BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI
WORK INTEGRATED LEARNING PROGRAMMES

COURSE HANDOUT

Part A: Content Design

Course Title	Mathematical Foundations for Data Science
Course No(s)	
Credit Units	4
Course Author	G Venkiteswaran
Version No	2
Date	15.09.2019

Course Description

Vector and matrix algebra, systems of linear algebraic equations and their solutions; eigenvalues, eigenvectors and diagonalization of matrices; graphs and digraphs; partially ordered sets and lattices; Boolean algebras and Boolean expressions;

Course Objectives

No	Objective- The course aims to
CO1	Introduce concepts in linear algebra and to use it as a platform to model physical problems.
CO2	Provide techniques for analytical and numerical solutions of linear equations and introduce the concept of convergence.
CO3	Utilize concepts of linear algebra and calculus in solving optimization problems.
CO4	Introduce some of the mathematical structures, concepts and notations used in discrete mathematics.
CO5	Introduce some concepts from graph theory, partially ordered sets, Boolean algebras.

Text Book(s)

No	Author(s), Title, Edition, Publishing House
T1	Erwin Kreyszig, Advanced Engineering Mathematics, Wiley India, 9 th Edition, 2011
T2	Kenneth H. Rosen, Discrete Mathematics and its Applications, Tata McGraw Hill, 7th Ed., 2011.

Reference Book(s) & other resources

No	Author(s), Title, Edition, Publishing House
R1	K Hoffman and R Kunze, Linear Algebra, Pearson Education, 2nd Edition, 2005.
R2	Kolman, Busby, Ross and Rehman, Discrete Mathematical Structures for Computer Science,

	Pearson Education, 6th Edition, 2017

Content Structure

No	Title of the module	References
M1	1. Matrices, System of equations, determinants and inverse of a matrix 1.1. Matrix Algebra-Row-reduced echelon form of a matrix, inverse of a matrix 1.2. System of linear equations, Consistency and inconsistency of system of linear equations	T1: Sec 7.1 – 7.3, 7.5, 7.8
M2	2. Vector spaces and Linear transformations 2.1 Vector space, subspace and span of a set, Linear dependence and independence of a set of vectors, basis and dimension 2.2. Linear transformation, rank and nullity	T1: Sec 7.4, 7.9, R1: Sec 3.2
M3	3. Eigenvalues, Eigenvectors and singular values 3.1. Eigenvalues 3.2. Eigenvectors 3.3. Singular value decomposition	T1: Sec 8.2, 8.3 and class notes
M4	4. Numerical linear algebra 4.1. Gauss elimination with partial pivoting and scaling 4.2. Iterative methods for solving linear system of equations	T1: Sec 20.1
M5	5. Matrix Eigenvalue Problems 5.1. Eigenvalue problems in linear system of equations 5.2. Power method for finding the dominant eigenvalue	T1: Sec 20.3, 20.8
M6	6. Linear and non-linear optimization 6.1 Basics of calculus 6.2 Linear optimization using simplex method and sensitivity 6.3 Non-linear optimization	Class notes
M7	6. Sets, Functions and Relations, Boolean Algebra 6.1 Introduction to set theory, set relations, set operators, cardinality of sets, Cartesian product of sets 6.2 Fundamentals of functions – range, domain, injection, surjection, bijection of functions 6.3 Fundamentals of relations, reflexive, symmetric and transitive properties in relations, representing relations, applications of relations, equivalence relations, partial order relations, lattices. 6.4 Boolean functions, representing Boolean functions	T2: Sec 2.1, 2.2, 2.3, 7.1 – 7.6, 10.1, 10.2
M8	7. Graph Theory 7.1 Introduction to graph theory, directed and undirected graphs, handshaking theorem, special graph structures, graph representations and isomorphism of graphs, connectedness, components, Euler, Hamilton paths and cycles	T2: Sec 8.1-8.5

Learning Outcomes:

No	Learning Outcomes
LO1	Students will be able to effectively use matrix algebra tools to analyse and solve systems of linear equations.
LO2	Students will be able to use some numerical methods to solve linear systems of equations
LO3	Students would be able to use methods in linear algebra to solve linear programming problems and methods in calculus to solve non-linear optimization problems.
LO4	Students will be able to work with some of the mathematical structures, concepts and notations used in discrete mathematics
LO5	Students will be able to apply the concepts of sets, functions, relations and graph theoretic concepts to problems in computer science

Part B: Contact Session Plan

Academic Term	II Semester 2019-2020
Course Title	Mathematical Foundations for Data Science
Course No	
Lead Instructor	

Course Contents

Contact Hours	List of Topic Title	Text/Ref Book/external resource
1	Introduction to matrices, row-reduced echelon form of a matrix, Consistency of linear systems and matrix inversion • Unary and binary operations and special matrices (orthogonal matrix, upper and lower triangular, diagonal and sparse) • Row reduction and determination of rank. Comparison to computation using determinants • Use of rank in determining the consistency and inconsistency of linear systems • Row reduction to determine the inverse of the matrix (the Gauss Jordan method) (this is to be used in Simplex method later on)	T1: Sec 7.1 – 7.3, 7.5, 7.8
2	Vector space, subspace and span, Linear dependence and independence, basis and dimension, Linear transformation, rank and nullity and the rank nullity theorem • Definition and examples of vector space (R^n, space of polynomials of finite degree, n x m matrices etc.,) • Determination of whether a non-empty set of a vector space is a subspace or not • Span of a finite set	T1: Sec 7.4, 7.9 R1: Sec 3.2

	• Linear dependence and independence (theory and couple of examples) • Basis and dimension of a finite dimensional vector space • Linear transformation T: V → W (definition and a couple of examples) • Range(T) and Ker(T) as subspaces of W and V respectively • Rank Nullity Theorem (statement without proof) with examples	
3	Eigenvalues and eigenvectors of a matrix with applications • Eigenvalues – definition and method of determination of eigenvalues • Eigenvectors – definition and methods of finding the eigenvectors • Greshgorin's result on the bounds for eigenvalues • Application in conic sections with examples • Similarity transformation with examples	T1: Sec 8.2 – 8.4
4	Singular value decomposition with examples (using MATLAB) and applications (Face recognition with SVD) • SVD of a matrix (derivation) • Exemplify using matlab for a couple of matrices and also show that the singular values are arranged in descending order. • Face recognition example.	Class notes
5	Gauss elimination with scaling and partial pivoting; LU factorization and related methods • Gauss elimination (with and without scaling and partial pivoting). Take an example to shown the role played by precision. • LU factorization, Cholesky and Crout's methods with examples	T1: Sec 20.1, 20.2
6	Iterative methods of solving linear systems; Matrix eigenvalue problems and Power method for finding the dominant eigenvalue • Write Ax = b in the form (L+D+U) x = b and work out the iterative scheme for Gauss Jacobi and Gauss Seidel iterations. • Introduce vector and matrix norms (row sum, column sum and Frobenius norms) and work out a few problems in Excel / Matlab • Explain the power method and work out a couple of problems.	T1: Sec 20.3, 20.8
7 -8	Application of linear algebra in optimization. Modelling linear programming problem and the basics of Simplex algorithm and sensitivity analysis. • Model a LPP in construction of buildings. • Model the currency conversion optimization problem. • Work out the graphical method of solution in the case of 2 variable case • Simplex method for simple cases	Class notes

	• Outline how Gauss Jordan produces the inverse matrix. • Graphical sensitivity analysis (Change in objective value coefficients and rhs of constraints)	
9	Calculus of one and several variables; Limits, continuity and differentiability; Maxima and minima of functions; Steepest gradient method for finding the maximum. Constrained optimization (Lagrange multipliers) • Review limits, continuity and differentiability (graphically and algebraically) • Maxima and minima in one variable • Steepest gradient method • Lagrange multipliers (for more number of constraints)	Class notes
10	Introduction to set theory, set relations, set operators, cardinality of sets, Cartesian product of sets • Definition and examples of set and set operations • Cartesian product of sets	T2: Sec 2.1, 2.2
11	Fundamentals of functions – range, domain, injection, surjection, bijection of functions • Definition and examples of functions • Writing the range, domain and codomain for a few well known functions (exponential, logarithmic and trigonometric functions) • One-one, onto and one to one relationship definition and examples (algebraic and graphical)	T2: Sec 2.3
12	Fundamentals of relations, reflexive, symmetric and transitive properties in relations, representing relations and applications • Definition and examples of relations • Reflexive, Symmetric and Transitive properties and examples • Closures of the above relations including Warshall's algorithm	T2: Sec 7.1, 7.2, 7.3
13	Representing relations, applications of relations, equivalence relations, partial order relations, lattices. • Representation of relations as matrices • Applications of relations • Equivalence relations with examples • Partial order and well ordering • Hasse diagrams • Lattices with examples using lub and glb concepts	T2: Sec 7.4, 7.5, 7.6
14	Introduction to graph theory, directed and undirected graphs, handshaking theorem, special graph structures, graph representations • Directed and undirected graphs with examples • Handshaking theorem connecting vertices and edges • Graph representation as a matrix and paths.	T2: Sec 8.1, 8.2

15	Isomorphism of graphs, connectedness, components, Euler, Hamilton paths and cycles • Graph isomorphism definition and examples (how to show two graphs are isomorphic / non-isomorphic) • Connectedness in graphs and components • Euler path and cycle (using degree of vertices) • Hamilton path and cycle (using degree of vertices)	T2: Sec 8.3, 8.4, 8.5
16	Boolean Algebra- Boolean Functions, Representing Boolean functions and functional completeness • Definition and examples of Boolean algebras • Representation of Boolean functions • Functional completeness (Complement, union, intersection) → (complement, union) {alternatively (complement, intersection) → NAND, NOR as a single operand	T2: Sec 10.1, 10.2

 # The above contact hours and topics can be adapted for non-specific and specific WILP programs depending on the requirements and class interests.

Lab Details

Title	Access URL
Lab Setup Instructions	Not applicable
Lab Capsules	Not applicable
Additional References	Not applicable

Select Topics and Case Studies from business for experiential learning

Topic No.	Select Topics in Syllabus for experiential learning	Access URL
1	Assignment - linear algebra topics	
2	Assignment- discrete structures topics	

Evaluation Scheme

Legend: EC = Evaluation Component

No	Name	Type	Duration	Weight	Day, Date, Session, Time
1	Assignment 1	Online		10%	22/05/20 9AM till 1/06/20 5PM
2	Assignment 2	Online		10%	07/08/20 9 AM till 17/08/20 5PM
3	Quiz 1	Online	30 min	5%	12/06/20 (9 PM to 10 PM)
4	Quiz 2	Online	30 min	5%	24/07/20 (9 PM to 10 PM)
5	Quiz 3	Online	30 min	5%	28/08/20 (9 PM to 10 PM)
6	Mid-Semester Exam	Closed book	90 min	30%	Regular 21/06/20(10 AM to 11.30 AM) Makeup 4/07/20 (10 AM to 11.30 AM)
7	Comprehensive Exam	Open book	150 min	40%	Regular 13/09/20(10 AM to 12.30 PM) Makeup 26/09/20(10 AM to 12.30 PM)

Important Information

Syllabus for Mid-Semester Test (Closed Book): Topics in Weeks 1-8

Syllabus for Comprehensive Exam (Open Book): All topics (in sessions 1 to 16) given in plan of study

Evaluation Guidelines:

1. EC-1 consists of two Assignments and two Quizzes. Announcements regarding the same will be made in a timely manner.
2. For Closed Book tests: No books or reference material of any kind will be permitted. Laptops/Mobiles of any kind are not allowed. Exchange of any material is not allowed.
3. For Open Book exams: Use of prescribed and reference text books, in original (not photocopies) is permitted. Class notes/slides as reference material in filed or bound form is permitted. However, loose sheets of paper will not be allowed. Use of calculators is permitted in all exams. Laptops/Mobiles of any kind are not allowed. Exchange of any material is not allowed.
4. If a student is unable to appear for the Regular Test/Exam due to genuine exigencies, the student should follow the procedure to apply for the Make-Up Test/Exam. The genuineness of the reason for absence in the Regular Exam shall be assessed prior to giving permission to appear for the Make-up Exam. Make-Up Test/Exam will be conducted only at selected exam centres on the dates to be announced later.

It shall be the responsibility of the individual student to be regular in maintaining the self-study schedule as given in the course handout, attend the lectures, and take all the prescribed evaluation components such as Assignment/Quiz, Mid-Semester Test and Comprehensive Exam according to the evaluation scheme provided in the handout.