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Group - 3C02

Assignment

Q1> Given a random sample $(x_1, x_2 - x_n)$

$u = \theta_1$ (mean) & $\sigma^2 = \theta_2$

Likelihood function $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \exp\left(-\frac{(x_i-\theta_1)^2}{2\theta_2}\right)$

to maximize take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2} \ln(2\pi\theta_2) - \frac{(x_i-\theta_1)^2}{2\theta_2} \right]$$

i> differentiate w.r.t θ_1 [For θ_1]

$$\frac{d}{d\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\theta = \sum_{i=1}^{n} \frac{x_c}{n} \quad [\text{mean}]$$

ii> differentiate w.r.t θ_2 [for θ_2]

$$\frac{d}{d\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[-\frac{1}{2\theta_2} + \frac{(x_i-\theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{2} \sum_{i=1}^{n} (x_i - \theta_1)^2 \quad [\text{Variance } \theta]$$

Q2 Binominal distribution $B(n, \theta)$ $\quad p = \theta$, $q = 1 - \theta$

PMF (Prob mass function)

$$f(x, m, \theta) = {}^nC_x \, \theta^x (1-\theta)^{m-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^mC_{x_i} \, \theta^{x_i} (1-\theta)^{n-x_i}$$

$$\ln L(\theta) = \sum_{i=1}^{n} [\ln {}^nC_{x_i} + x_i \ln \theta + m - x_i \ln(1-\theta)]$$

differentiate wrt θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta} \right] = 0$$

find θ

$$\sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n - x_i}{1-\theta} \right] = 0$$

$$\sum_{i=1}^{n} \left[\frac{(1+x_i) - (m - x_i)\theta}{\theta(1-\theta)} \right] = 0$$

$$\sum_{i=1}^{n} (1-\theta)\, x_i - (n - x_i)\,\theta = 0$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i n$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{m-n}} \rightarrow \text{MLE}$$