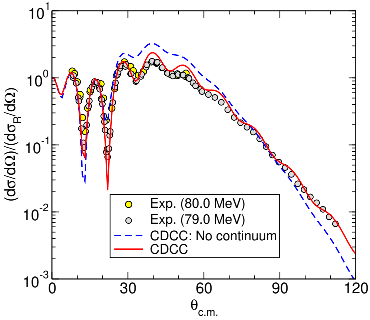
$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})\left[V_{pt}(\mathbf{R}+\frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R}-\frac{\mathbf{r}}{2})\right]\phi_{n'}(\mathbf{r})$$

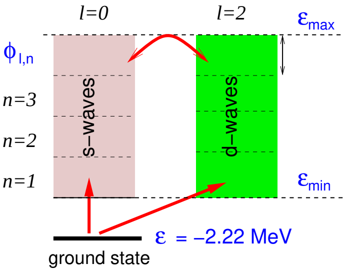
Application of the CDCC formalism: d+ ^{58}Ni

Coupled-Channels + Continuum discretization

$\Downarrow$

$\boxed{\text{Continuum-Discretized Coupled-Channels (CDCC)}}$



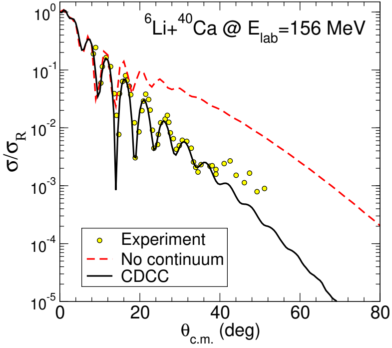


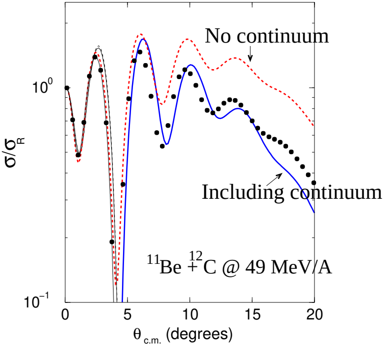
☞*Coupling to breakup channels has a important effect on the reaction dynamics*

Application of the CDCC method: ^{6}Li and ^{6}He scattering

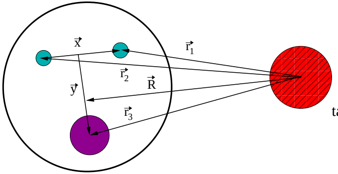
☞ The CDCC has been also applied to nuclei with a cluster structure:

- $^6\mathrm{Li}=\alpha + \mathrm{d}$ $\quad (S_{\alpha,d}=1.47$ MeV)
- $^{11}\mathrm{Be}={}^{10}\mathrm{Be} + \mathrm{n}$ ($S_n=0.504$ MeV)





Extension to 3-body projectiles

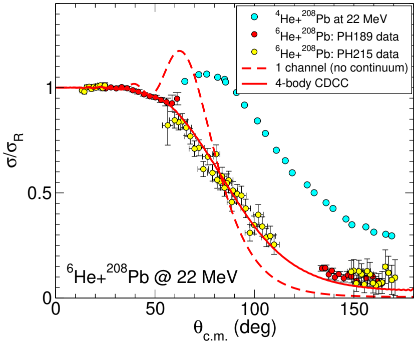


To extend the CDCC formalism, one needs to evaluate the new coupling potentials:

$$\boxed{V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\, \phi_n^*(\mathbf{x}, \mathbf{y}) \left\{ V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{\alpha t}(\mathbf{r}_3) \right\} \phi_{n'}(\mathbf{x}, \mathbf{y})}$$

☞ $\phi_n(\mathbf{x}, \mathbf{y})$ three-body WFs for bound and continuum states: hyperspherical coordinates, Faddeev, etc (difficult to calculate!)

Four-body CDCC calculations for ^{6}He scattering

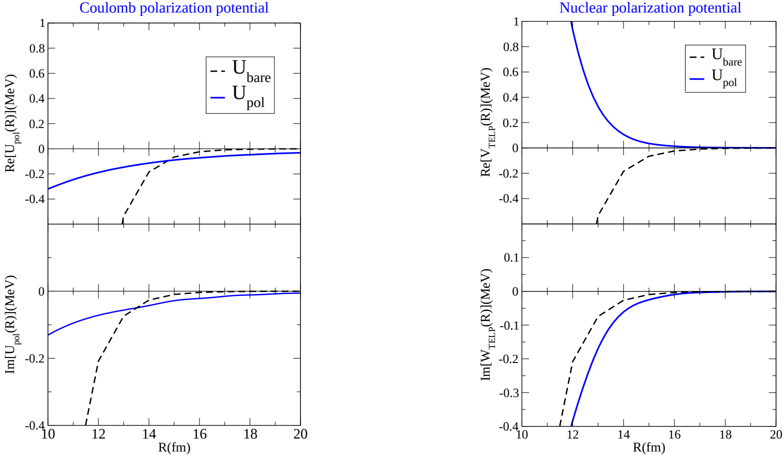


N.b.: 1-channel potential considers only g.s. → g.s. coupling potential:

$$\boxed{V_{00}(\mathbf{R}) = \int d\mathbf{r}\, \phi^*_{\text{g.s.}}(\mathbf{x},\mathbf{y}) \left\{V_{nt}(\mathbf{r}_1) + V_{nt}(\mathbf{r}_2) + V_{ct}(\mathbf{r}_3)\right\} \phi_{\text{g.s.}}(\mathbf{x},\mathbf{y})}$$

Data (LLN): Sánchez-Benítez et al, NPA 803, 30 (2008) L. Acosta et al, PRC 84, 044604 (2011)
Calculations: Rodríguez-Gallardo et al, PRC 80, 051601 (2009)

Polarization potential from CDCC calculations

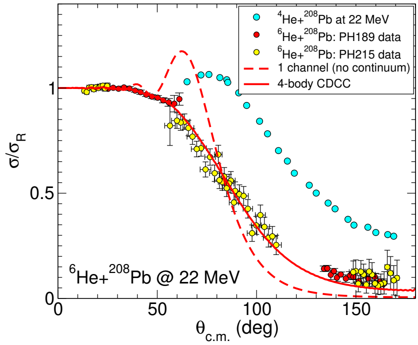


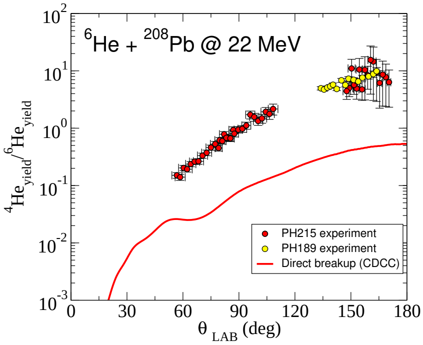
- Polarization potentials are long-ranged.
- Both nuclear and Coulomb couplings are important.

The problem of inclusive breakup

α production in ^{6}He scattering

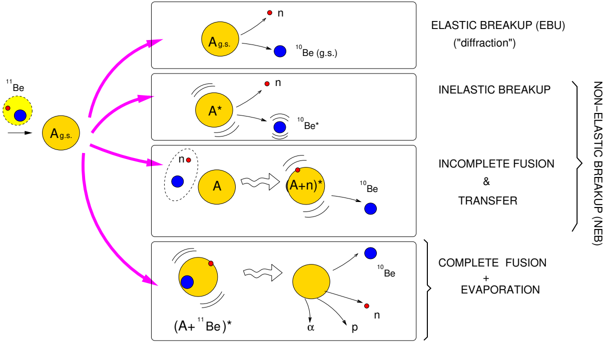
$$\boxed{^6\text{He} + ^{208}\text{Pb} \to \alpha + \text{X}}$$





☞CDCC reproduces elastic scattering, but not inclusive α's.

Evaluation of the inclusive breakup

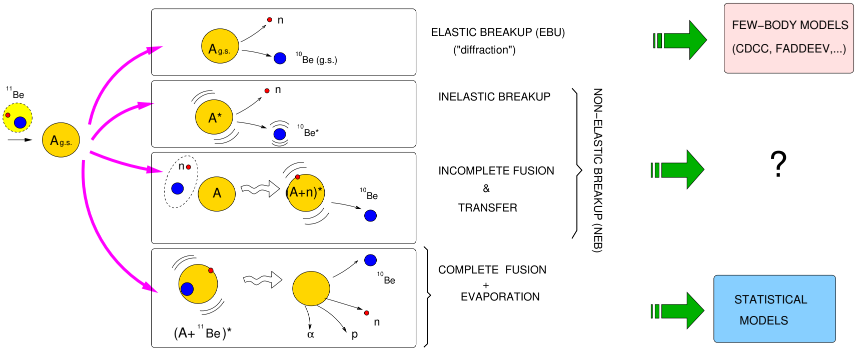


- For a reaction of the form $a(=b+x)+A \rightarrow b+$ anything

$$\boxed{\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}}$$

- CDCC provides only the EBU part (σ_{NEB} & σ_{CN} out of CDCC modelspace

Evaluation of the inclusive breakup



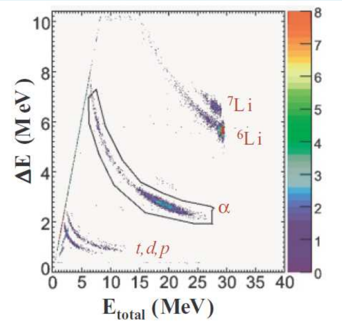
- For a reaction of the form $a(= b + x) + A \to b +$ anything

$$\boxed{\sigma_b = \sigma_{EBU} + \sigma_{NEB} + \sigma_{CN}}$$

- CDCC provides only the EBU part (σ_{NEB} & σ_{CN} out of CDCC modelspace

Evidence of NEB contributions in inclusive (^{6}Li,αX)

^{6}Li+^{209}Bi @ 32 MeV

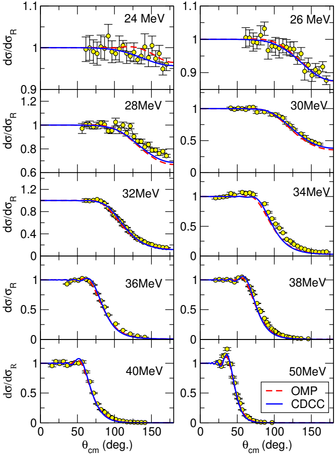


Santra *et al*, PRC85,014612(2008)

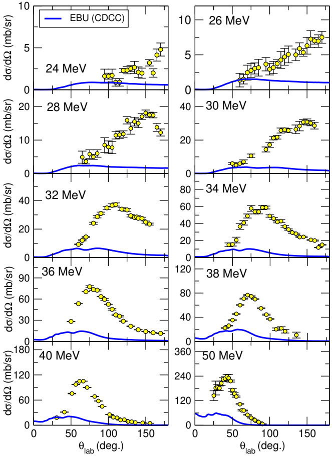
If elastic breakup were the dominant mechanism, we would expect $N_\alpha \approx N_d$ but, experimentally, one finds $N_\alpha \gg N_d$

Evidence of NEB contributions in inclusive ^{209}Bi(^{6}Li,α)X

Elastic scattering



Inclusive α's



Explicit evaluation of inclusive breakup in Ichimura-Austern-Vincent model

- Inclusive breakup:

$$\boxed{a(=b+x)+A \rightarrow b+(x+A)^*}$$

Inclusion of all relevant $x+A$ channels is not feasible in general $\Rightarrow$ use closed-form models

- Inclusive differential cross section: $\sigma_b^{\mathrm{BU}} = \sigma_b^{\mathrm{EBU}} + \sigma_b^{\mathrm{NEB}}$:
  - σ_b^{EBU} is breakup leaving A in g.s. (e.g. CDCC)
  - σ_b^{NEB} corresponds to non-elastic x+A processes and can be calculated as the absorption in the $x+A_{gs}$ channel:

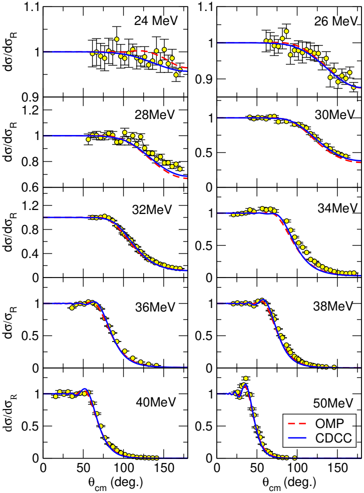
$$\boxed{\frac{d\sigma^{\mathrm{NEB}}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x^{(0)}|W_{xA}|\varphi_x^{(0)}\rangle} \quad \text{(optical theorem)}$$

where $\varphi_x^{(0)}$ describes $x-A$ scattering following $a \rightarrow b+x$ dissociation:

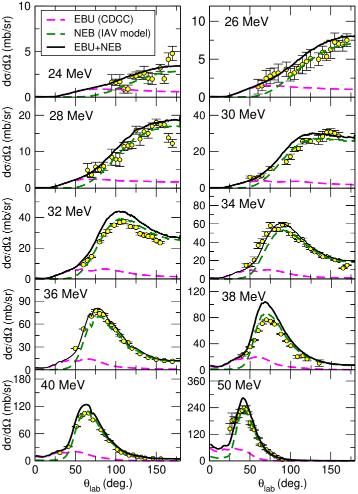
$$\boxed{[K_x + U_{xA} - E_x]\varphi_x^{(0)}(\mathbf{r}_x) = (\chi_b^{(-)}|V_{bx}|\chi_{aA}\phi_a\rangle}$$

Application to ^{209}Bi (^{6}Li+,α + X)

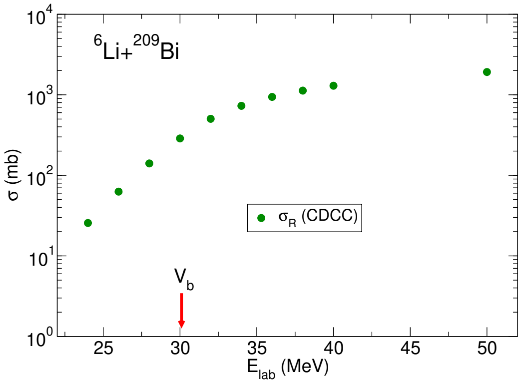
Elastic scattering



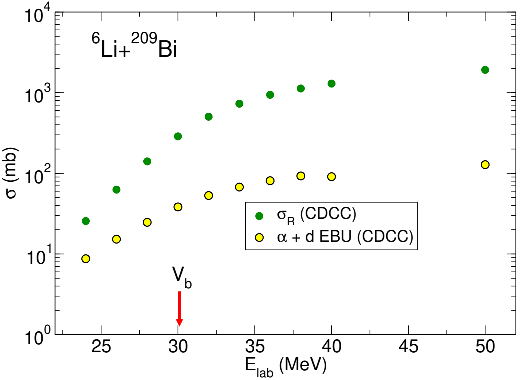
Inclusive α's



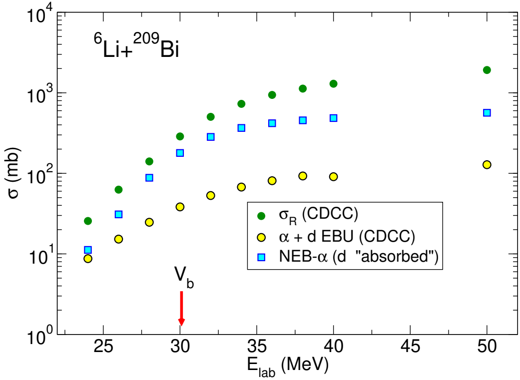
^{6}Li+^{209}Bi: incident energy dependence of cross sections



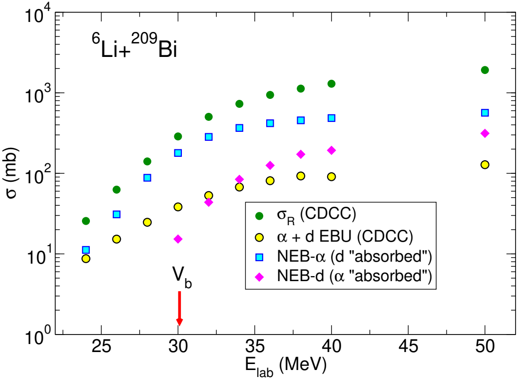
^{6}Li+^{209}Bi: incident energy dependence of cross sections



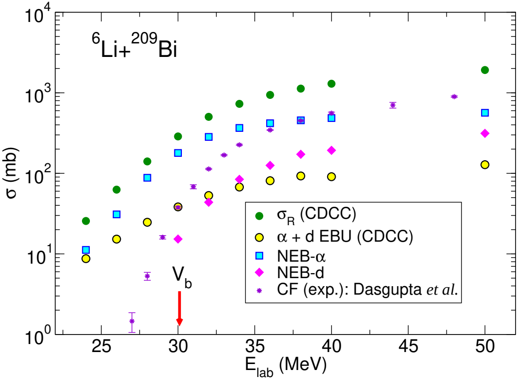
^{6}Li+^{209}Bi: incident energy dependence of cross sections



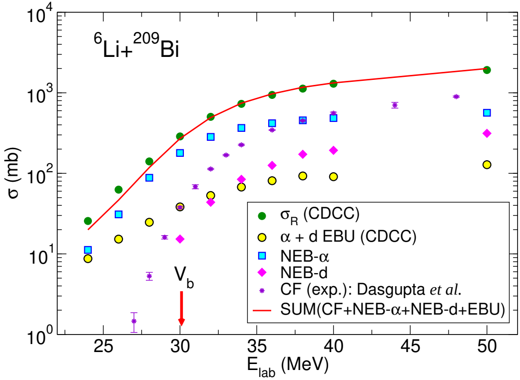
$^{6}Li+^{209}Bi$: incident energy dependence of cross sections



^{6}Li+^{209}Bi: incident energy dependence of cross sections



^{6}Li+^{209}Bi: incident energy dependence of cross sections



$$\boxed{\sigma_{reac} \approx \sigma_{\alpha+d}(EBU) + \sigma_{\alpha}(NBU) + \sigma_{d}(NBU) + \sigma(CF)}$$

Application to the ^{7}Be case

Data: Mazzocco *et al*: ☞ $\sigma_\alpha \approx 5\sigma_{3\mathrm{He}}$