

2.111/8.370/18.435 Problem Set 7 Solutions

Due: November 19, 2015

0. Survey RSA.

1. **Given** $U = \sum_j e^{i\phi_j} |j\rangle\langle j|$, $e^{i\phi_j}$'s are eigenvalues (ϕ_j 's are real), $|j\rangle$'s are corresponding eigenvectors. **Show that U is unitary.**

$$U^\dagger = \sum_j e^{-i\phi_j} |j\rangle\langle j|. \text{ Thus } U^\dagger U = \sum_{j,j'} e^{-i\phi_j} e^{i\phi_{j'}} |j\rangle\langle j'| = I.$$

2. **(Optional) Construct the output of quantum phase estimation when $k\phi_j/2\pi$ is not an integer. Provide a formula for the expected error in estimating $\phi_j/2\pi$.**

See Nielsen and Chuang p.223–224.