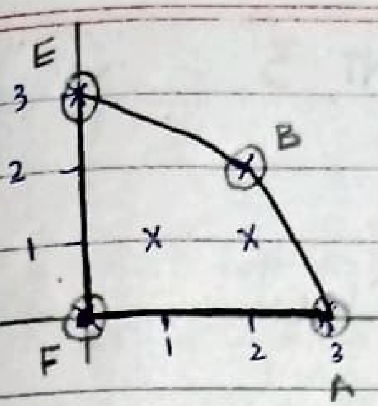
23K-0066
BS(AI)-5A

DAA ASSIGNMENT 3

Q1:

Step	Current point P	Candidate (q)	Checking point (r)	orientation (p,q,r)	More CCW?	New Candidate
1	F(0,0)	A(0,3)	B(2,2)	(0-0)(2-0) - (3-0)(2-0) = -6	No	-
2	F(0,0)	A(0,3)	C(1,1)	(0-0)(1-0) - (3-0)(1-1) = 0	No	-
3	F(0,0)	A(0,3)	D(2,1)	(0-0)(1-0) - (3-0)(2-0) = -6	No	-
4	F(0,0)	A(0,3)	E(3,0)	(0-0)(0-0) - (3-0)(3-0) = -9	No	-
5	A(0,3)	B(2,2)	C(1,1)	(2-0)(1-3) - (2-3)(1-0) = -3	No	-
6	A(0,3)	B(2,2)	D(2,1)	(2-0)(1-3) - (2-3)(2-0) = -2	No	-
7	A(0,3)	B(2,2)	E(3,0)	(2-0)(0-3) - (2-3)(3-0) = -3	No	-
8	A(0,3)	B(2,2)	F(0,0)	(2-0)(0-3) - (2-3)(0-0) = -6	No	-
9	B(2,2)	C(1,1)	D(2,1)	(1-2)(1-2) - (1-2)(2-2) = 2	Yes	D
10	B(2,2)	D(2,1)	E(3,0)	(2-2)(0-2) - (1-2)(3-2) = 1	Yes	E
11	B(2,2)	E(3,0)	F(0,0)	(3-2)(0-2) - (0-2)(0-2) = -6	No	-
12	B(2,2)	E(3,0)	A(0,3)	(3-2)(3-2) - (0-2)(0-2) = -3	No	-
13	B(2,2)	E(3,0)	C(1,1)	(3-2)(1-2) - (0-2)(1-2) = -3	No	-
14	E(3,0)	F(0,0)	A(0,3)	= (0-3)(3-0) - (0-0)(0-3) = -9	No	-
15	E(3,0)	F(0,0)	B(2,2)	(0-3)(2-0) - (0-0)(2-3) = -6	No	-
16	E(3,0)	F(0,0)	C(1,1)	(0-3)(1-0) - (0-0)(1-3) = -3	No	-
17	E(3,0)	F(0,0)	D(2,1)	(0-3)(1-0) - (0-0)(2-3) = -3	No	-

F → A → B → E → F



F → A → B → E → F

Q2)

Brute force :

```
bruteForceFF(P) :
    m = length(P)
    F = array of size m
    for j=0 to m-1:
        F[j] = 0
        for k = j down to 1:
            match = True
            for i=0 to k-1:
                if p[i] != p[j-k+1+i]:
                    match = False
                    break
            if match:
                F[j] = k
                break
    return F
```

P = ababaca

j	Substring P[0..j]	longest proper prefix = sufix	F[j]
0	a	–	0
1	ab	–	0
2	aba	a	1
3	abab	ab	2
4	ababa	aba	3
5	ababac	–	0
6	ababaca	a	1

Result: F = [0, 0, 1, 2, 3, 0, 1]

Time Complexity:

outer loop = $O(m)$

middle loop = $O(m)$ in worst case

inner loop = $O(m)$ in worst case

Total: $O(m^3)$ worst case

Optimized KMP:

Compute Failure KMP (P):

m = length (P)

F = array of size m

F[0] = 0

l = 0

```
for j = 1 to m-1 :
    while i > 0 and P[i] != P[j] :
        i = F[i-1]
    if P[i] == P[j] :
        i = i+1
    else :
        i = 0
    F[j] = i
return F
```

We compute F[j] using previously computed F values. We maintain a pointer i which is the length of the current longest prefix which is also a suffix for P[0...j-1]. For next j, we try to extend by comparing P[i] with P[j]. If they match, F[j] = i+1. If not we set i = F[i-1] and repeat until match or i = 0.

Time Complexity :

The while loop runs at most $O(m)$ total across all j because i increases at most by 1 per step, and decreases only via i = F[i-1] which is less than i.

So TC = $O(m)$

j	P[j]	comparison	i_{before}	i_{after}	F[j]
0	-	-	0	-	0
1	b	P[0] != P[1]	0	0	0
2	a	P[0] == P[2]	0	1	1
3	b	P[1] == P[3]	1	2	2
4	a	P[2] == P[4]	2	3	3
5	c	P[3] != P[5]	3	1	
~~6~~		P[1] != P[5]	1	0	
		P[0] != P[5]	0	0	0
6	a	P[0] == P[5]	0	1	1

F = [0, 0, 1, 2, 3, 0, 1]

Comparison:

	Brute-Force	Optimized KMP
Time complexity	$O(m^3)$	$O(m)$
Character comparisons	Upto $O(m^3)$	$O(m)$
Reuse of computed info	No reuse	Reuses F values via backtracking
Pratical efficiency	Slow for long patterns	Fast, used in real KMP matcher

Q3)

(a)

S = {1, 5, 6, 8} desired change is 13.

dp[i] → no. of ways to make amount i

dp[0] = 1 → base case (one way: Use no coins)

For each coin c update:

dp[i] += dp[i - c] for i = c to amount

coins / w	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	2	1+1 2	1+1 2	1+1 2	1+1 2	1+2 3	1+2 3	1+2 3	1+2 3
6	1	1	1	1	1	2	2+1 3	2+1 3	2+1 3	2+1 3	3+1 4	3+2 5	3+3 6	3+3 6
8	1	1	1	1	1	2	3	3	3+1 4	3+1 4	4+1 5	5+1 6	6+1 7	6+2 8

Final dp[13] = 8

There are 8 combinations to make amount 13 with coins {1, 5, 6, 8}

(b)

str1 = KITTEN str2 = SITTING

dp[i][j] → min operations to convert first i chars of str1 to first j chars of str2

$\left.\begin{array}{l}\text{dp[i][0] = i → delete all i chars}\\ \text{dp[0][j] = j (Insert j chars)}\end{array}\right\}$ Base cases

if str1[i-1] == str2[j-1]:
    dp[i][j] = dp[i-][j-1]
else:
    dp[i][j] = 1 + min(dp[i-1][j], dp[i][j-1],
                       dp[i-1][j-1])
                       (delete, insert, substitute)

	D	S	I	T	T	I	N	G
0	0	1	2	3	4	5	6	7
K	1	1	2	3	4	5	6	7
I	2	2	1	2	3	4	5	6
T	3	3	2	1	2	3	4	5
T	4	4	3	2	1	2	3	4
E	5	5	4	3	2	2	3	4
N	6	6	5	4	3	3	2	3

~~Final [illegible]~~

Edit distance = 3

Operations:

1) K → S (substitute)
2) E → I (substitute)
3) Insert G at end

(c)

length [ ] = {1, 2, 3, 4, 5, 6, 7, 8}
price [ ] = {1, 5, 8, 9, 10, 16, 18, 20}
Rod length : 8

dp [L] = max value for rod length L
dp [L] = max (price [i] + dp [L-i-1]) for i = 0 to L-1
or simpler : dp [L] = max (dp [L], price [i] + dp [L-i])
with 1-based indexing

dp [0] = 0
~~dp~~ L = 1 : max (1) = 1
L = 2 : max (5, 1 + dp[1] = 1+1 = 2) = 5
L = 3 : max (8, ~~1~~ + dp [2] = 1+5, 6 , ~~5~~ + dp [1] = 5+1 = 6)
= 8

L = 4 : max (9, 1 + dp [3] = 1+8 = 9, ~~5~~ + dp [2] = 5+5 = 10,
8 + dp [1] = 8+1 = 9) = 10

L = 5 : max (10, 1 + dp [4] = 1+10 = 11, 5 + dp [3] = 5+8 = 13,
8 + dp [2] = 8+5 = 13, 9 + dp [1] = 9+1 = 10) = 13

L = 6 : max (16, 1 + dp [5] = 1+13 = 14, 5 + dp [4] = 5+10 = 15,
8 + dp [3] = 8+8 = 16, 9 + dp [2] = 9+5 = 14,
10 + dp [1] = 10+1 = 11) = 16

L = 7 : max (18, 1 + dp [6] = 1+16 = 17, 5 + dp [5] = 5+13 = 18,
8 + dp [4] = 8+10 = 18, 9 + dp [3] = 9+8 = 17,
10 + dp [2] = 10+5 = 15, 16 + dp [1] = 16+1 = 17)
= 18

$L = 8 :\ \max(20,\ 1 + dp[7] = 1 + 18 = 19,$

$5 + dp[6] = 5 + 16 = 21,$

$8 + dp[5] = 8 + 13 = 21,$

$9 + dp[4] = 9 + 10 = 19,$

$10 + dp[3] = 10 + 8 = 18,$

$16 + dp[2] = 16 + 5 = 21,$

$18 + dp[1] = 18 + 1 = 19)$

$L = 8 :\ 21$

Rod length. L	Max value dp[L]	Optimal cuts
0	0	-
1	1	1
2	5	2
3	8	3
4	10	2+2
5	13	2+3
6	16	6
7	18	2+5 or 1+6
8	21	2+6 or 3+5

Maximum value = 21

Optimal cuts : (2,6) or (3,5) or (5,3) or (6,2)

all give 5+16 = 21 or 8+13 = 21