

Answer all the questions.

- 1) The analysis of the voltage around the three loops gives the following equations for the currents in different parts of the an electrical circuit (Figure 1):
 $20(i_1 - i_2) + 10(i_1 - i_3) = 0$;
 $25i_2 + 10(i_2 - i_3) + 20(i_2 - i_1) = 0$ and
 $30i_3 + 10(i_3 - i_2) + 10(i_3 - i_1) = 200$
 Find the currents by Gauss Siedal iterative scheme. (12)

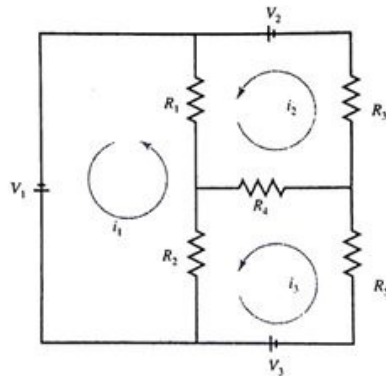


Figure 1

- 2) (a) When trying to find the acidity of a solution of magnesium hydroxide in hydrochloric acid, we obtain the following equation

$$f(x) = e^{-0.5x}(4 - x) - 2$$
 where x is the hydronium ion concentration. Find the hydronium ion concentration for a saturated solution (acidity equal to zero) using a suitable iterative scheme. (7M)
 (b) Find the solution for the steady state of the concentration, of two chemical species, in an oscillatory chemical system described by the nonlinear system:

$$x^2 - \sqrt{3}xy + 2y^2 - 10 = 0; \quad 4x^2 + 3\sqrt{3}xy + y^2 - 22 = 0; \quad (7M)$$

- 3) For a three mass-four spring system, between two fixed walls, the displacements for the masses can be expressed as the following homogeneous set of equations:

$$\left(\frac{2k}{m_1} - \omega^2\right)x_1 - \frac{k}{m_1}x_2 = 0; \quad -\frac{k}{m_2}x_1 + \left(\frac{2k}{m_2} - \omega^2\right)x_2 - \frac{k}{m_2}x_3 = 0 \text{ and}$$

$$-\frac{k}{m_3}x_2 + \left(\frac{2k}{m_3} - \omega^2\right)x_3 = 0$$

If all the masses $m_i = 1\text{kg}$ and all the spring constants $k = 20\text{ N/m}$, express the system in the matrix format as $[A] - \lambda[I] = 0$ where λ is the square of the angular frequency ω^2 . Employ the power method to determine the highest eigen value and its associated eigen vector. Explain how the smallest eigen value can be obtained by power method.

- 4) Using LU Decomposition method, solve
$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 11 \end{bmatrix} \quad (12)$$