Central Bank Digital Currency with Adjustable Interest Rate in Small Open Economies*

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Abstract

Open economy implications of central bank digital currencies (CBDCs) are a new and exciting area of research. In this paper, we assess the economic and welfare outcomes of monetary policies with an adjustable interest rate of a CBDC as a secondary instrument and a Taylor-type rule as a primary instrument in a small open economy. We develop a dynamic stochastic general equilibrium (DSGE) model, in which bank deposits and CBDC are competing media of exchange, and are both competing assets of domestic and foreign bonds. The model provides parity conditions between CBDC vis-à-vis its competing assets, which imply that the efficiency in transacting with CBDC plays an important role in determining both the interest rate differentials and the effectiveness of the adjustable CBDC interest rate as a monetary policy instrument. Our simulation exercises find that a counter-cyclical rule for the CBDC interest rate improves social welfare, despite the distributional effects among financially constrained and unconstrained agents. We also find that the adjustable CBDC interest rate provides a possibility for the central bank to achieve both objectives of monetary autonomy and exchange rate stability, in a way analogous to a sterilized foreign exchange intervention.

Keywords: central bank digital currency, monetary policy, blockchain, distributed ledger, small open economy

JEL codes: E4, E5, F3, F4

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1 Introduction

The proliferation of blockchain-based cryptocurrencies has the central banks on their toes. The coronavirus disease 2019 has further accelerated the process of digitizing traditional paper money and coins. The latest survey conducted by the Bank for International Settlements (BIS) found that "central banks collectively representing a fifth of the world's population are likely to launch retail CBDCs in the next three years" (Boar and Wehrli, 2021). At the same time, the BIS has proposed systems that use central bank digital currencies (CBDCs) in cross-border transactions, including a system being tested in conjunction with the central banks of Thailand, China and the United Arab Emirates (Auer et al. (2021)). The implications of CBDC in open economy contexts are now an entirely new and exciting area of research.

Following the increasing popularity of bitcoin, discussions on how central banks can adopt bitcoin's underlying technology have often involved the interest-bearing feature. Interest-bearing central bank money is not new, but such a feature was costly to implement until a robust ledger system and affordable data storage became available. In the low interest rate environment after the global financial crisis, lacking an interest rate on paper money and coins is thought to be the reason that the central bank's target interest rate could hardly go into the negative territory, thus hindering the potency of monetary policy. Even during normal time, the fixed, zero interest rate on money means that money supply always has to meet every dollar change in money demand, increasing the risk of hyperinflation. Bordo and Levin (2017) argued that an interest-bearing CBDC facilitates a price stability objective, allowing households and firms to make more informed decisions on future consumption and investments. Barrdear and Kumhof (2021) simulated a post-crisis scenario in which the central bank injected CBDC into the economy leading to a 3% permanent increase in GDP. Despite the various benefits which an interest-bearing feature may imply, to our best knowledge, no study has presented a thorough assessment on the monetary regimes without and with an adjustable interest rate on the CBDC. This forms the main motivation of this paper, and with CBDC getting increasing international attention, we further examine the implications for open economies.

A CBDC with an adjustable interest rate means that the central bank is equipped with an additional monetary policy instrument. Traditionally, the central bank controls money supply by setting the interest rate on government bonds. Because money always bears a zero interest rate, the bond interest rate alone measures the opportunity cost of holding money. Money demand is influenced by the interest from bonds, instead of the interest on money itself. Subsequently, money supply adjusts passively to meet money demand. With an adjustable interest rate on the CBDC, as the central bank sets the bond interest rate, it may simultaneously set the interest rate on CBDC, so as to prevent an excessive supply of money. In other words, a CBDC with an adjustable interest rate may complement the existing monetary policy framework as a secondary policy instrument.

The objective of this paper is to assess the policy outcomes when CBDC with an adjustable interest rate is used as a secondary monetary policy instrument. In order to do so, we develop an open-economy DSGE framework where bank deposits and CBDCs are competing media of exchange. Both deposits and CBDCs are part of interest-bearing assets held by households together with domestic and foreign bonds. Households' demand of assets and economic decisions are affected by central bank's decisions regarding the interest rates of CBDCs. In an otherwise identical environment, the central bank in this economy can choose to follow only a primary Taylor-type monetary rule and keep the CBDC interest rate at a constant value, eg, zero, (baseline regime); or augment the primary rule with a secondary monetary policy rule using either a CBDC interest rate rule (the price rule regime) or a CBDC quantity rule (the quantity rule regime). Because the model environment is otherwise identical across all the regimes discussed, estimated changes in economic stability and/or society welfare could be used to assess the monetary regimes adopted by the central bank. This framework thus serves as a toolkit for ranking the monetary policy regimes based on indicators widely used in practice.

We provide three major contributions to the research on CBDC. First, we are the first to examine CBDC in an open economy context. We allow trade in consumption goods and capital flows between home country and the rest the world. Special attention is paid to the parity conditions between the interest rates of foreign and domestic assets. In a conventional small open economy model with only domestic and foreign bonds, interest rate differentials induce exchange rate movements via the uncovered interest rate parity condition. In this paper, we derive the bilateral parity conditions to understand the richer dynamics as the interest-bearing feature is introduced to the CBDC.

Our second major contribution is in the welfare analyses for the CBDC monetary regimes. For the CBDC monetary regimes discussed, we rank their economic outcomes based on both economic stability and a consumption-equivalent welfare measure. We also discuss the the best CBDC regime to adopt when a country is subject to specific uncertainties (shocks). Earlier studies have not conducted welfare analyses for CBDC regimes. We evaluate the price rule and the quantity rule as outlined in Barrdear and Kumhof (2021), and find that both rules are welfare improving in a small open economy. We also find that there are distributional effects between financially constrained and unconstrained agents.

As the third contribution, we document the possibility that the added flexibility in monetary policy, due to CBDC, provides a way for the central bank to manage its exchange rate while keeping the autonomy in domestic money supply. In other words, the impossible trinity of monetary policy can be relaxed. In this model, we deliberately limit the circulation of CBDC within the geographical boundary of the issuing country. Such an assumption is necessary at the initial stage as the central banks roll out the retail CBDCs. It is in fact consistent with the ongoing proof-of-concept experiments in countries like China. We present, however, a more important policy implication for the emerging economies. We show that an interest-bearing CBDC that is confined within the country works in an analogous way to the sterilized foreign exchange intervention that is commonly used when a central bank aims to simultaneously stabilize the exchange rate while keeping monetary autonomy and free capital flow. With the CBDC, the central bank is able to maintain its exchange rate via the uncovered interest rate parity, and at the same time influence the domestic economic activity via one of the CBDC monetary regimes.

The remainder of the paper is organized as follows. Section 2 reviews related literature. Section 3 presents the DSGE framework. Section 4 discusses the parameters used in our model. Section 5 presents the equilibrium dynamics. Section 6 presents the welfare outcome of the CBDC regimes. Section 7 examines the interest rate parity conditions and the possibility of relaxing the impossible trinity. Section 8 concludes.

2 Related literature

There is a fast growing literature on the macroeconomics of cryptocurrencies (Yermack (2013); Danezis and Meiklejohn (2016); Chiu and Koeppl (2019); Niepelt (2018); Benigno (2019); Schilling and Uhlig (2019b,a); Uhlig and Xie (2021)). Our current paper is more broadly related to three strands of CBDC literature. The first strand examines the consequences of CBDC in the context of a competitive banking industry (Kumhof and Noone (2018); Meaning et al. (2018); Williamson et al. (2019); Brunnermeier and Niepelt (2019); Chiu et al. (2019); Schilling et al. (2020); Fernández-Villaverde et al. (2020); Andolfatto (2021)). Our paper also considers the context where CBDC and deposits are competing media of exchange. The second strand of literature pertains to the role of CBDC in the monetary policy setting. While some of these papers like Bordo and Levin (2017); Zhu and Hendry (2018); Davoodalhosseini (2018); Keister and Sanches (2019) use a new monetarist model, our paper is one of a handful that use a DSGE New Keynesian framework. Barrdear and Kumhof (2021) initiated the DSGE New Keynesian literature on CBDC by examining CBDCs as a secondary monetary policy instrument in a closed economy framework. Grossa and Schillerb (2020) analyze the impact of CBDC interest-bearing feature on bank funding in a New Keynesian setup. The third strand is part of a small but burgeoning literature on open economy implications of CBDC. To our knowledge, we are the first to examine CBDC in an open economy context. While our paper examines the implications of adjustable CBDC interest rate in a small open economy, Ferrari et al. (2020) investigate the cross-border spillover effects of CBDC using a two-country DSGE model. Auer et al. (2021) conceptually analyze the potential benefits of cross-border flow in multi-currency CBDC.

Our paper is also very closely related to the literature on exchange rate management and monetary policy trilemma such as Georgiadis and Zhu (2019); Han and Wei (2018); Ahmed (2021); Alba and Wang (2021). Studies like Adler and Tovar (2011); Ostry et al. (2012), find exchange rate deviations to be costly for emerging markets as much of their growth is export driven. A large body of work like Benes et al. (2015); Steiner (2017); Xie et al. (2017); Alla et al. (2020) advocate the use of sterilized foreign exchange intervention to relax the monetary policy trilemna. Our current work argues a novel idea to use CBDC as the channel to preserve domestic monetary autonomy while managing the exchange rate through uncovered interest parity.

3 Model

Our model features a small-open economy with deposits and CBDC being competing media of exchange. The domestic economy is a simplified variant of the Barrdear and Kumhof (2021) model. It retains the consumption loans in Barrdear and Kumhof (2021); Benes and Kumhof (2015); Jakab and Kumhof (2015); Kumhof and Wang (2018) as the means of credit creation in the economy. Without loss of generality, and for ease of identifying the transmission mechanism, we abstract from Barrdear and Kumhof (2021) the real-estate and capital-goods sectors. The domestic economy is small relative to the rest of the world in the sense that any disturbance in the domestic economy is negligible to the world, but not vice versa. The interactions between the domestic and the world economies are via internationally traded bonds held by agents in the economy and a production sector employing intermediate inputs of production from abroad, as in Gopinath et al. (2020). In addition to the policy interest rate, the central bank also has autonomy on either the interest rate of CBDC, or the supply of it, as modeled in Barrdear and Kumhof (2021).

3.1 Households

Agents in a household are either financially unconstrained or constrained. The number of unconstrained agents take up an ω fraction of total agents in a household. The financially unconstrained agent is similar to a representative agent in a conventional New Keynesian model. The agent places her savings in the form of deposits, which can be used as a medium of exchange, and government bonds, which is less liquid. She also has access to internationally traded assets such as foreign-currency-denominated bonds. The agent finances her consumption and savings using incomes from labor supply and interests from asset holdings. A financially constrained agent, on the other hand, does not have access to either domestic or foreign bonds. Her deposits are created by taking loans from the banking sector. With some probability, the agent defaults the loan by paying the collateral she pledges at the bank. The agent's consumption and deposits are financed by the loans, labor income, interest incomes, and profits from the banks and firms.

Banks are owned by the households. In each period, regardless of agent type, there is a probability ι that each agent in the household becomes a banker. The profits from bank operations are distributed among the constrained and unconstrained agents as dividends.

We denote the financially unconstrained and constrained agents with superscripts u and c, respectively. In any period t, an agent of type j living in household $h \in [0,1]$ derives utility from consumption bundle $c_t^j(h)$ and liquidity services $l_t^j(h)$, and disutility from labor supply $n_t^j(h)$. This is written as the following period utility function:

$$u^j\left[c_t^j(h), n_t^j(h), l_t^j(h); \psi_n^j, \psi_l^j\right]$$
$$= (1-\upsilon)\, z_{mu,t} \log\left[c_t^j(h) - \upsilon \bar{c}_{t-1}^j\right] - \psi_n^j \frac{n_t^j(h)^{1+\theta_n}}{1+\theta_n} + \psi_l^j \frac{l_t^j(h)^{1-\theta_l}}{1-\theta_l}, \quad (1)$$

for $j \in \{u, c\}$. The consumption bundle is a CES aggregate of goods from all sectors in the economy $c_t^j(h) = \left[\int_0^1 c_t^j(h,i)^{\frac{\theta_p-1}{\theta_p}} di\right]^{\frac{\theta_p}{\theta_p-1}}$, where θ_p is the elasticity of substitution between i varieties of goods. The agent's consumption also depends on general consumption habit, which is the mean consumption of all agents of the same type in the previous period, given by $\bar{c}_{t-1}^j = \int_0^1 c_{t-1}^j(h)dh$. $l_t^j(h)$ is the liquidity services derived from agent's holding of liquid assets, namely, deposits ($d_t^j(h)$) and/or CBDC ($m_t^j(h)$),

$$l_t^j(h) \equiv \left[\left(1-\gamma^j\right)^{1/\epsilon} d_t^j(h)^{1-1/\epsilon} + \gamma^{j\,1/\epsilon}\left(\psi_m m_t^j(h)\right)^{1-1/\epsilon}\right]^{\frac{\epsilon}{\epsilon-1}}. \quad (2)$$

where ψ_m is the weight of CBDC in liquidity services. It measures the transaction efficiency of CBDC relative to deposits. ψ_n^j and ψ_l^j are the weights of labor and liquid assets in utility preferences specific to agent types. θ_n and θ_l are the elasticity parameter. $z_{mu,t}$ is the exogenous shock to the agent's marginal utility of consumption. The lifetime utility of an agent of type j is expressed as the bellman equation:

$$U_t^j = u^j\left(c_t^j, n_t^j, l_t^j; \Theta^j\right) + \beta^j\, \mathbb{E}_t\left[U_{t+1}^j\right]. \quad (3)$$

where Θ^j is the set of parameters specific to the agent type. Note that we drop the index for household h in the remainder of this paper, as there is no heterogeneity among agents of the same type. We now describe the optimization problems of the unconstrained and the constrained agents in detail.

Unconstrained agent The unconstrained agent's asset holdings consist of bank deposits, domestic government bonds, and foreign bonds. Among these assets, only bank deposits provide her with liquidity services, via which she derives utility. As such, for an unconstrained agent, we have $l_t^u = d_t^u$ in Eq. (2). The agent's budget in period t is

$$c_t^u(1+\tau_c) + (b_t^u + d_t^u)\zeta_t - q_t b_t^{*u}$$
$$= w_t n_t^u(1-\tau_n) + r_{b,t} b_{t-1}^u + r_{d,t} d_{t-1}^u - q_t r_t^* \zeta_{t-1}^* b_{t-1}^{*u} + \Psi_t^u + \frac{1-\iota}{\omega}\Omega_t \quad (4)$$

On the left-hand side, the agent's expenditure includes consumption c_t^u plus the consumption tax at the rate τ_c, government bonds b_t^u, deposits at the bank d_t^u, and foreign bonds b_t^{*u}. q_t is the real exchange rate of home currency in terms of foreign currency, so that an increase in the value of q_t implies a lower purchasing power, or a real depreciation, of the domestic currency. Transaction costs incurred on account of changes in domestic financial assets (deposits and bonds) are denoted by $\zeta_t = \left[1 + \phi_{i,b}\left(\frac{b_t}{4gdp_t} - \frac{\bar{b}}{4\overline{gdp}}\right)\right]$ where gdp_t is gross domestic product defined in Eq. (44) and the variable with a bar indicates its steady state level. ζ_t is rebated back to the agents as a transfer Ψ_t^u.

On the right-hand side, the first term in agent's income is the payroll from work net of income taxes, in which w_t and τ_n are the real wage rate and the income tax rate, respectively. The second, third and fourth terms are the gross interests from government bonds, bank deposits, and foreign bonds, respectively. The ex-post real interest rate of an asset x, denoted by $r_{x,t}$, is associated with its nominal counterpart, $i_{x,t}$ via the Fisher's equation $r_{x,t} = \frac{i_{x,t-1}}{\pi_t}$ where π_t is the general price inflation. Analogously, the real foreign interest rate is given by $r_t^* = \frac{i_{t-1}^*}{\pi_t^*}$. A risk premium is involved in the interest rate of foreign bonds to prevent excessive borrowing from the rest of the world. This is defined as $\zeta_t^* = \exp\left[\varphi\left(\frac{q_t b_t^{*u}}{4gdp_t} - \frac{\bar{q}\bar{b}^{*u}}{4\overline{gdp}}\right)\right]$, following Schmitt-Grohé and Uribe (2003). The last term denotes the real lump-sum income Ω_t, which includes dividends, lump-sum tax, government transfers to agents and a portion of the bank's ad-hoc costs as in Barrdear and Kumhof (2021) (the details are discussed in Appendix A.1.2).

There is a total of six equations pinning down the dynamics relevant to unconstrained agents. The agent's first-order conditions are ones with respect to c_t^u, n_t^u, d_t^u, b_t^u, and b_t^{*u}. The budget constraint is involved to pin down the Lagrangian multiplier λ_{bt}^u. The first-order conditions for the other five choice variables are listed below:

$$c_t^u : u_{c,t}^u + \lambda_{b,t}^u (1+\tau_c) = 0 \tag{5}$$

$$n_t^u : u_{n,t}^u = \lambda_{b,t}^u w_t (1-\tau_n) \tag{6}$$

$$l_t^u : u_{l,t}^u + \lambda_{b,t}^u \zeta_t = \beta^u \mathbb{E}_t \left[\lambda_{b,t+1}^u \frac{i_{d,t}}{\pi_{t+1}}\right] \tag{7}$$

$$b_t^u : \lambda_{b,t}^u \zeta_t = \beta^u \mathbb{E}_t \left[\lambda_{b,t+1}^u \frac{i_{b,t}}{\pi_{t+1}}\right] \tag{8}$$

$$b_t^{*u} : \lambda_{b,t}^u = \beta^u \mathbb{E}_t \left[\lambda_{b,t+1}^u \frac{i_t^*}{\pi_{t+1}^*} \zeta_t^* \frac{q_{t+1}}{q_t}\right] \tag{9}$$

where $u_{c,t}^u, u_{n,t}^u$ and $u_{l,t}^u$ are marginal utilities with respect to c_t^u, n_t^u and l_t^u, respectively.

Constrained agent The constrained agent's budget in period t is given by

$$\begin{aligned} &c_t^c (1+\tau_c)(1+s_t^c) + (d_t^c + m_t^c)\zeta_t + \Gamma_t \mathfrak{c}_{t-1} \\ &\quad = \mathfrak{l}_t^c \left[1 - \phi_{\mathfrak{l}}\left(\mathfrak{l}_t^c - \mathfrak{l}_{t-1}^c\right)\right] + w_t n_t^c (1-\tau_n) + r_{d,t} d_t^c + r_{m,t} m_{t-1}^c + \Pi_t + \Psi_t^c + \frac{\iota}{1-\omega}\Omega_t. \end{aligned} \tag{10}$$

Instead of government and foreign bonds, the constrained agent has access to the CBDC denoted by m_t^c, with a gross real interest rate $r_{m,t}$. Changes in domestic asset ownership result in transaction cost ζ_t which is rebated back as Ψ_t^c. It also receives profits Π_t from firm ownership. The constrained agent takes up loans of amount $\mathfrak{l}_t$ in order to create deposits at the banks. $\phi_{\mathfrak{l}}(\cdot)$ is the quadratic adjustment cost for loans, defined by $\phi_{\mathfrak{l}}\left(\mathfrak{l}_t^c - \mathfrak{l}_{t-1}^c\right) \equiv \frac{\varphi_{\mathfrak{l}}}{2}\left(\mathfrak{l}_t^c - \mathfrak{l}_{t-1}^c\right)^2$. In order to obtain the loans, the constrained agent pledges collateral of amount $\mathfrak{c}_{t-1}$, consisting of fractions of both realized financial and real cash flows of the agent:

$$\mathfrak{c}_{t-1} = \kappa^r \left(r_{d,t} d_{t-1}^c + r_{m,t} m_{t-1}^c \right) + \kappa^r \frac{4(1-\tau_l) w_{t-1}}{\pi_t} n_{t-1}^c \tag{11}$$

where κ^r is the fraction of real cash flow that the agent pledges. In the event of default, this collateral is forfeited. $\Gamma_t \mathfrak{c}_{t-1}$ is the value of the pledged collateral with the bank in period t. The transaction cost s_t is given by $s_t = z_{md,t} A v_t + \frac{B}{v_t} - 2\sqrt{AB}$, following Schmitt-Grohé and Uribe (2007). $z_{md,t}$ is the money demand shock. An unexpected increase in $z_{md,t}$ leads to higher liquidity holdings as a saving vehicle. The velocity is $v_t = \frac{c_t^c(1+\tau_c)}{l_t^c}$ where l_t^c represents liquidity services as defined in Eq. (2). In the case of a constrained agent, since the liquidity services influence her economic decision via the transaction cost, we assume that liquidity services do not enter her utility function again. In other words, we set $\psi_l^c = 0$ in Eq. (1).

A constrained agent maximizes her lifetime utility Eq. (3) subject to her budget constraint and the loans available to her. The budget constraint is given by Eq. (10), in which the amount of collateral can be substituted by Eq. (11). The available loans come from the bank's zero-profit condition following Barrdear and Kumhof (2021) (the details are discussed in Appendix A.1.1). We use $\lambda_{b,t}^c$ and $\lambda_{0,t}^c$ to denote the Lagrangian multipliers for the constrained agent's budget constraint and the bank's zero-profit condition. The optimal choices of the constrained agent are pinned down by six first-order conditions, namely:

$$c_t^c : u_{c,t}^c + \lambda_{b,t}^c \left(1+\tau_c\right) \tau_t^{liq} = 0 \tag{12}$$

$$n_t^c : u_{n,t}^c = \lambda_{b,t}^c \left(1-\tau_l\right) w_t - \beta^c \kappa^r \mathbb{E}_t \left[\frac{\partial \mathfrak{c}_t}{\partial n_t^c} \lambda_{b,t+1}^c \left[\Gamma_{t+1} + \lambda_{0,t}^c \left(\Gamma_{t+1} - \xi \mathcal{G}_{t+1} \right) \right] \right] \tag{13}$$

$$d_t^c : \lambda_{b,t}^c \left(\zeta_t + \left(1+\tau_c\right) c_t^c s_{d,t} \right) = \beta^c \, \mathbb{E}_t \left[\frac{i_{d,t}}{\pi_{t+1}} \lambda_{b,t+1}^c \left[1 - \kappa^r \left((1+\lambda_{0,t}^c) \Gamma_{t+1} - \xi \mathcal{G}_{t+1} \right) \right] \right] \tag{14}$$

$$m_t^c : \lambda_{b,t}^c \left(\zeta_t + \left(1+\tau_c\right) c_t^c s_{m,t} \right) = \beta^c \, \mathbb{E}_t \left[\frac{i_{m,t}}{\pi_{t+1}} \lambda_{b,t+1}^c \left[1 - \kappa^r \left((1+\lambda_{0,t}^c) \Gamma_{t+1} - \xi \mathcal{G}_{t+1} \right) \right] \right] \tag{15}$$

$$\mathfrak{l}_t^c : \lambda_{b,t}^c \left(1 - \mathfrak{l}_t^c \frac{\partial \phi_{\mathfrak{l}} \left(\mathfrak{l}_t^c - \mathfrak{l}_{t-1}^c \right)}{\partial \mathfrak{l}_t^c} - \phi_{\mathfrak{l}} \left(\mathfrak{l}_t^c - \mathfrak{l}_{t-1}^c \right) \right) = -\beta^c \mathbb{E}_t \left[\lambda_{b,t+1}^c \lambda_{0,t}^c \frac{i_{l,t}}{\pi_{t+1}} \right] \tag{16}$$

$$\bar{\omega}_{t+1}^c : \mathbb{E}_t \left[\lambda_{0,t}^c \left(\Gamma_{t+1}' - \xi \mathcal{G}_{t+1}' \right) + \Gamma_{t+1}' \right] = 0 \tag{17}$$

where $u_{c,t}^c$ and $u_{n,t}^c$ are marginal utilities with respect to c_t^c and n_t^c, respectively; $s_{d,t}$ and $s_{m,t}$ are marginal transaction costs with respect to d_t^c and m_t^c, respectively; and Γ_{t+1}' and $\mathcal{G}_{t+1}'$ are first derivatives with respect to $\bar{\omega}_{t+1}^c$. $\bar{\omega}_{t+1}^c$ determines the bankruptcy of constrained agents that default on their loan interest payments. The details on $\bar{\omega}_{t+1}^c$, Γ_{t+1} and $\mathcal{G}_{t+1}$ are elaborated further in the appendix.

In Eq. (12), we use τ_t^{liq} to denote the mark up on transaction costs due to monetary frictions, i.e $\tau_t^{liq} \equiv 1 + s_t + s_{c,t}$. This mark up is also likened to a "liquidity" tax with a distortionary tax rate by Barrdear and Kumhof (2021). Henceforth, we also refer to τ_t^{liq} as the liquidity tax. Through the first order conditions, we see that τ_t^{liq} affects the marginal rate of substitution of consumption and labor. Hence, due to the monetary frictions, changes in the quantity of bank deposits and in the quantity of CBDC or in the quantity of monetary transaction balances get transmitted to the real economy.

Parity conditions Given the co-existence of multiple assets in the model, it is worth exploring the parity conditions among their interest rates. A conventional model for a small open economy provides the uncovered interest rate parity condition between domestic and foreign interest rates. In our model, the two additional domestic assets, bank deposits and CBDC, are at the same time competing media of exchange. The pair-wise parity conditions among the interest rates ought to account for their transaction utility and the substitutability between them.

The Lagrangian multiplier $\lambda^u_{b,t}$ is solved by Eq. (5), and subsequently substituted into Eqs. (7) to (9) to obtain the following parity conditions for the nominal interest rates:

$$\frac{i_{d,t}}{i_{b,t}} = 1 - \frac{(1+\tau_c)\, u^u_{l,t}}{u^u_{c,t}\zeta_t} \tag{18}$$

$$\frac{i^*_t\zeta^*_t}{i_{b,t}} = \frac{\mathbb{E}_t\left[\frac{u^u_{c,t+1}}{\pi_{t+1}}\right]}{\mathbb{E}_t\left[\zeta_t \frac{u^u_{c,t+1}}{\pi^*_{t+1}}\frac{q_{t+1}}{q_t}\right]} \tag{19}$$

Eq. (18) implies that the nominal return of bank deposits is lower relative to that of government bonds when the marginal utility of liquidity services is higher relative to the marginal utility of consumption. In other words, when bank deposits are used as a medium of exchange, the utility from their liquidity compensates the loss in interest earnings. Whereas, Eq. (19) is the conventional uncovered interest rate parity when log-linearized: when the risk-adjusted foreign interest rate is higher relative to the return of domestic government bond, the domestic currency is expected to appreciate to eliminate any arbitrage. Taken together, these two equations imply that the interest rate differential between foreign bonds and bank deposits are accounted for by both exchange rate movements and the utility from using bank deposits as a medium of exchange. An increase in foreign interest rate may result in either a real depreciation of the domestic currency, or bank deposits being a more favorable medium of exchange.

Eqs. (14) and (15) together give an intuitive parity condition between the interest rates of bank deposits and CBDC. Moving $i_{d,t}$ and $i_{m,t}$ to their respective left-hand sides and equating the resultant expressions, we obtain the following equation

$$\frac{i_{d,t}}{i_{m,t}} = \frac{\zeta_t + (1+\tau_c)\, c^c_t s_{d,t}}{\zeta_t + (1+\tau_c)\, c^c_t s_{m,t}} \tag{20}$$

which implies that the difference in the interest rates of bank deposits and CBDC is the result of the difference in their marginal transaction costs $s_{d,t}$ and $s_{m,t}$. Based on its definition, the transaction cost s_t is associated with the liquidity holding, l^c_t, which is defined in Eq. (2) as aggregate of bank deposits and CBDC, taking into account the transaction technology. From Eq. (2), the difference in the marginal transaction costs is a result of the substitutability between bank deposits and CBDC, indicated by ϵ, and the relative transaction technology ψ_m. In some cases, an interesting result can be derived. We summarize this result with the following proposition.

Proposition 1 There exists at least one set of parameters $\gamma^c = \tilde{\gamma}^c$, $\psi_m = \tilde{\psi}_m$, and $\epsilon = \tilde{\epsilon}$, such that the nominal interest rates of bank deposits and CBDC are always equal.

The simplest case is when $\psi_m = 1$ and ϵ is infinitely large, in which case bank deposits and CBDC are perfect substitutes in providing liquidity services and with the same transaction technology. In this scenario, the marginal transaction costs are identical between these two liquid assets. The parity condition Eq. (20) therefore says $i_{d,t} = i_{m,t}$. A more important policy implication from Proposition 1 is that the central bank does not always have autonomy on the CBDC interest rate. There exist scenarios in which CBDC interest rate has to be pegged to that of bank deposits. For CBDC to be used as an effective secondary monetary policy instrument, CBDC should at least possess different, if not superior, transaction technology over its competitors, or be an imperfect substitute of them. The central bank is then able to independently adjust the CBDC interest rate or supply.

Combining the three parity conditions Eqs. (18) to (20), we can derive the parity condition between the interest rates of foreign bonds and CBDC as:

$$\frac{i_t^* \zeta_t^*}{i_{m,t}} = \frac{\mathbb{E}_t \left[\frac{u^u_{c,t+1}}{\pi_{t+1}} \right]}{\mathbb{E}_t \left[\zeta_t \frac{u^u_{c,t+1}}{\pi^*_{t+1}} \frac{q_{t+1}}{q_t} \right]} \times \left[1 - \frac{(1+\tau_c)\, u^u_{l,t}}{u^u_{c,t} \zeta_t} \right]^{-1} \times \frac{\zeta_t + (1+\tau_c)\, c_t^c s_{d,t}}{\zeta_t + (1+\tau_c)\, c_t^c s_{m,t}} \tag{21}$$

which depicts that the interest rate differential between foreign bonds and CBDC is associated with CBDC's function as a medium of exchange, in addition to mere exchange rate movements as in the case of the conventional uncovered interest rate parity.

3.2 Firms

Firms are monopolistically competitive. Outputs are sold in both domestic and foreign markets. Each firm $i \in [0, 1]$ produces a unique good, $y_t(i)$, using imported foreign intermediate goods $k_t^*(i)$ and labor from the households $n_t(i)$, as in Gopinath et al. (2020):

$$y_t(i) = k_t^*(i)^{\alpha} \left(a_t n_t(i) \right)^{1-\alpha} \tag{22}$$

Firm i seeks to maximize its profit

$$\max_{k_t^*(i), n_t(i)} \Pi_t(i) = y_t(i) - w_t n_t(i) - q_t k_t^*(i) \tag{23}$$

subject to the demand function for its goods $y_t(i) = \left(\frac{p_t(i)}{p_t} \right)^{-\theta_p} y_t$, where y_t is the aggregate output in the economy, $p_t(i)$ is the price of goods sold by firm i, and p_t is the general price level defined as $p_t \equiv \left(\int_0^1 p_t(i)^{1-\theta_p} di \right)^{\frac{1}{1-\theta_p}}$.

Prices are sticky and standard as in Calvo (1983). In each period, a fraction $(1-\vartheta)$ of the firms get to reset the prices. The marginal cost is

$$\mu_t = \frac{1}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} \frac{w_t^{(1-\alpha)} q_t^{\alpha}}{a_t^{\alpha}} \tag{24}$$

The optimal price $\tilde{\pi}_t \equiv \frac{\tilde{p}_t}{p_t}$ follows

$$\tilde{\pi}_t = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \vartheta^s Q_{t,t+s} y_{t+s} \mu_{t+s} \left(\frac{p_{t+s}}{p_t} \right)^{1+\theta_p} \right]}{\mathbb{E}_t \left[\sum_{s=0}^{\infty} \vartheta^s Q_{t,t+s} y_{t+s} \left(\frac{p_{t+s}}{p_t} \right)^{\theta_p} \right]} \tag{25}$$

where $Q_{t,t+s} \equiv \beta_c^s \frac{\lambda^c_{b,t+s}}{\lambda^c_{b,t}}$ is the stochastic discount factor since constrained agents own and derive profits from the firms. The aggregate inflation follows as $1 = \vartheta \pi_t^{\theta_p - 1} + (1-\vartheta)\, \tilde{\pi}_t^{1-\theta_p}$.

3.3 Banks

The banks provide loans to the financially constrained agents and create deposits in agents' accounts (both types). A representative bank's balance sheet comprises of loans $\mathfrak{l}_t$, deposits $\mathfrak{d}_t$ and its post-dividend net worth $\mathfrak{n}_t$.

$$\mathfrak{l}_t = \mathfrak{d}_t + \mathfrak{n}_t \tag{26}$$

The bank's objective is to maximize its expected pre-dividend profits from lending activities elaborated as follows:

$$\max_{\mathfrak{l}_t, d_t} \mathbb{E}_t \left[r_{l,t+1} \mathfrak{l}_t \omega^b_{t+1} - r_{d,t+1} d_t - \mathcal{L}_{t+1} - \chi \mathfrak{l}_t \mathfrak{F}^b_t \left(\bar{\omega}^b_{t+1} \right) \right] \tag{27}$$

where $r_{l,t+1}$ is real wholesale lending rate and $r_{d,t+1}$ corresponds to the real deposit rate. $\mathcal{L}_{t+1}$ is the bank's loss from the loans, and $\chi \mathfrak{l}_t \mathfrak{F}^b_t \left(\bar{\omega}^b_{t+1} \right)$ is the penalty paid by the bank to the regulators in case of failing to meet the MCAR (solvency risk). $\bar{\omega}^b_{t+1}$ is the measure of the cut-off bankruptcy risk. The loss function and the penalties are modeled in the same way as Barrdear and Kumhof (2021). Hence, we leave the details in the Appendix[1].

The bank's objective function is maximized subject to its balance sheet constraint. The first-order condition of the maximization problem which determines the spread between deposit rate and wholesale lending rate is :

$$0 = \mathbb{E}_t \left[r_{l,t+1} - r_{d,t+1} - \chi \mathfrak{F}^b_t \left(\bar{\omega}^b_{t+1} \right) - \chi \mathfrak{f}^b_t \left(\bar{\omega}^b_{t+1} \right) \frac{r_{d,t+1}}{r_{l,t+1}} \frac{\mathfrak{l}_t}{\mathfrak{n}_t} \frac{1}{(1 - \Upsilon \zeta) \frac{\mathfrak{l}^c_t}{d_t}} \right] \tag{28}$$

The wholesale lending rate $r_{l,t+1}$ differs from the retail lending rate $r_{r,t+1}$ due to a measure of bankruptcy risk $\bar{\omega}^c_{t+1}$.

In period t, the bank obtains the pre-commitment from the borrowers (constrained agents) to be paid $i_{r,t}$ for loan issuance. However, only a proportion $\left(1 - \mathfrak{F}^c_t \left(\bar{\omega}^c_{t+1} \right)\right)$ of the borrowers pay the committed interest rate in period $t+1$ as the rest enter bankruptcy. In case of interest payment default from bankruptcy, the bank recovers a fraction $(1-\xi)$ of the pledged collateral. This gives rise to the following ex-ante condition that pins down the retail lending rate $r_{r,t+1}$:

$$\mathbb{E}_t \left[\left(1 - \mathfrak{F}^c_t \left(\bar{\omega}^c_{t+1} \right)\right) r_{r,t+1} \mathfrak{l}_t + (1 - \xi) \int_0^{\bar{\omega}^c_{t+1}} \mathfrak{c}_t \omega^c \mathfrak{f}^c_t (\omega^c) d\omega^c \right] = \mathbb{E}_t \left[\{ r_{l,t+1} \mathfrak{l}^c_t \right] \tag{29}$$

The bankruptcy risk cut-off measure $\bar{\omega}^c_{t+1}$ is explained further in the Appendix.

3.4 Government

The government who also plays the role of the central bank face the budget constraint:

$$b_t + m_t = r_t b_{t-1} + r_{m,t} m_{t-1} + g_t + trf_t - \tau^{ls}_t - \tau_n w_t n_t - \tau_c \left[(1 - \omega) c^c_t + \omega c^u_t \right] \tag{30}$$

where b_t and m_t are government supply of bonds and CBDC, respectively. This budget constraint means that the issuance of CBDC is financed by government bonds. We assume that the government

[1]See Appendix A.1.1 for more details

expenditure always equals a fraction of GDP so that:

$$g_t = \phi_g gdp_t \tag{31}$$

The government manages the domestic output gap through the following rule on government deficit:

$$deficit_t = \overline{deficit} - \phi_{deficit} \ln\left(\frac{y_t}{\overline{y}}\right) \tag{32}$$

where $deficit_t = \frac{(b_t+m_t)-\frac{(b_{t-1}+m_{t-1})}{\pi_t}}{4gdp_t}$. $\phi_{deficit}$ is the response coefficient which allows the deficit to fluctuate when there is an output gap.

The monetary policy of the government/central bank follows the conventional (nominal) interest rate rule of Barrdear and Kumhof (2021) which includes interest rate smoothing and deviations of annualized inflation from target inflation. The nominal policy rate $i_{b,t}$ follows the below Taylor rule

$$i_{b,t} = i_{b,t-1}^{\rho_i}\left(\frac{\zeta_t}{\beta_u}\right)^{1-\rho_i}\mathbb{E}_t\left[\left(\frac{\pi_t\pi_{t+1}\pi_{t+2}\pi_{t+3}}{\overline{\pi}^4}\right)^{\frac{\phi_{i,\pi}(1-\rho_i)}{4}}\right] \tag{33}$$

where $i_{b,t}$ is the nominal policy rate, ρ_i is the smoothing parameter and $\phi_{i,\pi}$ is the inflation feedback coefficient.

Given the policy interest rate Taylor rule in Eq. (33), the CBDC maybe used as an additional monetary policy instrument for price stability. As such, the CBDC monetary policy framework follows one of the below regimes.

Baseline regime The baseline CBDC regime assumes the central bank only follows the conventional interest rate rule in Eq. (33) for price stability. Hence, nominal CBDC interest rate is fixed at its steady level, $\overline{i_m}$:

$$i_{m,t} = \overline{i_m} \tag{34}$$

This regime is a special case of the price rule regime that will be elaborated soon. The central bank sets a fixed interest rate on CBDC, and allow the supply of CBDC to adjust itself to meet the demand for money. In a more specific case where $\overline{i_m} = 0$, this regime is equivalent to an economy with non-interest bearing cash.

Price rule regime In the CBDC price rule regime, the central bank use nominal CBDC interest rate as an additional instrument for price stability based on the below rule

$$i_{m,t} = {i_{m,t-1}}^{\rho_{i_m}}\left(i_{b,t}\frac{\overline{i_m}}{\overline{i_b}}\right)^{1-\rho_{i_m}}\mathbb{E}_t\left[\left(\frac{\pi_t\pi_{t+1}\pi_{t+2}\pi_{t+3}}{\overline{\pi}^4}\right)^{-\frac{\phi_{i_m,\pi}(1-\rho_{i_m})}{4}}\right] \tag{35}$$

where ρ_{i_m} is the smoothing parameter. $\overline{i_m}$ and $\overline{i_b}$ correspond to the steady state levels of the nominal CBDC and bond interest rates, respectively. $\phi_{i_m,\pi}$ is the price rule inflation feedback coefficient.

Under the price rule regime, the central bank has autonomy on the interest rate of CBDC. There are two components contributing to the movements of the CBDC interest rate. The first component maintains a constant spread with the policy interest rate $i_{b,t}$, ensuring that CBDC functions as a complement to government bonds as a saving vehicle. The second component in the price rule adjusts to the expected inflation. Should there be a tendency for the inflation to rise, the CBDC interest rate decreases, making CBDC a less attractive asset and hence removing CBDC liquidity from the economy. The resultant lower liquidity level is expected to reduce consumption, and hence bring inflation back to the target.

Quantity rule regime Under the quantity rule regime, the central bank adjusts the CBDC-to-GDP ratio for achieving price stability through

$$\frac{m_t}{4gdp_t} = \rho_m \left(\frac{m_{t-1}}{4gdp_{t-1}}\right) + (1-\rho_m)\left[\frac{\overline{m}}{4\overline{gdp}} - \frac{\phi_{m,\pi}}{4}\mathbb{E}_t\left[\ln\left(\frac{\pi_t\pi_{t+1}\pi_{t+2}\pi_{t+3}}{\overline{\pi}^4}\right)\right]\right] \tag{36}$$

where ρ_m is the smoothing parameter and $\phi_{m,\pi}$ is the inflation feedback coefficient.

The quantity rule follows the same logic as the price rule, but it controls the level of liquidity directly. The central bank maintains a specific level of CBDC supply in the economy. When there is a tendency for inflation to increase, the CBDC supply responds counter-cyclically to inflation dynamics, reducing the liquidity level in the economy. As a result, the central bank curbs the rising inflation.

It is worth stressing that while the adjustable interest rate of CBDC may serve as an additional policy instrument, its impacts on the economy are complicated. For example, Eq. (21) indicates that the effects of a change in the CBDC interest rate could range from its impact on exchange rate movements to demands for all assets accessible to the households. Our simulation exercises will explore the impacts of the above three CBDC regimes on the economy.

3.5 Aggregation and equilibrium

From the production function, the aggregate output of the economy is given by $\int_0^1 y_t(i)\,di = {k_t^*}^{\alpha} a_t {n_t}^{1-\alpha}$. Also, aggregating the demand function for type-i goods leads to $\int_0^1 y_t(i)\,di = D_t y_t$, where $D_t \equiv \int_0^1 \left(\frac{p_t(i)}{p_t}\right)^{-\theta_p} di$ is the price dispersion. The relationship between aggregate output and inputs of production is

$$y_t = a_t {k_t^*}^{\alpha} n_t^{1-\alpha} D_t^{-1}$$

where the law of motion for the price dispersion is $D_t = (1-\vartheta)\,\tilde{\pi}_t^{-\theta_p} + \vartheta \pi_t^{\theta_p} D_{t-1}$. The market clearing conditions of the model are given below:

$$\textbf{Goods:}\ \ (1-\omega)\,y_t = (1-\omega)\,c_t^c + \omega c_t^u + g_t + c_t^* \cdot \overline{gdp} + \zeta \mathcal{T}_t^{costs} \tag{37}$$

$$\textbf{Loans:}\ \ \mathfrak{l}_t = (1-\omega)\,\mathfrak{l}_t^c \tag{38}$$

$$\textbf{Deposits:}\ \ \mathfrak{d}_t = (1-\omega)\,d_t^c + \omega d_t^u \tag{39}$$

$$\textbf{Domestic bonds:}\ \ b_t = \omega b_t^u \tag{40}$$

$$\textbf{CBDC:}\ \ m_t = (1-\omega)\,m_t^c \tag{41}$$

$$\textbf{Labor:}\ \ (1-\omega)\,n_t = (1-\omega)\,n_t^c + \omega n_t^u \tag{42}$$

where c_t^* is the ratio of exports to steady state GDP and $\mathcal{T}_t^{costs}$ is the bank's adhoc costs as specified by Barrdear and Kumhof (2021) (elaborated in Appendix A.1.2). The balance of payments identity is:

$$q_t \omega b_t^{*u} = q_t \zeta_{t-1}^* r_t^* b_{t-1}^{*u} + q_t (1-\omega)\, k_t^* - c_t^* \cdot \overline{gdp} \tag{43}$$

where the trade balance measures the change in the net foreign liabilities position of the economy.

Finally, the gross domestic product gdp_t is defined as

$$gdp_t = (1-\omega)y_t - \zeta \mathcal{T}_t^{costs} - q_t\,(1-\omega)\,k_t^* \tag{44}$$

3.6 Exogenous shocks

There are 5 shocks in our model: shocks to the domestic labor productivity a_t, the marginal utility of consumption $z_{mu,t}$, the money demand $z_{md,t}$, the foreign interest rate i_t^* and the export demand normalized to the steady-state GDP c_t^*. The vector $\boldsymbol{e}_t$ contains the 5 shocks which follow the auto-regressive process of order one. Hence, we have:

$$\ln(\boldsymbol{e}_t) = \rho_{\boldsymbol{e}} \ln(\boldsymbol{e}_{t-1}) + (1-\rho_{\boldsymbol{e}})\ln(\overline{\boldsymbol{e}}) + \varepsilon_{\boldsymbol{e}}, \qquad \varepsilon_{\boldsymbol{e}} \sim N\left(0, \sigma_{\boldsymbol{e}}^2\right) \tag{45}$$

4 Parameterization

In this section, we summarize the calibration of the parameter values in the model. Section 4.1 reports the parameters and values directly calibrated from past literature. Section 4.2 identifies the parameter values with implied steady state implications. Section 4.3 reports the estimated coefficients. Finally, Section 4.4 presents the optimal CBDC monetary policy coefficient values.

4.1 Calibrated from literature

In Table 1, panel A shows the parameter values sourced from the benchmark model of Barrdear and Kumhof (2021). The parameters in panel B such as the non-linear pricing parameters, ϑ and θ_p and the risk premium coefficient, φ_e, are calibrated using small open economy literature (Galí and Monacelli, 2005; Benes et al., 2015). Finally the fiscal policy output gap coefficient is calibrated based on the estimate of Girouard and André (2006) for New Zealand which we consider as an example of a small open economy.

Table 1: Parameters directly calibrated from literature

	Description	Value	Source
	Panel A		
ψ_l^u	Weight of liquid asset in unconstrained agent utility preferences	0.0109	
ψ_l^c	Weight of liquid asset in constrained agent utility preferences	0	
θ_n	Labor supply elasticity	1	
ω	Share of unconstrained agents in the population	5%	Barrdear and Kumhof (2021)
υ	Consumption habit	0.7	
Υ	Minimum capital adequacy ratio	8%	
$\phi_{\mathfrak{l}}$	Loan adjustment cost coefficient	0.00001	
$\phi_{i,b}$	Elasticity of policy rate to government bonds-GDP ratio	0.005	
	Panel B		
ϑ	Fraction of firms with prices unchanged	0.75	Galí and Monacelli (2005)
θ_p	Elasticity of substitution between differentiated goods	6	Galí and Monacelli (2005)
φ_e	Risk premium coefficient	0.1	Benes et al. (2015)
$\phi_{deficit}$	Fiscal policy output gap coefficient	0.12	Girouard and André (2006)

4.2 Parameters with implied steady state implications

Here, we discuss the parameters with steady state implications. The steady state values in the last column of Table 2 are annualized unless mentioned otherwise. In line with Gopinath et al. (2020), we set the value of α to be 0.7. This results in labor income share to be about 82% of GDP in the steady state. Income and consumption tax rates (τ_l, τ_c) and coefficients on government spending (ϕ_g), real collateral (κ^r) and monetary transaction costs (A) are calibrated to match steady-state target ratios computed from the mean of New Zealand data from 2010 to 2018[2]. Lump-sum tax rate is set to equal 0.1 to match the assumption that lump-sum taxes contribute one-third of total tax revenue in the steady-state (Barrdear and Kumhof, 2021). The value of ι is calibrated using Barrdear and Kumhof (2021). The weight of labor hours in utility preferences, ψ_n, is calibrated based on the assumption that the unconstrained agent's labor supply is equal to one (Barrdear and Kumhof, 2021). The share of bank networth paid as dividends, δ_b, equals 5.73%. This implies a steady state buffer in bank capital adequacy at 2.5%.

The value of monetary transaction coefficient B results in the semi-interest elasticity of constrained agents' deposit demand at 6.74 in the steady-state. This is very close to the money demand interest elasticity estimate of Ball (2001). As unconstrained agents are the marginal depositors, Barrdear and Kumhof (2021) argue that unconstrained agents should exhibit high semi-interest elasticity of deposit demand with a value much greater than 50. The calibrated value of liquidity services elasticity θ_l satisfies this condition as the unconstrained agents' elasticity of deposit demand equal 110 in the steady state. The elasticity of substitution ϵ implies the constrained agents' interest elasticity of CBDC demand to equal 15 in the steady state. The calibrated parameter values of bank riskiness and bankruptcy cost imply MCAR violation rate at 25% and 2.5%, respectively, every quarter. The discount factor parameters β_c and β_u are calibrated to match the nominal policy and deposit rate at 3% and 2%, respectively in the steady-state. The interest rate targets match the mean of New Zealand data from 2010 to 2018. The values of χ and σ_c are calibrated in accordance with Barrdear and Kumhof (2021). We calibrate the CBDC transaction coefficient ψ_m to satisfy the condition that nominal CBDC rate falls in the interval between nominal policy rate and deposit rate; $\overline{i^d} < \overline{i^m} < \overline{i}$ (Barrdear and Kumhof, 2021). The calibrated value of $\psi_m = 1.2654$ implies CBDC rate to equal 2.92% steady state. Finally, CBDC coefficient in liquidity services, γ^c, implies the ratio of CBDC to GDP at 20% in the steady state. This target is close to the assumption of Barrdear and Kumhof (2021) where the CBDC used for consumption purposes equal 19.31% of GDP in the steady state.

Domestic good price, inflation, nominal exchange rate, real exchange rate and imported intermediate good price at the steady state equal 1, so $\overline{P} = \overline{\pi} = \overline{\Delta e} = \overline{q} = 1$. Steady state marginal cost $\overline{\mu}$ equals to $\frac{\theta_p - 1}{\theta_p}$. The steady state exports-to-GDP ratio is set at 30% in line with the estimation of Jacob and Munro (2018).

4.3 Estimation

We estimate the baseline monetary policy coefficients (see Eq. (33)), persistence and standard deviations of the exogenous shocks using Bayesian estimation (Herbst and Schorfheide, 2015). The estimated coefficients are reported in Table 3. The prior distribution of the policy rate smoothing coefficient ρ_i and the the AR(1) coefficients of the exogenous shocks is set as beta distribution with mean 0.7 so that posterior estimates lie within the range of $[0, 1]$. The prior mean of the standard deviation of all the shocks is assumed to be large at 100 (Justiniano and Preston, 2010). The standard deviations of

[2]Refer to Table 9 for data sources.

Table 2: Parameters with implied steady state implications

	Parameter		Implied steady state	
	Description	Value	Description	Value
α	Foreign intermediates share in production	0.7000	Labor income share of GDP	82%
τ_n	Income tax	0.2174	Income tax revenue/GDP	17.83%
τ_c	Consumption tax	0.0730	Consumption tax revenue/GDP	6.16%
ϕ_g	Government spending rule coefficient	0.1900	Government spending/GDP	19%
κ^r	Real collateral coefficient	0.95	Constrained agent loan/GDP	96.61%
A	Monetary transaction cost coefficient	0.7183	Constrained agent deposit/GDP	45.18%
τ_{ls}	Lump-sum tax	0.1000	Lump-sum tax/total tax revenue	30.77%
ι	Constrained agents' share of lump-sum income	0.9466	Unconstrained agent consumption/constrained agent consumption	2.3:1
ψ_n	Weight of labor hours in utility preferences	0.1356	Unconstrained agent labor supply	1
δ_b	Share of bank networth as dividends	0.0573	Buffer in bank capital adequacy	2.5%
B	Monetary transaction cost coefficient	0.9853	Constrained agent semi-interest elasticity of deposit demand	6.74
θ_l	Elasticity of liquidity services	1.1	Unconstrained agent semi-interest elasticity of deposit demand	110
ϵ	Elasticity of substitution between CBDC and deposits	2	Constrained agents semi-interest elasticity of CBDC demand	16
σ_b	Bank riskiness parameter	0.0129	Share of banks violating MCAR per quarter	25%
ξ	Bankruptcy cost parameter	0.0563	Loan default rate per quarter	2.3%
β^u	Unconstrained agent discount factor	0.9926	Policy rate	3%
β^c	Constrained agent discount factor	0.9676	Deposit rate	2%
χ	MCAR penalty coefficient	0.0117	Wholesale lending rate	6.67%
σ_c	Collateral asset value riskiness	0.6300	Retail lending rate	17.32%
ψ_m	CBDC transaction efficiency relative to deposits	1.2654	CBDC rate	2.92%
γ^c	CBDC coefficient in liquidity services	0.3244	CBDC/GDP	20%

all the exogenous shocks are assumed to follow an inverse gamma distribution. Following Jacob and Munro (2018), we set the distribution of policy rate inflation feedback coefficient as gamma distribution with prior value of 2. Using New Zealand's quarterly data on GDP, consumption, M1 money supply, CPI inflation and imports from 2010 to 2018, we specify five observables for Bayesian estimation. They are GDP (gdp), aggregate consumption (c), constrained agent liquidity services (l^c), inflation (π) and imports (k^*). We applied one sided hp-filter with $\lambda = 1600$ to secure the cyclical data for estimation in Dynare. The Monte-Carlo based optimization routine was utilized to tune the scale parameter so that we could achieve an acceptance ratio between 25% to 35%. We also set the Metropolis-Hastings algorithm with 200,000 replications to compute the posterior mode.

Table 3: Estimated coefficients of the baseline regime

Symbol	Description	Prior density	Prior mean	Posterior mean	HPD interval	Prior standard deviation
Panel A: Shock parameters						
ρ_a	Autoregressive coefficient of productivity shock	beta	0.7	0.8552	[0.7464, 0.9591]	0.1
ρ_{md}	Autoregressive coefficient of money demand shock	beta	0.7	0.7217	[0.5897, 0.8550]	0.1
ρ_{mu}	Autoregressive coefficient of marginal utility of consumption shock	beta	0.7	0.9655	[0.9512, 0.9782]	0.1
ρ_{i^*}	Autoregressive coefficient of foreign interest rate shock	beta	0.7	0.5374	[0.4413, 0.6420]	0.1
ρ_{c^*}	Autoregressive coefficient of export demand shock	beta	0.7	0.6031	[0.4857, 0.7308]	0.1
σ_a	Standard deviation of productivity shock	inverse gamma	0.5	0.0711	[0.0588, 0.0813]	100
σ_{md}	Standard deviation of money demand shock	inverse gamma	0.5	0.0694	[0.0588, 0.0797]	100
σ_{mu}	Standard deviation of marginal utility of consumption shock	inverse gamma	0.5	0.0854	[0.0666, 0.1039]	100
σ_{i^*}	Standard deviation of foreign interest rate shock	inverse gamma	0.5	0.0799	[0.0647, 0.0953]	100
σ_{c^*}	Standard deviation of export demand shock	inverse gamma	0.5	0.0788	[0.0628, 0.0942]	100
Panel B: Baseline regime monetary policy coefficients						
ρ_i	Policy rate smoothing coefficient	beta	0.7	0.6406	[0.6093, 0.6746]	0.05
$\phi_{i,\pi}$	Policy rate inflation feedback coefficient	gamma	2	2.0842	[1.9251, 2.2489]	0.1

The theoretical moments of certain variables in the estimated baseline model are able to match their empirical counterparts using New Zealand data. As shown in Table 4, the model implied standard deviations of GDP, consumption and inflation and growth in nominal exchange rate are very close to their empirical counterparts. In the case of persistence, the model implied autocorrelations of consumption, imports and nominal exchange rate growth very closely resembles AR(1) estimates based on New Zealand data.

Table 4: Business cycle properties of the baseline regime

	Panel A: Standard deviation		**Panel B: Auto-correlation**	
	Data	Model	Data	Model
GDP	0.06	0.05	0.83	0.72
Consumption	0.05	0.05	0.91	0.90
Imports	0.05	0.12	0.71	0.66
Inflation	0.01	0.02	0.24	0.40
Nominal depreciation	0.11	0.10	-0.09	-0.12

Note: The model implied standard deviations and first order auto correlations are computed using second order perturbations around the stochastic steady state.

Table 5 reports the variance decomposition of all the exogenous shocks in the baseline monetary policy regime. We find that domestic productivity shock contributes a large share in the volatility of GDP, consumption, liquidity services and inflation. The shock to marginal utility of consumption plays a strong role in determining the volatility of real exchange rate, foreign bonds and labor. Export demand shock contributes the biggest share to the volatility in imports and also plays a large role in the volatility of foreign bonds. Foreign interest rate shock also determines about 14% of the volatility in real exchange rate and foreign bonds. Finally, the contribution of the money demand shock is large for consumption and liquidity services.

Table 5: Variance decomposition of key variables in the baseline regime

	Shocks				
	Productivity	Export demand	Foreign interest rate	Money demand	Marginal Utility
GDP	44.71	19.53	5.75	10.35	15.05
Consumption	41.05	7.23	3.11	24.32	24.29
Liquidity services (constrained agent)	50.94	9.47	3.72	18.61	17.26
Imports	28.56	34.53	19.03	1.67	16.22
Inflation	78.64	2.95	7.82	9.51	1.07
Real exchange rate	14.73	11.99	14.15	3.78	78.01
Foreign bonds	1.07	28.19	14.83	3.81	77.41
Labor	34.91	2.85	4.74	9.44	48.06

4.4 Optimal coefficient values in regimes with adjustable CBDC interest rate

As explained previously, the monetary policy coefficients of the baseline regime are already estimated using data. This is intuitive as the baseline regime resembles the scenario of cash i.e. CBDC interest rate is fixed.

With baseline regime monetary policy coefficients estimated from the data, the question remains about the optimal value of the CBDC monetary policy regime coefficients i.e. the price rule and the quantity rule regime coefficients. We assume the central bank to be a social planner with the sole objective of maximizing the overall social welfare of the economy. She is faced with the pertinent question of determining the CBDC monetary policy coefficient values that would maximize the welfare of the small open economy. To achieve this objective, we assume that she will use CBDC monetary policy coefficient values that achieve the highest overall social welfare in the presence of all exogenous shocks (see Panel A of Table 3). We describe more about the overall social welfare measure below.

Following Schmitt-Grohé and Uribe (2007), the unconditional mean of a type-j agent's lifetime utility (see Eq. (3)) is considered as a measure of her welfare. We use DYNARE to compute the unconditional mean of an agent's life-time utility, $E_0 U_0^j$, through a second-order approximation of the model. To obtain a measure of overall social welfare, we use the population weighted average of the agents' unconditional mean of life-time utility. As such, overall social welfare, $\mathcal{W}_0$ is denoted by

$$\mathcal{W}_0 \equiv (1-\omega) E_0 U_0^c + \omega E_0 U_0^u \tag{46}$$

where $E_0 U_0^c$ and $E_0 U_0^u$ are the unconditional means of constrained and unconstrained agents life time utility, respectively. The social planner is assumed to use the overall social welfare measure, $\mathcal{W}_0$ to determine the optimal coefficients in the price and quantity rules. Towards this end, we conduct grid searches to obtain the optimal coefficient values that maximize the overall social welfare. First, we optimize the coefficients associated with the CBDC price rule regime (Eq. (35)) where the nominal CBDC interest rate ($i_{m,t}$) acts as a secondary monetary policy instrument in addition to the primary policy interest rate rule (Eq. (33)). We keep the coefficients ρ_i, $\phi_{i,\pi}$, $\phi_{i,b}$ and $\phi_{deficit}$ as in the baseline regime (see Table 1 and Table 3) and conduct a grid search for ρ_{i_m} and $\phi_{i_m,\pi}$ to maximize the social welfare of the economy, $\mathcal{W}$. We conduct a two dimensional grid search over the range $\rho_{i_m} \in [0, 0.9]$ and $\phi_{i_m,\pi} \in [0, 16]$. We find the optimal coefficients at $\rho_{i_m} = 0$ and $\phi_{i_m,\pi} = 8.1$.

The next step of welfare optimization is performed under the CBDC quantity rule regime (Eq. (36)) where the CBDC-to-GDP ratio ($\frac{m_t}{4gdp_t}$) acts as the secondary monetary policy instrument instead of the nominal CBDC interest rate . As in the optimization of the CBDC price rule regime, we keep the optimal values of coefficients ρ_i, $\phi_{i,\pi}$ and $\phi_{deficit}$ as in the baseline regime and conduct a grid search for the optimal value of coefficients ρ_m and $\phi_{m,\pi}$. We perform a two dimensional grid search over the range $\rho_m \in [0, 0.9]$ and $\phi_{m,\pi} \in [0, 40]$. We find the optimum at $\rho_m = 0$ and $\phi_{m,\pi} = 19.4$.

5 Equilibrium dynamics

Before examining the welfare implications of the CBDC regimes, we analyze the equilibrium dynamics in response to shocks. In this section, we report the simulation results for three monetary policy regimes; a) *Baseline regime*: The central bank uses only nominal policy rate as the monetary policy instrument. The nominal policy rate follows Eq. (33). The CBDC interest rate is fixed at the steady state (analogous to cash). b) *Price rule regime*: The central bank uses both policy rate and CBDC interest rate as monetary policy instruments. The CBDC interest rate is set in accordance with Eq. (35). c) *Quantity rule regime*: The policy rate and the CBDC quantity are used as monetary policy instruments. The CBDC-GDP ratio follows Eq. (36). The simulations use parameter values as reported in Section 4. The rest of the section discusses the simulation results when the small open economy with CBDC is faced with expansionary shocks on export demand, money demand, foreign interest rate and domestic productivity.

Export demand shock: Let us first consider the effects of a positive export demand shock equivalent to 1% of steady-state GDP. Fig. 1 shows the responses of the key variables depicted by the solid black line under the baseline regime, the long-dash blue line under the price rule and the dot-dash red line under the quantity rule.

First, we examine the case of baseline regime. In line with our expectation, the shock increases aggregate demand leading to an expansion of output (panel 1 in Fig. 1). Both labor and wage rise, which in turn increase the demand for consumption (panel 2, 3 and 4). Higher export also increases the current account which appreciates the real exchange rate (panel 5) through the balance of payments identity in Eq. (43). With the exchange rate appreciation, the cost of imported intermediate imports declines, which in turn reduces prices and inflation (panel 6). This causes the nominal policy interest rate to initially fall not only in the baseline regime but in the price and quantity regimes as well (panel 7).

The reduction in the policy rate reduces the return on government bonds, and unconstrained agents demand less government bonds (panel 12). The arbitrage by unconstrained agents reduces the nominal deposit rate (panel 8). Since the CBDC interest rate is fixed in the baseline regime, the nominal spread between policy and CBDC rate declines (panel 9). This implies a fall in the nominal spread between deposit and CBDC rate causing constrained agents to switch away from deposits (panel 10) to CBDC (panel 11). Consequently, the CBDC ratio increases.

The rise in aggregate demand also pushes up the demand for imported inputs for production so imports rise over time (panel 13). Furthermore, the real exchange rate appreciation reduces the income from foreign interest payments so unconstrained agents demand less foreign bonds. Hence, the foreign bond ratio gradually declines (panel 14).

The effects of the shock in the adjustable CBDC regimes (price rule and quantity rule) are similar. The CBDC monetary policy instruments respond to deflation either through an increase in CBDC rate or an increase in CBDC-GDP ratio. In the price rule regime, the larger decline in the spread between policy and CBDC rate makes agents switch towards more CBDC (panel 11). Whereas in the quantity rule regime, the central bank removes some of the CBDC from circulation. Hence, CBDC ratio rises more in adjustable CBDC interest rate regimes. In fact, the price rule regime shows a larger rise in CBDC ratio. This expansion in CBDC also means that monetary transaction costs are smaller for the agents to consume. This is evident from the lower liquidity tax (panel 16). As a result, consumption rises more in the adjustable CBDC interest rate regimes over time (panel 4).

The rise in consumption demand due to lower consumption liquidity tax improves aggregate demand. This causes the inflation dip to be lower when CBDC interest rate is adjustable. Hence, the response of inflation is comparatively stable with adjustable CBDC interest rate regimes. This advantage arises at the cost of more volatile consumption (see panel 4).

Money demand shock: We now consider the effects of a money demand shock which increases the demand for monetary transaction balances from constrained agents. Fig. 2 shows the responses of key variables. With this shock, constrained agents prefer to use money balances as safe saving vehicles than for spending on consumption. As a consequence, consumption decline (panel 4 in Fig. 2) and liquidity rise (not shown) following the shock. The shock also implies that consumption liquidity tax rise (panel 16) as agents have lower preference to use liquidity services for consumption.

The higher demand for liquidity can be satisfied through both CBDC and bank loans that result in additional bank deposits. As argued by Barrdear and Kumhof (2021), banks immediately respond to money demand since the banking sector is modeled using the financing through the money creation channel (Jakab and Kumhof, 2015). Banks can expand both loans and deposits by charging higher lending rates against collateral. This is evident from Fig. 2 where we find the spread between policy rate and lending rate to slightly drop following the shock (Panel 15 in Fig. 2). Although bank loans/deposits can cater to the liquidity demand, the usage of bank loans to satisfy the liquidity requirements comes with monetary transaction costs.

The decline in aggregate demand on account of agents' preference to save money results in deflation

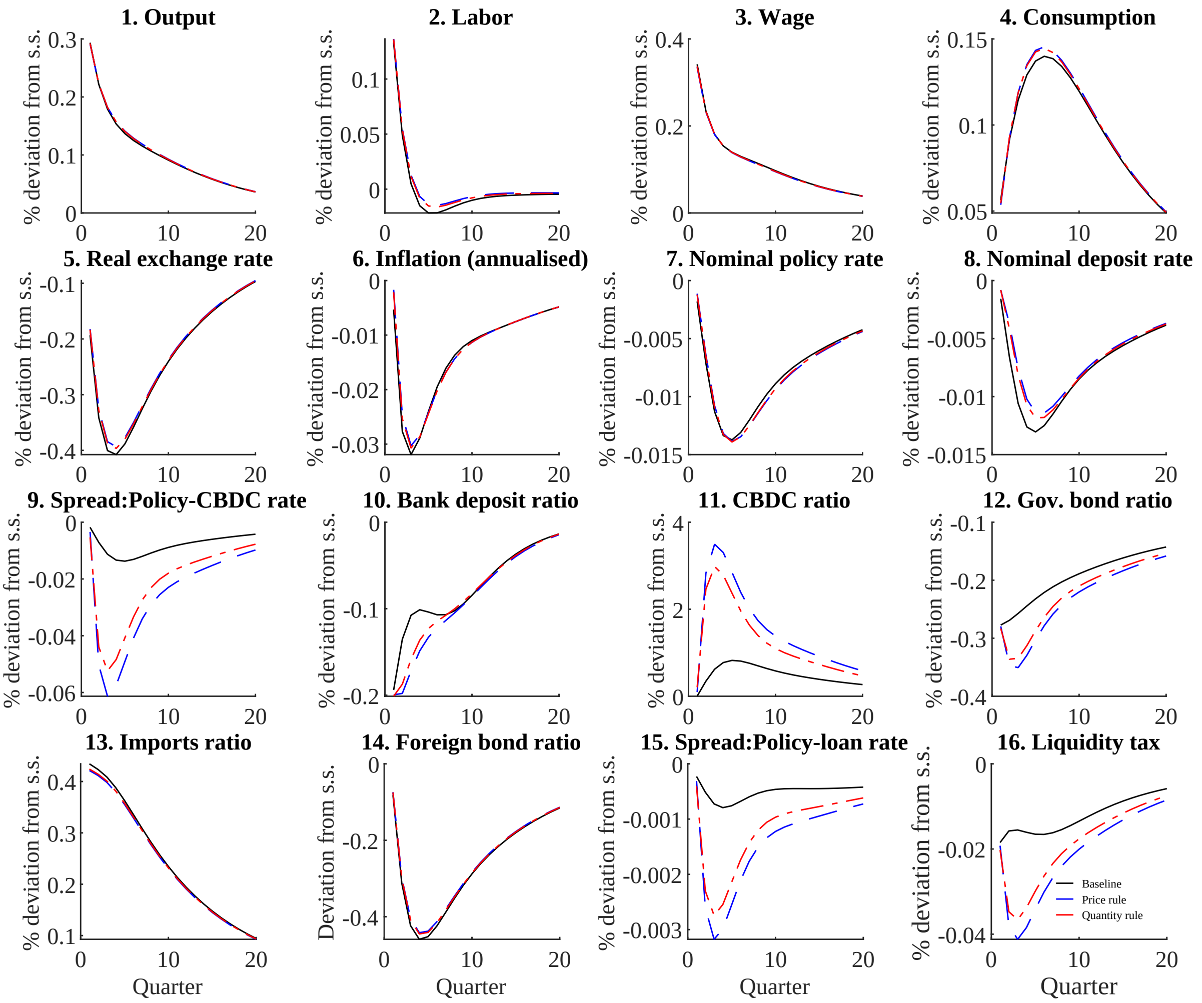


Figure 1: Response of key variables to a positive 1 standard deviation export demand shock normalized to GDP. Vertical axes indicate percentage deviation from steady state (except for foreign bond ratio). Horizontal axis indicate quarters after shock. The variables with ratio in the title indicates that it is shown as an annualized ratio to GDP. Table 10 provides more information about the ratios construction.

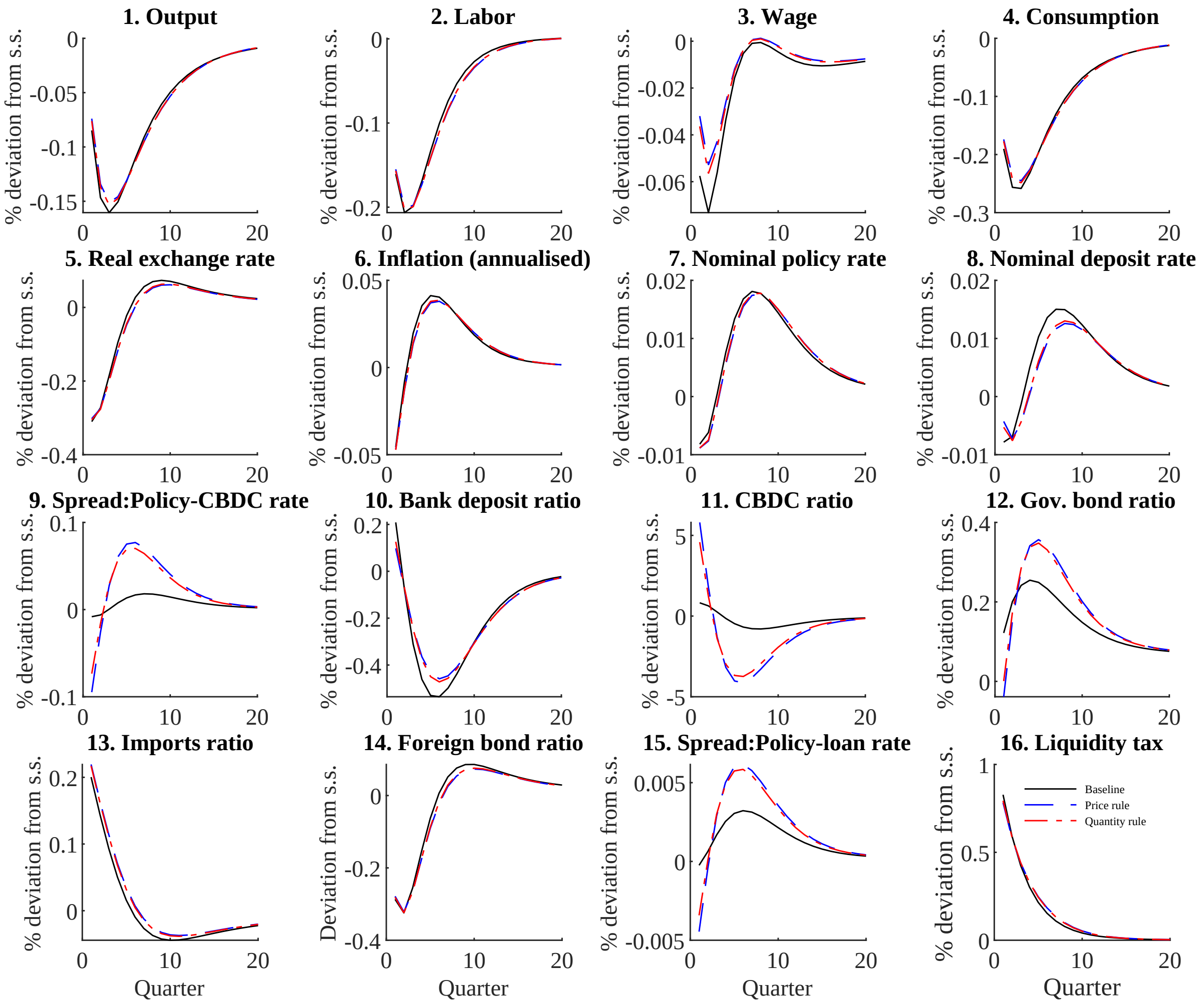


Figure 2: Response of key variables to a positive 1 standard deviation money demand shock. Vertical axes indicate percentage deviation from steady state (except for foreign bonds). Horizontal axis indicate quarters after shock. The variables with ratio in the title indicates that it is shown as an ratio to GDP. Table 10 provides more information about the ratios construction.

(panel 6). In the quantity rule regime, the central bank increases the quantity of CBDC as a response to deflation. Whereas in the price rule regime, the central bank reduces the nominal CBDC interest rate. This causes CBDC to rise more in the price rule regime leading to arguably lower monetary transaction cost. In other words, constrained agents in the price rule regime possess a larger share of CBDC than deposits resulting in lower monetary transaction costs in comparison to the other regimes. Consequently, the rise in liquidity tax is slightly smaller (panel 16) and the drop in consumption is less (panel 4).

Foreign interest rate shock: Fig. 3 shows the impulse responses of the key variables following a positive foreign interest rate shock. The exogenous rise in foreign interest rate increases the interest payments from holding foreign bonds. Hence, unconstrained agents increase their holding of foreign bonds. The rise in foreign interest rate also depreciates the exchange rate through the uncovered interest parity condition (panel 5). Firms now face higher marginal cost as the imported inputs for production becomes more expensive. This has a feedback effect on the domestic good prices, causing inflation to initially rise in the economy (panel 6). The depreciation in exchange rate improve the economy's trade balance. As a result, output expands (panel 1) leading to a rise in domestic consumption (panel 4) and labor (panel 2). The expansion of output also cause the inflation to dip drastically more than the initial rise (panel 6).

The monetary policy instruments become active to help the economy move back to price stability. Nominal policy interest rate initially rises in all the three regimes (panel 7). The higher inflation also reduces the nominal CBDC interest rate for the price rule regime (through the downward adjustment in the CBDC interest rate itself) and the quantity rule regime (through downward adjustment in the CBDC ratio). Hence, the spread between policy and CBDC rate initially widens when CBDC interest rate is adjustable (panel 9). The dip in CBDC ratio (and the resulting rise in liquidity tax, see panel 16) causes consumption to initially rise by less in the regimes with adjustable interest rate. However, owing to eventual deflation, CBDC ratio rises more than the initial dip (panel 11). Such drastic changes in the CBDC ratio contributes to overall consumption volatility in the adjustable CBDC interest rate regimes.

Bank deposits also increase following the foreign interest rate shock (panel 10). This can be explained by the decisions of the unconstrained agents who are the marginal investors in our economy. Their portfolio choices are highly elastic to interest rate changes. In the Barrdear and Kumhof (2021) model, unconstrained agents hold bonds and deposits as assets. In contrast, the portfolio of unconstrained agents in our model comprises foreign bonds in addition to deposits and government bonds. Thus, unconstrained agents arbitrage deposit rate to both policy rate and foreign interest rate in our model. The parity condition in Eq. (20) makes deposit as favorable medium of exchange when foreign interest rate endogenously rise. When CBDC interest rate is adjustable, the foreign interest rate shock causes more volatility in the bank deposits (panel 10) and CBDC ratio (panel 11). As a result, consumption response is also found to be more volatile when CBDC interest rate is adjustable (panel 4).

Productivity shock: Fig. 4 shows the impulse responses of key macroeconomic variables following a positive productivity shock. The productivity shock increases the supply of domestic (consumption) goods (panel 4), which in turn increases aggregate output (panel 1). The expansionary productivity shock culminates in a drop in labor hours (panel 2). This can be explained by the channels of habit formation in the utility function and the sticky prices. As argued by Liu and Phaneuf (2013), both afore-mentioned factors make output adjustment lag behind productivity improvement. This causes the demand for labor from firms to decline. This decline in labor demand more than offsets the increase in labor supply leading to a drop in aggregate labor (panel 2).

With inflation lower than its target, the nominal policy interest rate declines (panel 7). With the CBDC as secondary policy instrument, the CBDC interest rate rises when inflation is below its target under the price rule regime while the CBDC ratio increases when inflation is below its target under the quantity rule. The spread between policy and CBDC rate drops sharply when CBDC interest rate is adjustable (panel 9). In the price rule regime, CBDCs with higher interest become more attractive to constrained agents while government bonds are less attractive to unconstrained agents as nominal interest rate drops. To meet the changing demands of agents, the central bank increases CBDCs (panel 11) and reduces government bonds. The rise in CBDC is tremendously steep in the regimes with adjustable CBDC interest rate. This lowers monetary transaction cost in spending, leading to a drastic drop in consumption liquidity tax (panel 15). As a consequence, consumption rises more in the adjustable CBDC interest rate regimes (panel 4).

The steep drop in consumption liquidity tax also impacts the constrained agents' marginal rate of substitution (MRS) between consumption and labor hours. As argued by Barrdear and Kumhof (2021), this is evident from the first order conditions of the constrained agents with respect to labor supply and

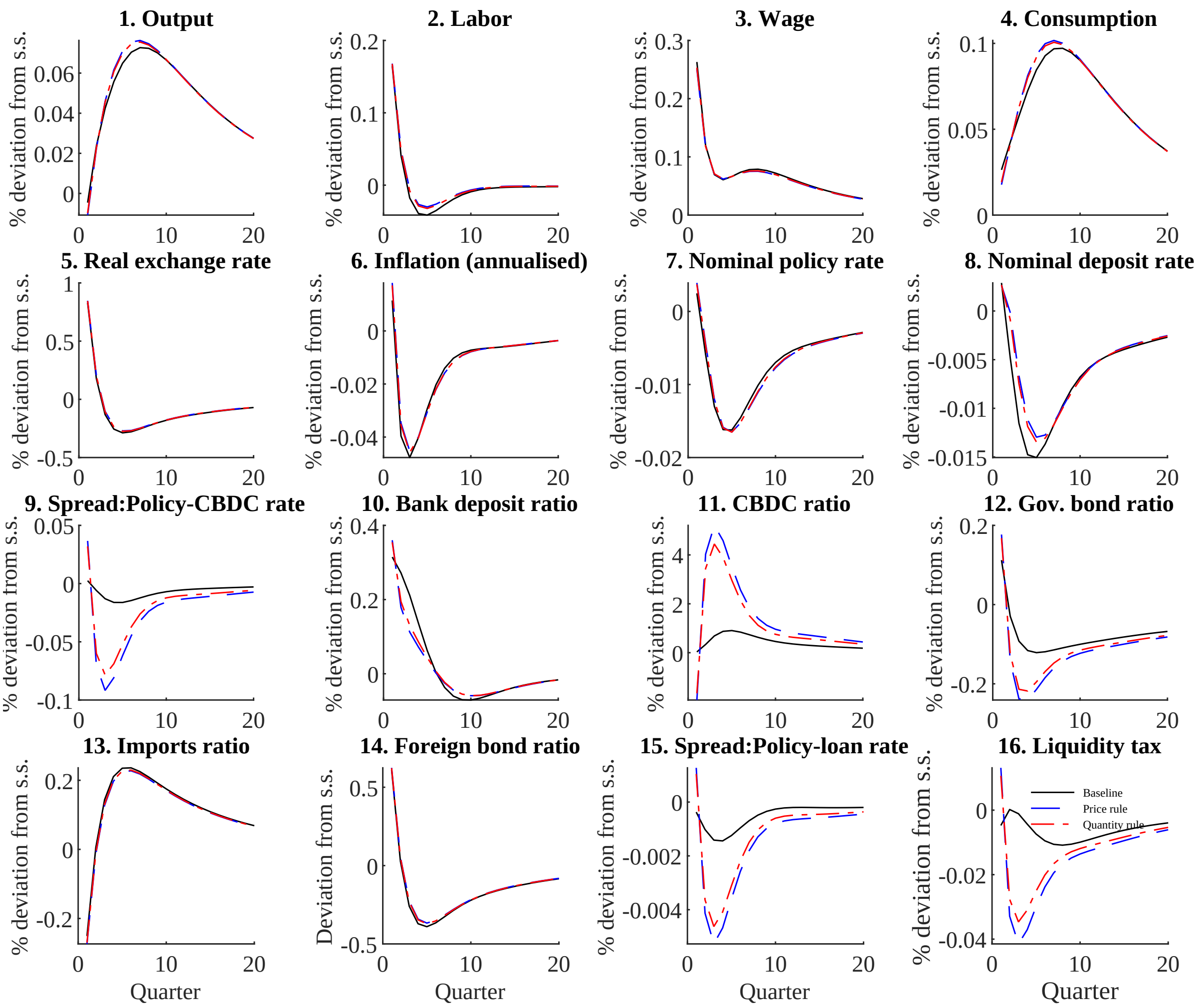


Figure 3: Response of key variables to a positive 1 standard deviation foreign interest rate shock. Vertical axes indicate percentage deviation from steady state (except for foreign bonds). Horizontal axis indicate quarters after shock. The variables with ratio in the title indicates that it is shown as a ratio to GDP. Table 10 provides more information about the ratios construction.

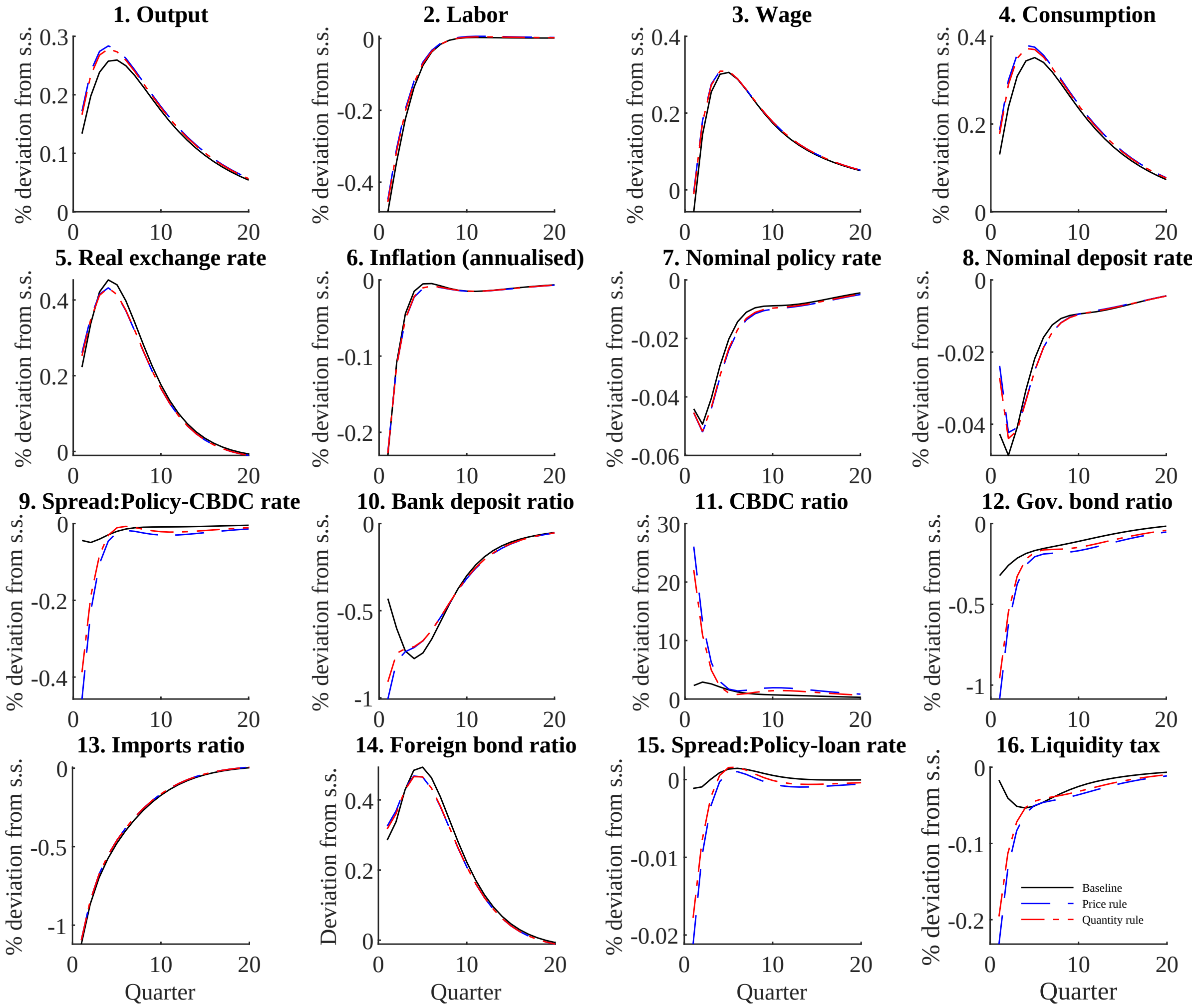


Figure 4: Response of key variables to a positive 1 standard deviation productivity shock. Vertical axes indicate percentage deviation from steady state (except for foreign bonds). Horizontal axis indicate quarters after shock. The variables with ratio in the title indicates that it is shown as a ratio to GDP. Table 10 provides more information about the ratios construction.

consumption (see Eq. (12)). Hence, we see the drop in labor hours (panel 2) is slightly less when CBDC interest rate is adjustable.

Marginal utility of consumption shock: Fig. 5 shows the impulse responses of key macroeconomic variables following a positive marginal utility of consumption shock. This shock increases the marginal utility from consumption which causes both agent types to demand more consumption goods (panel 4). This has an expansionary impact on output (panel 1), leading to an increase in labor hours (panel 2).

Since firms are owned by constrained agents, the exogenous rise in marginal utility of consumption lowers the firms' stochastic discount factor in the pricing equations (Amato and Laubach, 2004). As a consequence, inflation drops following the shock (panel 6) . The response of the CBDC policy instruments to deflation causes CBDC ratio to rise steeply in the regimes with adjustable CBDC interest rate (more so in the price rule regime, see panel 11). As a consequence, the drop in liquidity tax is also found to be steep in the adjustable CBDC interest rate regimes (more so in the price rule regime, see panel 16). As in the case of the productivity shock, the steep drop in liquidity tax has implications on the constrained agents

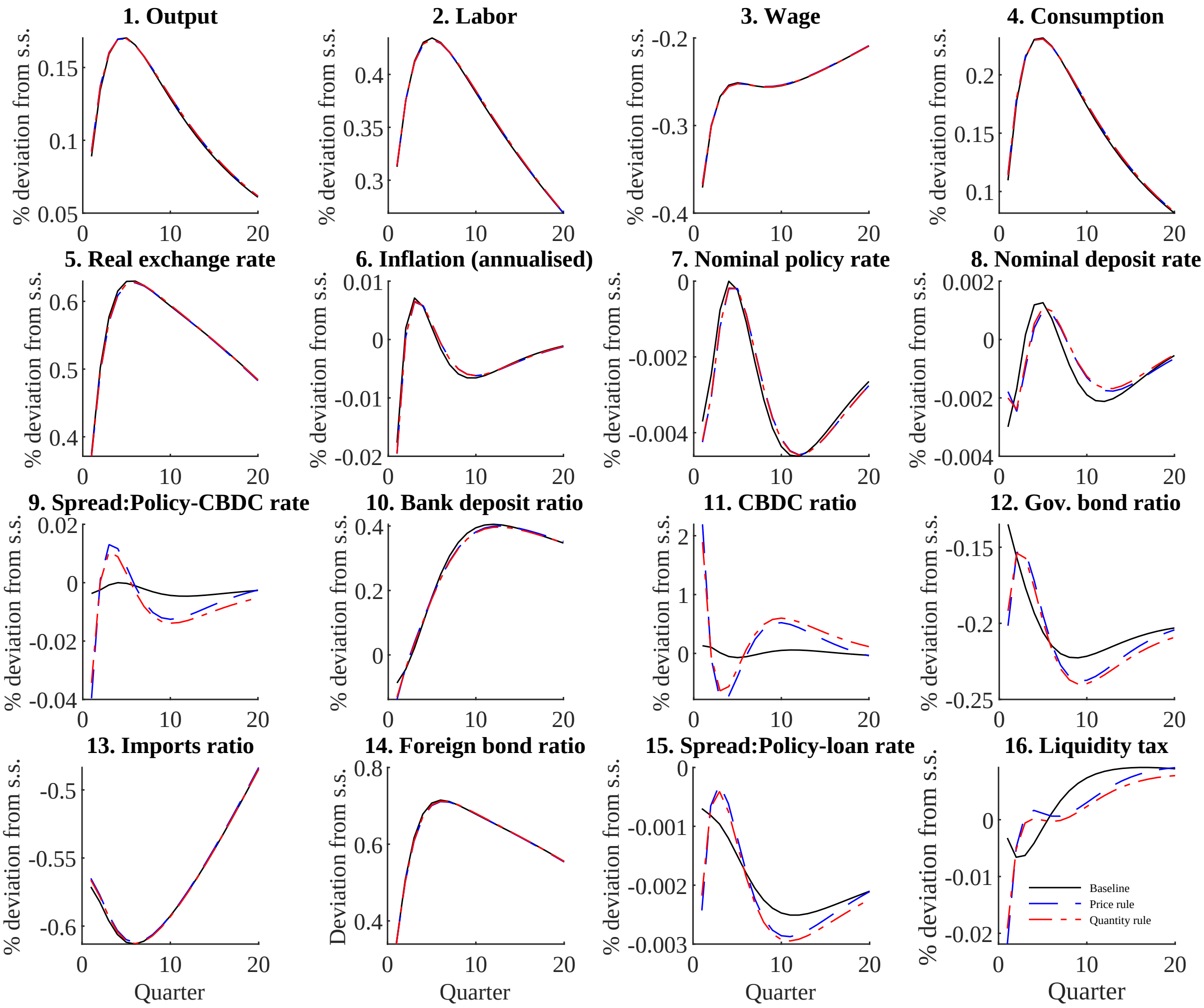


Figure 5: Response of key variables to a positive 1 standard deviation marginal utility of consumption shock. Vertical axes indicate deviation from steady state. Horizontal axis indicate quarters after shock. The variables with ratio in the title indicates that it is shown as a ratio to GDP. Table 10 provides more information about the ratios construction.

labor supply decisions (via the presence of liquidity tax in the marginal rate of substitution between consumption and labor). Although not evident at the first order simulations in Fig. 5, the impact of the liquidity tax drop is profound for labor volatility and welfare gain. This would be discussed further in Section 6.

6 Welfare analyses

This section examines the welfare implications of the adjustable CBDC interest regimes in comparison to the baseline regime. Section 6.1 computes the welfare gains associated with the CBDC monetary policy regimes. Section 6.2 provides the volatility of the key economic variables in each of the monetary policy regimes. Finally, Section 6.3 examines the contribution of each of the shocks in the CBDC monetary policy welfare gain/losses.

6.1 Welfare gain with CBDC monetary policy regimes

In this section, we compare the welfare between CBDC monetary policy regimes (at optimal coefficient values) and the baseline regime. To this end, we use the concept of compensating consumption variation (Kumhof and Laxton, 2009) to compute the welfare gains/losses when the central bank switches to the CBDC monetary policy regimes from the baseline regime. The compensating consumption variation of the price rule regime, η^j_{price}, can be defined as the percentage of consumption goods that the agent of type j would be willing to forgo to remain indifferent between baseline regime and price rule regime:

$$\eta^j_{price} = 100\left(1 - exp\left(\frac{(\beta^j - 1)\left(EU^j_{price} - EU^j_{baseline}\right)}{1-\upsilon}\right)\right),\ j \in c, u \tag{47}$$

where $E_0U^j_{0,price}$ and $E_0U^j_{0,baseline}$ denote the unconditional means of lifetime utility of agent j when she faces the price rule regime and baseline regime, respectively. In a similar manner, the compensating consumption variation of the quantity rule regime, η^j_{qty}, is given by.

$$\eta^j_{qty} = 100\left(1 - exp\left(\frac{(\beta^j - 1)\left(EU^j_{qty} - EU^j_{baseline}\right)}{1-\upsilon}\right)\right),\ j \in c, u \tag{48}$$

The overall society welfare gain/loss of a CBDC policy regime is measured by the population weighted average of compensating consumption variation associated with the respective CBDC policy (Bilbiie, 2008; Kumhof and Laxton, 2013). In the equation below, the welfare gains associated with price and quantity rule regimes are denoted by $\hat{\eta}_{price}$ and $\hat{\eta}_{qty}$, respectively. A negative value of $\hat{\eta}$ indicate a welfare loss.

$$\hat{\eta}_{price} \equiv (1-\omega)\eta^c_{price} + \omega\eta^u_{price} \tag{49}$$

$$\hat{\eta}_{qty} \equiv (1-\omega)\eta^c_{qty} + \omega\eta^u_{qty} \tag{50}$$

Table 6 shows the optimal CBDC policy coefficients and the respective welfare gains of CBDC monetary policy regimes in comparison to the baseline regime. As the welfare gain values (compensating consumption variation) are positive for society, we can infer that overall social welfare improves when the central bank switches to CBDC monetary policy regimes from the baseline regime. Although both CBDC policy regimes are welfare improving, the price rule regime shows a higher welfare gain. However, a breakdown of the welfare estimates of constrained and unconstrained agents in Table 6 shows that there are distributional consequences associated with CBDC monetary policy regimes. Constrained agents witness welfare gain under the quantity rule and an even larger welfare gain under the price rule. However, unconstrained agents face welfare loss in both regimes, particularly more in the price rule regime. This may be explained by the fact that constrained agents face monetary transactions costs in consumption spending whereas unconstrained agents do not. As the central bank operates through the price rule and the quantity rule to stabilize inflation, the resulting changes in monetary transaction costs (see Section 5 for transmission details) of constrained agents improve efficiency and stability while using CBDC for consumption spending compared to deposits. The same does not apply for unconstrained agents as they use only deposits for consumption spending.

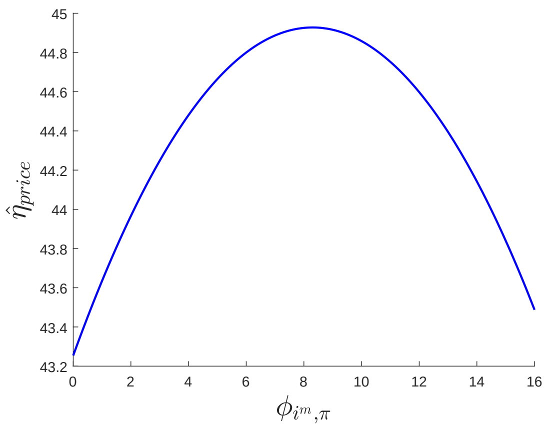
To summarize, Table 6 shows that both CBDC monetary policy regimes are welfare improving. The welfare gain of constrained agents is the highest under the price rule regime with the nominal CBDC interest rate (i_t^m) as the central bank's secondary policy instrument. The welfare loss of the unconstrained agents is also worst under the price rule regime. However, the large improvement in constrained agents' welfare more than compensates for the loss of unconstrained agents' welfare so society's welfare gain is highest under price rule regime.

Table 6: Welfare gain with CBDC monetary policy regimes

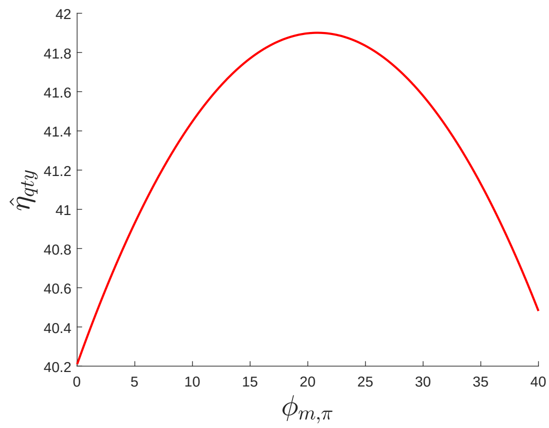
	ρ_i	$\phi_{i,\pi}$	$\phi_{deficit}$	ρ_{i_m}	$\phi_{i_m,\pi}$	ρ_m	$\phi_{m,\pi}$	Welfare Gain Society ($\hat{\eta}$)	Constrained agent (η^c)	Unconstrained agent (η^u)
Price rule	0.6406	2.0842	0.12	0	8.1	-	-	44.93	52.33	-95.67
Quantity rule	0.6406	2.0842	0.12	-	-	0	19.4	41.89	48.30	-79.79

The notations and the values of the parameters follows as in Section 4. Welfare gain is computed using second-order perturbations around the stochastic steady state in DYNARE.

We next examine the social welfare implications of the CBDC monetary policy regimes in comparison to the baseline regime when the value of CBDC inflation feedback coefficient changes. In Fig. 6, panel A plots the price rule inflation feedback coefficient, $\phi_{i^m,\pi}$ in the horizontal axis and the corresponding social welfare gain, $\hat{\eta}_{price}$ in the vertical axis. Panel B of Fig. 6 plots the quantity rule inflation feedback coefficient, $\phi_{m,\pi}$ in the horizontal axis and the corresponding social welfare gain, $\hat{\eta}_{qty}$ in the vertical axis. Panel A and B in Fig. 6 reveal three interesting points. First, welfare gain exists in both price and quantity rule regimes even when inflation feedback coefficient is 0. This highlights the novel finding that adjustable CBDC interest is welfare improving, irrespective of whether CBDC is used as a inflation stabilizing monetary policy instrument or not. Second, social welfare gain improves when CBDC inflation feedback coefficient increases from 0 to the optimal value (see Table 6) and falls thereafter. This happens in both price rule and quantity rule regimes. Third, the price rule regime is accompanied by higher welfare gains compared with the quantity rule regime. This is in line with the findings from Table 6.



(a) Price rule



(b) Quantity rule

Figure 6: Welfare gain in CBDC monetary policy regimes

6.2 Volatility

In addition to welfare gains, the volatility implications measured by the standard deviations of the key variables under the three regimes are shown in Table 7. In the simulation, the economy is subject to all the five shocks. Table 7 shows the standard deviation of liquidity services to increase when the economy moves from the baseline regime to the adjustable CBDC interest bearing system of the price rule and the quantity rule. This is intuitive as Section 5 shows the equilibrium quantity of CBDC to fluctuate more when CBDC interest rate is adjustable. Liquidity services have a direct bearing on consumption spending patterns. Hence, the volatility of consumption also increases when CBDC interest rate is adjustable. That said, Table 7 shows labor to be more stable when CBDC interest rate is adjustable. This can be explained through the liquidity tax changes (from CBDC movements in the adjustable CBDC interest rate regimes) that affect the constrained agents' marginal rate of substitution between consumption and labor (MRS), causing labor to be more stable. This is especially true for productivity and marginal utility of consumption shocks, where the drop in liquidity tax is steep for adjustable CBDC interest rate regimes (see panel 16 in Fig. 4 and Fig. 5). With labor largely being driven by productivity and marginal utility of consumption shocks (see Table 5), the steep decline in liquidity tax under these shocks contributes to lower labor volatility (via the impact through MRS)[3]. The stability in labor is the main driving force for the constrained agents' welfare gain (see Table 6) as her utility function comprises of labor in addition to consumption. On the other hand, unconstrained agents' utility function comprises of liquidity services (deposits) in addition to labor and consumption. When CBDC interest rate is adjustable, the increase in volatility associated with both consumption and liquidity services (see Table 7) contributes to the welfare loss of unconstrained agents (see Table 6).

Inflation is more stable when CBDC interest rate is adjustable. This highlights CBDC to be effective as a secondary monetary policy instrument to achieve the price stability objective. Real exchange rate is also less volatile when CBDC interest rate is adjustable. With stable inflation, the nominal policy rate (that responds to inflation through Eq. (33)) is more subdued in adjustable CBDC interest regimes. Through uncovered interest parity condition in Eq. (19), a more stable policy rate implies a more stable response in the real exchange rate. The volatility of imported intermediate production input also declines as the economy moves to price and quantity rule regimes. This is intuitive as the real exchange rate dynamics have an important bearing on dynamics of imports.

Table 7 also shows the standard deviations of key variables such as labor, inflation, real exchange rate and imports are lower under the price rule than the quantity rule. These findings are consistent with Poole (1970) who finds interest rate rules to perform better than quantity rules as monetary policy instruments.

Table 7: Standard deviation of key variables

	Baseline	Price rule	Quantity rule
Liquidity	0.0525	0.0583	0.0569
Consumption	0.0521	0.0540	0.0536
Labor	0.0672	0.0651	0.0655
Inflation	0.0186	0.0183	0.0185
Real exchange rate	0.1234	0.1225	0.1226
Imports	0.1193	0.1165	0.1169

Volatility moments are derived using second-order perturbations around the stochastic steady state in DYNARE. The simulations use parameter values as reported in Section 4.

[3]In Appendix A.2, Table 11 shows the labor volatility in the context of productivity shock to decline substantially when CBDC interest rate is adjustable.

6.3 Contribution of individual shocks

Our welfare analyses described above aim to identify the policy responses that deliver the best welfare gains for the society, when all shocks are simultaneously present. It is possible that these optimal policies may deliver better welfare outcomes under certain shocks than the others. As such, it is helpful to examine the welfare gains of the productivity shock, money demand shock, marginal utility of consumption shock, the foreign interest rate shock and the export demand shock one at a time.

Comparing across the society's welfare gains with the price rule and the quantity rule in Table 8, we find that society enjoys welfare gains with CBDC monetary policy regimes only under productivity and marginal utility shocks (η columns with magenta shock heading in Table 8). Society also enjoys welfare gains under foreign interest rate shock (only in quantity rule regime). In contrast, both constrained and unconstrained agents suffer welfare losses in the price rule regime when subject to an exogenous rise in foreign interest rates.

The economy also suffers welfare losses in both price and quantity rule regimes in the context of export demand shock. Such welfare losses can be attributed to consumption volatility due to larger CBDC issuance (Section 5). Agents also suffer a welfare loss in the context of the money demand shock. With this shock, agents prefer to use liquidity as saving vehicles as opposed to consumption but their preferences are inhibited under the adjustable CBDC interest rate regimes. This is evident from the responses on the money demand shock discussed in Section 5 where consumption drops by less when CBDC interest rate is adjustable (owing to larger CBDC issuance), reducing the ability of the agents to save more.

There are three points which we would like to highlight from Table 8 about the agents' welfare. First, we find welfare gain/loss of society is driven by constrained agents' welfare gain/loss. This is intuitive as 95% of the population comprises of constrained agents. Second, both agents do not simultaneously experience welfare gain in any of the shocks. Constrained agents' welfare gain is accompanied by unconstrained agents' welfare loss and vice versa. The only exception lie with the foreign interest rate shock when both the agent types suffer welfare loss under the price rule regime. Fourth, the price rule regime increases the disparity between constrained and unconstrained agents. When agent of type j, $j \in [c, u]$ suffer a welfare loss/gain, the magnitude of agent j's welfare loss/gain is larger in the price rule regime.

Table 8: Welfare gains and sources of shocks

	Productivity			Foreign interest rate			Export demand			Money demand			Marginal Utility		
	$\hat{\eta}$	η^c	η^u	$\hat{\eta}$	η^c	η^u	$\hat{\eta}$	η^c	η^u	$\hat{\eta}$	η^c	η^u	$\hat{\eta}$	η^c	η^u
Price rule	6.81	8.37	-6.08	-0.13	-0.05	-1.67	-7.95	-10.12	13.09	-1.35	-1.76	2.98	39.8	53.6	-115.15
Quantity rule	4.33	5.22	-2.17	0.14	0.26	-1.55	-6.70	-8.61	12.37	-1.14	-1.48	2.48	37.69	50.38	-102.77

$\hat{\eta}$, η^c and η^u pertains to welfare gain of society, constrained agents and unconstrained agents, respectively. A negative value indicates welfare loss. Welfare gain is computed using second-order perturbations around the stochastic steady state in DYNARE.

7 Sensitivity analyses

This section conducts sensitivity analysis to examine how certain parameters can improve the effectiveness of the price rule regime. Section 7.1 analyzes the role of imperfect elasticity of substitution between CBDC and deposits in providing more autonomy to the central bank to distinguish CBDC interest rate from the deposit rate. Section 7.2 discusses how the central bank in price rule regime can preserve the monetary autonomy while simultaneously managing exchange rate.

7.1 Elasticity of substitution between CBDC and deposits

In the previous section, we found that the overall welfare of the society improves the most in price rule CBDC regime. In this section, we explain how the effectiveness of the central bank's CBDC price rule regime depend on its ability to autonomously set the CBDC interest rate from the deposit rate. A central bank would only have some autonomy on the CBDC interest rate when the CBDC is not a perfect substitute for bank deposits. By perfect substitute, we mean that both currencies deliver the same utility to households. Barrdear and Kumhof (2021) assume that CBDC and bank deposits are not perfect substitutes because CBDC features better transaction facilities than bank deposits. Such assumption may or may not hold in reality. The central bank therefore needs to be aware of the consequence should consumers be indifferent between the two media of exchange.

The log-linear approximations of Eq. (20) at ζ_t=1 yield the following relationship between the nominal rates on deposits and CBDC.

$$\tilde{i}_{m,t} - \tilde{i}_{d,t} = \left[\frac{\mathcal{A}}{\epsilon} + (\mathcal{B} - \mathcal{A})\left(\mathcal{O} + \frac{1}{\epsilon}\right)\mathcal{S}^{d}_{liq}\right]\tilde{d}^{c}_{t} - \left[\frac{\mathcal{B}}{\epsilon} + (\mathcal{A} - \mathcal{B})\left(\mathcal{O} + \frac{1}{\epsilon}\right)\mathcal{S}^{m}_{liq}\right]\tilde{m}^{c}_{t} + (\mathcal{A} - \mathcal{B})\,\tilde{c}^{c}_{t} \tag{51}$$

where $\mathcal{A} = \frac{(B - A\overline{v}^2)\left(\frac{(1-\gamma)\overline{l^c}}{\overline{d^c}}\right)^{\frac{1}{\epsilon}}}{1 + (B - A\overline{v}^2)\left(\frac{(1-\gamma)\overline{l^c}}{\overline{d^c}}\right)^{\frac{1}{\epsilon}}}$, $\mathcal{B} = \frac{(B - A\overline{v}^2)\left(\frac{\gamma\overline{l^c}}{\psi_m\overline{m^c}}\right)^{\frac{1}{\epsilon}}}{1 + (B - A\overline{v}^2)\left(\frac{\gamma\overline{l^c}}{\psi_m\overline{m^c}}\right)^{\frac{1}{\epsilon}}}$, $\mathcal{O} = \frac{2A\overline{v}^2}{B - A\overline{v}^2}$, $\mathcal{S}^{d}_{liq} = (1-\gamma)^{\frac{1}{\epsilon}}\left(\frac{\overline{d^c}}{\overline{l^c}}\right)^{\frac{\epsilon-1}{\epsilon}}$ and $\mathcal{S}^{m}_{liq} = \gamma^{\frac{1}{\epsilon}}\left(\frac{\overline{m^c}}{\overline{l^c}}\right)^{\frac{\epsilon-1}{\epsilon}}$. The log-deviations from the steady state are identified with a tilde above the variable. The elasticity of substitution between CBDC and deposits equals ϵ. Finally, $\mathcal{S}^{d}_{liq}$ and $\mathcal{S}^{m}_{liq}$ correspond to the steady state share of deposits and CBDC, respectively, in the constrained agents liquidity services function l^c_t.

In the case of CBDC and deposits being perfect substitutes, then $\epsilon \to \infty$. This would reduce Eq. (51) to the below equation.

$$\tilde{i}_{m,t} = \tilde{i}_{d,t} \tag{52}$$

It is evident from Eq. (52) that the nominal CBDC interest rate would simply reflect the changes in nominal deposit rate when CBDC and deposits are perfect substitutes. This would constrain the central bank from using the nominal CBDC rate or the CBDC-to-GDP ratio as secondary monetary policy instruments. Hence, the elasticity of substitution between CBDCs and deposits, ϵ, has an important bearing on the effectiveness of the CBDCs as secondary monetary policy instruments. We explore more on this by performing sensitivity analyses on the economy's responses using a marginal utility of consumption shock.

Fig. 7 shows the impulse responses for ϵ equals 0.1, 2, 4.5 and 12 in the CBDC price rule regime with all other calibration parameters and policy coefficients remaining the same as in Section 4. $\epsilon = 2$ is the benchmark that was followed in all our previous section results. In Fig. 7, we find the spread between nominal CBDC and deposit rate to rise following the shock. The spread is wider and more volatile with low substitutability ($\epsilon = 0.1$) than high substitutability ($\epsilon = 12$). A lower substitutability between CBDCs and deposits ($\epsilon = 0.1$) makes the CBDC rate more independent from the deposit rate as explained earlier, causing a wider spread between the CBDC and deposit rate[4].

As argued by Barrdear and Kumhof (2021), the "liquidity benefits" of CBDC rise relative to deposits when CBDC and deposits are imperfect substitutes (low ϵ). In other words, low ϵ implies that a small change in CBDC is sufficient to help the economy cope with effects of the shock. Thus, the impact on

[4]For the parameters used in this paper, there is indeterminacy for ϵ beyond 12.

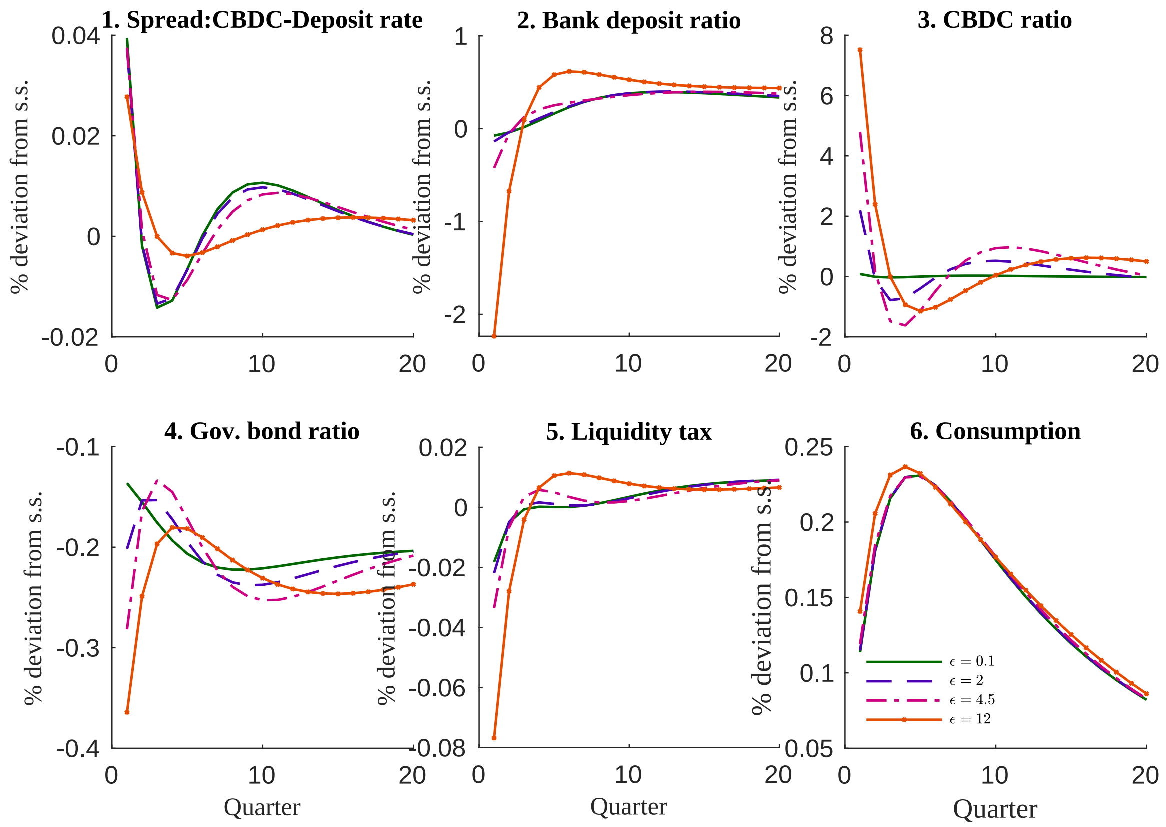


Figure 7: **Effect of substitution elasticity between CBDC and deposits, ϵ in the price rule regime**: Responses following a positive one standard deviation marginal utility to consumption shock.

bank deposits, CBDC and government bonds are less severe with lower elasticity of substitution. We also see the drop in the consumption liquidity tax to be relatively small when CBDC is complementary with deposits ($\epsilon = 0.1$). As a consequence, the rise in consumption is smaller when $\epsilon = 0.1$.

7.2 Exchange rate management

We know that the central bank in the price rule regime utilize both policy rate and CBDC rate for inflation stabilization. In this section, we consider the price rule regime scenario where the policy rate is used to stabilize exchange rate in addition to inflation. Does a mandate on nominal exchange rate smoothing or "leaning against the wind" (Benes et al., 2015) in the CBDC price rule regime give an option for the central bank to manage exchange rate while preserving its autonomy in domestic money supply (CBDC)? To address this question, we change Taylor rule in Eq. (33) to:

$$i_t = i_{t-1}^{\rho_i} \left(\frac{\zeta_t}{\beta^u}\right)^{1-\rho_i} \mathbb{E}_t \left[\left(\frac{\pi_t \pi_{t+1} \pi_{t+2} \pi_{t+3}}{\overline{\pi}^4}\right)^{(1-\rho_i)\frac{\phi_{i,\pi}}{4}}\right] \left(\frac{\Delta e_t}{\overline{\Delta e}}\right)^{(1-\rho_i)\phi_{i,\Delta e}} \tag{53}$$

where $\Delta e_t = \frac{e_t}{e_{t-1}}$, e_t is the nominal exchange rate and $\overline{\Delta e}$ refers to the steady state value of Δe_t. $\phi_{i,\Delta e}$ pertains to the exchange rate feedback coefficient in the Taylor rule. In all our previous section results, $\phi_{i,\Delta e}$ was assumed to be 0 with inflation stability as the sole objective of central bank (inflation targeting framework). When $\phi_{i,\Delta e} \to \infty$, the central bank follows a fixed exchange rate system where the nominal exchange rate is fixed via policy rate. This would reduce Eq. (53) to $\Delta e = 1$. When $0 < \phi_{i,\Delta e} < \infty$, authorities smooth exchange rate but do not target a specific exchange rate level (managed float).

With a non-zero $\phi_{i,\Delta e}$ in the price rule regime, the central bank utilizes a) the policy rate to manage exchange rate and b) the CBDC interest rate to stabilize inflation. To examine whether the central bank can preserve its autonomy in domestic money supply while managing exchange rates, we conduct a sensitivity analysis on the value of $\phi_{i,\Delta e}$. Specifically, our aim is to analyze the scenarios in the price rule regime when the central bank switches from inflation targeting ($\phi_{i,\Delta e} = 0$) to managed float ($\phi_{i,\Delta e} = 6$) and fixed exchange rate regime ($\phi_{i,\Delta e} = \infty$). We conduct the sensitivity analysis through an exogenous rise in nominal CBDC rate which is tantamount to an expansionary monetary policy shock. To incorporate the CBDC interest rate shock, we convert Eq. (35) in log terms to obtain:

$$\ln(i_{m,t}) = \rho_{i_m} \ln(i_{m,t-1}) + (1-\rho_{i_m}) \left[\ln(i_{b,t}) + \ln\left(\frac{\overline{i_m}}{\overline{i_b}}\right) - \frac{\phi_{i_m,\pi}}{4} \mathbb{E}_t \left[\ln\left(\frac{\pi_t \pi_{t+1} \pi_{t+2} \pi_{t+3}}{\overline{\pi}^4}\right)\right]\right] + \varepsilon_{i_m} \tag{54}$$

where ε_{i_m} is the exogenous shock to nominal CBDC interest rate.

Fig. 8 shows the impulse responses for $\phi_{i,\Delta e}$ equals 0, 6 and ∞ in the CBDC price rule regime with all other parameters remaining the same as in Section 4. Following an exogenous rise in CBDC interest rate (see panel 1 in Fig. 8), agents find CBDC more attractive. They switch to CBDC by exchanging government bonds, causing aggregate CBDC quantity to rise (panel 2). As expected, this leads to an expansion of domestic consumption (panel 3).

The rise in CBDC interest rate also causes the spread between policy and CBDC rate to narrow (panel 4). As agents increase CBDC by exchanging domestic bonds, the transaction cost ζ_t (see Eq. (53)) decline, leading to a fall in nominal policy rate. When $\phi_{i,\Delta e}$ is non-zero, the central bank stems the nominal exchange rate volatility (panel 6) through nominal policy rate which causes the policy rate to rise on impact. In fact, the nominal policy rate rise more as $\phi_{i,\Delta e}$ increase. In line with our expectation, the rate of nominal exchange rate depreciation is very subdued for non-zero values of $\phi_{i,\Delta e}$. Such a policy

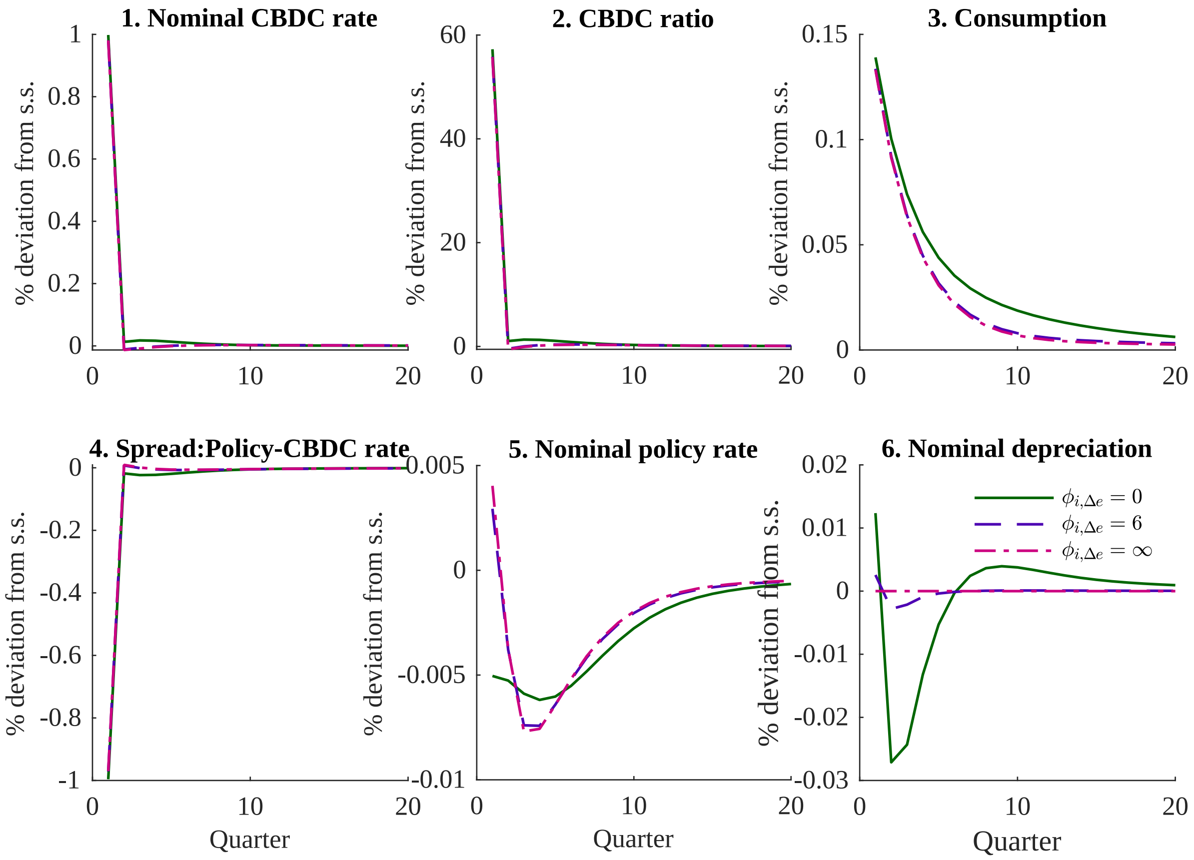


Figure 8: **Effect of Taylor rule exchange rate feedback coefficient $\phi_{i,\Delta e}$ in the price rule regime**: Responses following a positive one standard deviation CBDC interest rate shock.

rate tightening causes domestic consumption to increase by less when $\phi_{i,\Delta e}$ is non-zero.

The interesting point to note from Fig. 8 is the behavior of the CBDC ratio which is similar for all values of $\phi_{i,\Delta e}$. This supports our argument that central bank retains its autonomy over its domestic money supply while simultaneously managing exchange rate via Eq. (53).

8 Conclusion

In this paper, we explore the implications of an adjustable interest rate on CBDC in a small open economy setting. The model assumes that bank deposits and CBDC are competing media of exchange and allows simulation of both fixed and adjustable CBDC interest rates in a *ceteris paribus* environment. In the case when the CBDC bears a fixed and zero interest rate, it resembles an economy with physical cash circulating in it. Whereas, when the interest rate of the CBDC is allowed to adjust, the central bank may use this interest rate as a secondary monetary policy instrument to complement the existing Taylor rule.

For CBDCs with adjustable interest rates, the model establishes interest parity conditions among deposits and CBDCs and domestic and foreign bonds. We find that the interest rate differential between the interest rates of foreign bonds and CBDC is explained by both exchange rate movements and utilities derived from using deposits and CBDCs as media of exchange. An important policy implication we obtain is that the adjustable interest rate of CBDC does not always work as a monetary policy instrument. The CBDC needs to distinguish itself from bank deposits in factors such as transaction technology, so as to provide the central bank with autonomy on the CBDC interest rate. Furthermore, the CBDC as a secondary monetary instrument provides a possibility for the central bank to have independence on monetary policy even as it fixes the exchange rate.

The main simulations of this paper assess the welfare and economic outcomes of policy rules of the CBDC interest rate. Both the price-based and quantity-based policy rules generally deliver better social welfare than the fixed-interest regimes. We find distributional effects that agents who use CBDC are better off than those who do not use it. The welfare and economic outcomes also differ across different sources of uncertainties.

We also provide two sets of sensitivity analyses. In the first analysis, we test the proposition on the potency of the CBDC interest rate as a policy instrument in relation to the substitutability between CBDC and bank deposits. The simulated outcomes are consistent with the proposition, in that when the two media of exchange are more substitutable, the interest spread between CBDC and bank deposits narrows. In the second analysis, we show that the central bank is able to achieve exchange rate stability and monetary independence simultaneously. This is possible provided that the circulation of CBDC is confined within the geographical boundary of the issuing country. It is analogous to a sterilized foreign exchange intervention.

It is important to note that the simulated outcomes only provide limited information for policy makers. We reckon that the area of research on CBDC, especially on issues related to open economies, is still new. While the adjustable interest rate of CBDC gives more flexibility to the central bank in designing monetary policy, the interactions among economic variables become more complicated too. Careful and thorough analyses are needed before implementation of any new policy framework. There are issues which are equally important but are outside the scope of this paper. A lot of future research is needed.

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A Appendix

A.1 Model equations

A.1.1 Banks

The banking sector follows Benes and Kumhof (2015); Jakab and Kumhof (2015); Kumhof and Wang (2018) and Barrdear and Kumhof (2021). The short description of the banking sector given below adopts the notations of Barrdear and Kumhof (2021) for easy cross reference.

The nominal wholesale lending rate, retail lending rate and deposit rate are denoted by $i_{l,t}$, $i_{r,t}$ and $i_{d,t}$, respectively. For any interest rate $i_{x,t}$, the real interest rate is defined as $r_{x,t} = \frac{i_{x,t-1}}{\pi_t}$.

- Solvency risk: The realized gross loan returns of banks in period $t+1$ is $r_{l,t+1}\mathfrak{l}_t\omega^b_{t+1}$ where ω^b_{t+1} is a measure of solvency risk that follows log-normal distribution. Thus, we have $ln(\omega^b_{t+1}) \sim N\left(0, \sigma^2_b\right)$ where the probability density function and cumulative density function of ω^b_{t+1} are denoted by $\mathfrak{f}^b_t(\omega^b_{t+1})$ and $\mathfrak{F}^b_t(\omega^b_{t+1})$, respectively. If ω^b_{t+1} falls below the cut-off level $\bar{\omega}^b_{t+1}$, banks are unable to satisfy the MCAR requirements leading to a penalty payment of $\chi\mathfrak{l}_t\mathfrak{F}^b_t\left(\bar{\omega}^b_{t+1}\right)$. The realized cut-off condition in period t, $\bar{\omega}^b_t$ is given by

$$\bar{\omega}^b_t = \frac{r_{d,t}\mathfrak{d}_{t-1} + \mathcal{L}_t}{(1-\Upsilon\zeta^c)r_{l,t}\mathfrak{l}_{t-1}} \tag{A.1.1.1}$$

where $\mathcal{L}_t$ denotes the realized bank loan loss in period t. After accounting for loan loss, penalty payment (if any) and dividend payouts, the post-dividend net-worth of the banks can be expressed as

$$\mathfrak{n}_t = r_{l,t}\mathfrak{l}_{t-1} - r_{d,t}\mathfrak{d}_{t-1} - \mathcal{L}_t - \delta_b\mathfrak{n}_t - \chi\mathfrak{l}_t\mathfrak{F}^b_t\left(\bar{\omega}^b_{t+1}\right) \tag{A.1.1.2}$$

where δ_b is the proportion of net-worth paid out as dividends. Here, we note that banks expected loss in period $t+1$ is zero $\Longrightarrow \mathbb{E}_t\left[\mathcal{L}_{t+1}\right] = 0$.

- Bankruptcy risk: To secure loans, borrowers (constrained agents) pledge collateral $\mathfrak{c}_t$ with the bank. On account of the bankruptcy risk measure ω^c_{t+1}, the value of the collateral in period $t+1$ change to $\omega^c_{t+1}\mathfrak{c}_t$ where ω^c_{t+1} follows a log-normal distribution. Hence, we have $ln(\omega^c_{t+1}) \sim N\left(0, \sigma^2_c\right)$ where the probability density function and cumulative density function of ω^c_{t+1} are denoted by $\mathfrak{f}^c_t(\omega^c_{t+1})$ and $\mathfrak{F}^c_t(\omega^c_{t+1})$, respectively.

  Borrowers commit to payment of nominal retail interest rate $i_{r,t}$ in period t as they secure the loan contract. This makes the expected profits of banks for period $t+1$ as zero. However, borrowers who experience ω^c_{t+1} below the cut off level $\bar{\omega}^c_{t+1}$ are unable to pay the committed retail interest rate and enter into bankruptcy in $t+1$. This makes the realized ex post profits different from zero. The cut-off level $\bar{\omega}^c_t$ below which the borrowers are unable to meet their interest payment obligation is given by

$$\bar{\omega}^c_t = \frac{r_{r,t}\mathfrak{l}^c_t}{\mathfrak{c}_{t-1}} \tag{A.1.1.3}$$

Using the above, Eq. (29) can be re-written as

$$\mathbb{E}_t\left[\mathfrak{c}_t\left(\Gamma_{t+1}-\xi\mathcal{G}_{t+1}\right)-r_{l,t+1}\mathfrak{l}_t^c\right]=0 \tag{A.1.1.4}$$

where $\Gamma_{t+1} \equiv \bar{\omega}_{t+1}^c \int_{\omega_{t+1}^c}^{\infty} \mathfrak{f}_t^c(\omega_{t+1}^c) + \int_0^{\bar{\omega}_{t+1}^c} \omega_{t+1}^c \mathfrak{f}_t^c(\omega_{t+1}^c) d\omega_{t+1}^c$ is the banks realized share of the pledged collateral value. The monitoring costs incurred by the bank is denoted by $\xi\mathcal{G}_{t+1} \equiv \xi \int_0^{\bar{\omega}_{t+1}^c} \omega_{t+1}^c \mathfrak{f}_t^c(\omega_{t+1}^c) d\omega_{t+1}^c$. Eq. (A.1.1.4) shows that expected bank loss in period $t+1$ is 0. That said, the realized loan loss in period t is non-zero and given by

$$\mathcal{L}_t=(1-\omega)\left[\mathfrak{c}_{t-1}\left(\Gamma_t-\xi\mathcal{G}_t\right)-r_{l,t}\mathfrak{l}_{t-1}^c\right] \tag{A.1.1.5}$$

In the presence of solvency risk and bankruptcy risk as detailed above, the ad-hoc costs $\mathcal{M}_t$ faced by the bank in period t comprise penalty payments and monitoring costs:

$$\mathcal{M}_t=\chi\mathfrak{l}_t\mathfrak{F}_t^b\left(\bar{\omega}_{t+1}^b\right)+(1-\omega)\xi\mathcal{G}_t\mathfrak{c}_{t-1} \tag{A.1.1.6}$$

A.1.2 Households

The lump-sum income Ω_t in both constrained and unconstrained agents budget constraint is given by

$$\Omega_t=\delta^b\mathfrak{n}_t+trf_t-\tau_t^{ls}+(1-\varsigma)\mathcal{T}_t^{costs} \tag{A.1.2.1}$$

where τ_t^{ls} is the lump-sum tax, trf_t is the transfer from the government to agents and $\mathcal{T}_t^{costs}$ comprise bank's adhoc costs in Eq. (A.1.1.6) and transaction cost s_t^c faced by the constrained agents in consumption spending. Hence, we have

$$\mathcal{T}_t^{costs}=\mathcal{M}_t+(1-\omega)c_t^c(1+\tau_c)s_t^c \tag{A.1.2.2}$$

. The remaining portion ζ of $\mathcal{T}_t^{costs}$ enters the goods market clearing condition Eq. (37).

A.1.3 Firms

The firms' maximization problem yields the following condition:

$$(1-\alpha)\frac{y_t(i)}{h_t(i)}=w_t \tag{A.1.3.1}$$

$$\frac{w_t h_t(i)}{q_t k_t^*(i)}=\frac{1-\alpha}{\alpha} \tag{A.1.3.2}$$

$$\mu_t=\frac{1}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}\frac{w_t^\alpha q_t^{(1-\alpha)}{}^{\alpha}}{a_t} \tag{A.1.3.3}$$

The optimal price is

$$\tilde{\pi}_t=\frac{\theta_p-1}{\theta_p}\frac{\mathbb{E}_t\left[\sum_{t=0}^{\infty}\vartheta^t Q_{0,t}\mu_t\left(\frac{p_t}{p_0}\right)^{\theta_p}y_t\right]}{\mathbb{E}_t\left[\sum_{t=0}^{\infty}\vartheta^t Q_{0,t}\left(\frac{p_t}{p_0}\right)^{\theta_p-1}y_t\right]} \tag{A.1.3.4}$$

$$1=\vartheta\pi_t^{\theta_p-1}+(1-\vartheta)\tilde{\pi}_t^{1-\theta_p} \tag{A.1.3.5}$$

The recursive non-linear price setting equations are given below:

$$F_{1,t} = y_t + \mathbb{E}_t \left[\vartheta Q_{t,t+1}(\pi_{t+1})^{\theta_p} F_{1,t+1}\right] \tag{A.1.3.6}$$

$$F_{2,t} = y_t \mu_t + \mathbb{E}_t \left[\vartheta Q_{t,t+1}(\pi_{t+1})^{1+\theta_p} F_{2,t+1}\right] \tag{A.1.3.7}$$

$$\tilde{p}_t F_{1,t} = \frac{\theta_p}{\theta_p - 1} F_{2,t} \tag{A.1.3.8}$$

where $F_{1,t} \equiv \mathbb{E}_t \left[\sum_{s=0}^{\infty} \vartheta^s Q_{t,t+s} y_{t+s} \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{\theta_p}\right]$ and $F_{2,t} \equiv \mathbb{E}_t \left[\sum_{s=0}^{\infty} \vartheta^s Q_{t,t+s} y_{t+s} \mu_t \left(\prod_{k=1}^{s} \pi_{t+k}\right)^{1+\theta_p}\right]$.

The price dispersion:

$$D_t = \int_0^1 \left(\frac{p_t(i)}{p_t}\right)^{-\theta_p} di \tag{A.1.3.9}$$

$$D_t = (1 - \vartheta)\, \tilde{p}_t^{-\theta_p} + \vartheta \pi_t^{\theta_p} D_{t-1} \tag{A.1.3.10}$$

Aggregate output is

$$y_t D_t = a_t {k_t^*}^{\alpha} \left(a_t n_t\right)^{1-\alpha} \tag{A.1.3.11}$$

Foreign intermediate good price in domestic currency, q_t, is in other words the real exchange rate in our model. This is obtained by:

$$\frac{q_t}{q_{t-1}} = \Delta e_t \frac{\pi_t^*}{\pi_t} \tag{A.1.3.12}$$

where $\Delta e_t = \frac{e_t}{e_{t-1}}$ with e_t as the nominal exchange rate. The foreign price inflation π_t^* is exogenously determined.

A.2 Tables

Table 9: Data

Data description	Mnemonic	Source
Gross domestic product	gdp_{obs}	Stats NZ
Tax on personal income as % of GDP	$\frac{\tau_l wn}{gdp}$	OECD data
Tax on goods and services % of GDP	$\frac{\tau_c c}{gdp}$	OECD data
Government spending	g	Stats NZ
Personal consumer loan	$\mathfrak{l}_t^c$	Reserve Bank of New Zealand
Policy rate	i	Reserve Bank of New Zealand
Deposit rate	i^d	Reserve Bank of New Zealand
Final consumption expenditure	c_{obs}	OECD Stat
M1 money supply	l_{obs}^c	Federal Reserve St Louis
Consumer price index inflation	π_{obs}	Stats NZ
Imports of goods and services	k_{obs}^*	OECD Stat

GDP, consumption expenditure, M1 money supply, inflation and imports are seasonally adjusted data. Using CPI data, we obtained macroeconomic aggregates in real terms. Variables with subscript *obs* are used for Bayesian calibration. We transformed the data in logarithmic terms before HP filtering.

Table 10: Annualized terms

Figure header	**Value**
Inflation (annualized)	$\pi_t \pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}$
Spread: Policy-CBDC rate	$\frac{i_{b.t}}{i_{m,t}}$
Bank deposit ratio	$\frac{d_t}{4gdp_t}$
CBDC ratio	$\frac{m_t}{4gdp_t}$
Gov. bond ratio	$\frac{b_t}{4gdp_t}$
Imports ratio	$\frac{k_t^*}{4gdp_t}$
Foreign bond ratio	$\frac{b_t^*}{4gdp_t}$
Spread: Policy-loan rate	$\frac{i_{b.t}}{i_{l,t}}$

Table 11: Volatility of labor and consumption with individual shocks

	Labor volatility		
	Baseline	Price rule	Quantity rule
Productivity	0.0397	0.0364	0.0370
Marginal utility of consumption	0.0466	0.0465	0.0465
Export demand	0.0113	0.0114	0.0114
Foreign interest rate	0.0146	0.0143	0.0144
Money demand	0.0206	0.0204	0.0206

	Consumption volatility		
	Baseline	Price rule	Quantity rule
Productivity	0.0334	0.0367	0.0360
Marginal utility of consumption	0.0257	0.0257	0.0257
Export demand	0.0140	0.0146	0.0145
Foreign interest rate	0.0092	0.0099	0.0098
Money demand	0.0257	0.0244	0.0247

Volatility moments are derived using second-order perturbations around the stochastic steady state in DYNARE. The simulations use parameter values as reported in Section 4.