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(DSA Theoretical) —

* Data Structures

— Way of organizing data in a computer memory (cache, main, secondary)
So memory can be used efficiently in both of time / Space.

* Organization of data.

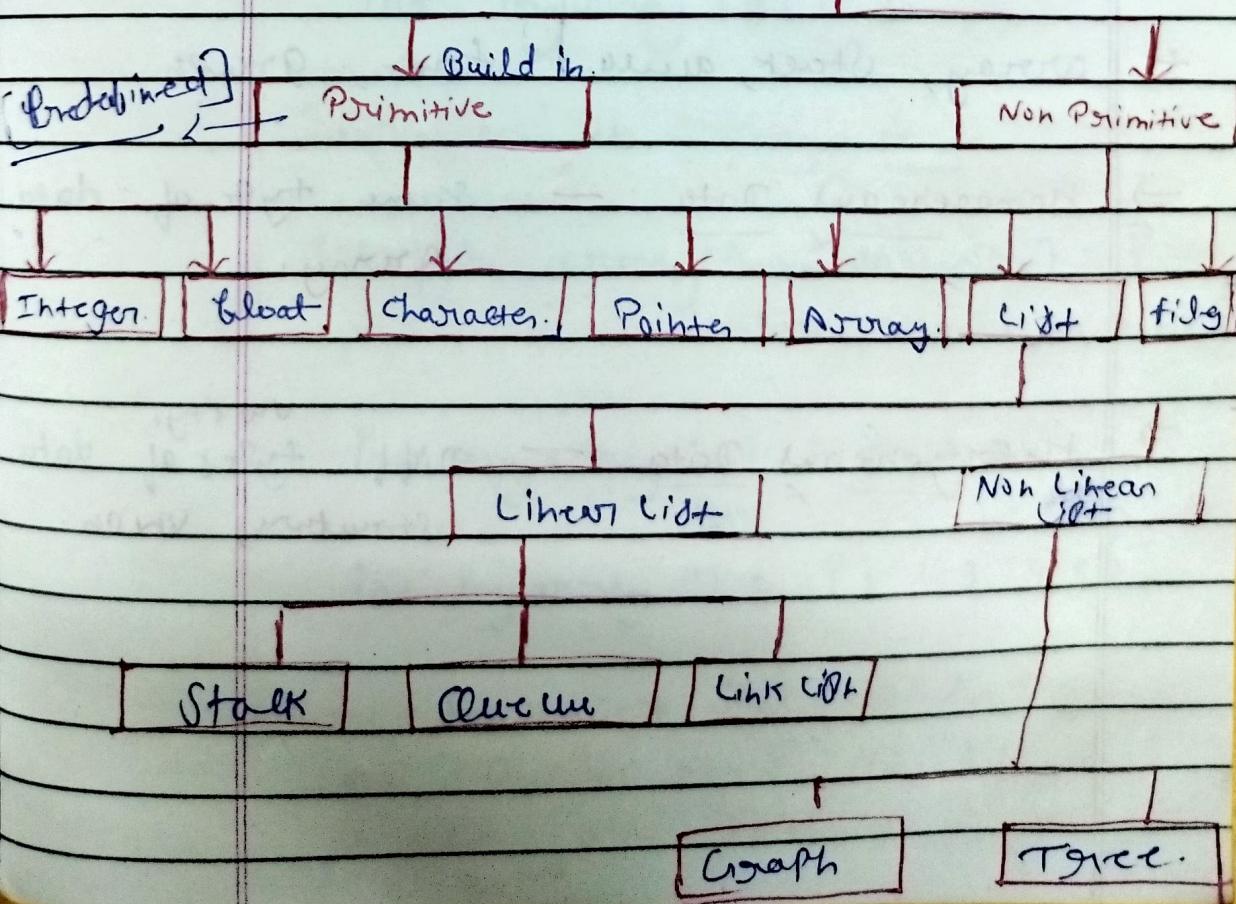
* Accessing Method.

* Degree of association. with Node having more than one related to it

* Processing Methods. (Operation on it)

* Effect of Data Structure —

Data Structure



Linear Data's

- * Data element arrange in linear order. each and every element attached to its previous/ next adjacent.

* Single Level.

* Implementation easy
Comparison to non linear.

* Data elements can be traversed in a single run only.

* array, Stack, queue.

⇒ Homogeneous Data : — same type of data.
Array.

⇒ Heterogeneous Data : — diff types of data.
structure, which.

Non Linear Data's

non-linear data structure attached in hierarchical manner.

Multiple Level.

Complex.

Can't traversed in single run.

tree, graph.

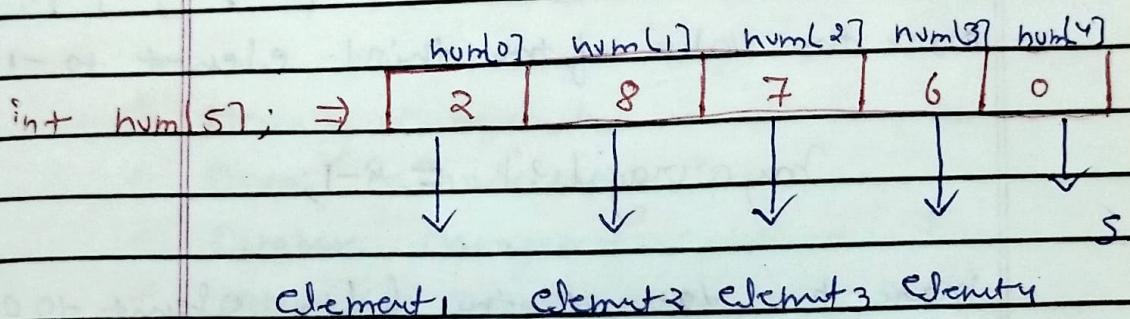
Collection of homogeneous memory locations.

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* Array : — array Store collection of elements of same type stored at contiguous memory locations. and can be accessed using an index.



Declare an array in C : —

datatype arrayName [array size];
int myarray [5];

Initialize an array in C : —

datatype arrayName [array size] = {
value₁, value₂, ..., value_n};

int myarray [5] = {1, 2, 3, 4, 5};

int myarray [] = {1, 2, 3, 4, 5};

Here, we haven't specified the size .
the compiler know its size is 5
as we are initializing it with 5
elements.

Merge P.B. —

How Java code executes —

Applications of Array —

- * Memory Management.
- * Data Representation.
- * Database Management.
- * Implementing Data Structures
- * Caching & Buffering.

Types of Indexing —

- o zero-based indexing — (Default).
- i one - " - "
- n n-based indexing —

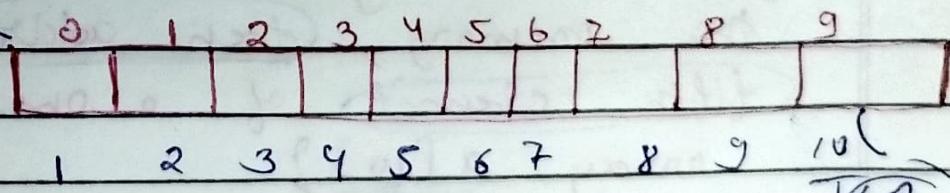
Size of an Array —

$$(9 - 0) + 1 = 10$$

Number of elements = (Upper bound - Lower bound) + 1

- Lower bound - index first element.
- Upper bound - index last element.

L.B.



array start at 0 start at 1 work
0 1 2 3 4 5 6 7 8 9 10

U.B.

4 bits

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Size = num of element * size of each elem

$$10 \times 4B = 40 \text{ Bytes}$$

* find Address of the element at k^{th} index

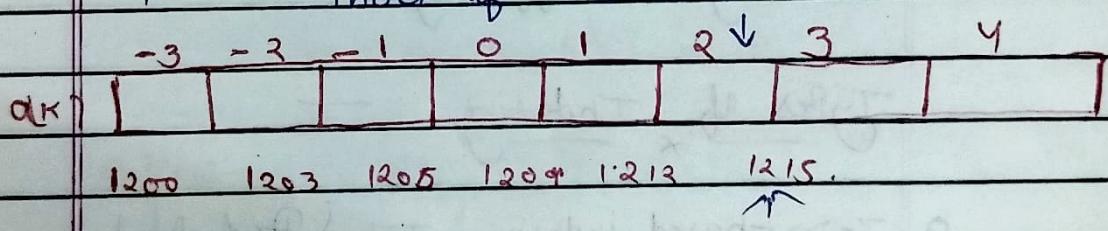
$$a[k] = B + w * k$$

$$a[k] = B + w * (k - \text{lower bound})$$

B - is the base address of the array $= 1200$

w - size of each element. $= 3$

k - index of the element.



$$1200 + 3(-5)$$

$$\underline{1200 + 15 = 1215}$$

* Let the base address of the first element of the array is 250 and each element of the array occupies 3 bytes in the memory, then address of the fifth element of a one-dimensional array $a[10]$

$$a[k] = B + w(k - L)$$

in bytes.

MongoDB Compat :- System at server
 out connect out at

Input

$$250 + 3 [4 - 0]$$

$$250 + \cancel{3} \times 13 = 263$$

Q.

An array has been [-6 6] of elements where array element takes 4 bytes, if the base address of the array is 3500 find the address of array (a)?

$$3500 + 4 [0 - (-6)]$$

$$3500 + 24 = 3524$$

Q.

Two - Dimensional array

2D array is organized as matrix which can be represent as the collection of rows/ columns.

	Column 0	Column 1	Column 2
Row 0	x[0][0]	x[0][1]	x[0][2]
Row 1	x[1][0]	x[1][1]	x[1][2]
Row 2	x[2][0]	x[2][1]	x[2][2]

Initialization of array

data-type array name [row] [columns];
 int disp[2][4] = {

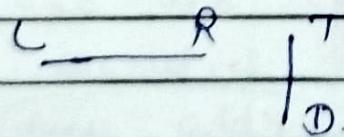
{10, 11, 12, 13},

{14, 15, 16, 17};

};

or

int disp[2][4] = {10, 11, 12, 13, 14, 15, 16, 17};



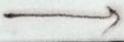
* Row Major implementation of 2D array

Array are arranged sequentially row by row.

(Default)

1	2	3
4	5	6
7	8	9

R M O



1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

* Column major implementation —

Array are arranged sequentially column by column.

00 ¹	01 ²	02 ³
10 ⁴	11 ⁵	12 ⁶
20 ⁷	21 ⁸	22 ⁹

0 ¹	10 ²	20 ³	01 ⁴	11 ⁵	21 ⁶	02 ⁷	12 ⁸	22 ⁹
----------------	-----------------	-----------------	-----------------	-----------------	-----------------	-----------------	-----------------	-----------------

Row Major implementation of 2D array

$$\text{Address of } a[i][j] = B + w[(v_2 - l_2 + 1)(i - l_1) + (j - l_2)]$$

B - Base address

w - Size of each elements.

l_1 - Lower bound of rows.

v_1 - Upper bound of rows.

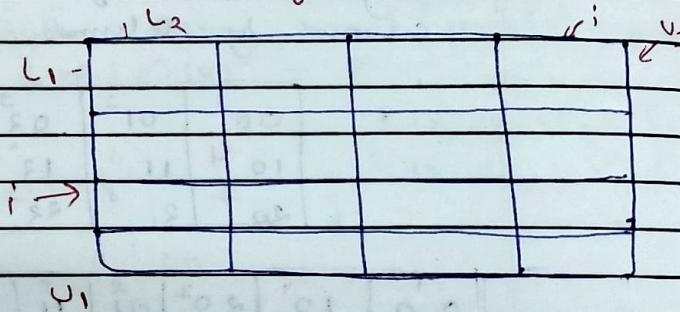
l_2 - Lower bound of columns.

v_2 - Upper bound of columns.

$(v_2 - l_2 + 1)$ = number of columns.

$(i - l_1)$ = num of rows before i .

$(j - l_2)$ = num of element before j .



Row wise -

$$\text{Address of } a[i][j] = B + w * [(v_2 - l_2 + 1)(i - l_1) + (j - l_2)]$$

Column wise -

$$\text{Address of } a[i][j] = B + w * [(v_1 - l_1 + 1)(i - l_1) + (j - l_2)]$$

3-Dimensional array

- z - x -

$A[(l_1) \dots (l_m)], (l_{m+1}) \dots (l_n), (l_{n+1}) \dots (l_k)$

location of $A[i, j, k] =$

$$3 + (i - l_1)(l_2 - l_3 + 1) + (k - l_3)$$

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31/2 ~~Matrix~~ ~~Content~~
~~Elmt~~ ~~E~~

Sparse Matrix —

A matrix is large num of its element are zero.

0	0	3	0
0	0	0	8
1	0	3	0
0	0	7	0

* Advantage : —

* Storage Efficiency : —

* Computational Speed : —

Representation : —

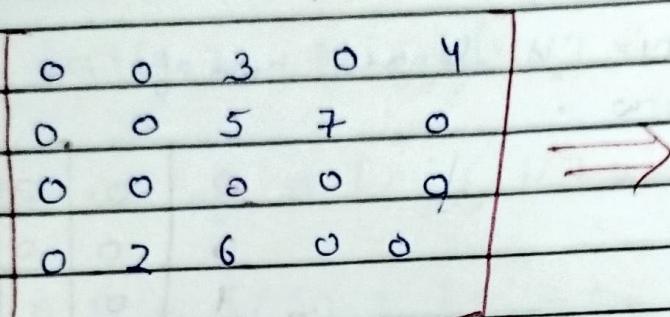
* Array Representation.

* Linked list Representation.

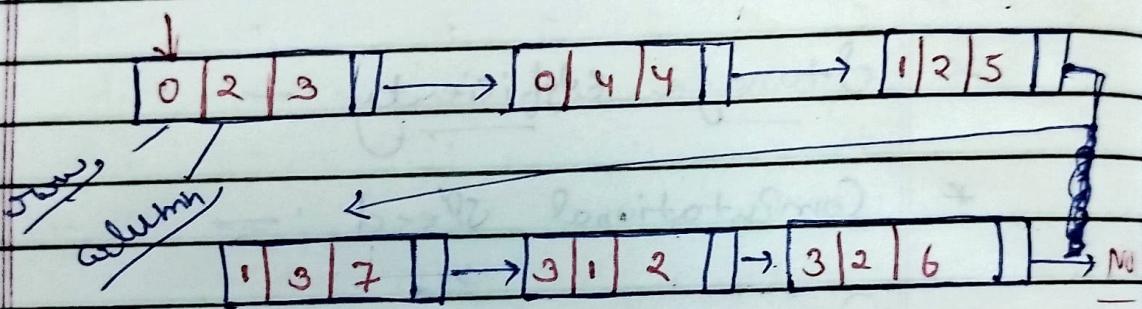
→ Array Representation : — 2D array is used to represent a sparse matrix.

	0	1	2	3	4		Row	0	0	1	1	3	3
0	0	0	(3)	0	4		Column	2	4	2	3	1	2
1	0	0	5	7	0		Value	3	4	5	7	2	6
2	0	0	0	0	0								
3	0	2	6	0	0								

Linked List Representation



Start :



Problem with Array

* fixed size / Reallocation : — have fixed sized, lead to memory waste if the allocated size is larger than the actual data.

* Inefficient insertion / deletion : — Adding / removing elements in the middle of the array requires shifting the remaining elements, resulting higher time complexity ($O(n)$) compare to linked list.

* Less flexible : — can only store elements of the same datatype..

Heterogeneous data type.

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* Linked list → dynamic data structure that

Consists of elements called nodes, which are connected in a linear sequence.

* Each node contains two parts:

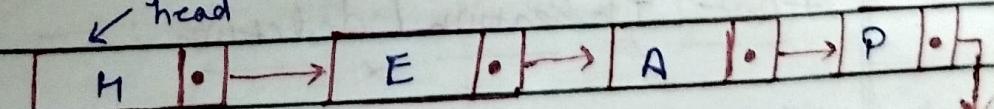
* data.

* Reference to the next node.

→ first part is the information part of the node, which can store any type of the information such as integers, characters, objects.

→ Second part called linked field / next pointer field, contains the address of the next node of the list

head



data Next.

A special case is the list that has no nodes, such a list is called null list / empty list.

Denoted by - Null pointer in the variable start / first / head.

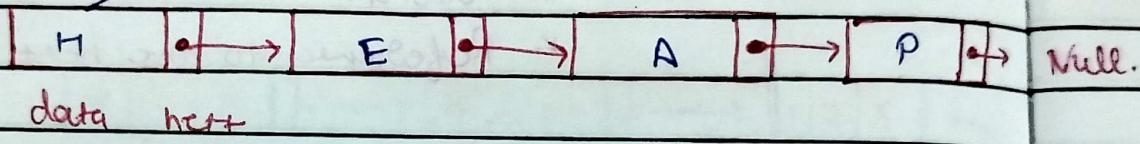
Implementation of Link list

Struct node
?

int data;

Struct Node *next;

Head



Advantages of Link list

* Dynamic size / efficient memory usage

Linked list can easily grow / shrink, allowing for efficient memory allocation.

* fast Insertion / deletion

Versatility — We can use various type (singly, doubly, circular) and can store elements of diff data type objects.

linked list * Space ~~amount~~ &
* internal/external fragmentation n^2 - help.

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Disadvantages

* Slow access time — have higher
time complexity.
Compare to array.

* Memory overhead — Each node in linked
list requires additional
memory to store the reference/pointer
to the next node.

* Pointer manipulation —

Aspect

Array

Linked List

Memory Allocation

Contiguous.

Non-contiguous.

Size flexibility

fixed size

Dynamic size, can
grow/shrink.

Access time

$O(1)$ direct

$O(n)$ required
traversal.

Insertion/
Deletion.

$O(n)$ worst case
Shifting may be
required.

$O(1)$

Memory
efficiency.

More memory efficient

Extra memory
Pointers.

```
#include <stdio.h>
#include <stdlib.h>
// Define Node Structure.
```

```
typedef struct Node
{
    int data;
    struct Node *next;
}
```

```
Node *createNode (int data)
```

```
{
```

```
    Node *newNode = (Node *) malloc (sizeof(Node));
    if (!newNode)
```

```
{
```

```
    printf ("Memory error in");
    exit (1);
}
```

```
}
```

```
newNode->data = data;
```

```
newNode->next = NULL;
```

```
return newNode;
```

```
S
```

```
int main()
```

```
{
```

```
    Node *head = createNode (10);
```

```
    head->next = createNode (20);
```

```
    head->next->next = createNode (30);
```

```
    printf ("Linked List: ");
```

```
    traverseList (head);
```

```
    return 0;
```

- Q) Write a C-Style Pseudocode for traversing a link list iteratively where pointer head have the address of the first node of the list.

Void traverseList (Node *head)

{

 Node *current = head;

 while (current != Null)

 {

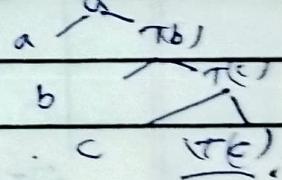
 printf ("→", current → data);

 current = current → next;

}

 printf ("Null\n");

}



- Q) Write a C - Style Pseudocode for traversing a link list recursively, where pointer head have the address of the first node of the list?

Void traverseListRecursive (Node *current)

 if (current == Null)

 {

 printf ("Null\n");

 return;

 }

printf ("→", current → data);

traverseListRecursive (current → next);

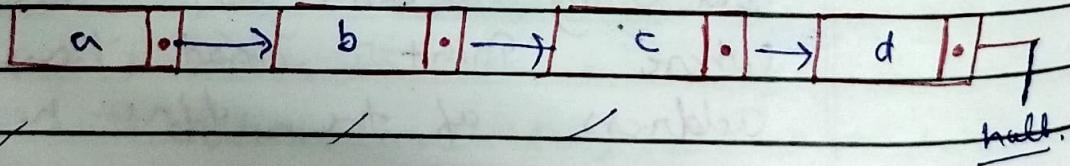
}

Q Write a C - Style Pseudocode for searching a key in a link list iteratively where Pointer head have the address of the first node of this list?

```

Node * SearchKeyIterative(Node *head, int key)
{
    Node * current = head;
    while (current != NULL)
    {
        if (current->data == key)
            return current;
        current = current->next;
    }
    return NULL;
}

```



Q Write a C-Style Pseudocode for searching a key in a link list recursively, where pointer head have the address of the first node of the list?

Node * searchKeyRecursive (Node * current, int key)
?

if (current == NULL)

?

| return NULL;

3

if (current → data == key)

?

| return current;

3

return searchKeyRecursive (current → next,
key);



s(a, c)

s(b, c)

s(c, c)

Q C-Style Pseudocode for inserting a node with a key in the starting of Link-List:

Void insertBeginning (Node **head, int key)

? Node * newNode = CreateNode (key);

| newNode → data = data;

| newNode → next = *head;

| *head = newNode;

* Write a C-Style Pseudocode for inserting a node with a key after a location in Link-list?

Void insertAfter(&Node *PprevNode , int key)
{

 if (Pprev Node == NULL)

 {

 printf("The given Previous node
 can't be Null lh");

 return;

 }

 Node *new Node = CreateNode (key);

 new Node → data = data;

 new Node → Next = Pprev Node → Next;

 Pprev Node → Next = new Node;

}

* C-Style Pseudocode for deleting a node from the starting of Link-list.

Void deleteAtBeginning (Node **head)

{

 if (*head == NULL)

 {

 printf("List is already empty. lh");

 return;

 }

 Node *temp = *head;

 *head = (*head) → Next;

 free (temp);

- ② write a C - style Pseudocode for deleting a node after a given location from the Starting of link-list

Void delete After (Node * PrevNode)
?

if (PrevNode == NULL || PrevNode->next ==
NULL)
?

| printf ("the given node is null or
| there's no node after it to delete. l");
| returning
S

Node * temp = PrevNode->next;

PrevNode->Next = temp->next;

free (temp);

S

#

* Write a C-Style Pseudocode for reversing a Link-list in a iterative?

Void reverseListIterative (Node **thead)
 ?

Node * Pprev = NULL;

Node * current = *thead;

Node * next = Null;

While (current != Null)

?

next = current → Next;

current → next = Pprev;

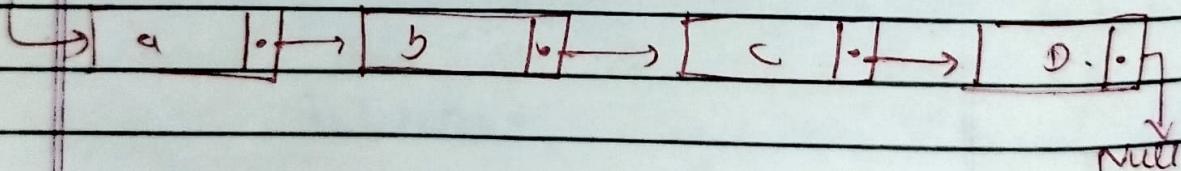
Pprev = current;

current = next;

S

*thead = Pprev;

}



Write a C-style Pseudocode for
Inverting a Link-List in a recursive
Node * reverseList Recursive (Node * head)
?

if ($\text{head} == \text{NULL}$ || $\text{head} \rightarrow \text{next} ==$
 null)

?

return head;

}

Node * next = reverseList Recursive
 $(\text{head} \rightarrow \text{next});$

$\text{head} \rightarrow \text{next} \rightarrow \text{next} = \text{head};$

$\text{head} \rightarrow \text{next} = \text{null};$

return next;

}

The following C function takes a single linked list of integers as a parameter and rearrange the elements of the list. The function is called with the list containing the integers 1, 2, 3, 4, 5 in the given order. What will be the contents of the list after the function execution.

Struct node.

?

int value;

Struct node *next;

S:

void rearrange(Struct node *list)

?

Struct node *p, *q;

int temp;

if ((list) == list -> next)

return;

p = list;

q = list -> next;

while(q)

?

temp = p -> value;

p -> value = q -> value;

q -> value = temp;

p = q -> next;

q = p? p -> next();

, ,

The following function takes a singly-linked list as input arguments. It modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank. Choose the correct alternative to replace the blank line.

~~typedef struct node~~

int value;

struct node *next;

{ Node,

Node *move_to_front (Node *head)

{

Node *p, *q;

if (head == NULL) { head = next =

null; }

return head;

q = NULL; p = head;

while (p->next != NULL)

{

q = p;

p = p->next;

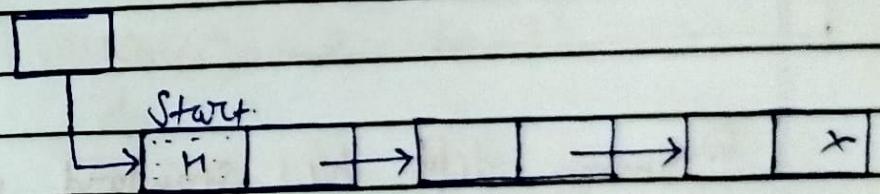
} q->next = NULL; p->next = head;

head = p;

return head;

Header linked list

- * is a variation of standard link list that includes a special node, called the header node, at the beginning of the list.
- * header node doesn't store any actual data; instead, it serves as a reference point that simplifies some operations on the linked list like - inserting, deleting element at the beginning of the list.



```
Void traverseHeaderLinkedList (Node *header)  
{
```

```
    if (header == NULL)
```

```
    {
```

```
        printf ("List is empty.\n");  
        return;
```

```
S
```

```
    Node *temp = header->next;
```

```
    while (temp != NULL)
```

```
{
```

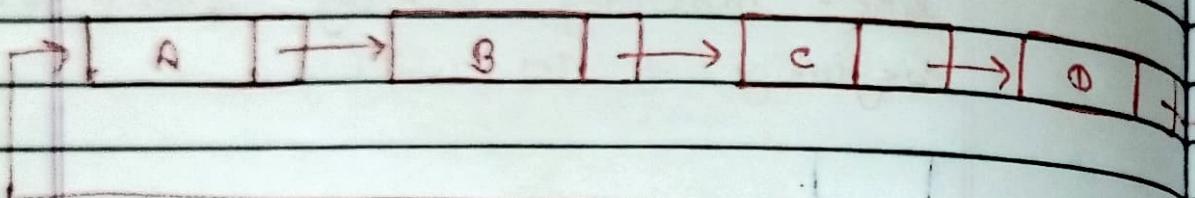
```
    printf ("%d - ", temp->data);
```

```
    temp = temp->next;
```

Print ("Null"):

Circular Linked List —

- * Variation of Singly linked list.
- * In which the last node in the list points back to the first node.
- * Creating loop in circular structure.



Primary diff b/w standard singly linked list and singly circular linked list —

- * in singly circular linked list last node's 'Null' pointer refers to the first node in the list, rather than being 'NULL'.

* features —

- * We used to implement data structures like queues or circular buffers. When elements are added to the end and removed from the front, with constant time complexity for both operations.

* Traversal of tree list requires a stopping condition, such as iterating until you reach the starting node again. or use counter to limit the number of iterations to avoid infinite loops.

Void traverseCircularLinkedList (Node *head)

{

if (head == NULL)

{

Printf ("List is empty\n");

S

return;

Node *temp = head;

do

{

Printf ("%d -> ", temp->data);

temp = temp->next;

J

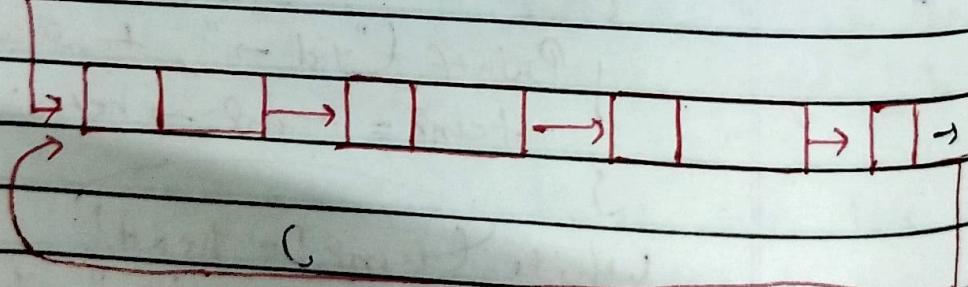
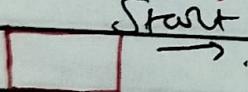
While (temp != head).

Printf ("%d (%head)\n"; head->data);

J

~~(X)~~ Header Circular link list :-

- * Variation of Singly Circular linked list that included a special node, called the Header node, at the beginning of the list.
- * The header node does not store any actual data.
- * The header node's primary purpose is to eliminate the need for special cases when performing certain operations like inserting / deleting elements at the beginning or end of the list.



[Circular header Node]

Void traverseHeader Circular List (Node *header)

{

if (header → next == header)

{

| printf ("List is empty\n");

|
| return;| }
| }

| Node *temp = header → next;

while (temp != header)

{

| printf ("→ d = ", temp → data);

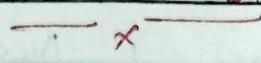
| temp = temp → next;

|
| }

| printf (" HEADER ");

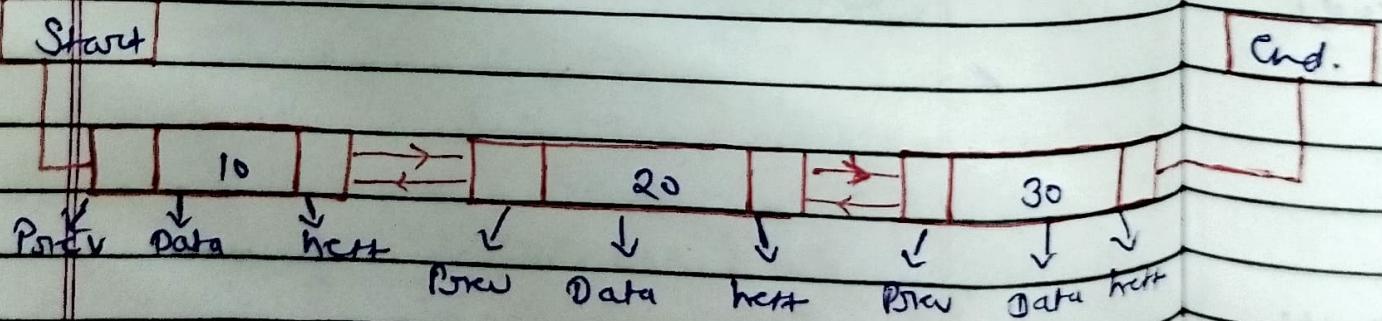
|
| }

② Doubly link list

→ 

* In which each ~~node~~^{node} contain data element and two pointers, one pointing to the previous node (Previous Pointer) and other pointing to the next node (the next pointer) in the sequence.

- * The bidirectional linking allows for easier traversal and manipulation of the list in both forward/backward direction.
- * as well as simplifying some operations such as insertion and deletion of nodes at any position in the list.



`#include <stdio.h>`

`#include <stdlib.h>`

`typedef struct Node;`

`{`

`int data;`

`struct Node *Prev;`

`struct Node *Next;`

`} Node;`

`Node *CreateNode (int data)`

`{`

`Node *newNode = (Node *)malloc (`

`sizeof (Node))`

`if (!newNode)`

`{`

`printf ("Memory error !");`

`exit (1);`

`}`

`newNode->data = data;`

`newNode->Prev = NULL;`

`newNode->Next = NULL;`

`return newNode;`

`}`

`Void InsertAtBeginning (Node **head, int data)`

`{`

`Node *newNode = CreateNode (data);`

`if (*head != NULL)`

`{ (*head)->Prev = newNode;`

`}`

`newNode->Next = *head;`

`*head = newNode;`

Void insertAtEnd (Node **head, int data)

{

Node * newNode = CreateNode(data);
if (*head == NULL)

{

* head = newNode;
return;

}

Node * temp = *head;

while (temp -> next != NULL);

{

temp = temp -> next;

}

temp -> next = newNode;

newNode -> prev = temp;

}

Void deleteAtBeginning (Node **head)

{

if (*head == NULL)
return;

Node * temp = *head;

* head = (*head) -> next;

if (*head != NULL)

{

(*head) -> prev = NULL;

}

free (temp);

void deleteAtEnd (Node **head)

?

if (*head == NULL)

return;

Node *temp = *head;

while (temp->next != NULL)

?

| temp = temp->next;

3

if (temp->prev != NULL)

?

| temp->prev->next = NULL;

5

else

?

| *head = NULL;

3

free (temp);

void deleteByPointer (Node **head, Node *loc)

?

if (*head == NULL || loc == NULL)

return;

if (loc->prev == NULL)

?

| *head = loc->next;

3

else ?

| loc->prev->next = loc->next;

(loc->next != NULL)

?

$\text{loc} \rightarrow \text{next} \rightarrow \text{prev} = \text{loc} \rightarrow \text{prev}$

}
free (loc);

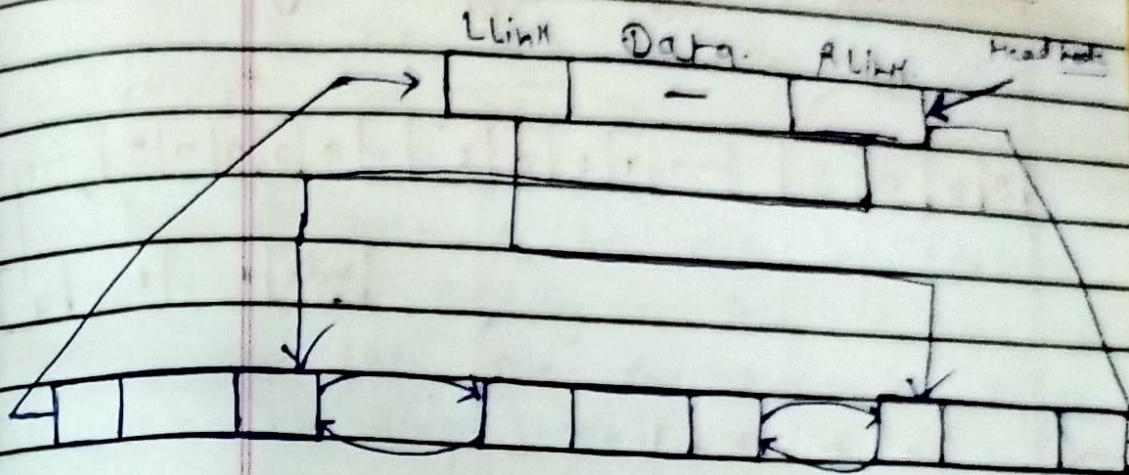
}

Key features of doubly linked list

- * Each node had two Pointers: 'next' Pointing to the Subsequent node and 'Previous' Pointing to the Preceding node.
- * first node Previous Pointers and the last node next Pointers are set to the NULL indicating the beginning and end of the list, respectively.
- * Doubly linked list allow for easier traversal and manipulation in both forward / backward direction compared to singly linked list.
- * Consumes more memory than singly linked lists due to addition Previous pointer.

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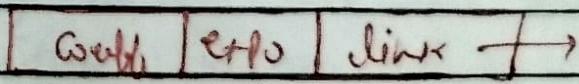
Header Circular Doubly linked List



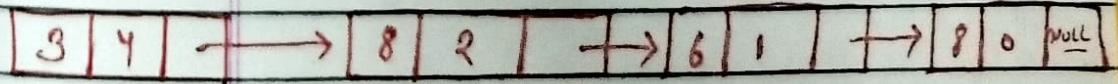
Polynomial Representation Using Linked List

- * Each node should consist of three elements namely Coefficient, exponent and a link to the next term.
- * The Coefficient field holds the value of the coefficient of a term, the exponent field contains the exponent value of that term and link field contains the address of the next term in the polynomial.

Ex →

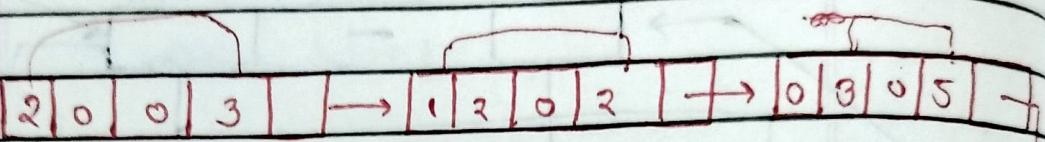


$$3n^4 + 8n^3 + 6n^2 + 8$$



Ex - 2

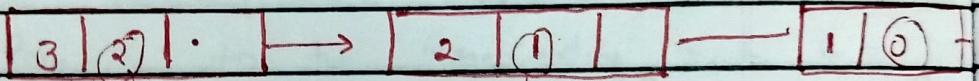
$$3n^2 + 2ny^2 + 5y^3 + 7yz$$



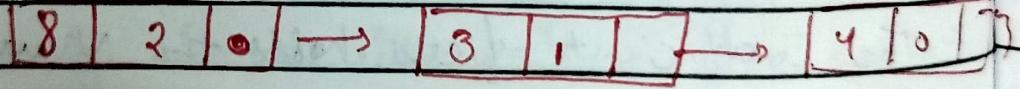
null [] [] []

③

$$3n^2 + 2n + 1$$



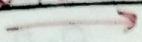
$$5n^2 + 1n + 3$$



$$8n^2 + 3n + 4$$

L.

Stack

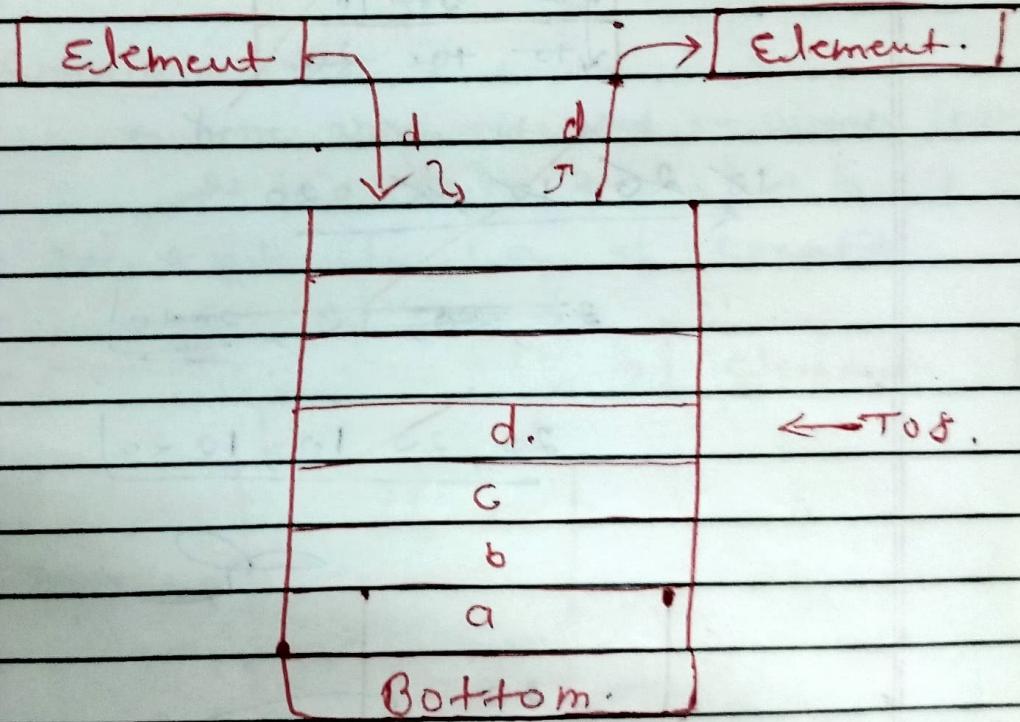


(LIFO)

First In Last Out

A Stack is a non Primitive linear data structure.

- * addition of new data item / deletion of existing data item is done from Only one end known as top of Stack (TOS)

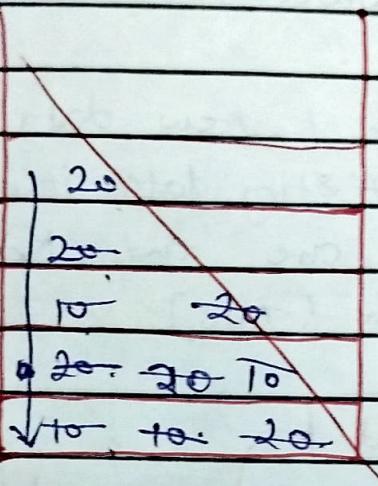


- * Most frequently accessible element in the Stack is the top most element whereas the least accessible element is the bottom of the Stack.

2:4:46

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Push(10), Push(20), Pop, Push(10), Push(20), Push(10), Push(20), Push(20), Push(20). the sequence of values popped out is.



10, 20, 10, 20, 20

20, 20, 10, 20, 10

20, 20, 10, 10, 20

20, 20, 10, 20

20

20

10

20

10

Push

Pop

10, 20, 10, 20

20, 20, 10, 10, 20

$C \rightarrow C \rightarrow C$

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Applications of Stack

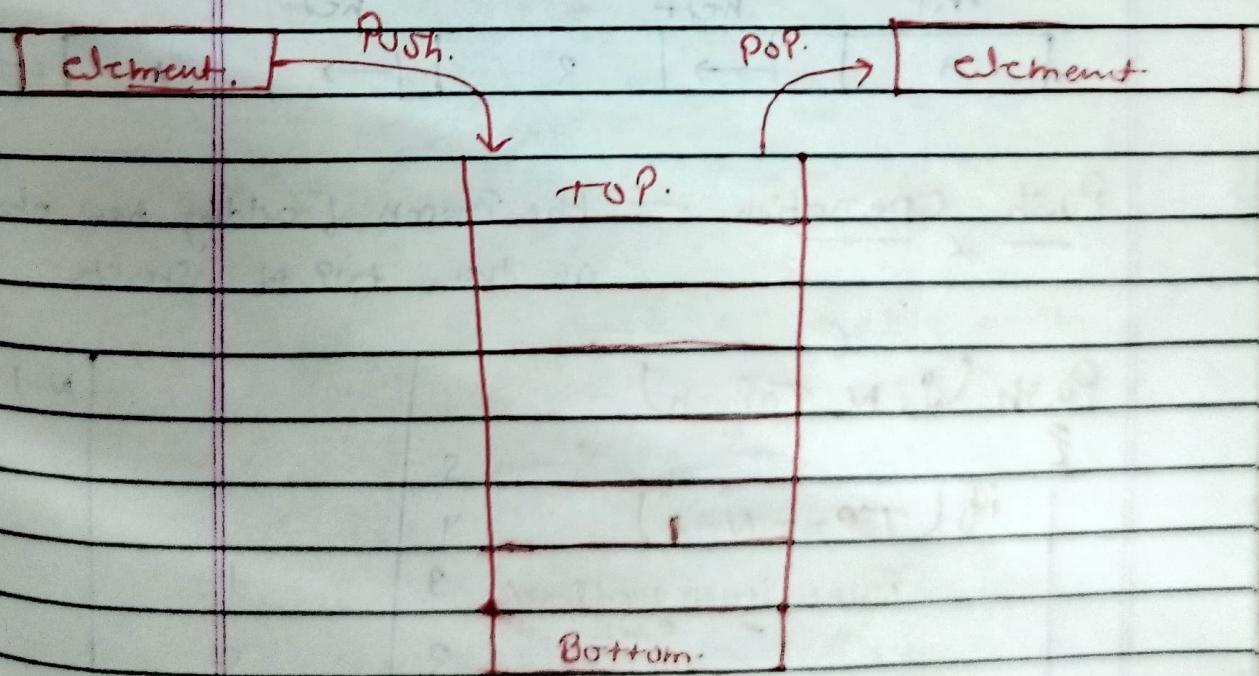
Balanced Parenthesis.

- * Expression Parsing :- help evaluate/ check Programming expression.
- * Backtracking :- used in algorithm like - Eight Queen Problem, maze - finding.
- * Function Call :- maintains fun detail during call.
- * Undo feature :-
- * Syntax Checking :-

Stack Implementation :- two ways.

Static Implementation :-

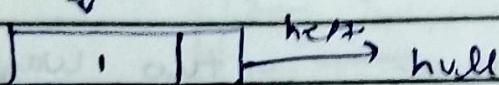
- * Here array is used to create Stack.
It is a simple technique but is not a flexible way of creation.



Dynamic Implementation

It is also called linked list representation and uses Pointers to implement the stack type of data structure.

TOP



TOP



TOP

next

next

next

null

Push Operation :— the process of adding new data on the top of stack.

Push (s, n, TOP, n)

\exists

if ($\text{TOP} == N - 1$)

Print Stack overflow,
 exit .

$\text{TOP} = \text{TOP} + 1$

$\text{Stack}[\text{TOP}] = n$

exit

\exists

s

y

z

l

b

a

-1

$N-1$

d

c

b

a

Pop operation → delete element from
top of the stack.

Pop (S, N, TOP)

?

if ($TOP = -1$)

?

3

Point underflow and exit. 2

$y = S(TOP)$

?

$TOP = TOP - 1$

0

return (y) and exit

-1

typedef struct

Stack

?

int arr[MAX_SIZE];

int *TOP;

Stack;

void initialize (Stack *S)

?

$S \rightarrow TOP = -1$

int isEmpty (Stack *S)

?

$\text{return } S \rightarrow TOP == -1;$

J

int isFull (Stack *S)

?

$\text{return } S \rightarrow TOP == MAX_SIZE - 1;$

S

void Push (Stack *S, int item)

{

if (isfull(S))

{

| Print ("Stack is full\n");
| return;

}

S-> arr [++(S-> top)] = item;

}

int Pop (Stack *S)

{

if (isEmpty(S))

{

| Print ("Stack is empty\n");

| exit (1);

return S-> arr [(S-> top) --];

}

#include < stdio.h >

#include < stdlib.h >

#define MAX_SIZE 100

int main()

{

Stack S;

| initialize (&S);

| return 0;

}

Linked List

Date: / /
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typedef struct Node

int data;

struct Node * next;

S Node;

typedef struct

?

Node * top;

S struct;

Void initialize (Stack * s) // overflow

?

| S -> top = NULL;

?

Int isEmpty (Stack * s)

?

| return S -> top == NULL;

?

Void Push (Stack * s , int item)

?

Node * newNode = (Node *)

malloc (sizeof (Node));

if (newNode == NULL)

?

| printf ("Stack overflow !");

| exit (1);

5

newNode → data = item;

newNode → next = J → top;

J → top = newNode;

5

int Pop (Stack * S)

?

if (isEmpty (S))

?

| Print ("Stack Underflow ! ");
exit (0);

5

Nodes * temp = J → top;

int poppedData = temp → data;

J → top = J → top → next;

free (temp);

return (poppedData);

5

#include <stdio.h>

#include <stdlib.h>

int main ()

?

Stack S;

initialize (8S);

return;

5

Void reverseString (char str[])

?

int length = strlen (str);
Stack S;

initialize (S);

for (int i = 0; i < length; i++)

?

, Push (S, str[i]);

S

for (int i = 0; i < length; i++)

?

str[i] = pop (S);

S

S

Push.

a b c d

d	
c	
b	
a	

Pop.

d, c, b, a.

int main()

?

char str[] = "Hello, World";

printf ("Original String : %s\n", str);

reverseString (str);

printf ("Reverse String : %s\n", str);

return 0;

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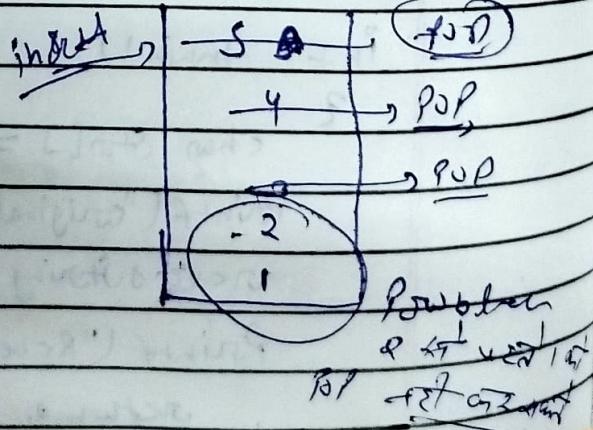
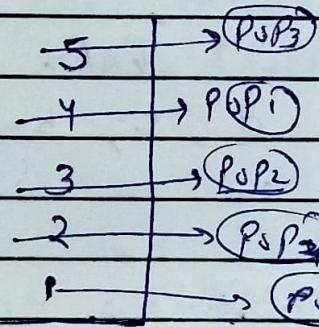
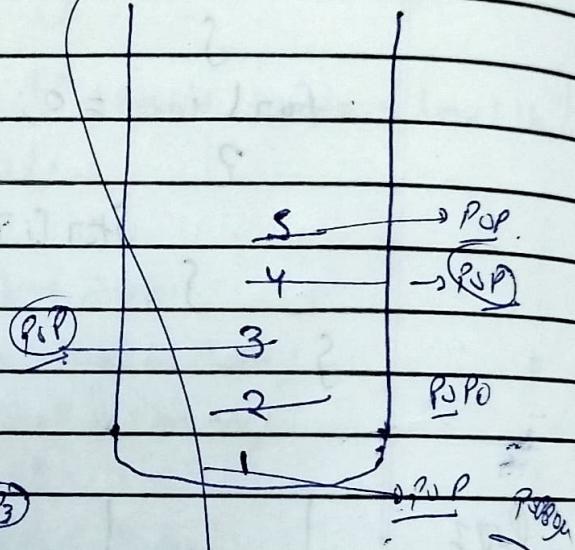
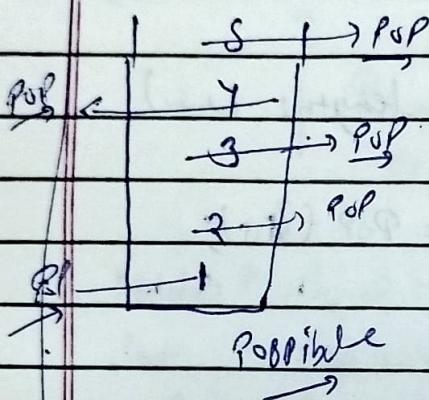
* if the input sequence is 1, 2, 3, 4, 5 then identify the wrong stack permutation. Possible (POP) sequence.

3, 5, 4, 2, 1

2, 4, 3, 5, 1

4, 3, 5, 2, 1

4, 3, 5, 1, 2



Notation

Infix notation : — Operator is written in b/w the Operands e.g. A+B.

Prefix notation : — Operator is written before the Operands. also called Polish notation
+ AB.

Postfix : — Operator are written after the operands. also called Suffix & Reverse Polish notation.

AB+

* Postfix notation is suitable for a Calculation.

* Any expression entered into the Computer is first converted into Postfix notation stored in Stack and then calculate.

Ex

~~a + b * c / d ^ e ^ f ^ d - c~~, Convert Prefix Postfix.
~~a + b * c / d ^ e ^ f ^ d - c~~
~~a + b * c / d ^ e ^ f ^ d - c~~
~~a + b * c / d ^ e ^ f ^ d - c~~

A, I, *, +, -, =

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$a + b * c / d \wedge e \wedge f * d - c$ convert it
into both Prefix & Postfix.

Postfix

$a + b * c / d \wedge e \wedge f * d - c$

$a + b * c / d \wedge e \wedge f * d - c$

$a + b * c / \underline{d e f g \wedge h \wedge i} * d - c$

$a + \cancel{b c} \times / \underline{d e f g \wedge h \wedge i} * d - c$

$a + b c \times / \underline{d e f g \wedge h \wedge i} * d - c$

$a + b c \cancel{\times} \underline{d e f g \wedge h \wedge i} / d \times d - c$

~~$a b c \times d e f g \wedge h \wedge i / * d \times d - c$~~

$a + b c \times \cancel{d e f g \wedge h \wedge i} / d \times + - c$

$a b c \times \cancel{d e f g \wedge h \wedge i} / d \times + c -$

Prefix

Left →

$a + b * c / d \wedge e \wedge f * d - c$

$a + b * c / \underline{d \wedge e \wedge f} * d - c$

$a + \cancel{b c} / \underline{d \wedge e \wedge f} * d - c$

$a + \cancel{1} * \cancel{b c \wedge d \wedge e \wedge f} * d - c$

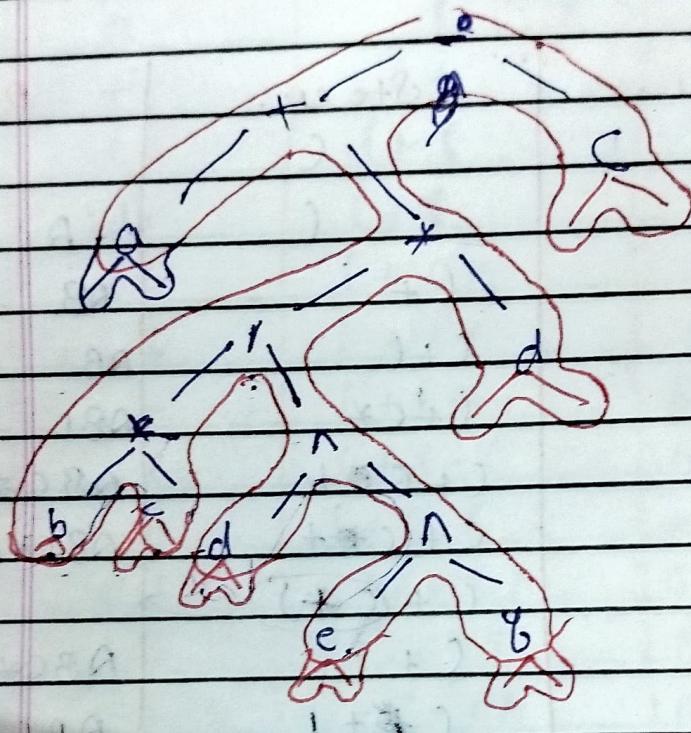
$a + \cancel{1} * \cancel{b c \wedge d \wedge e \wedge f} d - c$

$\rightarrow a + \cancel{b c \wedge d \wedge e \wedge f} d - c$

Tree at

left →

$$a + b * c / d \wedge e \wedge f * d - c$$



Prefix $- + a * / b c \wedge d \wedge e \wedge f * d - c$

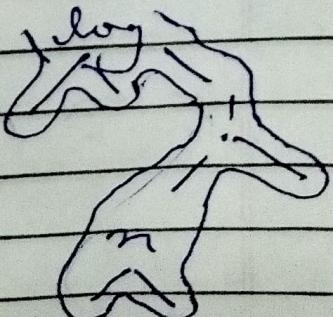
Infix $- a + b * c / d \wedge e \wedge f * d - c$

Postfix $a b c * d e f * d \wedge \wedge + c -$



Consider an expression $\log(n!)$, Convert it into both Prefix and Postfix notation.

$\log ! n$



Prefix $-\log ! n$

Infix $\log n !$

Postfix $\log n ! - \log$

Lc88 at high ⌈ right ⌋
 + * ()

but
 high ⌈ low ⌋
 * + ⌈ ⌋

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Stack ← : —

(A + (B * C + D) / E)

Character	Stack	Postfix
((
A	(A
+	(+	AB
((+ (AB
*	(+ C *	ABC
+	(+ C * +	ABC *
D	C + C * +	ABC * D
)	(+ C *)	
I	C +	ABC * D +
E	C * + I	ABC * D +
)	C	ABC * D + E
		ABC * D + E I +

C +
tigris

$$A + (B * C - (D / E \wedge F) * H)$$

Character	Start	Postfix
A	([ext])	A
+	(+	A
((+ (A
B	(+ ()	AB
*	(+ () *	AB
C	(+ () *	ABC
-	(+ () -	ABC *
D	(+ () - (ABC * D
I	(+ () - ()	ABC * D I
E	(+ () - ()	ABC * D E
N	(+ () - () N	ABC * D E F
F	(+ () - () N	ABC * D E F
)	(+ () -	ABC * D E F A I
*	(+ () - *	ABC * D E F A I
H	(+ () - *	ABC * D E F A I H
J	(+ () - *	ABC * D E F A I H *

Evaluation of Arithmetic operation :- ABC + DEF = ?

$$10 \times 5 + 100 - 5 \div 7 \times 3$$

$$50 + 10 - 5 \div 7 \times 3$$

$$50 + \boxed{10 - 5} + 1$$

$$60^{\vee} - s + 1$$

$$\underline{55} + 1$$

① M M a t

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Evaluate the Postfix expression —

$$8 \ 2 \ 3 \ * \ 1 \ 1 + 4 \ 1 \ * \ 3 \ 1 +$$

$$14 + 2 = 16$$

(2)		
3		4 1 2
4		
1		4 * 1
-		
4		
(14)		8 + 6 = 14
5		
1		6 1 1 = 6
8		
3		
2		2 * 3 = 6
(8)		

Evaluate the Prefix expression —

$$+ + . 8 \ 1 * 2 \ 3 \ 1 \ 1 + 4 \ 1 \ 3.$$

R → L

$$\frac{14}{8}$$

$$14 + 2 = 16$$

6

$$6 1 1 = 6$$

3

$$2 \times 3 = 6$$

1

$$4 \times 2 = 8$$

4

$$4 \times 1 = 4$$

1

Aly → ① Add eight Parenthesis "()" to P.
 [This act as a sentinel]

- ② Scan P from left to right and repeat Step 3 and 4 for each element of P until the sentinel ")" is encountered.
- ③ If an operand is encountered, Put it on Stack.
- ④ If an operator \oplus is encountered then.
 - ⑤ Remove the top two elements of Stack where A is the top element B is the next to top element.
 - ⑥ Evaluate $B \oplus A$.
 - c Place the result of (b) back on STACK
 [End of its structure]
 [End of Step 2 loop]
- ⑦ Set value equal to top element on STACK.
- ⑧ END.

Recursion — When function call itself.

$$f(n) = f(n-1) + f(n-2)$$

* Base Case: — Provides a direct solution without further recursive call.

* Recursive Case: — fun calls itself to address smaller instances of problem.

* Call Stack: — ^{2nd fun ↗ 3rd fun ↗} _{1st fun ↘ 2nd ↘ 3rd ↘} call chain at rightmost call stack. ^{end of ↘}

Deep recursion cause a stack overflow error.

int factorial (int n)

?

if (n == 0)

?

| return 1; // Base Case: factorial of 0

]

else

?

| return n * factorial (n-1); // Recursive Case.

5

$f(4)$

$$4 \times 6 f(3) =$$

24

$f(3)$

$f(2)$

$f(1)$

$f(0)$

$$3 \times 2 f(2) = 6$$

$$2 \times 1 f(1) = 2$$

$$1 \times 0 f(0) = 1$$

Iteration :-

* Process of Repetitively executing a set of statements, as long as Specified Condition remains true.

Loop type :-

for loop: Used for Known num of repetition.

while loop: Runs as long as Condition is true

do-while: execute at least one before checking the Condition.

Control Statements:-

- Break: Exit the loop.

- Continue: Skip to the next iteration.

Nested loop: — Loops within loops

int factorial (int n)

?

int result = 1;

for (int i = 1; i <= n; i++)

?

result *= i;

S

return result;

S

Aspect

Recursion

Iteration

Basic

function calls itself

Use loop to repeat

concept

to solve sub problem.

create code block.

Memory usage

Uses more memory.

Uses less memory.

Termination

Requires base case to prevent infinite loop.

Condition

Performance

May be slower due to overhead of fun call.

typically faster
due to direct level
machinice.

① Direct Recursion

A function calls

itself directly.

* Tail Recursion

→ End Print Call after

size of stack

used call after Pending with

* Head Recursion* Indirect Recursion

Two or more functions call each other in a cyclic manner.

far ex — fun A calls fun B, and fun B calls fun A.

find the O/P of —

Tail Recursion

Void main()

?

fun(4);

?

Void fun (int x)

?

if ($n > 0$)

?

printf ("%d", n);

fun (n - 1);

?

call size n - 1

?

Head Recursion

Void main()

{

fun(4);

{

Void fun(int n)

{

if ($n > 0$)

{

fun($n - 1$);

printf("y.d", n);

{

f(4)

f(3)

f(2)

f(1)

f(0)

3

2

1

(*)

Find the output : —

Void main()

{

fun(3);

{

Void fun(int n)

{

if ($n > 0$)

{

printf("y.d", n);

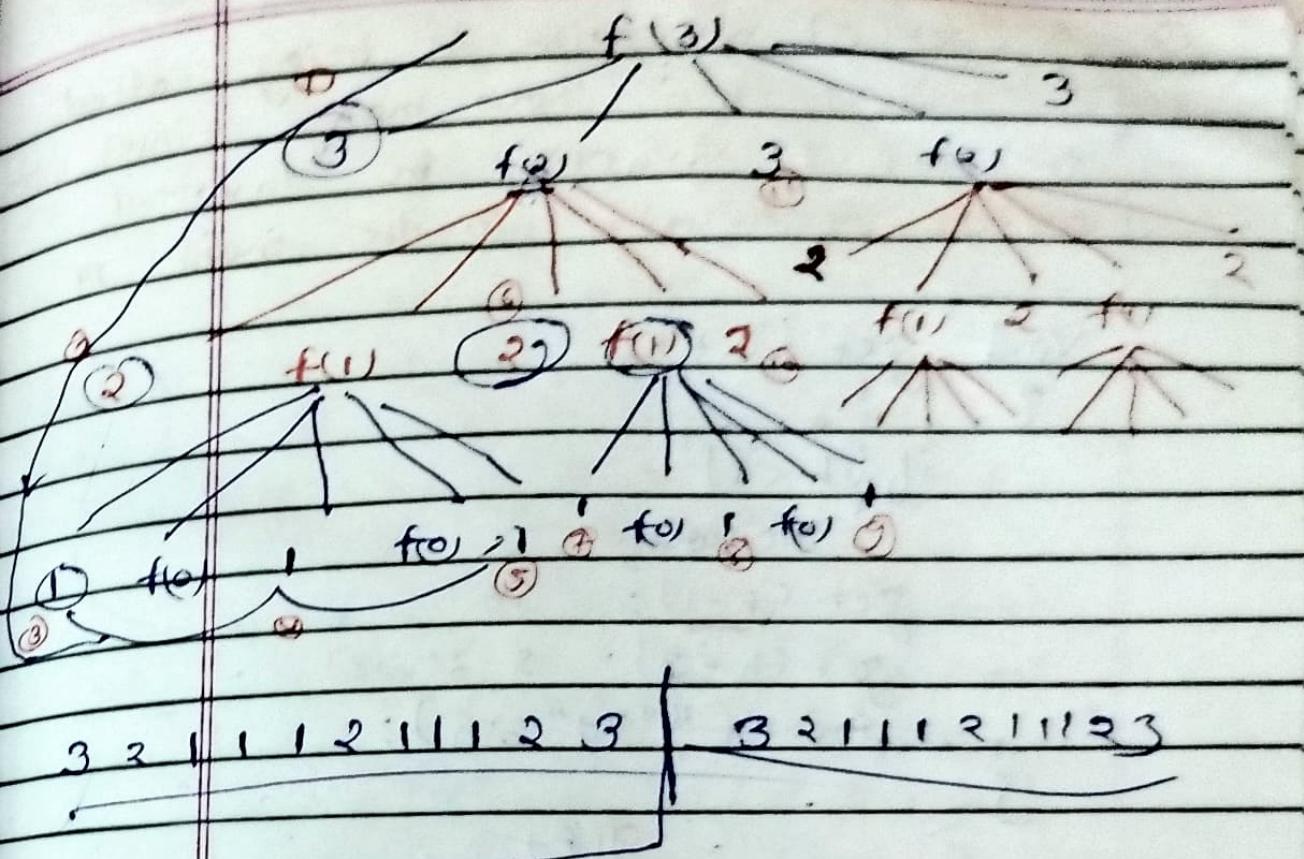
fun($n - 1$);

printf("y.d", n);

fun($n - 1$);

printf("y.d", n);

{



Q) find the output of following code n=5;

int n(int n)

{

 if (n < 3)

 return 1;

 else

 return n(n-1) + n(n-1) + 1;

$$f(3) = 3$$

$$f(4) = f(3) + f(3) + 1 = 7$$

$$f(5) = f(4) + f(4) + 1$$

$$7 + 7 + 1$$

$$f(5) = 15$$

$$\begin{aligned} f(5) &= f(4) + f(4) + 1 \\ f(4) &= f(3) + f(3) + 1 \\ f(3) &= f(2) + f(2) + 1 \end{aligned}$$

$$\begin{aligned} f(2) &= f(1) + f(1) + 1 \\ f(1) &= 1 \end{aligned}$$

$$= 3$$

3:15:12

call findans(1)

Done	11
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Q. If factorial function is being called
is more than how many times
the factorial function be invoked
before returning to the main().

Void get (int n)

?

if ($n < 1$)

return;

get ($n - 1$); $S - 1$ get ($n - 3$); $S - 3 = 2$

printf ("xd", n);

S

g(5)

g(4)

g(2)

5

Point

call findans(1)

$$g(-2) = 1$$

$$g(-1) = 1$$

$$g(0) = 1$$

$$g(1) = 3 =$$

$$g(2) = g(1) + 1 + 3 = 6$$

$$g(3) = g(2) + g(1) + g(0)$$

$$3 + 1 + 1 = 5$$

$$g(5) = g(4) + g(3) + g(2)$$

$$11 + 5 + 1$$

$$= 17 \text{ Ans}$$

$$g(3) = g(2) + g(1) + g(0)$$

$$5 + 1 + 1 = 7$$

$$g(4) = g(3) + g(2) + g(1)$$

$$7 + 3 + 1 = 11$$

(*) The Fibonacci numbers commonly denoted f_n , form a sequence, called the Fibonacci Sequence, such that each term is the sum of the two preceding ones, starting from 0 to 1.

$$f(0) = 0 \quad \text{if } n = 0, \text{ then } f(n) = 0;$$

$$f(1) = 1 \quad \text{if } n = 1, \text{ then } f(n) = 1$$

$$\text{if } n > 1, \text{ then } f(n) = f(n-1) + f(n-2).$$

$$f(n) = f(n-1) + f(n-2)$$

$$f(2) = f(1-1) + f(1-2)$$

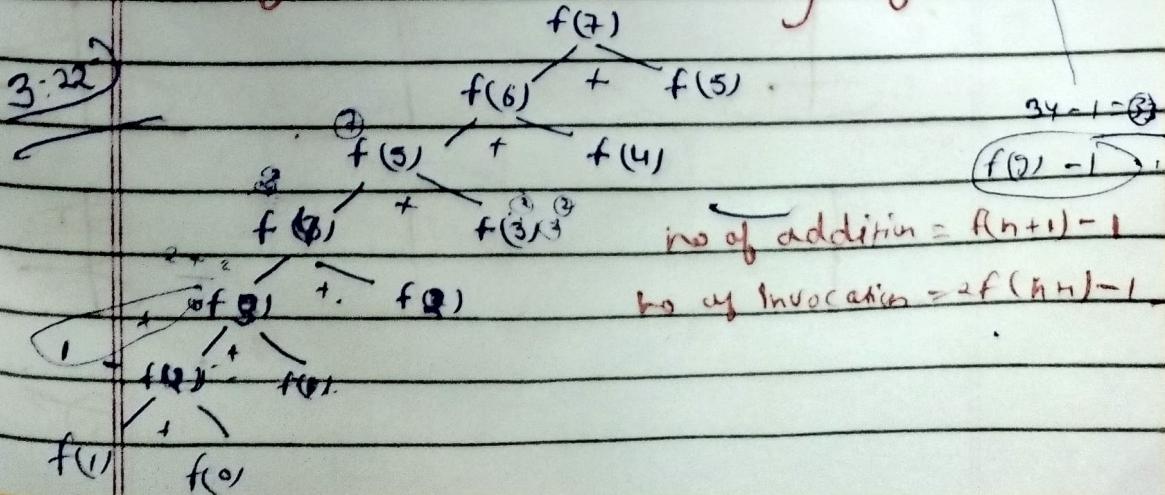
$$f(1) + f(0)$$

$$f(2) = 1 + 0 = 1$$

$$f(3) = f(2) + f(1) = 2$$

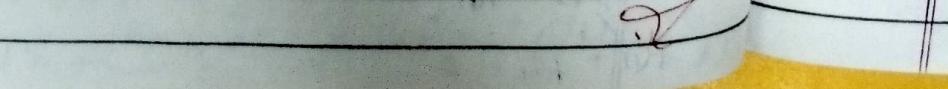
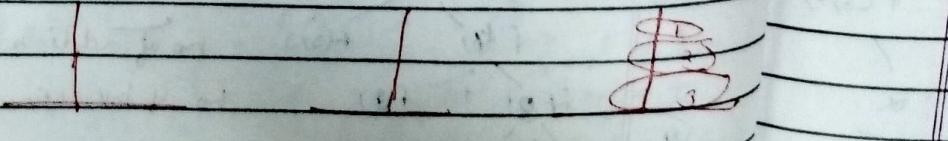
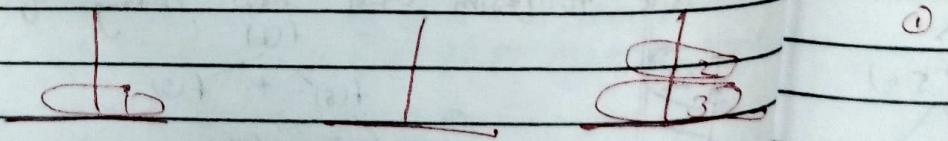
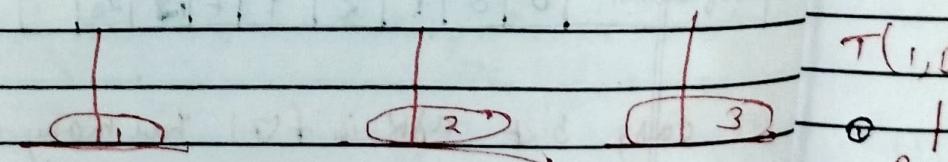
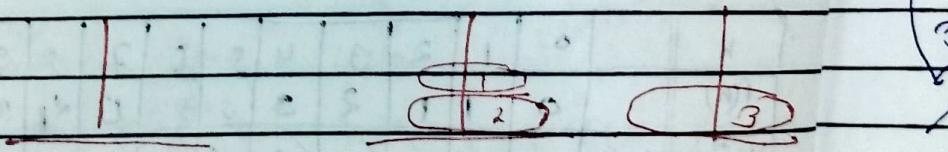
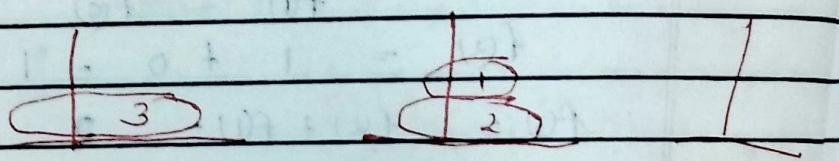
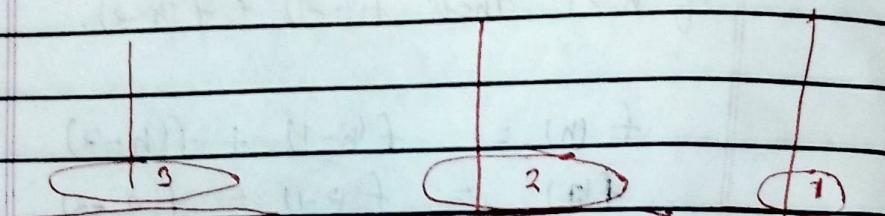
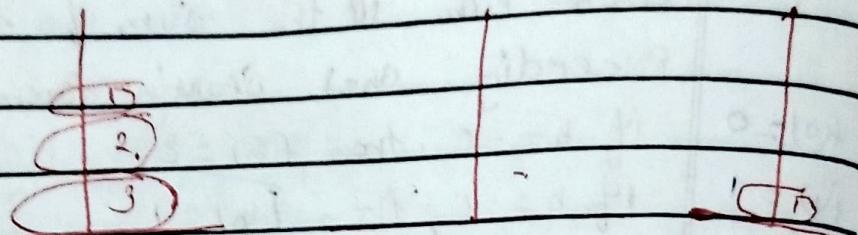
n	0	1	2	3	4	5	6	7	8	9	10	11
$f(n)$	0	1	1	3	3	5	8	13	21	34		
No of invocation	1	1	2	3	5	9	17	25	41			
No of addition	0	0	1	2	4	7	12	20				

it's easy, but ask in $f(7)$ how many addition are required and how many $f(n)$ calls



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Tower of Hanoi → / Brahma: —



Tower (N, B, A, E)

?

if ($h = 1$)
 ?

$B \rightarrow E$
return;

tower ($n-1, B, E, A$);

$B \rightarrow E$

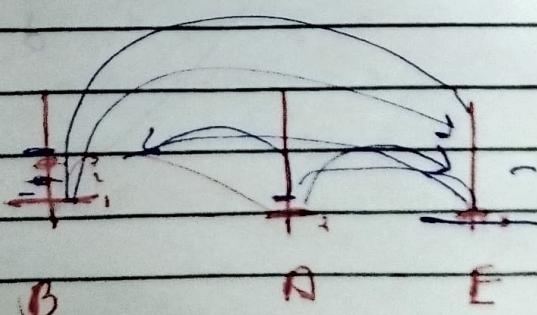
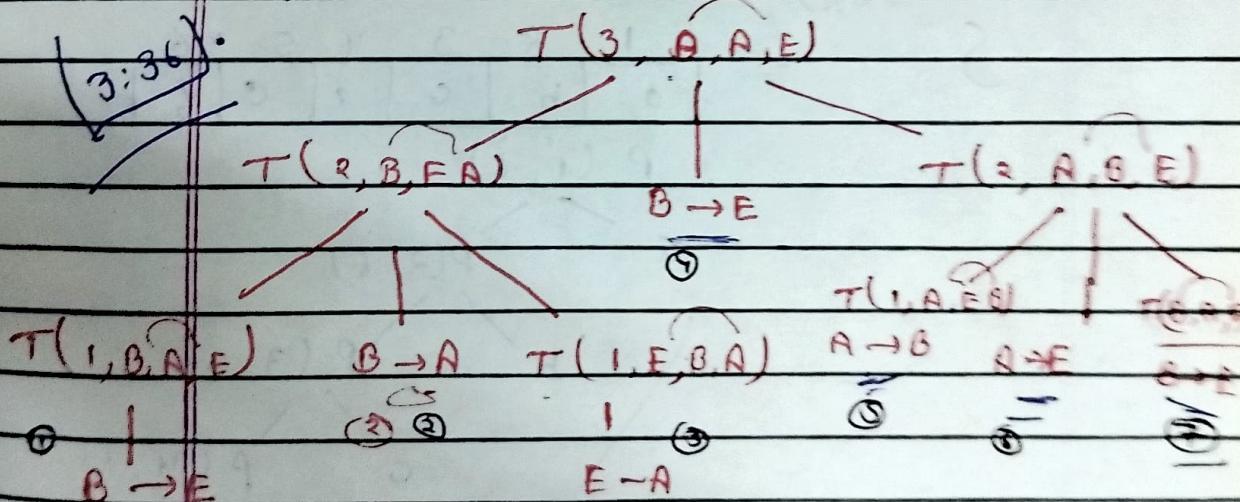
tower ($n-1, A, B, E$)

Return;

total disk moves = $2^h - 1$

" num' of fun call = $2^{h+1} - 1$

How many invocation are require for the
first disk to move = h .



$$2 = 3$$

$$3 = 7$$

$$h = 2^h - 1$$

~~visit function~~

Q. find the SIF -

visit Printarray(a, i, j)

?

if ($i == j$)

?

{ printf("x", a[i]);

return;

}

else

?

{ printf("x.d", a[i]);

Printarray(a, i+1, j)

}

S

	1	2	3	4	5	6
	a	b	c	a	c	b

P(i, 6)

a

P(2, 6)

b

P(3, 6)

c

P(4, 6)

d

P(5, 6)

e

P(6, 6)

f

~~read~~ → find the op

Void Print array (a, i, j)

{
if (i == j)
?

| printf ("d", a[i]);
| return;

}

else

{

Print array (a, i+1, j)

Print ("d", a[i]);

}

5

1 2 3 4 5 6

a	b	c	d	e	f
---	---	---	---	---	---

P(1, 6)

P(2, 6) a

P(3, 6) b

P(4, 6) c

P(5, 6) d

P(6, 6) e

g

Q) Void Point_Something (a, i, j)

{

if (i == j)

{

Printf ("y.d", a[i]);

return;

}

else

{

if (a[i] < a[j])

Print_Something (a, i+1, j);

else

Print_Something (a, i, j-1);

}

1 2 3 4 5 6

S	50	40	20	90	160	30
---	----	----	----	----	-----	----

P (1, 6)

1

P (1, 5)

P (2, 5)

P (3, 5)

1

P (4, 5)

P (5, 5)

② Preorder

void

{

}

what this function is doing :-

void what (Struct Bnode *t)
?

if (+)
?

what ($\leftarrow \text{LC}\right);$

printf ("y-d", $\leftarrow \text{data}\right);$

what ($\leftarrow \text{RC}\right);$

g

(\oplus)

1

2

3

4

5

w(1)

w(3)

w(2)

w(5)

w(4)

x

x

x

x

x

3

x

x

w(4)

w(5)

② Preorder —

Linear traversal.

void what (Struct Bnode *t)

?

if (+)

printf ("y-d"; $\leftarrow \text{data}\right);$

what ($\leftarrow \text{LC}\right);$ L

what ($\leftarrow \text{RC}\right);$ R

)

Bst nodes —

o $\text{Void } \overrightarrow{\text{what}}(\text{Street Node } *t)$

?

if (t)

?

$\text{what } (t \rightarrow \text{LC});$

$\text{what } (t \rightarrow \text{RC});$

$\text{printf}("y.d"; t \rightarrow \text{data});$

g

find what function doing --

(*)

void what (Struct Node *c)
?

if (+)
?

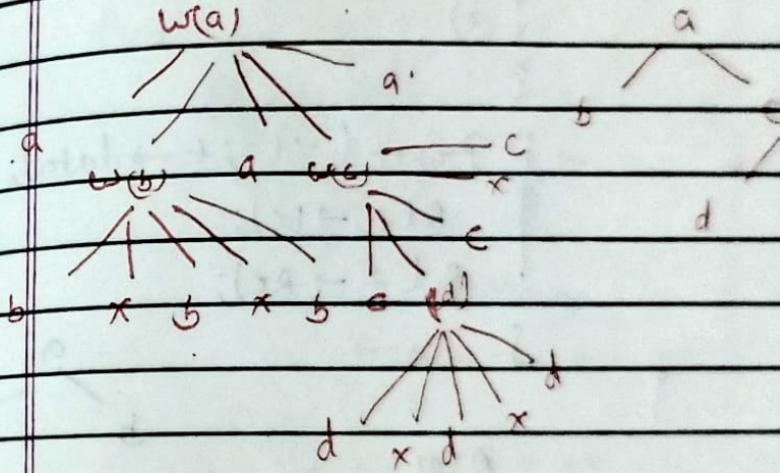
Printf ("yd", + → data);

what (+ → Lc);

Printf ("xd", + → data);

what (+ → Rc);

Printf ('yd', + → data);



abbb c ddd cc a.

find what this function is doing

Void A (Struct Bnode * t)

?

if (t)

?

B (t → LC);

printf ("%d"; t → data);

B (t → RC);

5

Void B (Struct Bnode * t)

?

if (t)

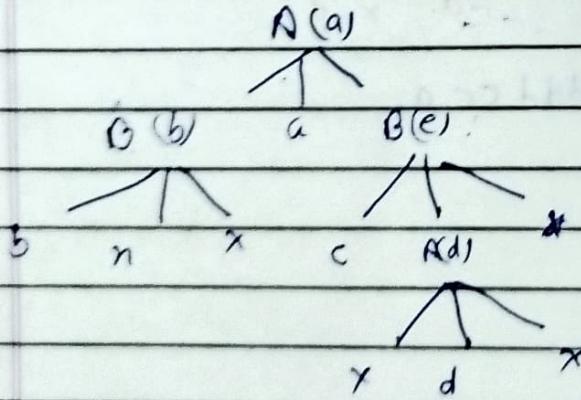
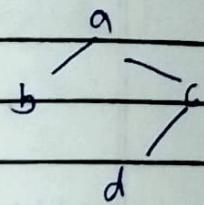
?

printf ("%d"; t → data);

A (t → LC);

A (L t → RC);

5



b a c d)

Q What does following C program print $n = 25$;

Void fun (int n)
{

if ($n == 0$)

return;

printf ("%d", n * 2);

fun (n / 2);

1000 ↴

↓

f(25)

f(12)

f(6)

f(3)

f(1)

f(0)

Q Recursive fun (n, y) - What is the value of fun (4, 3).

int fun (int n, int y)

{

if ($n == 0$)

return y;

return fun (n - 1, n + y);

f(4, 3)

/ \

f(3, 7)

/

f(2, 10)

/

f(1, 12)

/

f(0, 13)

(13)

Q

int fun (int n, int y)

? 2 0 = 0

if (y == 0)

 return 0;

 return (n + (fun(

 n, y - 1));

A) $n+y$

B) $n+n^y$

C) n^y

D) 0

Important \rightarrow P.T.O.

Date:	11
Page:	

Queue

list

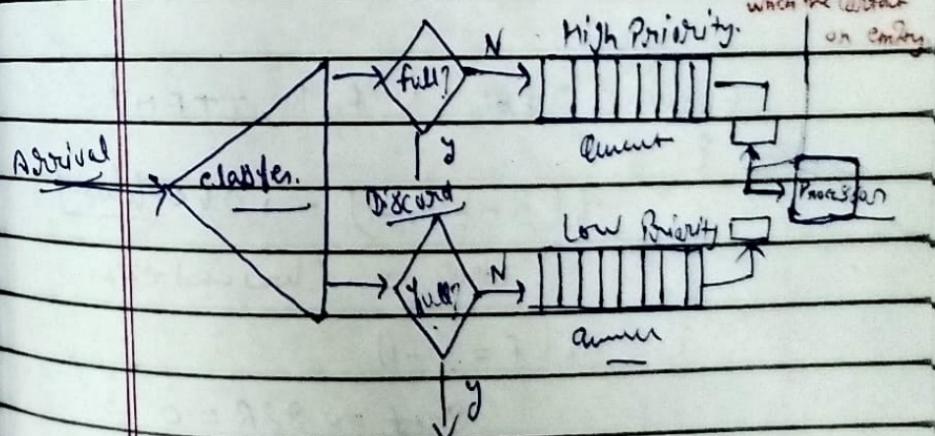
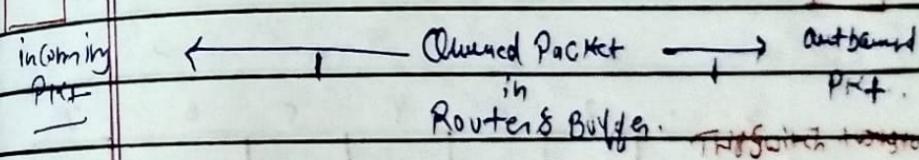
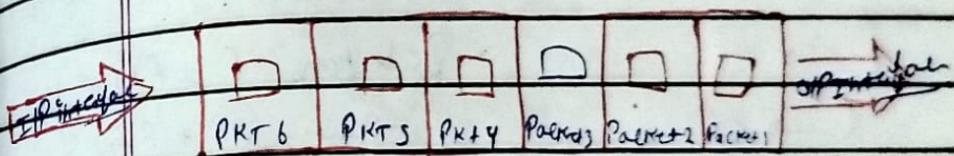
Queue is a linear list of elements
in which deletions can take place
only at one end called front.
and

~~deletion~~ insertion can take place only
at the end called rear.

the terms front and rear are used
in describing a linear list.

(4:6)

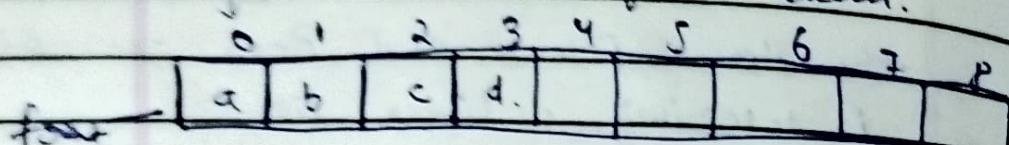
" A queue is a low primitive linear data structure
if it is a homogeneous collection of elements.



Mostly each of array queue will be maintained by a linear array QUEUE and two pointer variable.

Front - Containing location of front char.

Rear - Containing location of rear char
of the Queue.



$$f \rightarrow -1$$

Data after $f = f + 1$

$$A \rightarrow -1$$

initially $R + R + 1$

Iteration -



Enqueue (QUEUE, N, f, R, ITEM)

?

if ($R == N - 1$) → overflow

writing overflow and exit.

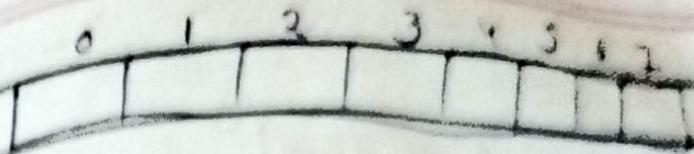
if ($f == -1$)

Set $f = 0$ & $R = 0$

else

$R = R + 1$

queue[R] = ITEM.



Decompile (Queue, N, & R, ITEM)

?

if ($f == -1$)

return underflow error

ITEM = Q[REAR]

R (f = -1)

Sc+F = -1 & R = -1

else

f = F + 1

type defn true Return item

?

int arr[MAX_SIZE];

int front;

int rear;

S Queue;

void initialize (Queue *q)

?

q → front = -1;

q → rear = -1;

)

int * isEmpty (Queue *q)

?

return (front == -1);

S

int is full (Queue *q)

?

return (rear == MAX_SIZE - 1);

S

void enqueue (Queue *q, int item)

{

if (isfull(q))

{

printf ("queue is full !\n");

return;

}

if (isempty(q))

{

q->front = 0;

}

q->arr [++(q->rear)] = item;

}.

int dequeue (CircularQueue *q)

{

if (isempty(q))

{

printf ("Queue is empty !\n");

exit (1);

}

int dequuemItem = q->arr [q->front];

If (q->front == q->rear)

{

q->front = -1;

q->rear = -1;

}

else

q->front = (q->front + 1) / ~~max size~~

3

return dequation;

3

#include < stdio.h >

" " < stdlib.h >

define MAX size 100

int main ()

?

Queue q;

initialize (8);

return;

3

Ques with the help of linked list -

———— + ————— x —————

typedef struct Node

?

int data;

struct Node * next;

Node;

typedef struct

?

Node * front;

Node * rear;

5 Ques;

Implementation / deletion game and 2nd pt State
" " diff end 2nd " Queue

Date:	/ /
Page:	

Void enqueue (Queue *q, int item)

{

Node *newNode = (Node *) malloc(sizeof(Node));
if (newNode == NULL)

{

| printf("Queue overflow\n");
| return;

}

newNode -> data = item;

newNode -> next = NULL;

if (is Empty (q))

{

q -> front = newNode;

}

else

{

| q -> rear -> next = newNode;

}

q -> rear = newNode;

}

int deQueue(Queue * q)

?

if (isEmpty(q))

?

| Printf("Queue Underflow");
| EXIT(0);

S

Node * temp = q->front;

int deQueueItem = temp->data;

q->front = q->front->next;

if (q->front == NULL)

?

| q->rear = NULL;

S

free(temp);

return deQueueItem;

S

#include < stdio.h >

#include < stdlib.h >

int main()

?

Queue q;

initialize(&q);

return 0;

Q Consider the following sequence of operations on an array stack.

$\text{Push}(54)$; $\text{Push}(53)$; $\text{Pop}()$; $\text{Push}(55)$;
 $\text{Push}(62)$; $S = \text{Pop}()$;

Q Consider the following sequence of operations on an empty queue. $\text{Enqueue}(21)$;
 $\text{enqueue}(24)$; $\text{dequeue}()$; $\text{enqueue}(28)$; $\text{enqueue}(29)$;
 $q = \text{dequeue}()$. The value of $1 + q$ is

62	Pop
55	
52	
54	$\cancel{\text{Pop}}$

$$S = 62$$

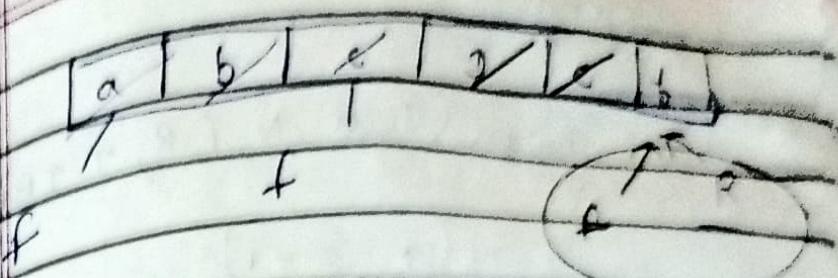
~~121 24 28 32~~

$$q = 24$$

$$S = 63$$

$$q = 24$$

$$\overline{8-6}$$



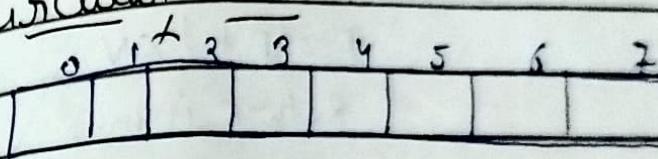
next free

Problem

so we use

circular
queue

Circular Queue —



Enq a, b, c

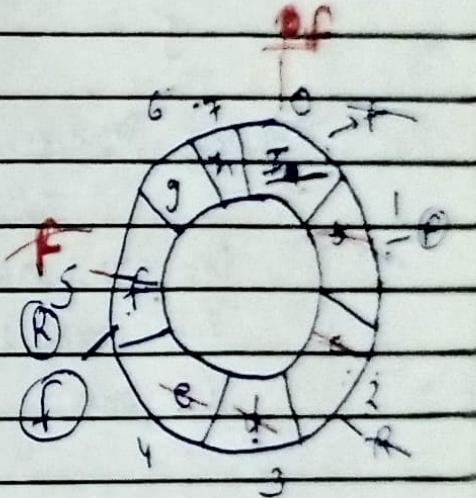
Deq 1 element

Enq d, e, f.

Enq g, h, i

Deq 4 element

Enq j, k, l, m, n



Inception :—

Enhanced (QUEUE, N, F, R, ITEM)

?

if ($f == 0$ && $R == N - 1$) :: ($f == R + 1$)

Write overflow and exit.

If ($f == -1$)

Set $f = 0$ && $R = 0$

else $R = \frac{8 \times 8}{(R + 1) \times N}$

Queue (R) = Item.

Deletion :-

Dequeue (array, N, F, R, ITEM)

?

if $F == -1$

 write Under yellow and green

ITEM = QUEUE [F]

if $F == R$

 Set $F = -1$ & $R = -1$

else

$F = (F + 1) \% N$

Return item.

}

typedef Struct

?

int arr [MAX_SIZE];

int front;

int rear;

}

Circular Queue;

Void initialize (CircularQueue *q)

?

 q → front = -1;

 q → rear = -1;

}

int isEmpty (CircularQueue *q)

{

| return $q \rightarrow front = -1$;

}

int isFull (CircularQueue *q)

{

| return $(q \rightarrow rear + 1) \% \text{MaxSize} = q \rightarrow front$;

}

void enqueue (CircularQueue *q, int item)

{

| if (!isFull(q))

{

| | printf ("Queue is full\n");

| | return;

}

| if (!isEmpty(q))

{

| | $q \rightarrow front = 0$;

| | $q \rightarrow rear = 0$;

S

else

{

| | $q \rightarrow rear = (q \rightarrow rear + 1) \% \text{MaxSize}$

S

$q \rightarrow arr[q \rightarrow rear] = \text{item};$

S

int deQueue (CircularQueue *q)

?

if (isEmpty (q))

?

| printf ("queue is empty\n");
| exit (1);

S

int deQueueItem = q->arr [q->front];

if (q->front == q->rear)

?

| q->front = -1;

| q->rear = -1;

S

else

?

| q->front = (q->front + 1) % MAX;

S

| return deQueueItem;

S

int main()

?

| CircularQueue q;

| initialize (&q)

S return 0;

Queue Using Circular List

typedef Struct Node;

{ int data; }

Struct Node * front,

S Node;

typedef Struct

{

Node * front;

Node * rear;

S circularQ;

Void initialize (CircularQ * q)

{

| q → front = NULL;

| q → rear = NULL;

S

int isEmpty (CircularQ * q)

{

| return q → front == NULL;

S

Enqueue an item to the queue -

Void enqueue (CircularQueue *q, int item)

{

Node *newNode = (Node *) malloc (sizeof(Node));
if (newNode == NULL)

{

| printf ("Queue overflow !");
| return;

}

newNode -> data = item;

newNode -> front = NULL;

if (IsEmpty (q))

{

q -> front = newNode;

q -> rear = newNode;

newNode -> next = newNode;

}

else

{

newNode -> next = q -> front;

q -> rear -> next = newNode;

q -> rear = newNode;

}

}

Dequeue an item from the queue

int dequeue (CircularQueue *q)

{

if (isEmpty (q))

{

| printf ("Queue Underflow\n");

| exit (1);

}

Node *temp = q->front;

int dequedItem = temp->data;

if (q->front == q->rear)

{

| q->front = NULL;

| q->rear = NULL;

} else

{

| q->front = q->front->next;

| q->rear->next = q->front;

}

| free (temp);

| return dequedItem;

}

#include <stdio.h>

#include <stdlib.h>

int main()

{

| CircularQueue q;

| initialize (&q);

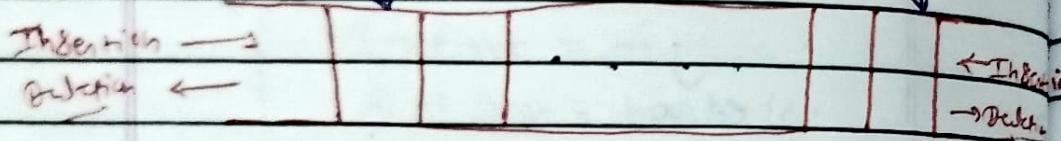
giant \Rightarrow at insertion and deletion,
and \Rightarrow stack \Rightarrow ~~stack~~ \Rightarrow ~~one of two~~ \Rightarrow ~~on 3rd~~

Dequeue — insertion / deletion operation
are performed at either end of
the queue we can insert an element
from the rear end or the front end
also deletion is possible from either end.

- ④ Dequeue ~~can~~ can be used both as a ~~list~~
and as a queue.

Implementation of Dequeue —

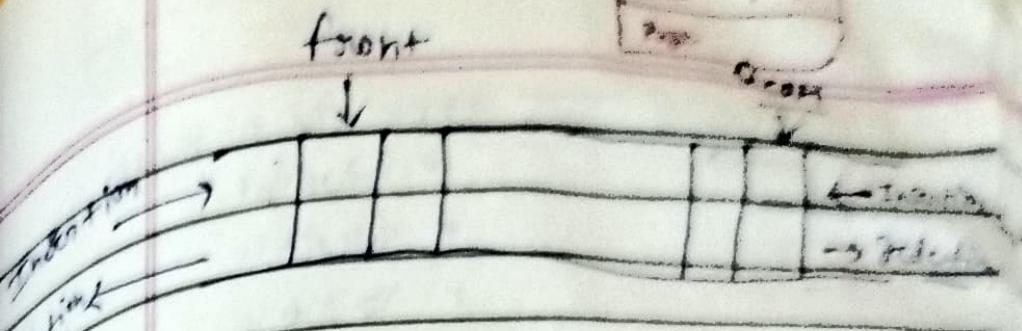
- + using a doubly linked list
- + using a circular array.



Types of Dequeue —

- Input-restricted dequeue — element can be added at only one end but we can delete element from both ends.

- Output-restricted dequeue — deletion takes place at only one end but allows insertion at both ends.



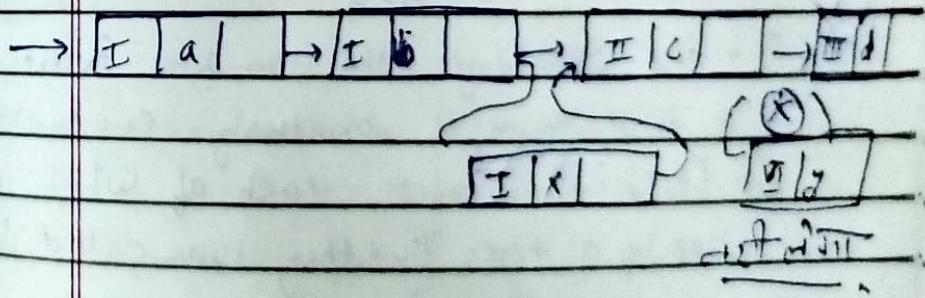
Priority Queue

- * is a collection of elements such that each element has been assigned a priority.

We have some rules for dealing with the elements.

- An element of higher priority is processed before any element of lower priority.
- Two elements with the same priority are processed according to the order in which they were added to the queue.

Class I, II III

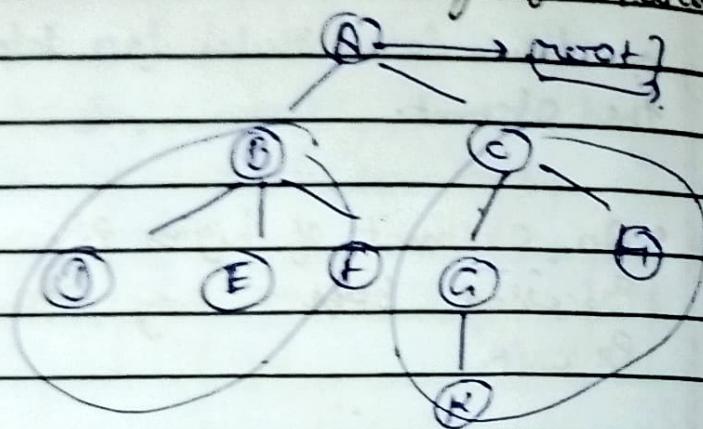


Tree → tree is a must

- * finite
- * hierarchical
- * flexible
- * variable
- * non-linear
- * advanced

data structure, It represents
hierarchical relationship existing b/w
several data item.

Used in wide range of application.



A tree is a finite set of one or more data item (node) such that:

- There is a special data item called root of the tree.

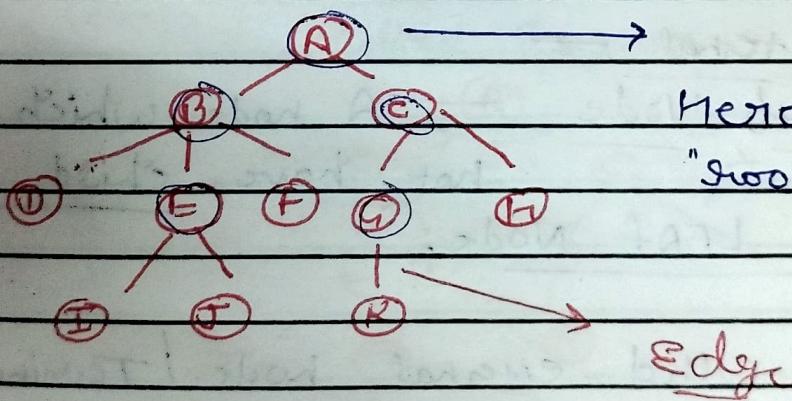
And,

- the remaining data item are partitioned into num of mutually exclusive (disjoint) subset, each of which is itself a tree and they are called subtree
- Every node (except root) is conn'g by a

directed edge from exactly one other node. A direction: Parent to Child.

Terminology

- * Root: — * first/TOP most Node is called root Node.
- * we always have exactly one root node in every tree.
- * we can say that root node is the origin of tree data structure.



Here 'A' is the "root" node.

Edge

- * Edge: — Connection link b/w any two nodes is called as Edge.

Parent of child & 1
Connect root child.

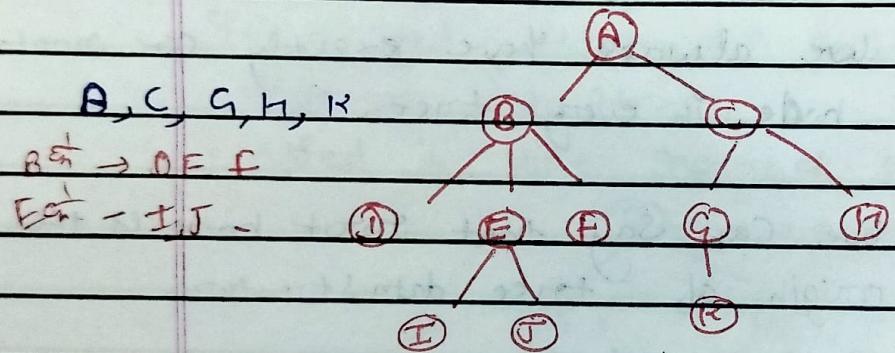
- * Parent: — The node which is Predecessor of any node is called as Parent node.

Any node which has children/child called Parent.

Ex: — A, B, C, E, G.

* Child Node :— The node which is descendant of any node is called as child node.

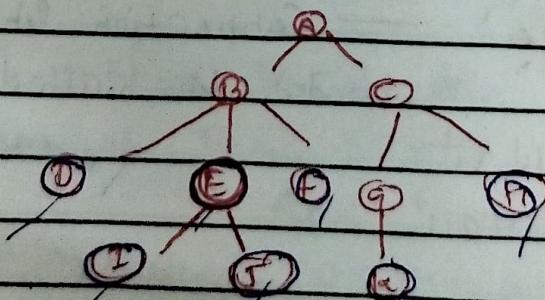
* A node which has the link from its Parent node is called as child node.



External :—

* Leaf Node :— A node which does not have child is called as leaf node.

* Also called external node / Terminal node.



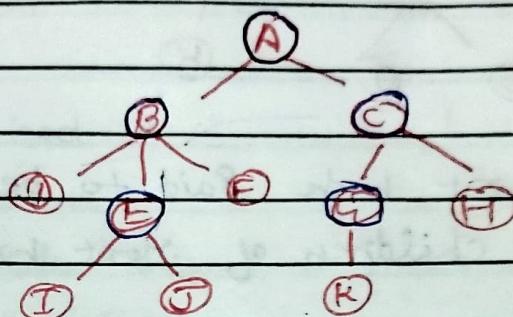
Here is D, I, J, K, E, & H are leaf node.

* A node without successor is called leaf node.

Leaf node or BCT Internal node

Date: - / /
Page: -

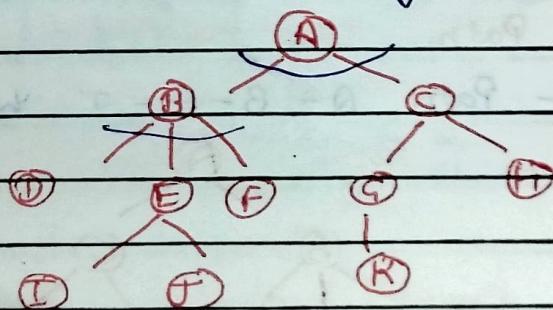
- * Internal Node — * A node which has at least one child is called as INTERNAL Node.



A, B, C, G, E

- * Node other than leaf node called internal

- * Degree : — Total number of children it called degree.

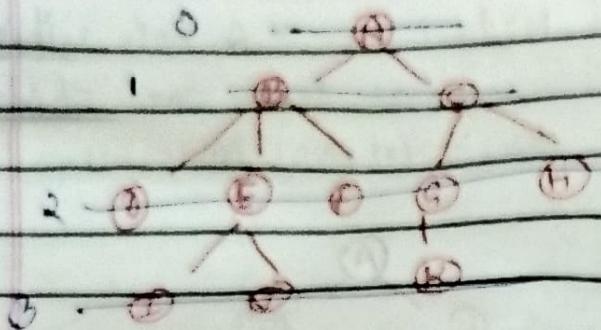


Degree of A is 2.

Degree of B is 3

" " F is 0.

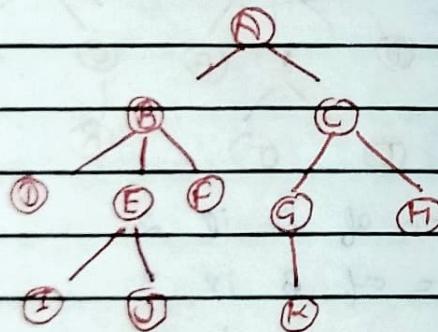
- * Level / Depth / Height — In tree each step from top to bottom is called as a level and the level count start with 0 and increment by one at each level.



- * Root node - Said to be at Level 0 and the children of root node are at Level 1.
- * Path - The Sequence of Nodes / Edges from one node to another node. It is called as Path.

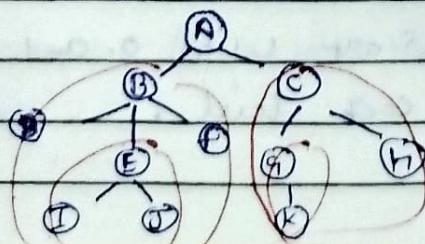
Length of a Path is total num of edge in that Path.

- Path A-B-E-J has length 4.



A Path is the Sequence of nodes and edges b/w two nodes.

- * Subtree - Tree inside tree is called



Binary Tree

A Tree in which any node can have maximum two children (Left + Right)

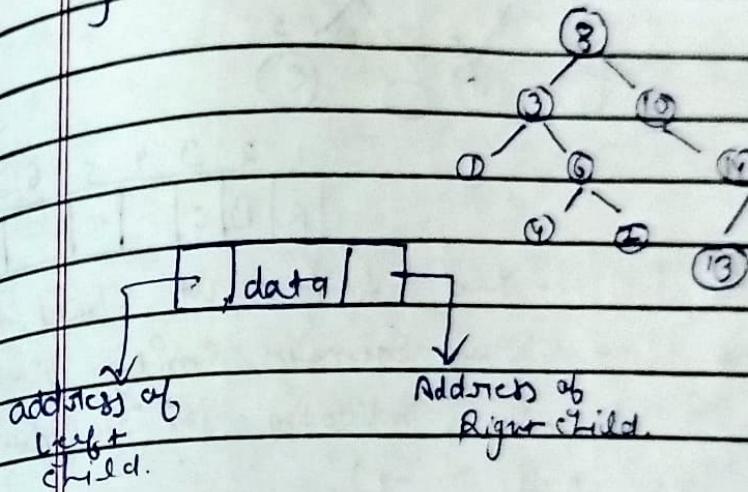
Struct Node?

int data;

Struct Node * left;

Struct Node * right;

S



Q Let T be a binary search tree with nodes. The minimum and maximum possible height of T are -

- (A) 4 and 15 respectively.
- (B) 3 and 14 respectively.
- (C) 4 and 14 respectively.
- (D) 3 and 15 respectively.

Binary tree representation using array

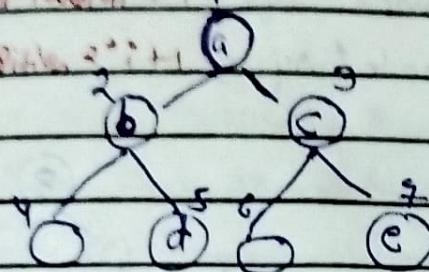
Binary tree can be represented using array

Root is at index 1

for any given node at position i

- * left child is at position $2^i + 1$

- * right child is at position $2^i + 1 + 1$



1	2	3	4	5	6	7
A	B	C	D	E		

If node does not have left / Right child, the position in the array remains empty. It is filled with a special value indicating it's vacant (like null or -1).

Linked Representation of binary tree —

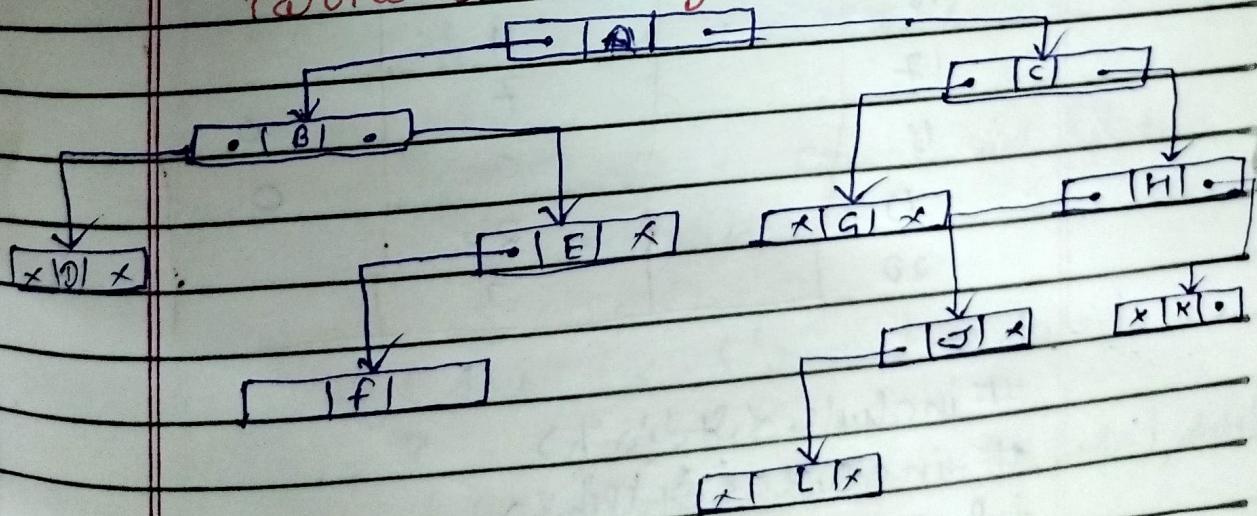
* Consider a binary tree T which uses three parallel arrays, INFO, LEFT, RIGHT and a pointer variable ROOT

* first of all, each node N of T will point to a location R such that:

① INFO [R] contains the data at node N .

② LEFT [R] " " location of the left child N .

- ⑥ $\text{RIGHT}[k]$ contains the location of the right child of node N .
3. ROOT will contain the location of the root R and T.
4. if any subtree is empty, then the corresponding pointer will contain null value
5. if the tree itself is empty, then Root will contain the null value.
- ⑥ INFO may actually be a linear array of records or a collection of parallel arrays.



	INFO	left	Right
1	K	0	0
2	C	3	6
3	G	0	0
4		14	
5	A	10	3
6	H	12	1
7	L	0	0
8		9	
9		4	
10	B	18	13
11		19	
12	F	0	0
13	E	12	0
14		15	
15		16	
16		11	
17		7	0
18		0	0
19		20	
20		0	

```
#include <stdio.h>
```

```
#include <stdlib.h>
```

```
typedef struct Node
```

```
{
```

```
int data;
```

```
struct Node *left;
```

```
struct Node *right;
```

```
};
```

Node * createNode (int data)

{

Node * newNode = (Node *) malloc (sizeof (Node));

if (!newNode)

{

}

| printf ("Memory error \n");

| exit (1);

{

newNode->data = data;

newNode->left = NULL;

newNode->right = NULL;

return newNode;

{

void insert (Node ** root, int data)

{

if (*root == NULL)

{

| *root = createNode (data);

| return;

{

if (data < (*root)->data)

{

| insert (&(*root)->left), data);

{

else

{

| insert (&(*root)->right), data);

{

{

Traversal of binary tree

The process of visiting (Checking / updating) each node in a tree data structure exactly once is called tree traversal.

They may be traversed in depth-first or breadth-first order.

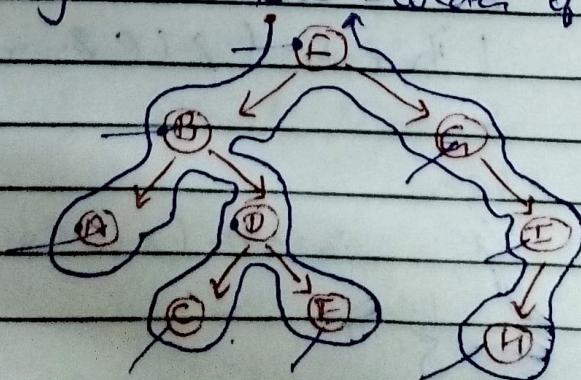
Way of traversing in depth-first

- * In-order ~~L Root R~~
- * Pre-order Root ~~L R~~
- * Post-order L R Root

* Pre-order (Root LR)

Pre-order : f, B, A, D, C, E, G, T, H.

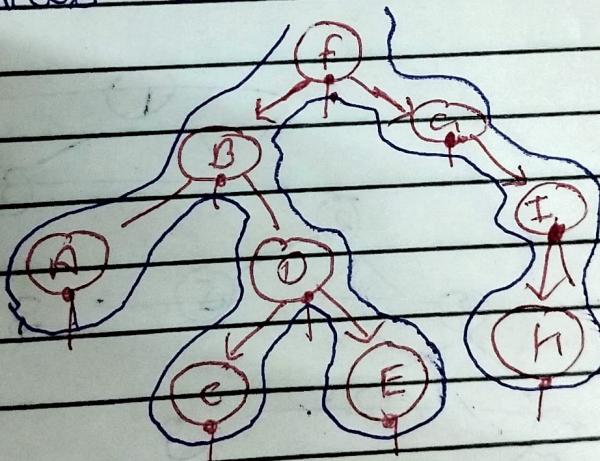
- * Check if the current node is empty / Null.
- * Display the data part of the root
- + Traverse the left subtree by recursively calling the Pre-order function.
- + Traverse the right subtree by recursively calling the Pre-order function.



* in-order (L Root R)

in-order A, B, C, D, E, F, G, H, I

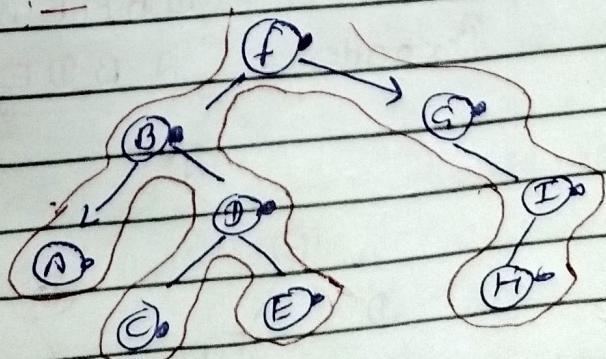
- check if the current node is empty or null.
- Traverse the left subtree by recursively calling the in-order function.
- Display the data part of the root/node ^{current}.
- Traverse the right subtree by recursively calling the in-order function.
- in binary search tree, in-order traversal retrieves data in sorted order.



A, B, C, D, E, F, G, H, I

* Post order :-

- +



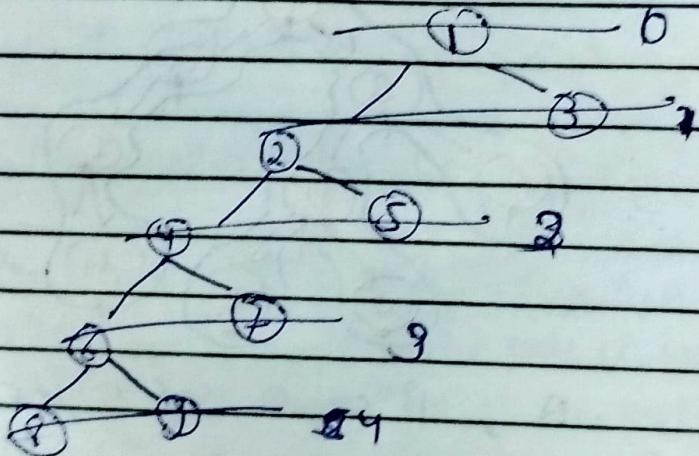
A, C, E, D, B, H, I, G, F

Inorder \rightarrow Sam, L to R.
 Post O \rightarrow R to L.
 Preo \rightarrow L to R

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The Post order traversal of a binary tree is 8, 9, 6, 7, 4, 5, 3. The Inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is .

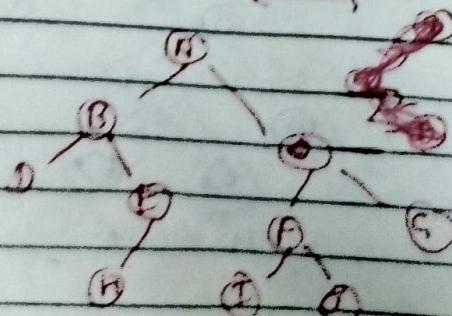
Post \rightarrow Inorder



Q

Inorder: D B H E A T F J G C

Preorder: A B D E H C F I J G



Parent node left & right side size
Right side BST.

Date: 11

Page:-

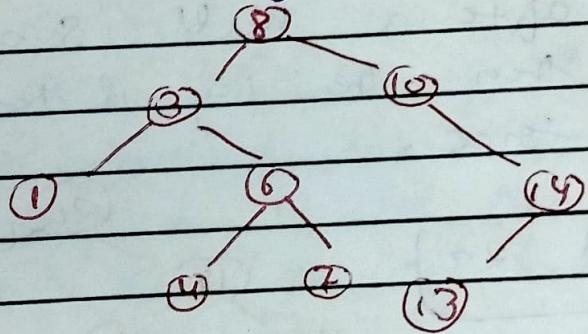
Binary Search tree / BST / ordered tree
sorted tree

Left subtree of a node contains a key less than node's key.

or

Right subtree of node contains a key greater than node's key.

Left / Right sub tree must each also be a binary search tree



In Binary Search Tree inodes traversal there is sequence.

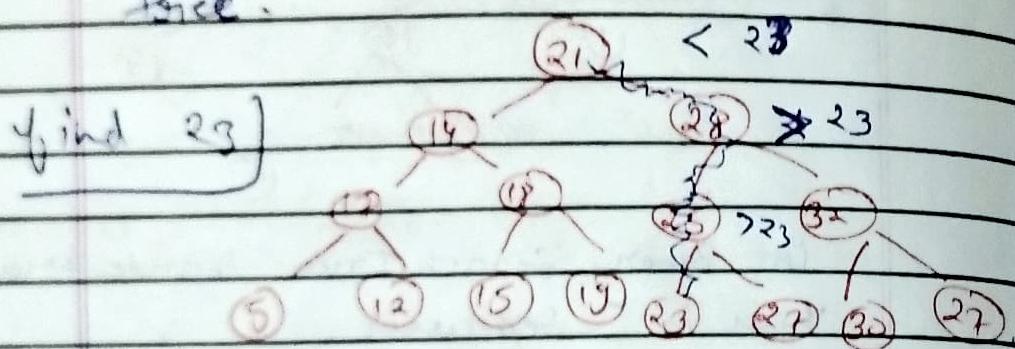
1, 3, 4, 6, 7, 8, 10, 13, 14)

Searching — we start by examining the root node.

- * if tree is null the key we are searching for does not exist in the tree, otherwise
- * if the key equals that of the root, the search is successful and we return the node.

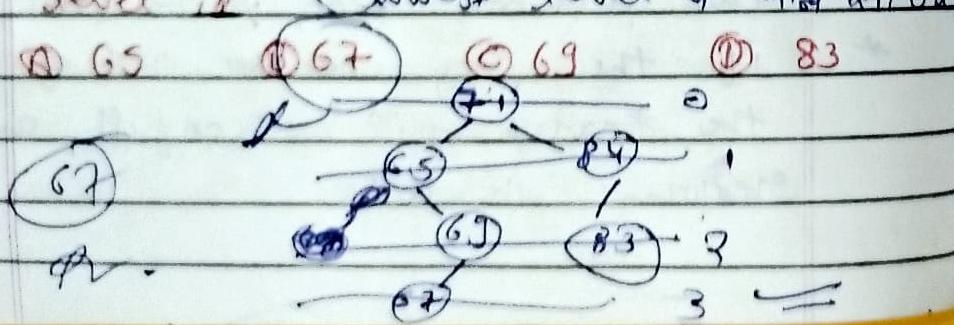
- if the key is less than that of the root node, we search the left subtree
- if the key is greater than that of the root we search the right subtree
- this process is repeated until the key is found or the remaining subtree is null.

if the searched key is not found after a null subtree is searched then the key is not present in the tree.



- Q While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is (lowest level of a non-leaf node)

① 65 ② 67 ③ 69 ④ 83



Key

Greater

② J

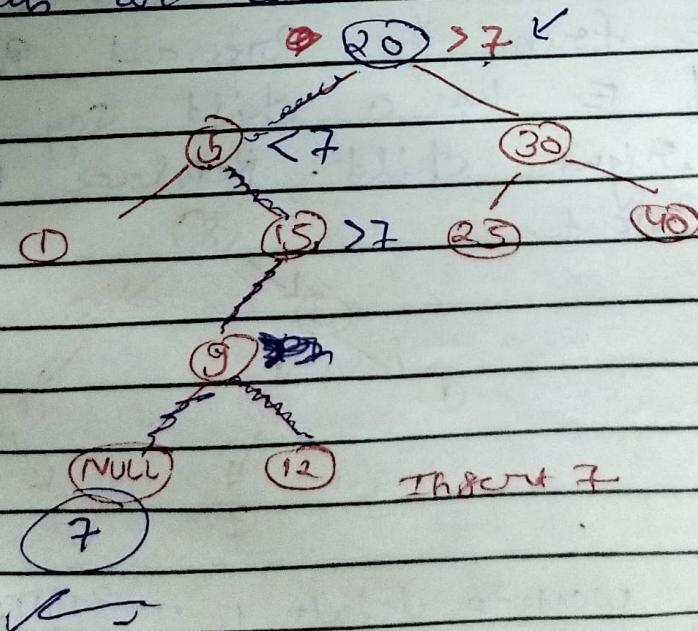
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In-Sertion — if the Key is not equal
the root we search the
left or right subtree as before.

Eventually, we will reach an external
node and add the new key value
pair (here enclosed as a record
'new Node') as its right or left
child depending on the node's key.

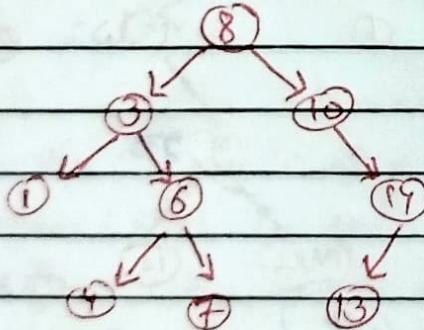
In other words we examine the root
and recursively insert the new node
to the left subtree if its key is
less than that of the root or the
right subtree if its key is greater
than or equal to the root.



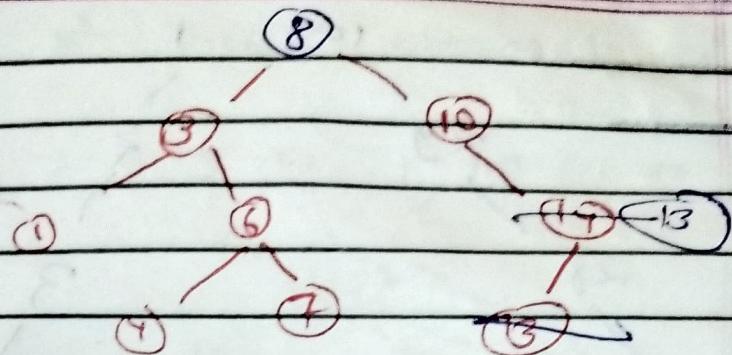
Deletion

- Deleting a node with no children:
Simply remove the node from the tree.
- Deleting a node with one child. Draw a new node and replace it with its child.
- Deleting a node with two children:
Call the node to be deleted D.
Do not delete D.

Instead, choose either its in-order Predecessor node or its in-order Successor node as replacement node.
E. Copy the user values of E to D.
- if E does not have a child Simply remove E from its Previous Parent &
if E has a child say F, it is a right child. Replace E with f at E's Parent.



Imagine we want to delete 4 - ~~3 1 12~~ Node ~~4 1 7~~
and child ~~4 1 7~~ at 3rd delat ~~4 1 7~~
easy ~~4 1 7~~ ~~4 1 7~~.

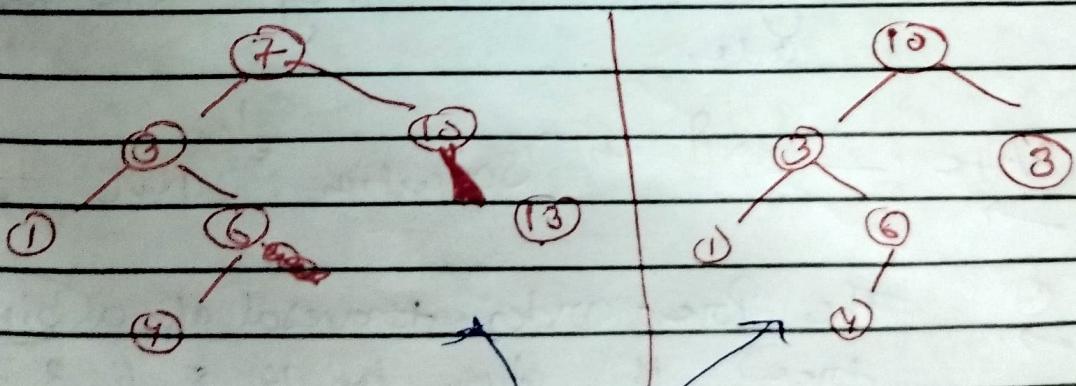


~~31) BT edit 14 at delete 8 -> 8' at
14 at 13 is at 13 is at 13, 14 at
replace at 14 8'!~~

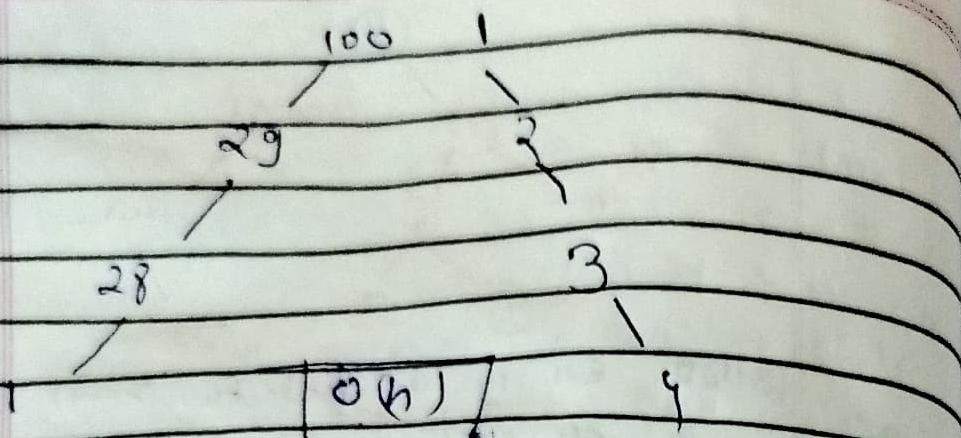
if we want to delete: 8 — predecessor
inorder n'th element?

Shorted form - 1 3 6 7 8 10 13

inorder Predecessor \rightarrow find at 9th replace
inorder Successor \rightarrow find at 10th replace



Algorithm	Average	Worst case
Space	$O(n)$	$O(n)$
Search	$O(\log n)$	$O(n)$
Insert	$O(\log n)$	$O(n)$
Delete	$O(\log n)$	$O(n)$



it can save lot of time if we use stack, cause of the reason that it has big difference of time.

The major advantage of binary search trees over other data structures is that the updated sorting algorithms and search algorithms such as in-order traversal can be very effective they are also easy to code.

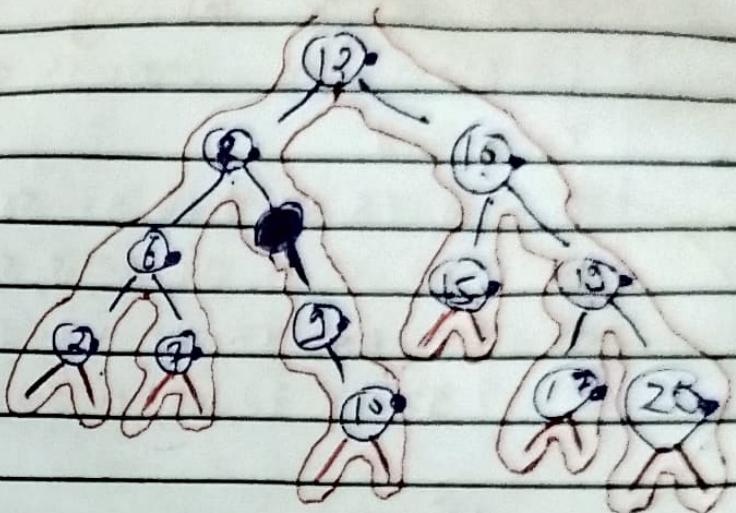
Note: — ~~Dept of~~ Solution ^{1.} AVL tree.

Q) The Post-order traversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20. Then the Post-order traversal of this tree is.

- (A) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20
- (B) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 15
- (C) 1, 3, 5, 7, 8, 15, 19, 25
- (D) 4, 6, 7, 9, 18, 20, 25.

BST \leftarrow in-order traversal sequence

2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20



Prec \rightarrow 13, 8, 6, 3, 7, 9, 10, 16, 15,
19, 12, 20

Post \rightarrow 2, 6, 7, 8, 10, 12, 15, 16, 17, 19, 20

Pre \rightarrow 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12

Q Which of the following is/are correct
in-order traversal sequence of binary
search tree(s) 1, 3, 5, 7, 8, 15, 19, 20.

① 5, 8, 9, 12, 10, 15, 20.

2, 7, 10, 8, 14, 16, 20.

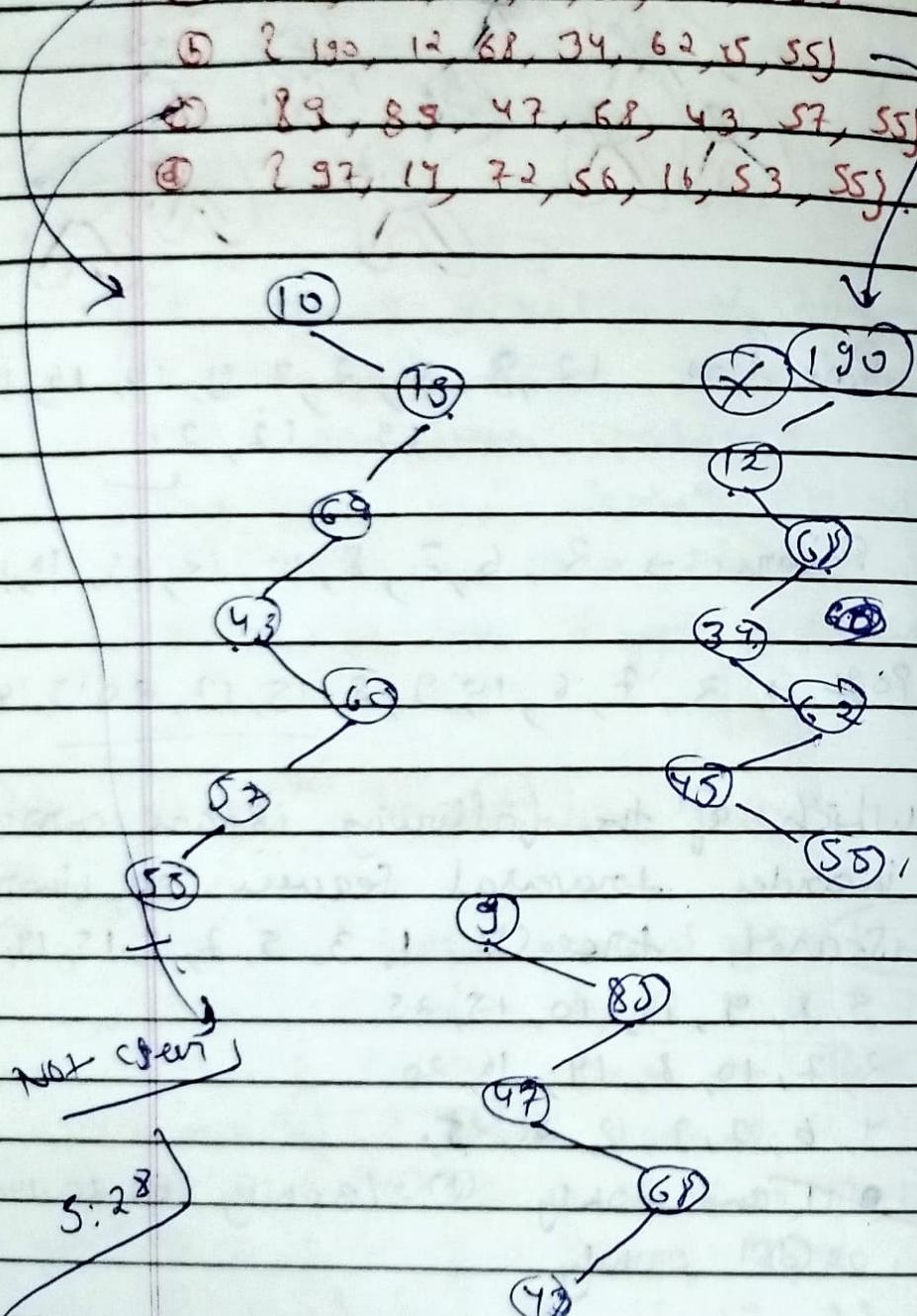
4, 6, 7, 9, 18, 20, 25.

④ ~~1 and 4 only.~~ ⑤ 2/3 only ⑦ 2 and 4 only.

⑥ 20 only.

Q Suppose that we have numbers between 1 and 100 in a binary search tree and want to search for the number which of the following sequence cannot be search of node examined.

- (1) 210, 75, 64, 43, 60, 57, 55, 5
- (2) 2190, 12, 68, 34, 62, 15, 55, 1
- (3) 89, 85, 47, 68, 43, 57, 55, 1
- (4) 297, 17, 72, 56, 16, 53, 55, 1



Date: / /
Page: /

Node * SearchBST (Node * root, int data)
?

if ($\ast \text{root} == \text{NULL}$ || $\text{root} \rightarrow \text{data} == \text{data}$)
?

| return root;

{

if ($\text{data} < \text{root} \rightarrow \text{data}$)

?

return SearchBST ($\text{root} \rightarrow \text{left}$,
 data);

{

return SearchBST ($\text{root} \rightarrow \text{right}$, data);

{

Void insertBST (Node ** ~~root~~, int data)
?

if ($\ast \text{root} == \text{NULL}$)

?

| $\ast \text{root} = \text{createNode} (\text{data})$;

| return;

if ($\text{data} < (\ast \text{root}) \rightarrow \text{data}$)

?

| insertBST ($\delta((\ast \text{root}) \rightarrow \text{left})$, data);

else if ($\text{data} > (\ast \text{root}) \rightarrow \text{data}$)

?

| insertBST ($((\ast \text{root}) \rightarrow \text{right})$, data);

else ?

Print ("Element already exist in BST")

Node* deleteBST (Node* root, int data)

?

if (root)

return root;

if (data < root->data)

?

if (root->left = deleteBST (root->
data),
root->right = deleteBST (root->
right, data))

?

else if (data = root->data)

?

root->right = deleteBST (root->
right, data);

?

else

?

if (root->left)

?

Node* temp = root->right;

free (root);

return temp;

?

else if (root->right)

?

Node* temp = root->left;

free (root);

?

return temp;

Node* temp = findValueNode (root->right);

root->data = temp->data;

root->right = deleteBST (root->
right, temp->data);

?

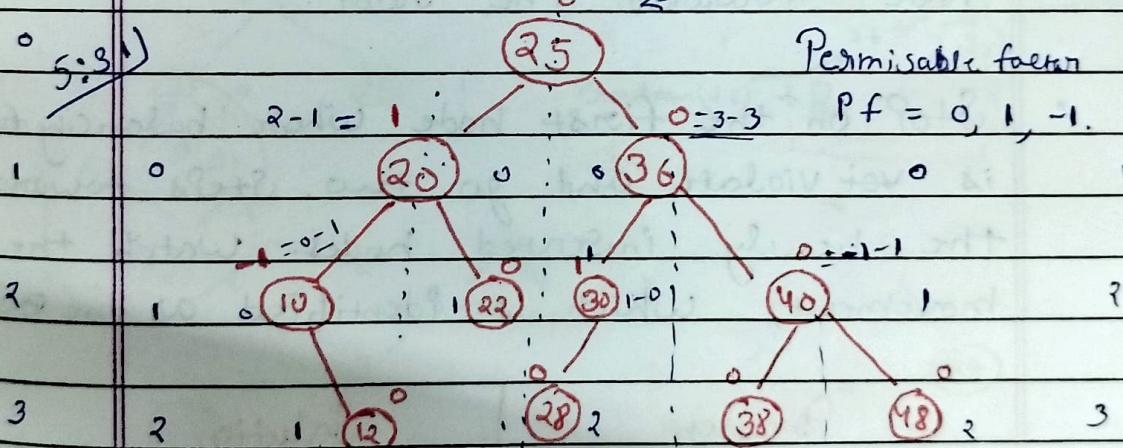
return root;

?

AVL Tree — An AVL tree (named after inventors Adel'son-Velsky and Landis) is a self-balancing binary search tree.

- * The heights of the two child subtrees of any node differ by at most one.
- * If at any time they differ by more than one, rebalancing is done to restore this property.

$$| - 3 - 3 = 0$$



Left subtree / Right subtree height difference calculate in $\frac{1}{2}$!

Balance factor

In binary tree the balance factor of a node N is defined to be the height difference between its two child subtrees.

balance factor (N) = Height (Left subtree (N)) - Height (Right subtree (N))

A binary tree is defined to be an AVL tree if the invariant Balance factor (NIE) $-1, +1$ holds for every node N in the tree.

Insertion in an AVL tree

- Insert a node similarly as we do in BST.
- After the insertion start checking the balance factor of each node in a bottom up fashion that is from newly inserted node towards the root.
- Stop on the first node whose balancing factor is violated and go two steps towards the newly inserted nodes. Watch the movement, which is identified as the Problem.

Problem	Solution
✓ LL	R \leftarrow
\rightarrow RR	①
LR	LR
RL	RL

AVL tree & Bst & at BST.

5:36

Date _____
Page _____

- Q Consider an empty, AVL tree and Insert the following nodes in sequence.

21, 26, 30, 9, 4, 19, 28, 18, 15, 10, 2, 3, 7, 1

$L=0$
 $R=2 \Rightarrow -2$ Problem.

$L=3$
 $R=1$

$L=0$

$R=1$

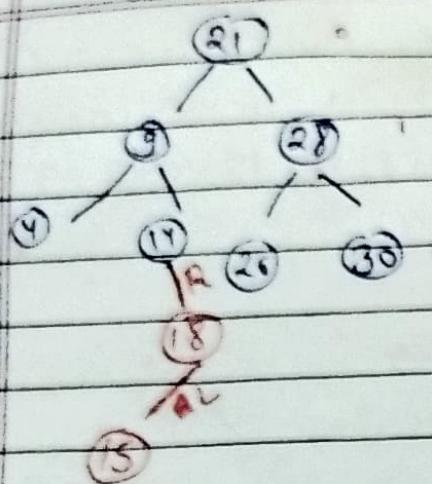
$L=1$

$R=0$

$L=2$

$R=1$

$L=3$



$$LHS = 3$$

$$RHS = 2$$

$$LHS = 4$$

$$RHS = 2$$

$$RL =$$

RA

नीचे से R का रूप

$$=$$

L → R

5

14

15

18

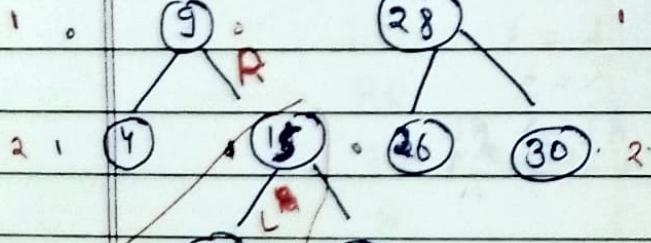
14
R → L

⇒

13

15
14
18

21

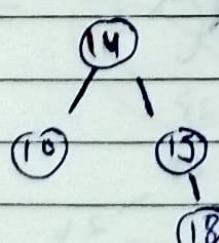


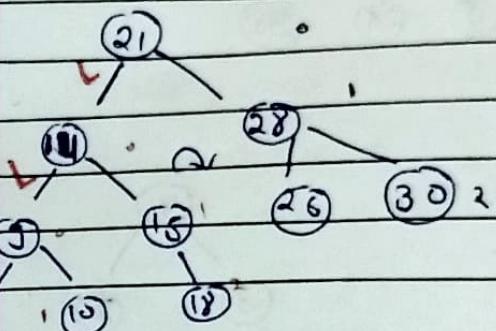
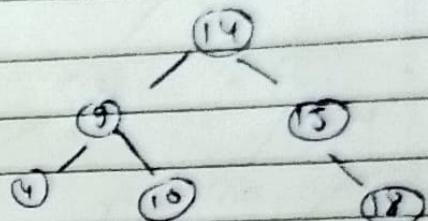
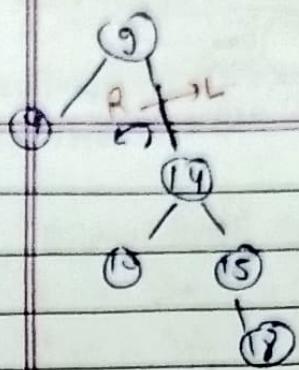
Problem (9, 21) —

$$LHS = 1$$

$$RHS = 3 = -2$$

$$LR = \boxed{4} R$$



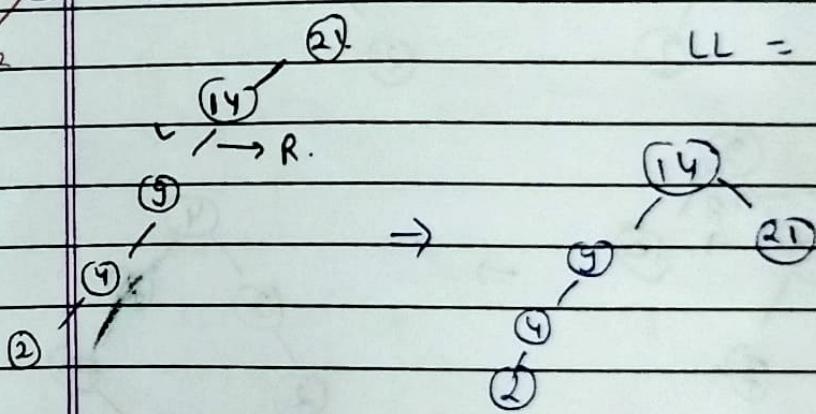


Problem 2

$LHS = 4$

$RHS = 2 = 3$

$LL = R$

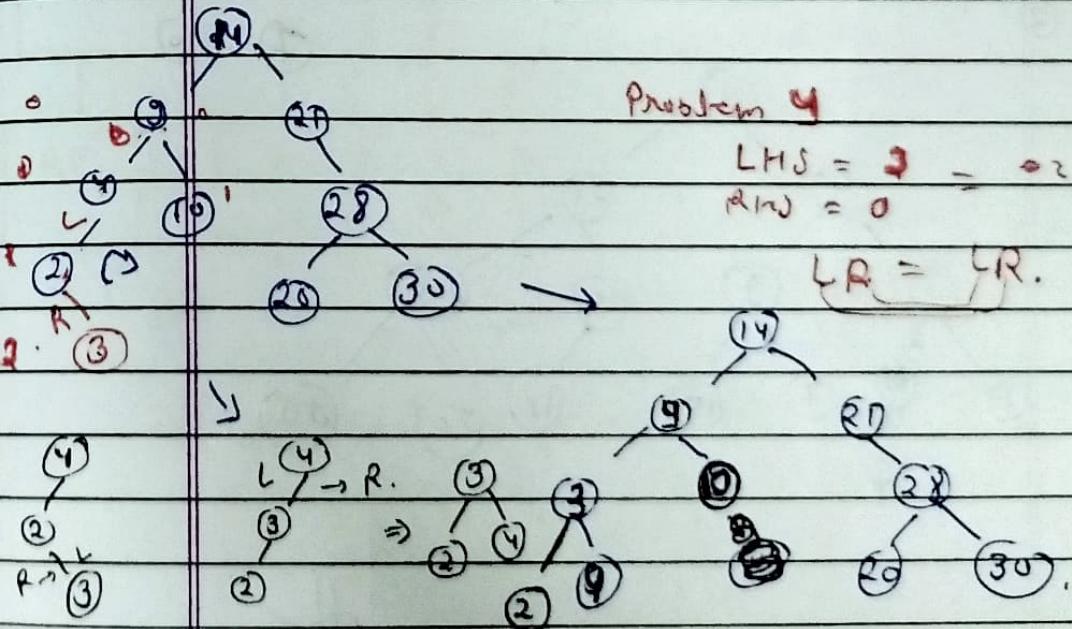


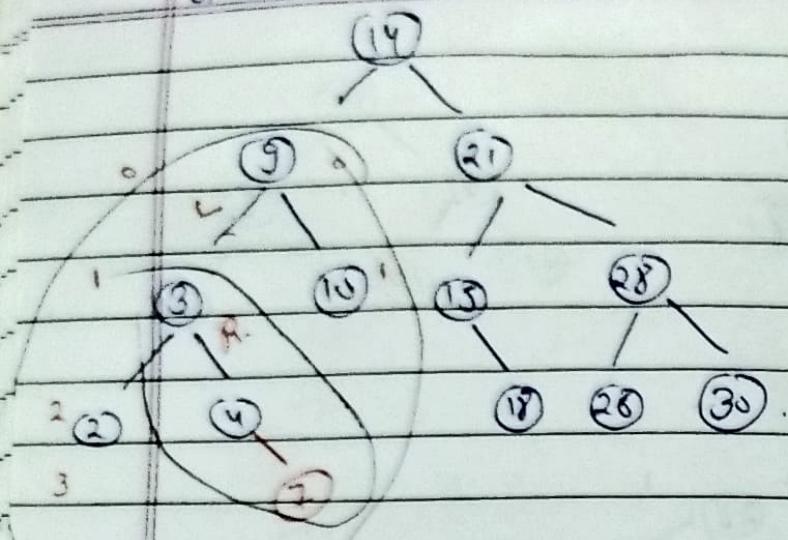
Problem 4

$LHS = 3 = 0$

$RHS = 0$

$LR = LR.$



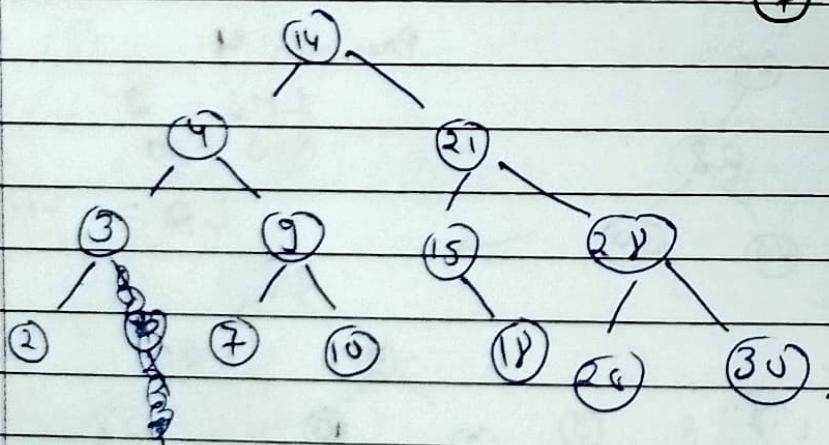
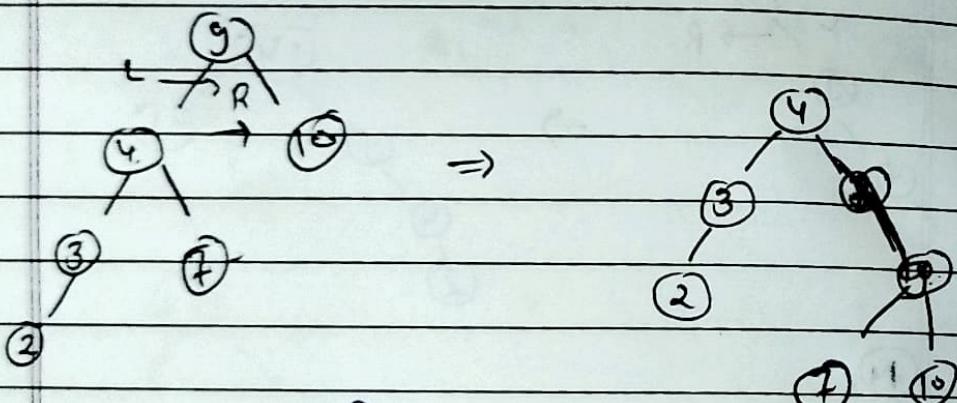
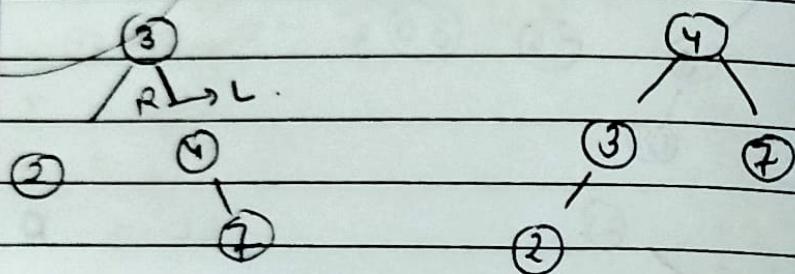


Problem 9

L-3

$$R = -$$

$$LR = LR$$



* no LL, RR, LR, RL \rightarrow at any edit edit according
Rotation w/JIT & T

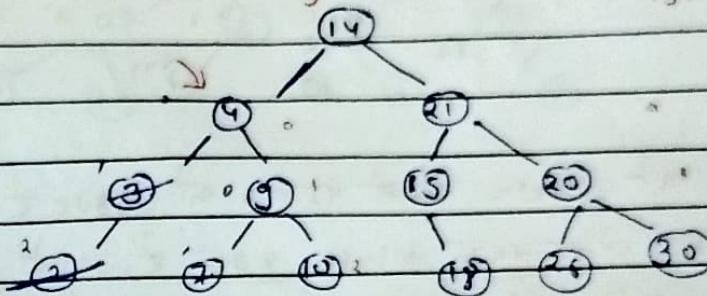
7) tree walk case at $\frac{2}{3}$ of
day (h) time $\frac{1}{3}$ JIT



Deletion in an AVL tree :

Q

Delete the following node in sequence : 3, 10, 18, 2, 17, 15.



AVL
Deletion

L

R

L₀

L₁

L₋₁

R₀

R₁

R₋₁

RR

RL

RR

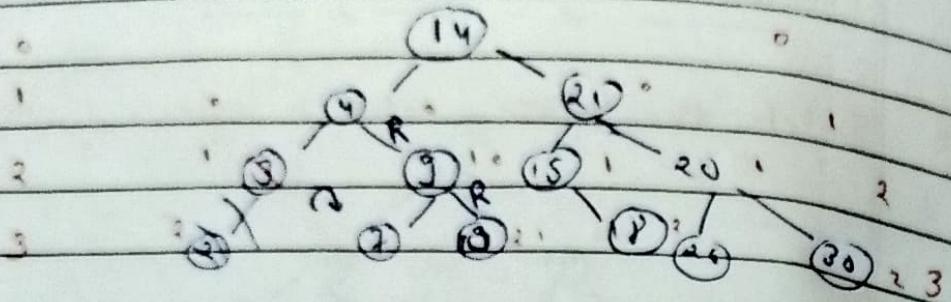
LL

LL

LR

(One more time screen) \rightarrow

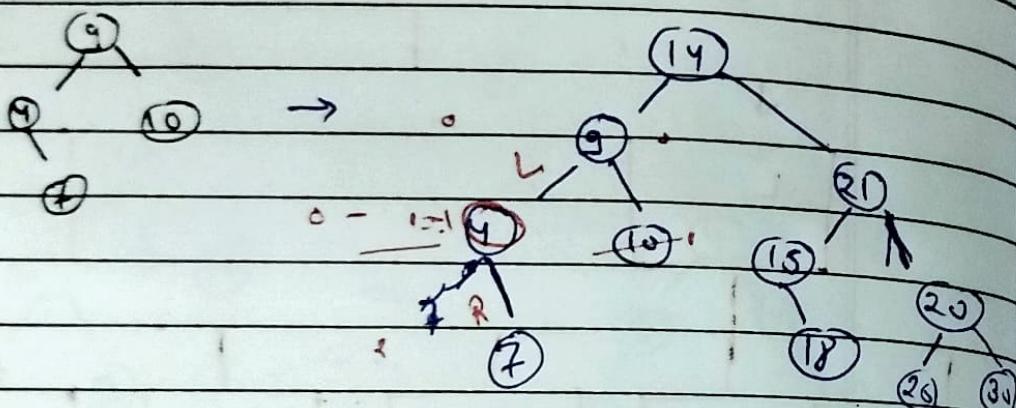
2 3 10 18 4 9 14 3 25



→ 3 at delete and 4 at 1880C $\frac{1}{2}$,

→ 30 at delete $\frac{1}{2}$ 311 $\frac{1}{2}$ Reverse side.
at Balance factor check what?

→ 9 at balancing f o $\frac{1}{2}$ L \rightarrow RR Problem.

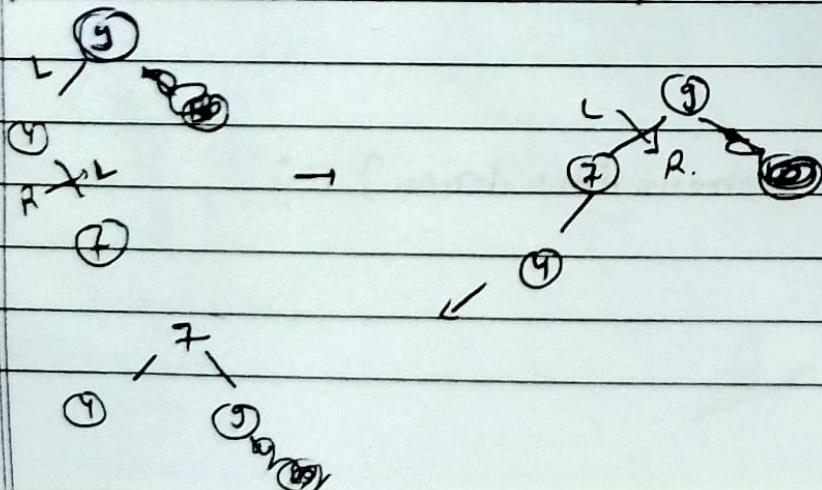


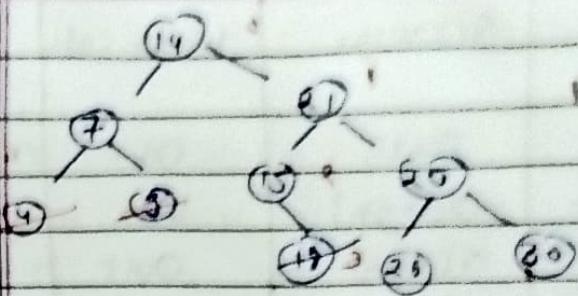
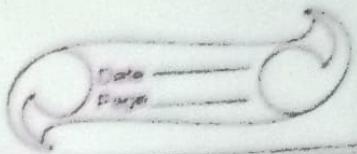
→ 10 at remove on 2+ 1 on 1 Problem.

→ 9 at 1st at 1st R.R. type of deletion $\frac{1}{2}$.

→ 30 at delete $\frac{1}{2}$ 311 $\frac{1}{2}$ 311 at reverse side at min.

LR Problem.





$$1 - 2 = E \text{, Problem 1}$$

18 delete and \rightarrow wif Problem 17

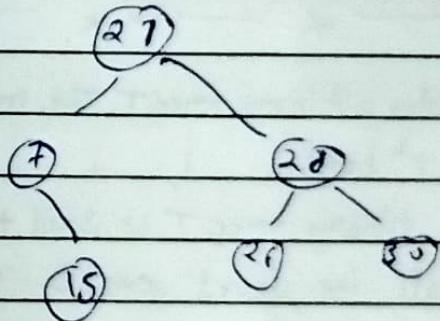
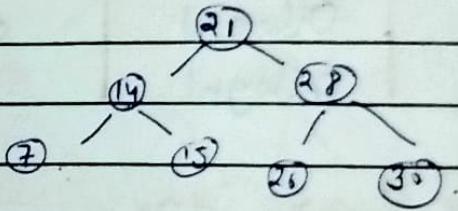
4 " " " " " " "

\rightarrow 9 \leftarrow \rightarrow 14 \leftarrow Premium

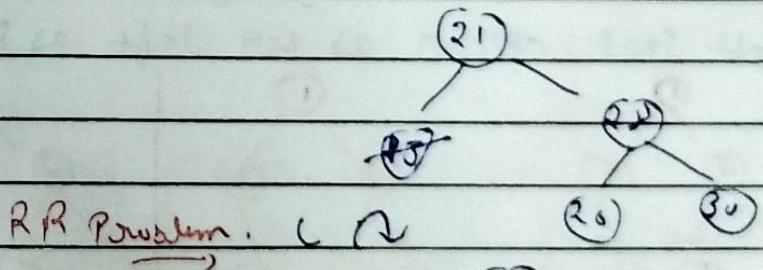
\rightarrow 14 \leftarrow \rightarrow 14 \leftarrow L-type wif deletion

\rightarrow 14 \leftarrow delete \rightarrow 14 \leftarrow \rightarrow Premium

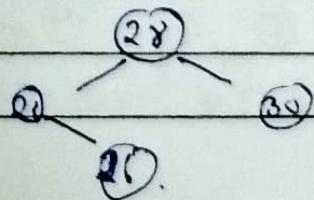
$$1 - 2 = -1 \text{ नीर्माण L-1 Problem 2 } \rightarrow \text{ 17 } \rightarrow \text{ 22 }$$



R₁



RR Problem. C R



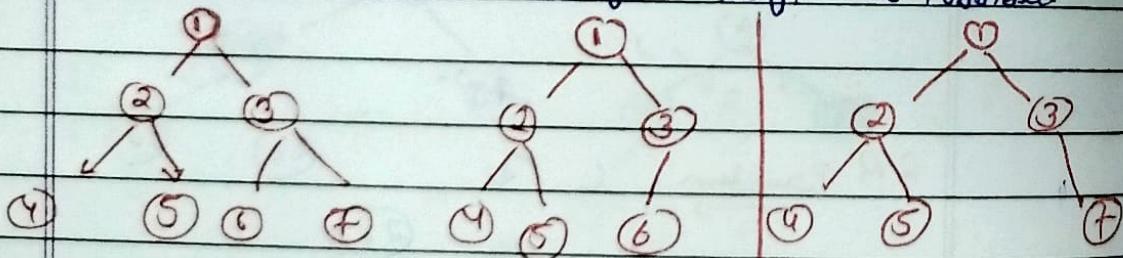
Algorithm	Average	Worst Case	
Space	$O(n)$	$O(n)$	
Search	$O(\log n)$	$O(n)$	
Insert	$O(\log n)$	$O(n)$	
Delete	$O(\log n)$	$O(n)$	Binary Search tree

Algorithm	Average	Worst Case	
Space	$O(n)$	$O(n)$	
Search	$O(\log n)$	$O(\log n)$	AVL tree
Insert	$O(\log n)$	$O(\log n)$	
Delete	$O(\log n)$	$O(\log n)$	

* Complete Binary Tree : — L to R, T to D.

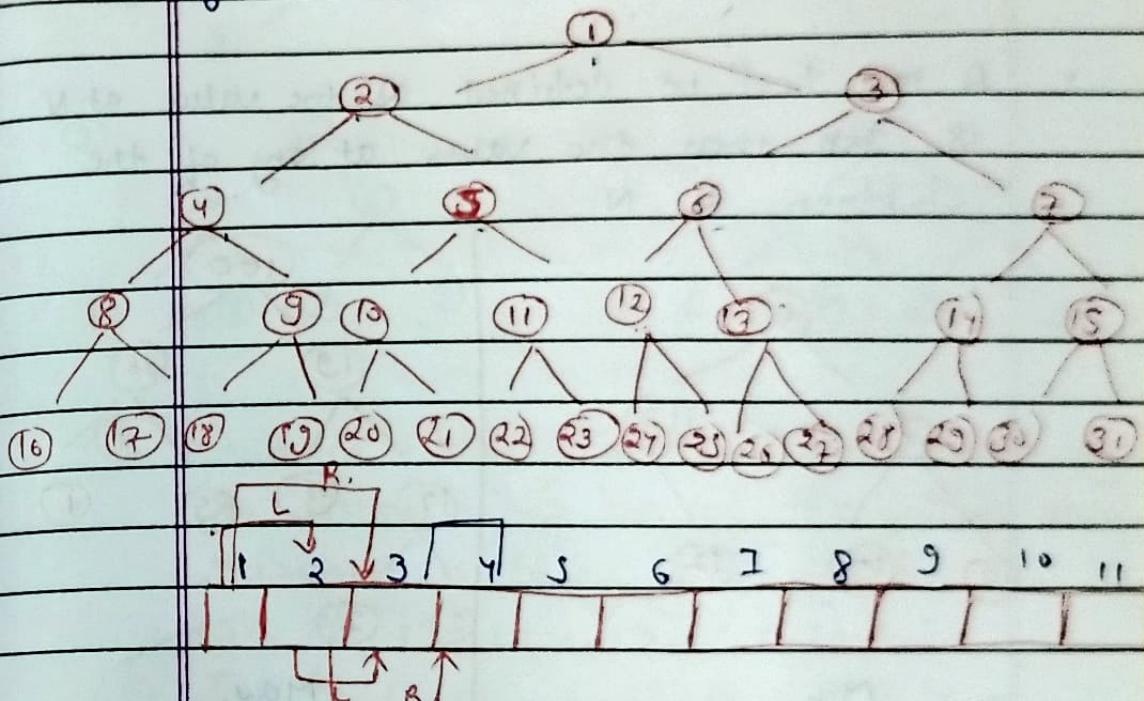
* Consider a binary tree T, The maximum num of nodes at height i is 2^i nodes.

* The binary tree T is said to be complete binary tree, if all its level except possibly the last, have the maximum num of nodes and if all the nodes at the last level appear as far left as possible.



That is not
Complete binary
tree.

- * If it consider as array.
- * One can easily determine the children and Parent of a node K in ~~any~~ a complete tree T .
- * Specially the Left / Right Children of the node K are $2 \times K$, $2 \times K + 1$ and the Parent of K is the node lower bound ($\lceil K/2 \rceil$).



$$22^{\text{th}} \text{ Parent} = \frac{22}{2} = \underline{\underline{11}}$$

$$\text{Left child of } K = 2K$$

$$\text{Right child of } K = 2K + 1.$$

$$\text{Left child of } 7 = 14.$$

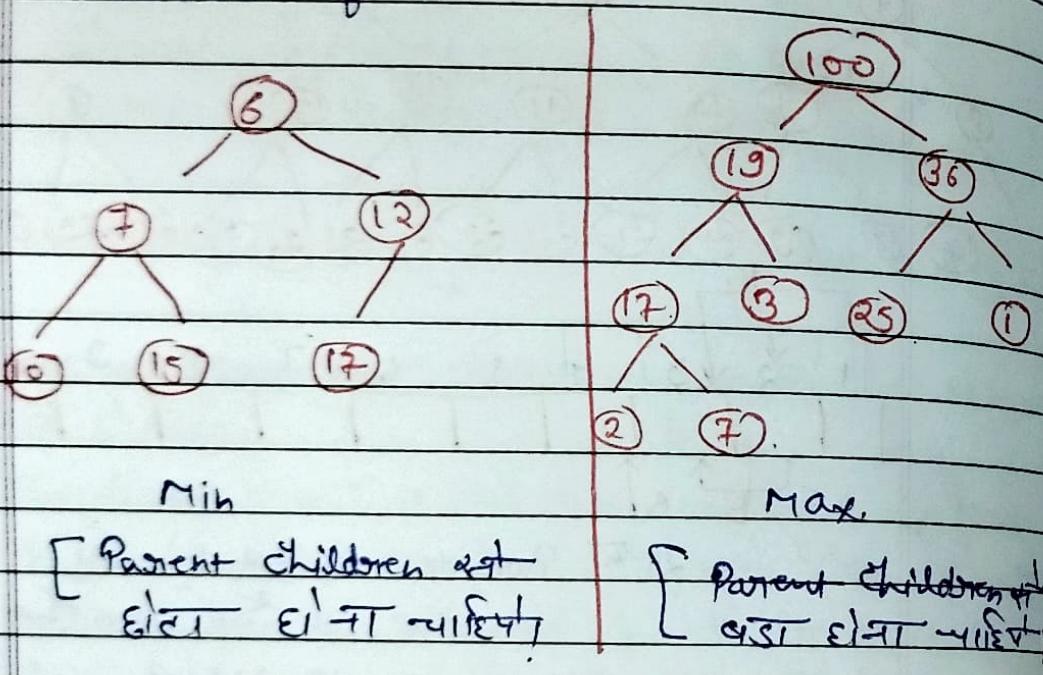
$$\text{Right child of } 7 = 14 + 1 = 15.$$

~~2nd~~ Pointer based ~~char~~ of requirement or
~~char~~

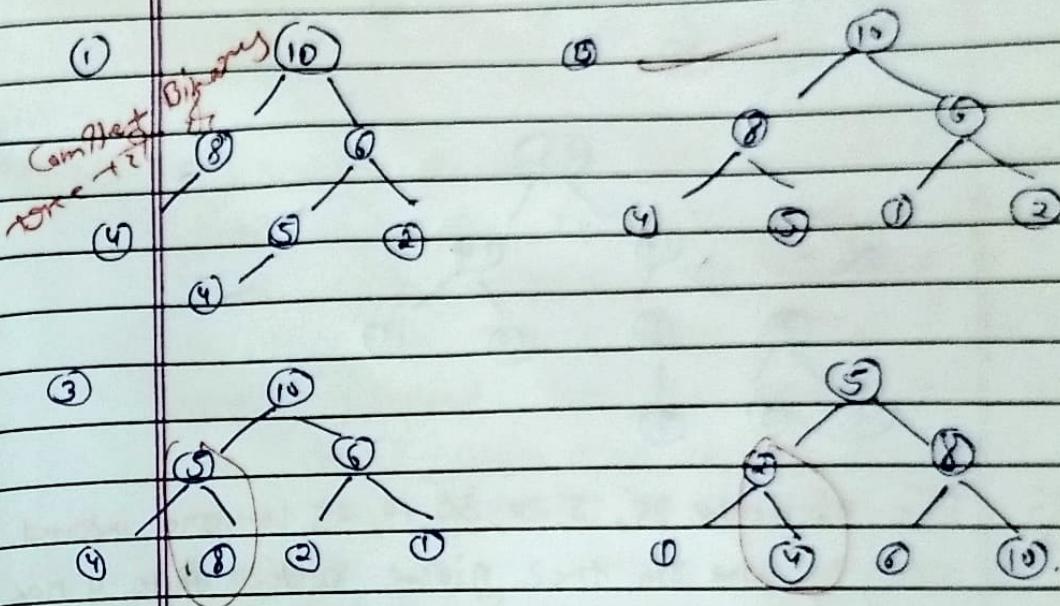
* Heap :-

Suppose H is a complete binary tree with N elements, H is called a heap, if each node N of H has following Properties:

- The value at N is greater than to the value at each of the children of N then it is called max heap.
- A min heap is defined as the value at N is less than the value at any of the children of N.

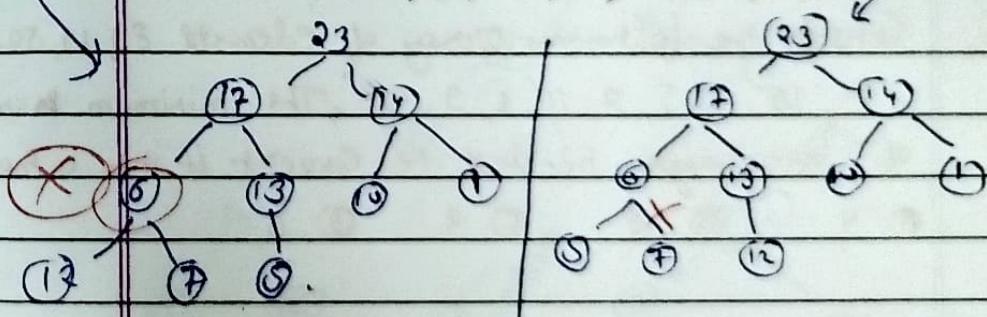


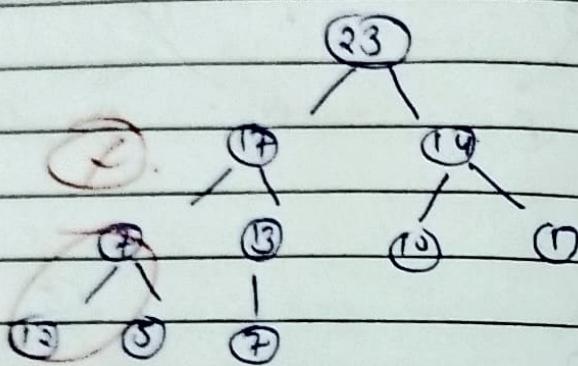
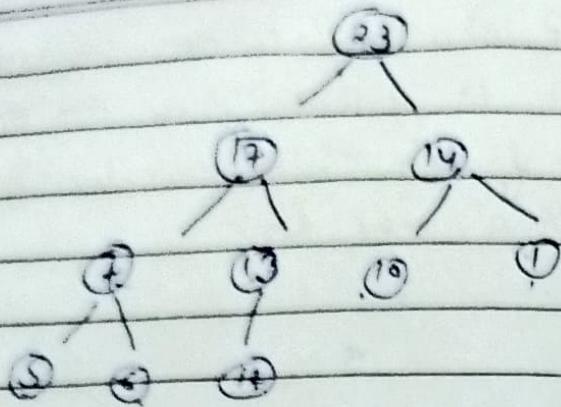
Q A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?



Q Consider a binary max-heap implemented using an array. Which one of the following arrays represents a binary max-heap?

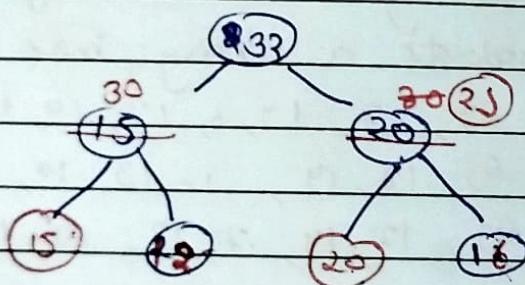
- (A) 23, 17, 19, 6, 13, 10, 1, 12, 7, 5.
- (B) 23, 17, 14, 6, 13, 10, 1, 5, 7, 12.
- (C) 23, 17, 19, 7, 13, 10, 1, 5, 6, 12.
- (D) 23, 17, 14, 7, 13, 10, 1, 12, 5, 7.





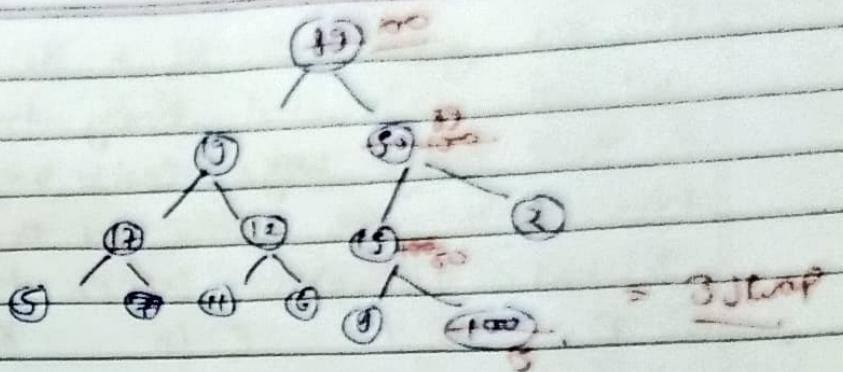
Q) The elements 38, 15, 28, 36, 13, 25, 16 are inserted one by one in the given order into a Max Heap. The resultant Max Heap is.

A) ~~38 42 28 36 15 25 16~~ check ~~38 42 28 36 15 25 16~~ Max heap is

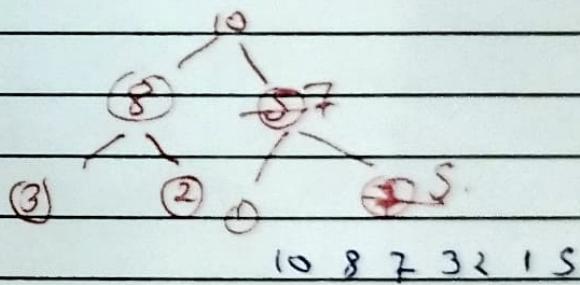


Q) Consider the following array of Elements 89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100. The minimum num of interchanges needed to convert it into a max-heap

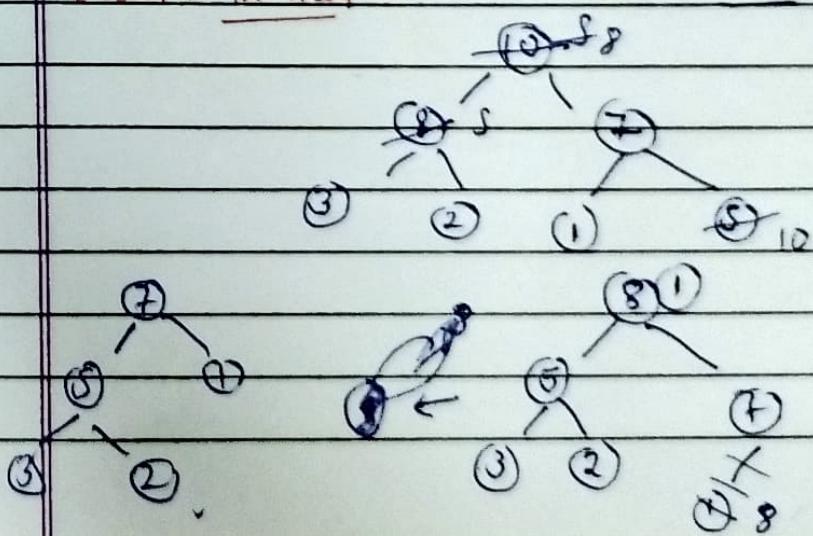
- (A) 4 (B) 5 (C) 2 (D) 3.



- (C) A Priority queue is implemented as a Max heap initially. It has 5 elements. The Level-order traversal of the heap is 10, 8, 5, 3, 2. Now new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:
- (A) 10, 8, 7, 3, 2, 1, 5
 - (B) 10, 8, 7, 3, 2, 1, 5
 - (C) 10, 8, 7, 1, 2, 3, 5
 - (D) 10, 8, 7, 5, 3, 2, 1.



Delete in heap →



6910:28

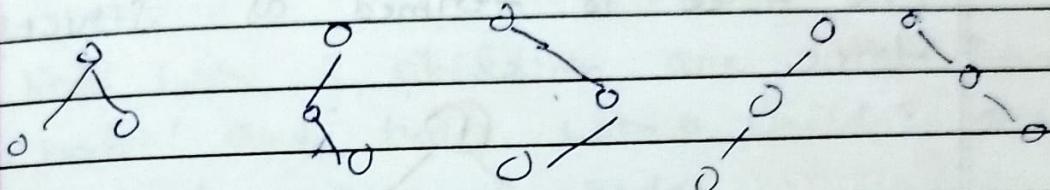
Date _____
Page _____

- * We are given a set of n distinct elements
And an unlabeled binary tree with n nodes.
is how many ways can we populate the
tree with the given set so that it
becomes a binary search tree?

- (A) D (B) I (C) (b) (D) $O(n^n)$ which

$$\frac{2^n C_n}{n+1} =$$

- (Q) The maximum number of binary trees that can be formed with three unlabeled nodes is: —
- (A) 1 (B) 5 (C) 4 (D) 3



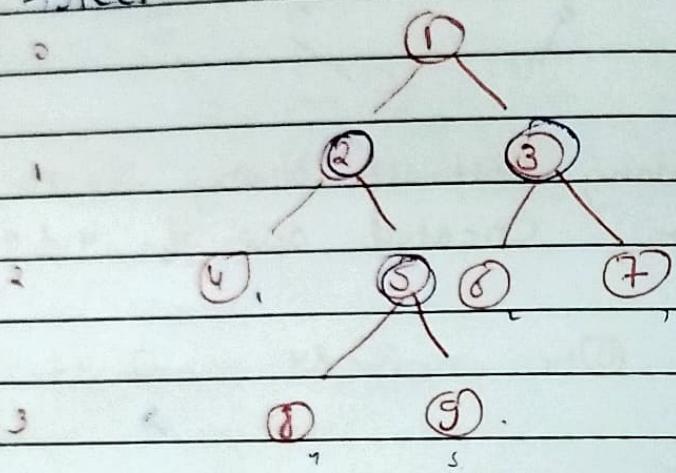
6: B,

- (Q) How many distinct binary search trees can be created out of 4 distinct keys?

- (A) 4 (B) 14 (C) 24 (D) 43

Strictly binary tree

If every non-leaf node in a binary tree has exactly one child or right subtree, the tree is formed as Strictly binary tree.



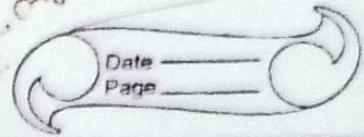
A Strictly binary tree with n leaves always contain $2n-1$ nodes.

If every non-leaf node in a binary tree has exactly two children, the tree is known as Strictly binary tree.

$$2 \times 3 - 1 = 6 - 1 = 5$$

6.15

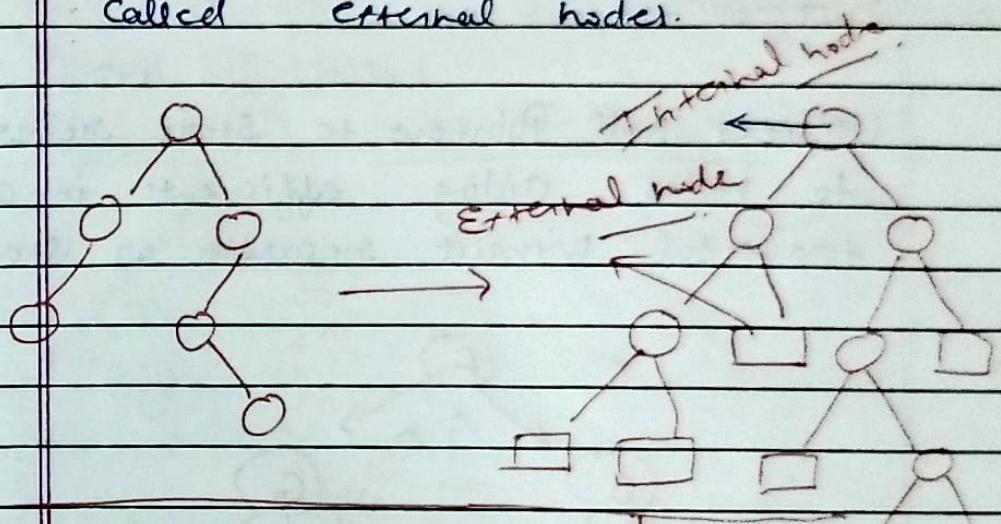
if it starts entry
at ex pointer, then it is
node.



Extended binary tree → 2-tree

if each node has either 0 or 2 children.

Nodes with 2 children are called internal nodes and nodes with 0 children are called external nodes.



Features	Strictly Binary tree	Extended Binary tree
Definition	Every non-leaf node has exactly two children.	Every node has either 0 or 2 children.
Node with one child	No nodes with only one child.	No nodes with only one child.

Special Node	Node.	May have external (dummy) nodes to make it full.
--------------	-------	--

Typical Usage	Less common in practical application.	Used in Sciences like Huffman coding trees where it's beneficial to consider the tree as full.
---------------	---------------------------------------	--

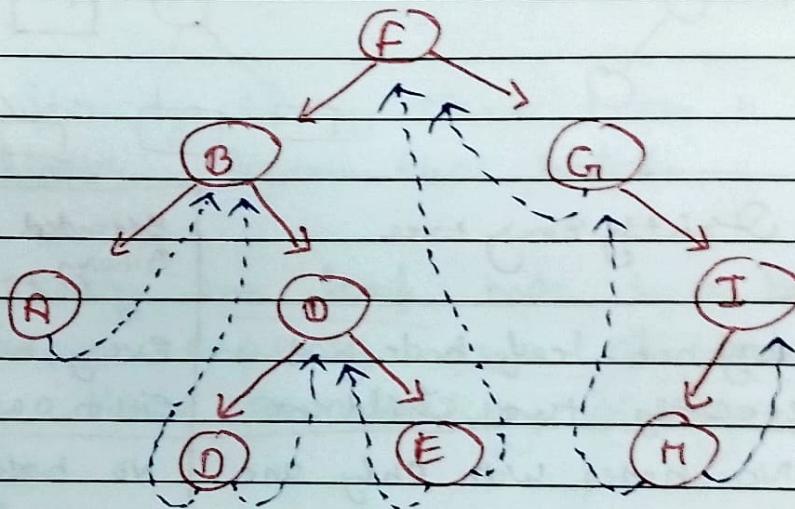
(x)

Threaded binary tree

- A threaded binary tree is a modified binary tree that uses null pointers to link to the next node in an in-order sequence, optimizing in-order traversal.

Purpose: →

Utilizes null pointers to store references to nodes, aiding efficient in-order traversal without recursion or stacks.

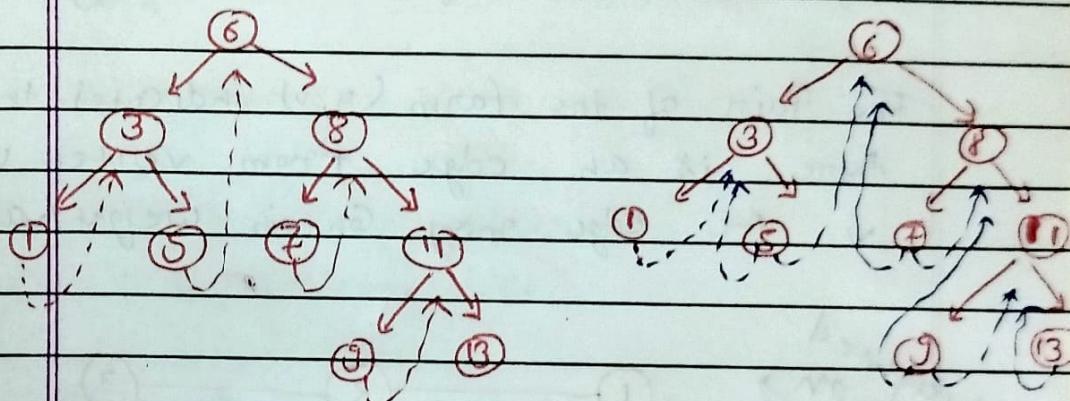


3/1/12
child at in-order traversal point
at grandparent

3/1/12 Right child has threads in sequence 3/1/12
Grand!

Types:

- Single Threaded: Nodes threaded towards either in order Predecessor / Successor.
- Double Threaded: Nodes threaded towards both Predecessor and Successor.

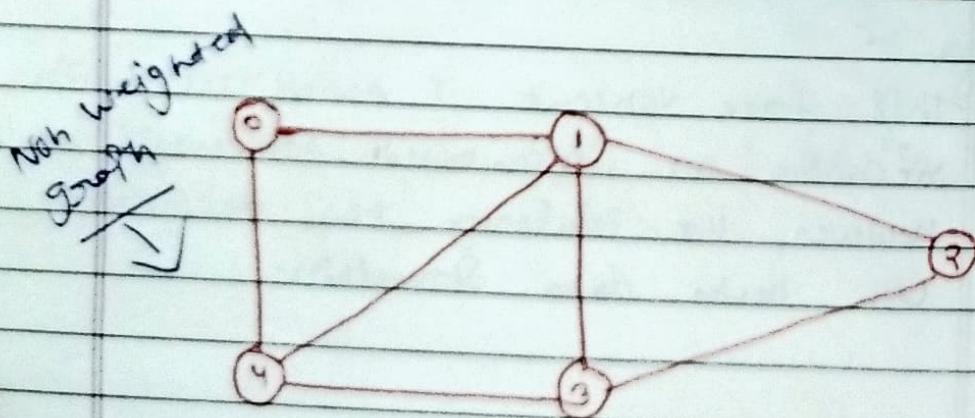
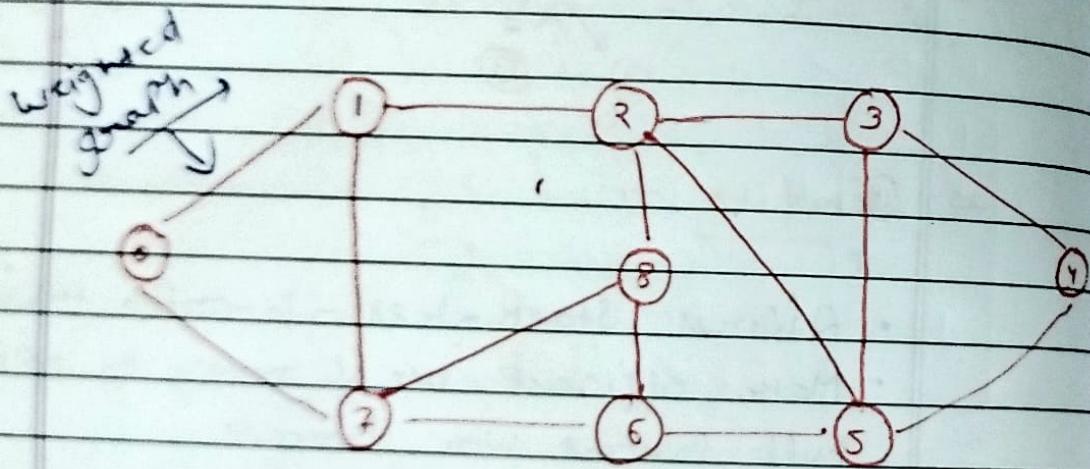


* Benefits:

- Allows stack-less -in-order traversal
- Makes efficient use of memory by replacing null pointers with threads.
- This tree variant is beneficial when recursive or stack-based traversals aren't however, its popularity has decreased with new data structures.

Graph

- Graph is the data structure that consists of following two components.
- A finite set of ordered pairs of the form (u, v) called as edge. The pair is ordered because (u, v) is not same as (v, u) in case of a directed graph (digraph).
- The pair of the form (u, v) indicates that there is an edge from vertex u to
 - The edge may contain weight/value/cap.



Graphs are used to represent many real life applications. Graphs are used to represent networks. The network may include roads in a city or telephones between.

Graphs are also used in social networks like LinkedIn, Facebook, in Facebook each person is represented with a vertex (node) each node is a structure and contains information like Person Id, name, gender.

Representation of graph in memory

- * Adjacency Matrix
- Adjacency List.

There are other representations also like Incidence Matrix and incidence list. The choice of the graph representation is situation specific. It totally depends on the type of operations to be performed and ease of use.

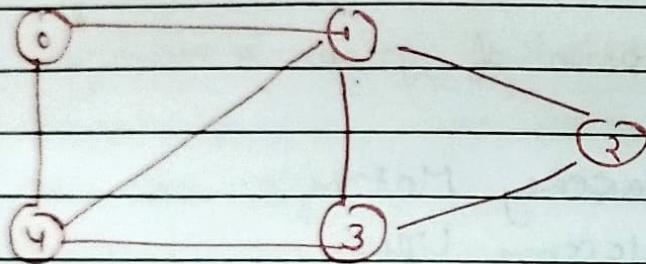
Adjacency Matrix

Adjacency Matrix is a 2D array of size $V \times V$ where V is the number of vertices in a graph. Let the 2D array be $\text{adj}[i][j]$, a slot $\text{adj}[i][j] = 1$ indicates

that there is an edge from vertex i to vertex j.

Adjacency matrix for Undirected graph is always symmetric.

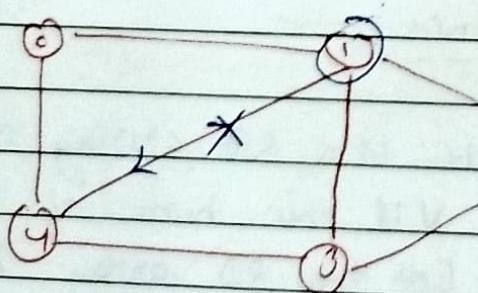
Adjacency Matrix is also used for directed graphs, if $adj[i][j] = w$, then there is an edge from vertex i to vertex j with weight w.



	0	1	2	3	4	
0	-	-	-	-	-	1
1	1	-	-	-	-	1
2	-	1	-	-	-	1
3	-	-	1	-	-	1
4	1	-	-	-	1	-

5x5.

for directed graph.



	0	1	2	3	4	
0	-	-	-	-	-	1
1	1	-	-	-	-	1
2	-	1	-	-	-	1
3	-	-	1	-	-	1
4	1	-	-	-	1	-

Incidence Matrix

(a) Representation of undirected graph

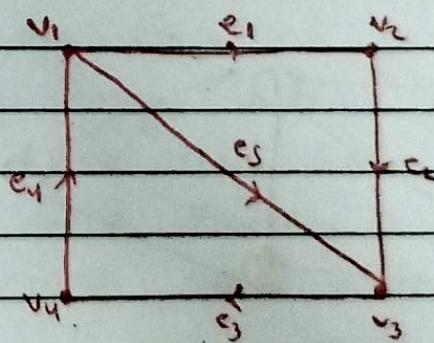
Consider a undirected graph $G = (V, E)$ which has n vertices and m edges all labelled. The incidence matrix $I(G) = [b_{ij}]$, is then $n \times m$ matrix.

- Where $b_{i,j} = 1$ when edge e_j is incident with v_i = 0 otherwise.

(b) Representation of directed graph

The incidence matrix $I(D) = [b_{ij}]$ of digraph D with n vertices and m edges is the $n \times m$ matrix in which

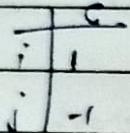
- $b_{i,j} = 1$ if arc j is directed away from vertex v_i
- $= -1$ if arc j is directed towards vertex v_i
- $= 0$ otherwise



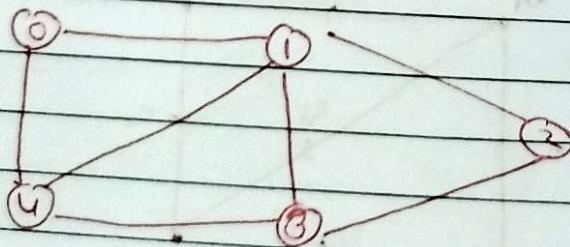
The matrix of the graph is:-

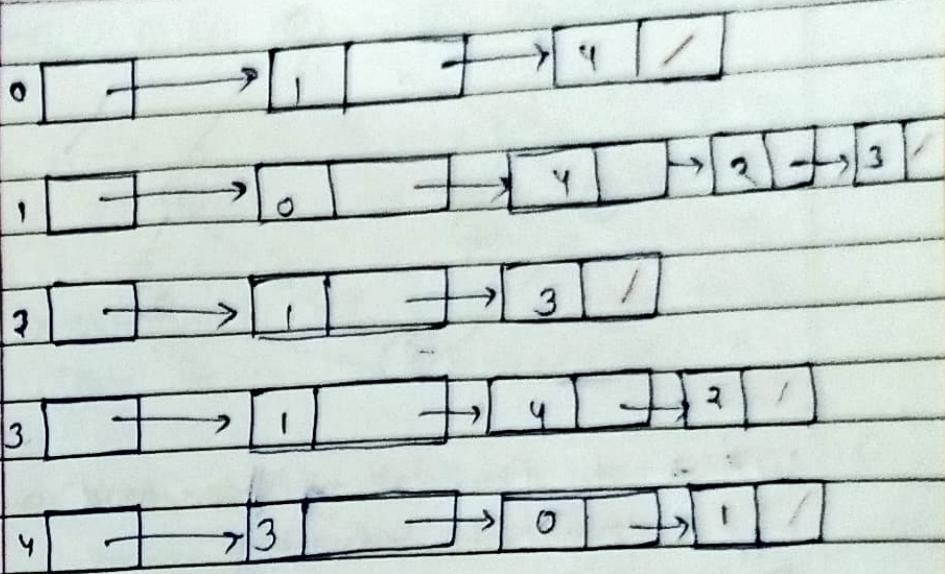
$$T(D) = \begin{bmatrix} v_1 & e_1 & e_2 & e_3 & e_4 & e_5 \\ v_2 & -1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & -1 & 1 & 0 & -1 \\ v_4 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

$i \rightarrow j$



Adjacency List: — An array of list is used. Size of array is equal to the num of vertices. Let the array be $\text{array}[]$. An entry $\text{array}[i]$ represents the list of vertices adjacent to the i th vertex. This representation can also be used to represent a weighted graph. The weights of edges can be represented as list of pairs.





a. for non-weighted graph

[Info Adj-list]

b. for weighted graph

[weight] [Info] [Adj-list]

Graph Traversal : —

— * —

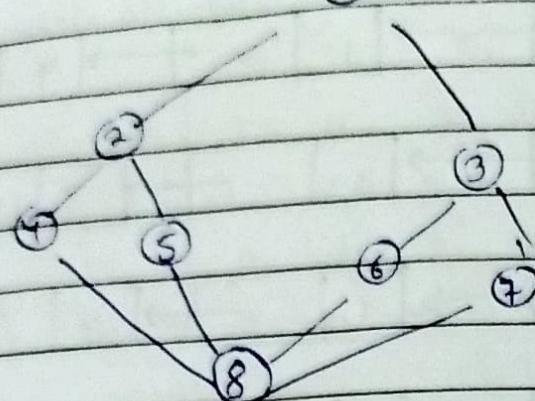
- Traversal means visiting all the nodes of a graph.
- Depth first traversal (or search) for a graph is similar to Depth first traversal of a tree.
- The only catch here is unlike tree, graph may contain cycles. So we may come to the same node again.
To avoid processing a node more than once, we use a Boolean visited array.

QPS
QFD

DFS
Breadth first search

Date
Page

①



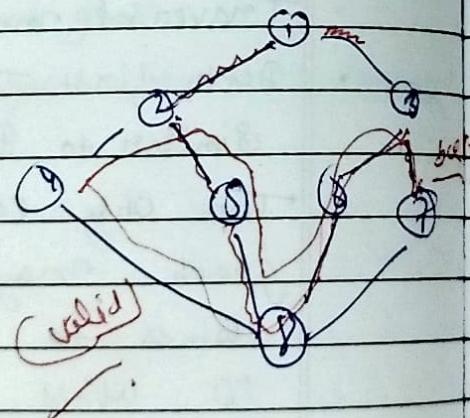
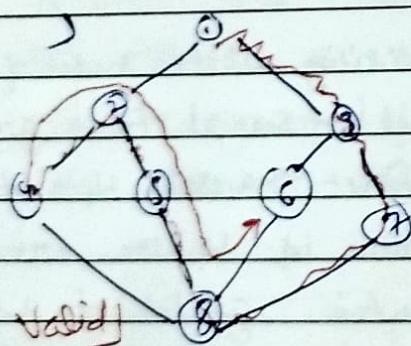
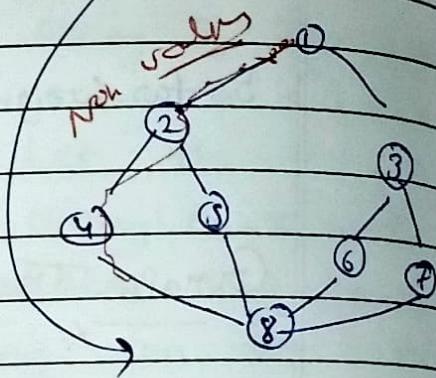
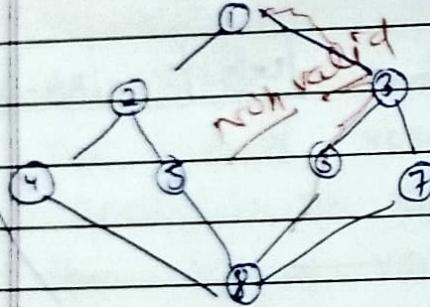
Q Which of the following are valid and invalid
DFS traversal sequence

④ 1, 3, 7, 8, 5, 2, 4, 6

⑤ 1, 2, 5, 8, 6, 3, 7, 4

⑥ 1, 3, 6, 7, 8, 5, 2, 4

⑦ 1, 2, 4, 5, 8, 6, 7, 3.



The Dfs algorithm works as follow.

- Start by Putting any one of the graphed vertices on top of a Stack.
- Take the top item of the Stack and add it to the visited list.
- Create a list of that vertex's adjacent nodes. Add the ones which aren't in the visited list to the top of Stack.
- Keep repeating steps 2 and 3 until the Stack is empty.

Dfs(v)
→

?

visited(v) = 1

for all n adjacent to v

?

{ if (n is not visited)
 Dfs(n)

S

S

Importance of Dfs:

- Testing whether graph is Connected.
- Computing a Spanning forest of G .
- Computing the Connected Components of G .
- Computing a Path b/w two vertices of G or figuring out if such Path exists.
- Computing a cycle in G or figuring out if no such cycle exists.

Application:-

- Finding Connected Components.
- Topological Sorting.
- finding 2 - (edge or vertex) - Connected Components.
 ,, ,, ,, ,,
- find bridges of graph - B.
- finding Strongly Connected Component.

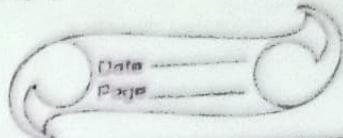
BFS — Breadth first search :-

- Breadth first Search for a graph is similar to binary Dfs. The only catch here unlike trees, graph may contain cycles so we may come to the same node again.

To avoid processing a node more than once we use a Boolean visited array. Simplicity

BSF

→ Children by children
nearby.



• 30.4

It is assumed that all vertices are
reachable from the starting vertex
i.e. the graph is connected.
Just print + 3rd child print

Q

BFS

7
1 3 2 4

① 1, 3, 2, 5, 4, ~~6, 8~~

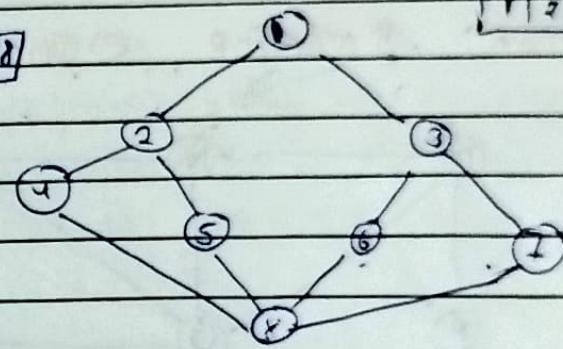
② 1, 3, 2, 5, 4, 7, 6, 8

⑤ 1, 3, 2, 7, 5, 6

⑦ 1, 2, 3, 7, 5, 6, 8

1 2 3 4 5 6 7 8

1 1 2 3



BSF (v)

?

Visited (v) = 1

insert [v, Ø]

while (Ø != Ph)

?

u = Delete (Ø);

for all n adjacent to u

?

if (n is not visited)

?

visited (n) = 1

insert (n, Ø)

j

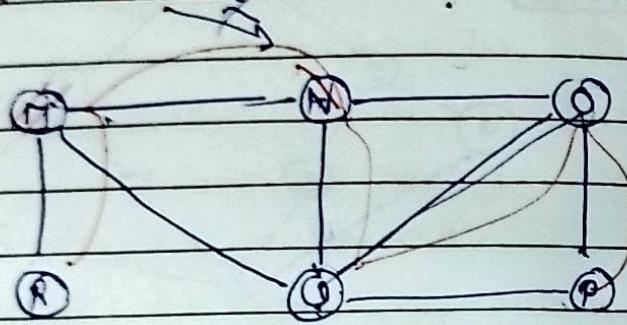
5

Time complexity $O(|V| + |E|)$.

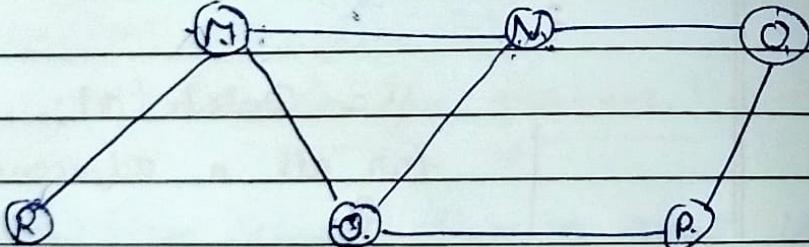
23/08/23

- Q) BFS has been implemented using a queue data structure which one of the following is a possible order of visiting the nodes in a graph above.

A) MNPQR B) NOMPQR C) OMNRPQ D) POONMR



A) MNOPQR B) NOMPQR C) OMNRPQ
D) OMNPQR



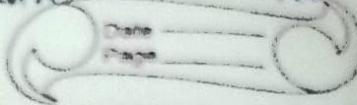
Importance of BFS

- * One of the Single Source Shortest Path algorithm, so it is used to compute the shortest Path.
- * It is also used to solve Puzzles such as the Rubik's Cube.
- * BFS is not only the quickest way of solving the Rubik's Cube, but also the most optimal way of solving it.

Applications →

- Copying garbage Collection.
- finding the shortest Path b/w nodes w.r.t with Path length measured by number of edges.
- Ford - fulkerson method for Computing the maximum flow in a flow net.
- Serialization/Deserialization of binary tree vs serialization in sorted order, allow the tree to be inserted in the efficient manner.
- Testing bipartiteness of a graph.

→ taking an array of size 6 & time complexity of O(n) for search operation
elements are 10, 20, 30, 40, 50, 60
Search operation is at index 3



→ Matrix :-

Main idea of data structure is to help us store the data. But most common operation on any data structure is not insert / delete but actually search, even for insertion / deletion search is the optimised.

In any of the data structure the search time first depends on the number of elements which data structure contains and then on type of structure.

g -

Unsorted array - $O(n)$

Sorted array - $O(\log n)$

Link list - $O(n)$

BST - $O(n)$

BST - $O(n)$

AVL - $O(\log n)$.

By our own analysis we get insertion generate one address

$$1+1+2+1+2+2+0+1+n=1$$

$$11 = 1+1 = 2$$

→ So hashing is a technique where search time is independent of the num of items in which we are searching a data value.

→ The basic idea is to use the key ~~is key~~ to find the address in the memory to make searching easy for us to do. Then we, will have Address Card. We consider any other key and convert it into a smaller geometrical form as index in a table called hash table.

The values are then stored in hash table by using that key you can access the element in O(1) time.

Key

values

graphical
chart

	000			X
John Smith	→ 001	Lisa Smith	S21-8978	•
Lisa Smith	002	:	:	:
Sam Smith	1 S1	John Smith	S21-1234	•
	1 S2			X
	1 S3	Ted Baker	447-4785	•
	1 S4			X
Sandra Lee	2 S3			*
Ted Baker	2 S4	Sam Doe	S21-S030	•
	2 S5			*

Q. Given the following hash function
hash(K) = (3K₁ + 2K₂ + 5K₃) mod 11
With elements 3 and 5 and 6. With what
following statements are true?

- Q 9, 2, 1983, 7/97 talk to the student
- Q 19, 21, 5/97 talk to the same value
- Q All elements talk to the same value
- Q each element talk to the diff values

b) Collision — It is possible that two
→ diff set of keys K₁ and K₂
will yield the same hash address.
This situation is called Collision.

The technique is to reduce collision is
called Collision resolution.

data → 31-41 hash fun 4-131-7

Collision → 7

Characteristics:-

- + Easy to Compute and understand.
- + Efficiently Computable - it must take less time
- + Should uniformly distribute the key.
- + Must have low Collision rate.

Most Popular hash function

Douglas-Wasserman method

The size of the sum of items in the table is estimated. That sum is then used as a divisor into each original value or key to extract a quotient and remainder.

The remainder is the loaded value. (Since this method is liable to produce a sum of collisions, any search algorithm would have to code to recognize a collision and offer an alternate search mechanism.)

$$H(k) = k \text{ mod } m$$

$$H(k) = k \text{ mod } m + 1$$

Mid Square Method.

Folding Method.

etc to complete.

Collision Resolution Technique

Open Addressing / Closed Hashing

All elements are stored in the hash table. i.e. collision is resolved by Rehashing through alternate location with its key in a particular sequence.

When searching for an element, we check one by one examine table slots until the desired element is found or it is clear that the element is not in the table. So at any point, size of table must be greater than or equal to total number of keys.

3 types of Probing :-

- * Linear.
- * Quadratic.
- * Double.

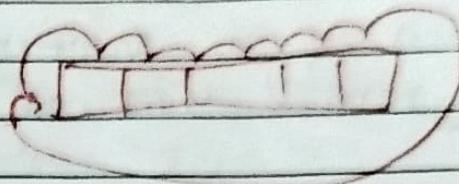
Linear Probing :-

In linear probing method, in case of a collision we find out the next free open and store the key that is causing collision in it.

method -

$$h(k, i) = (h^k(i) + i) \bmod m;$$

for $i = 0, 1, \dots, m-1$.



- Q A hash table contains 10 buckets and uses linear probing to resolve collision. The key values are integers and the hash function used is $h_k(x) = x \bmod 10$. If the values ~~43~~³, 165, 62, 123, 142 are inserted in the table, in what location would the key value 142 be inserted.

(A) 2 (B) 3 (C) 4 (D) 6

$$\overbrace{43}^{43 \% 10 = 3} \quad 10 = 3$$

0		
1		
2	62	142
3	43	← 123 collision
7	123	← 165 collision
5	165	Collision
8	142	
9		

data clustering of Problem 8.1
first data goes first just etc
- mostly

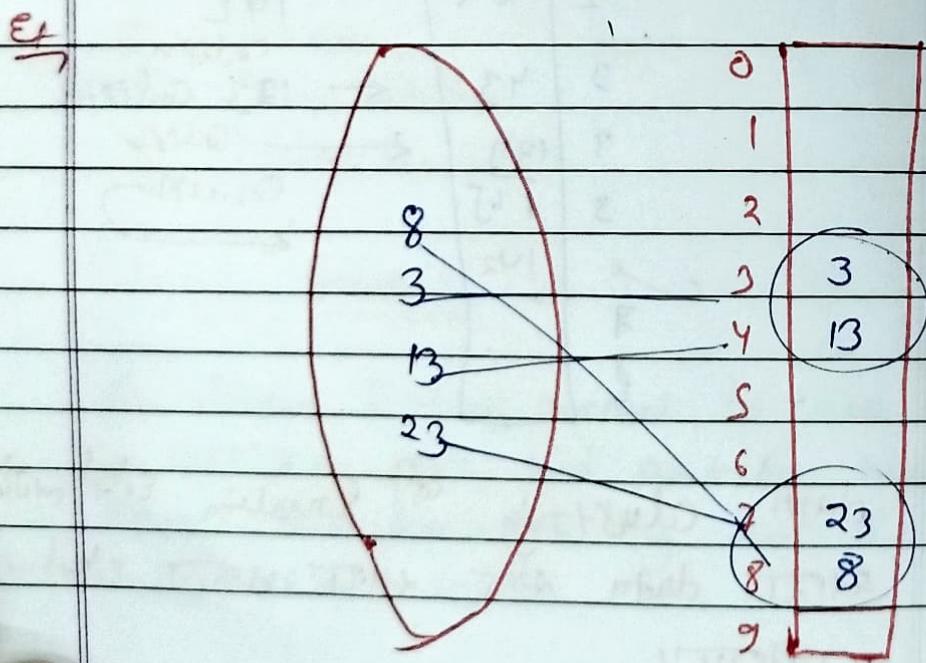
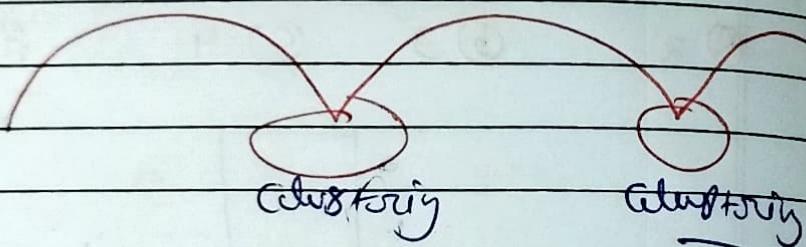
Anatolic Problem

Quadratic Resolving Operation by taking the original hash index and adding successive values of an arbitrary quadratic polynomial until an open slot is found.

Quadratic Probing uses a hash function
of the form

$$y_h(k, i) = (y'_h(k) + f(i^2)) \bmod m$$

Where, η^i is an auxiliary function
and $i = 0, 1, \dots, m-1$.



8, 3, 13, 23

$$h(8) = [(h(8) + f(3))] \bmod 10 = 8 \text{ so}$$

it get location 8.

3 will be placed at $h(3) = [h(3) + f(3)] \bmod 10 = 3$
 no collision so it is " " " 3

13 " " " , $h(13) = [h(13) + f(3)] \bmod 10 = 2$
 collision occurs so we increase the value of i.

$h(13) = [h(13) + f(13)] \bmod 10 = 4$, no
 collision so it " " " 4 " .

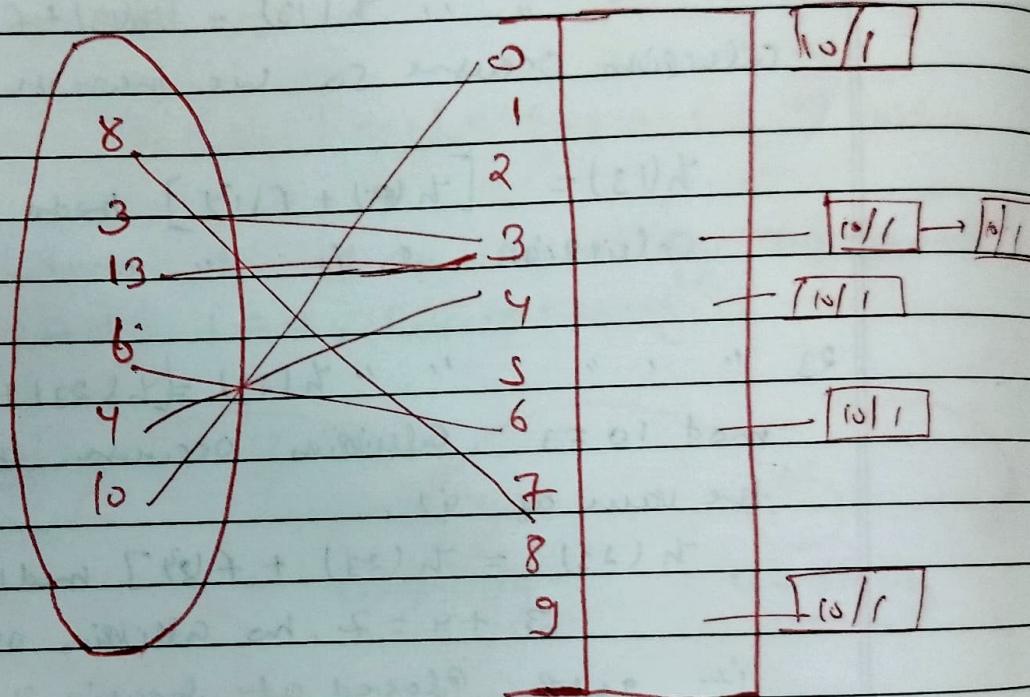
23 " " " " $h(23) = [h(23) + f(3)] \bmod 10 = 6$
 mod 10 = 3 collision occurs increase
 the value of i.

$h(23) = h(23) + f(3) \bmod 10 = 7$
 $3 + 4 = 7$, no collision occurred so
 it gets placed at location 7.

Quadratic probing avoid clustering of
 elements and help improve the search
 time.

Chaining :-

The idea is to make each cell of hash table point to a linked list of records that have same hash value. In Chaining, we place all the elements that hash to the same slot into the same linked list.



Double Hawking : — Hawking $c = \sqrt{e}$
— $\sqrt{2} \approx 1.414 \cdot 314\ldots$

Hawking $c = \sqrt{e}$ to measure collision.