

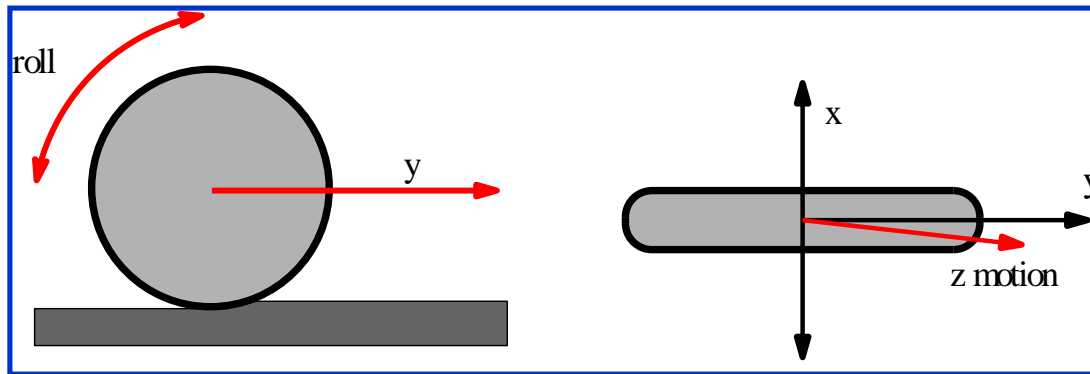
# Probabilistic Robotics

## **Wheeled Locomotion**

# Locomotion of Wheeled Robots

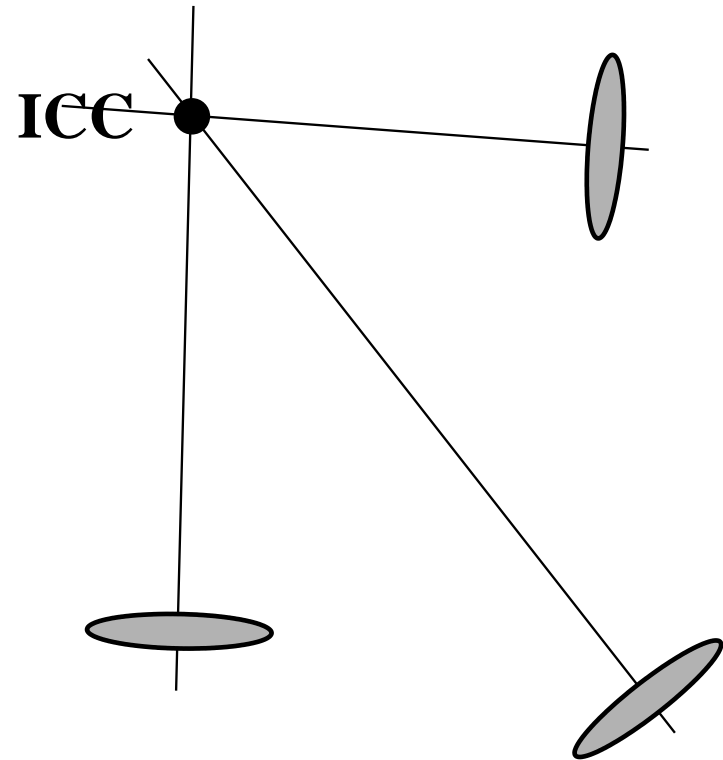
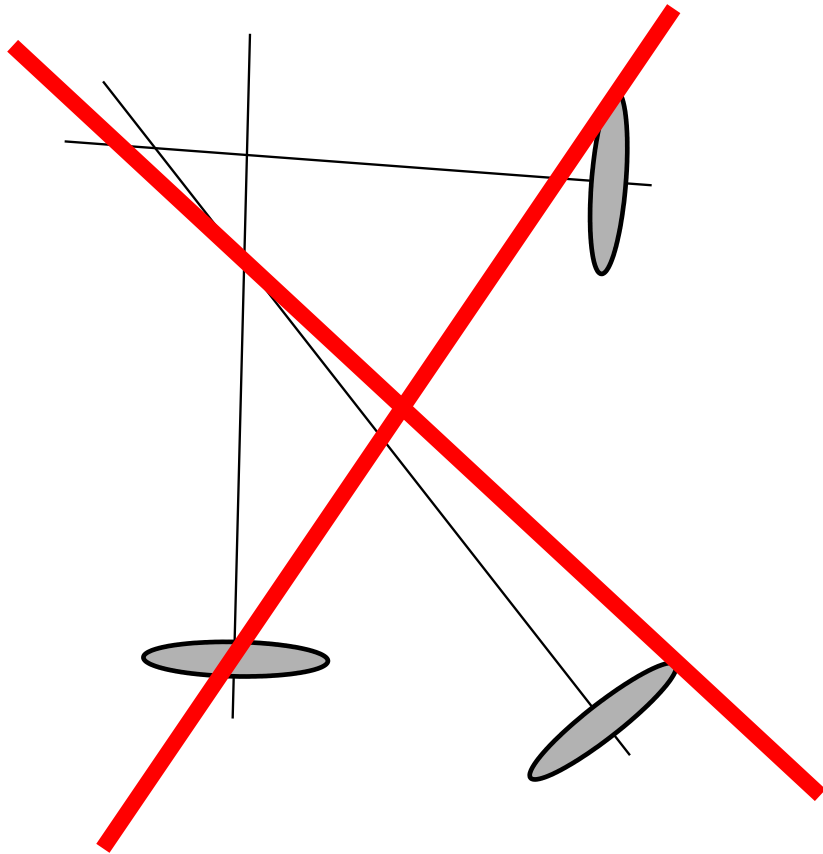
Locomotion (Oxford Dict.):

Power of motion from place to place



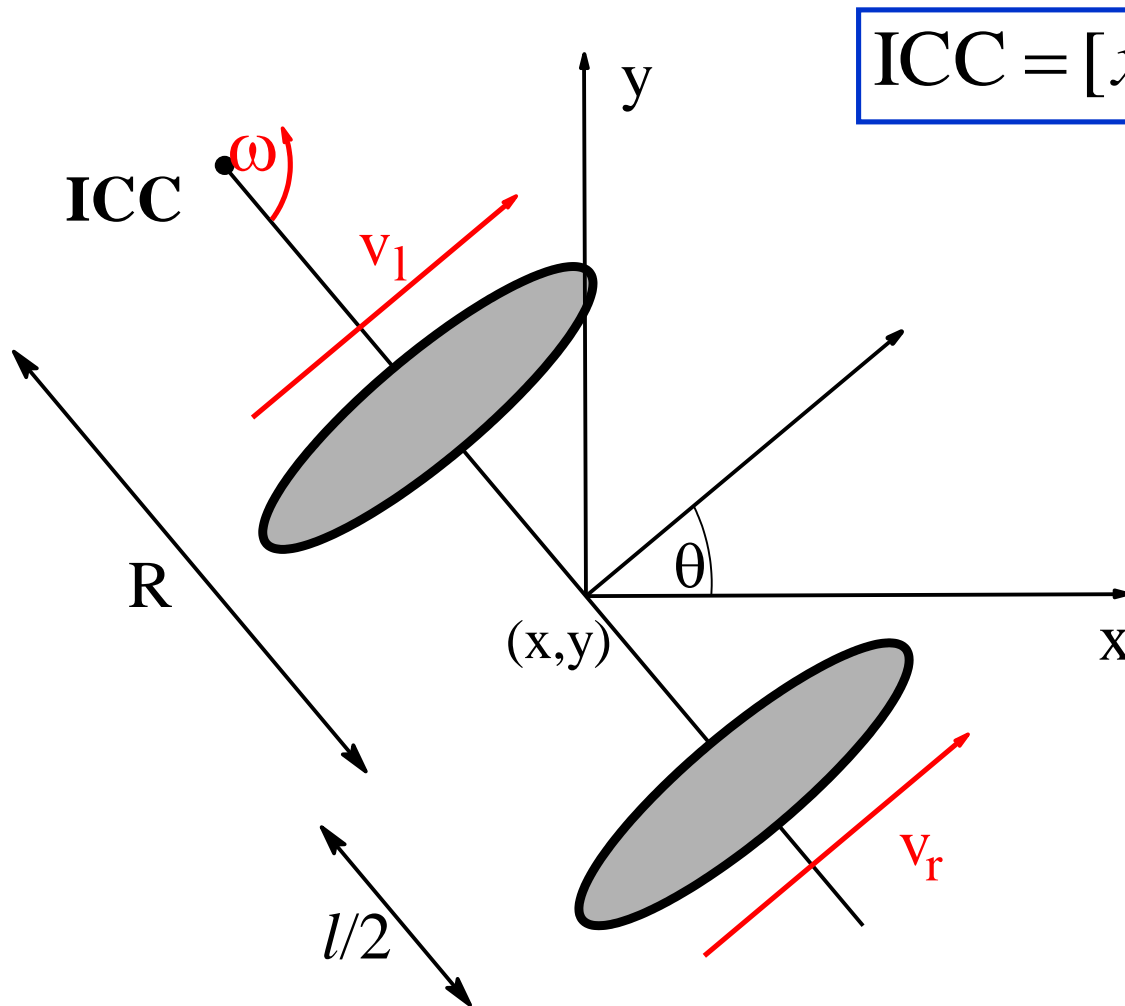
- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- Mecanum wheels, XR4000

# Instantaneous Center of Curvature



- For rolling motion to occur, each wheel has to move along its y-axis

# Differential Drive



$$\text{ICC} = [x - R \sin \theta, y + R \cos \theta]$$

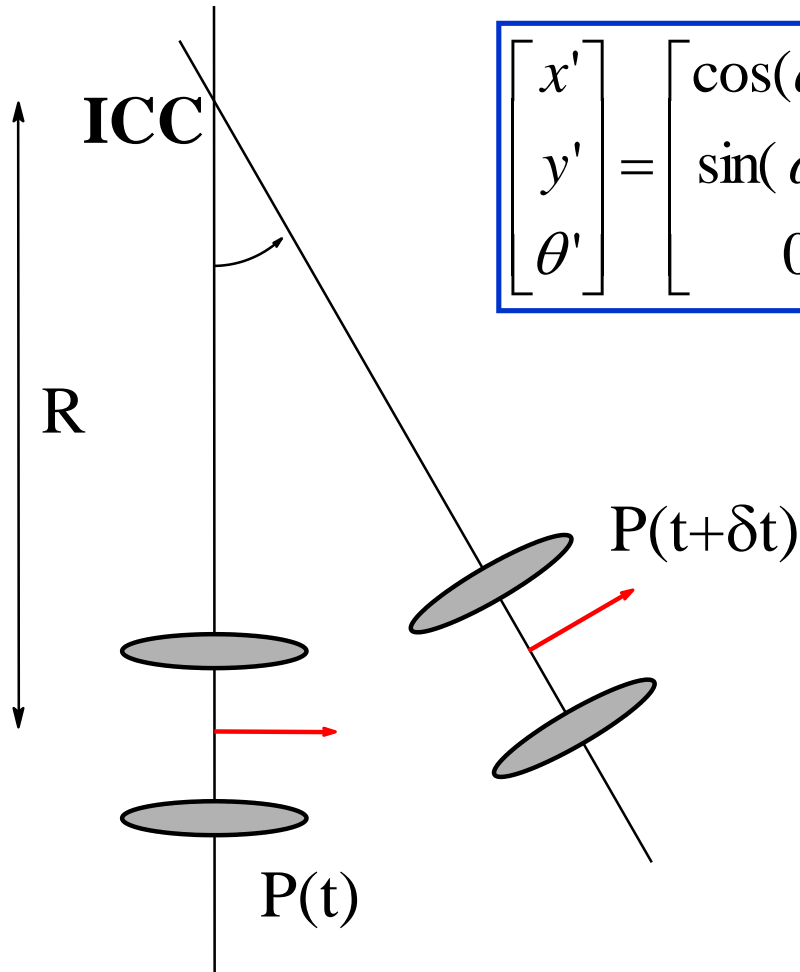
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2} \frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

# Differential Drive: Forward Kinematics



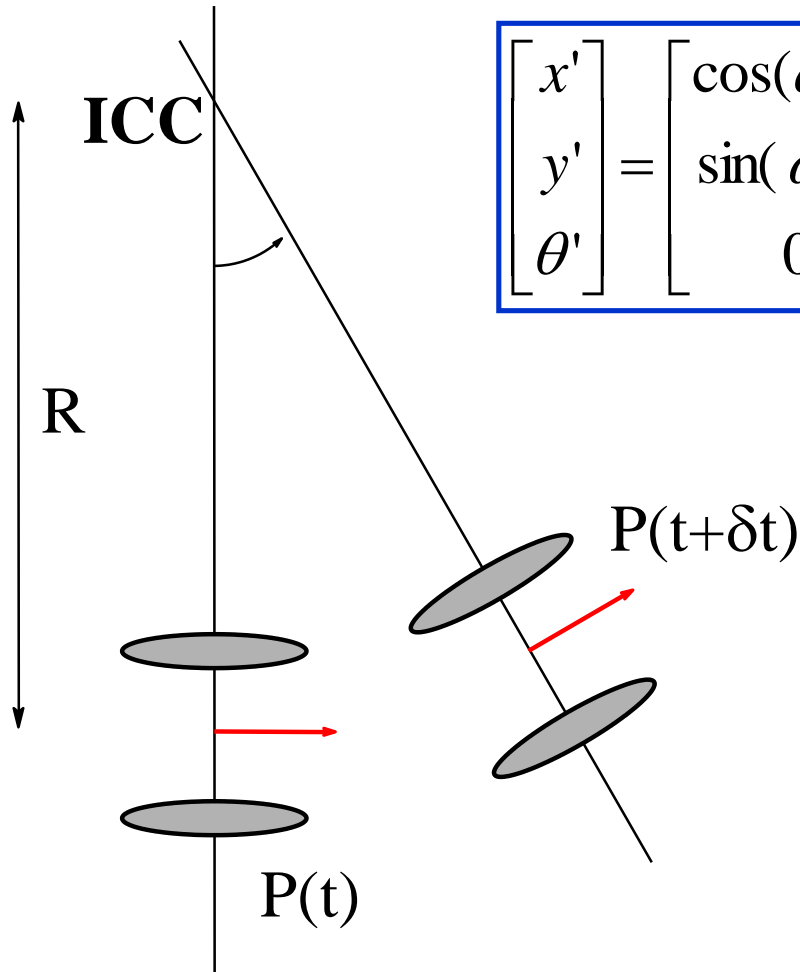
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \text{ICC}_x \\ y - \text{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \text{ICC}_x \\ \text{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$

$$\theta(t) = \int_0^t \omega(t') dt'$$

# Differential Drive: Forward Kinematics



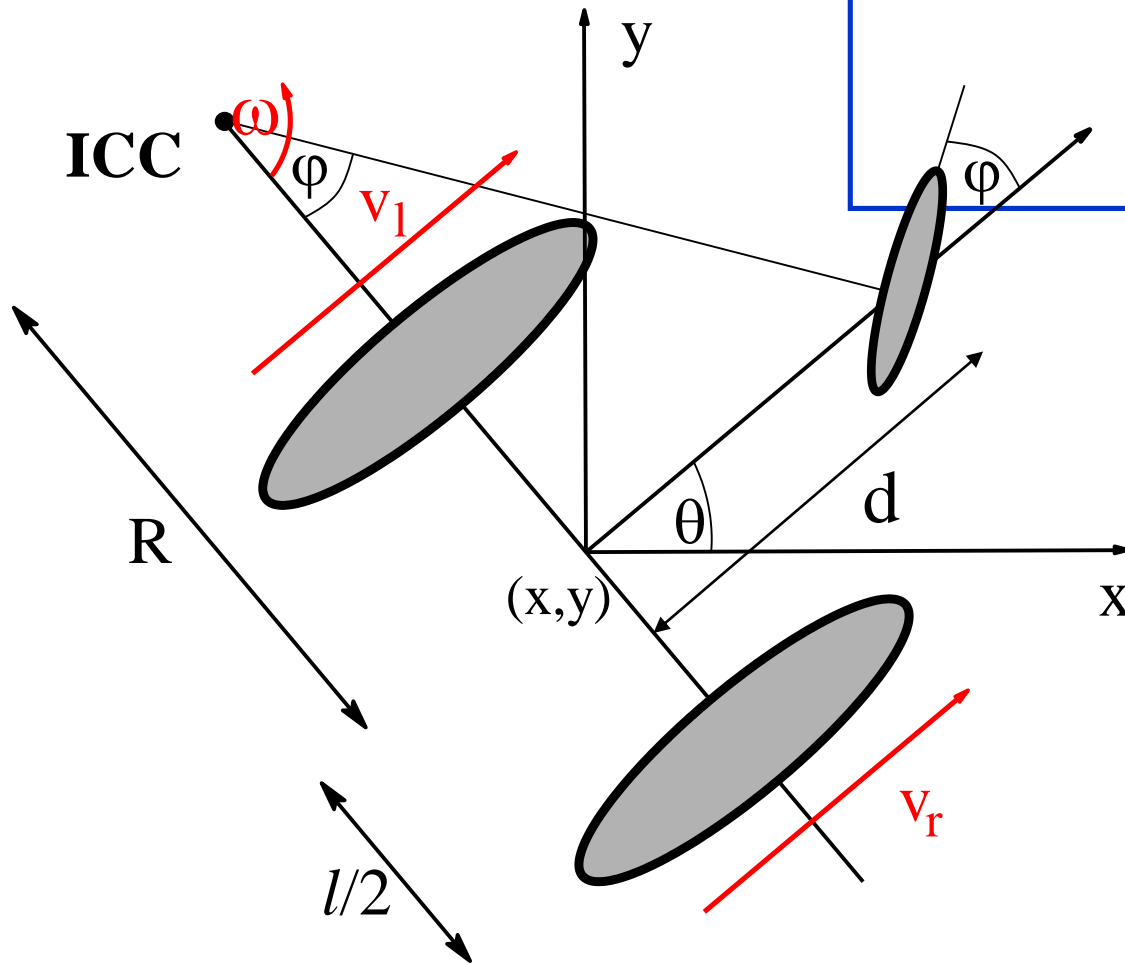
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \text{ICC}_x \\ y - \text{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \text{ICC}_x \\ \text{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \frac{1}{2} \int_0^t [v_r(t') + v_l(t')] \cos[\theta(t')] dt'$$

$$y(t) = \frac{1}{2} \int_0^t [v_r(t') + v_l(t')] \sin[\theta(t')] dt'$$

$$\theta(t) = \frac{1}{l} \int_0^t [v_r(t') - v_l(t')] dt'$$

# Ackermann Drive



$$\text{ICC} = [x - R \sin \theta, y + R \cos \theta]$$

$$R = \frac{d}{\tan \varphi}$$

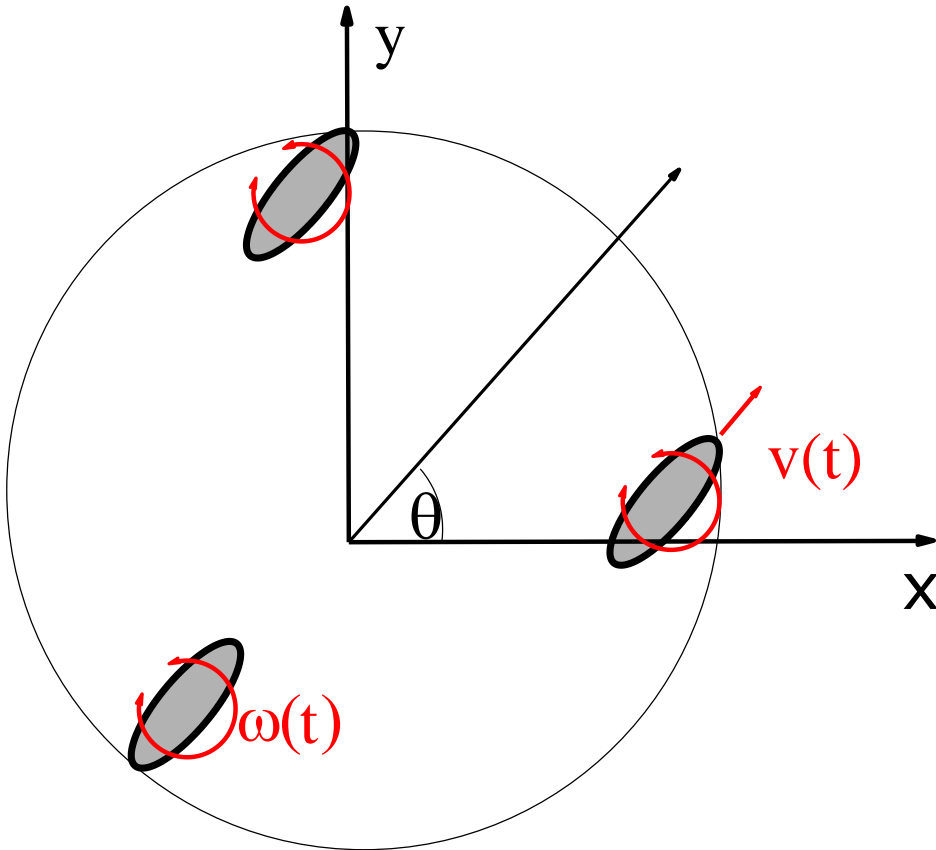
$$\omega(R + l/2) = v_r$$

$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2} \frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

# Synchronous Drive



$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$

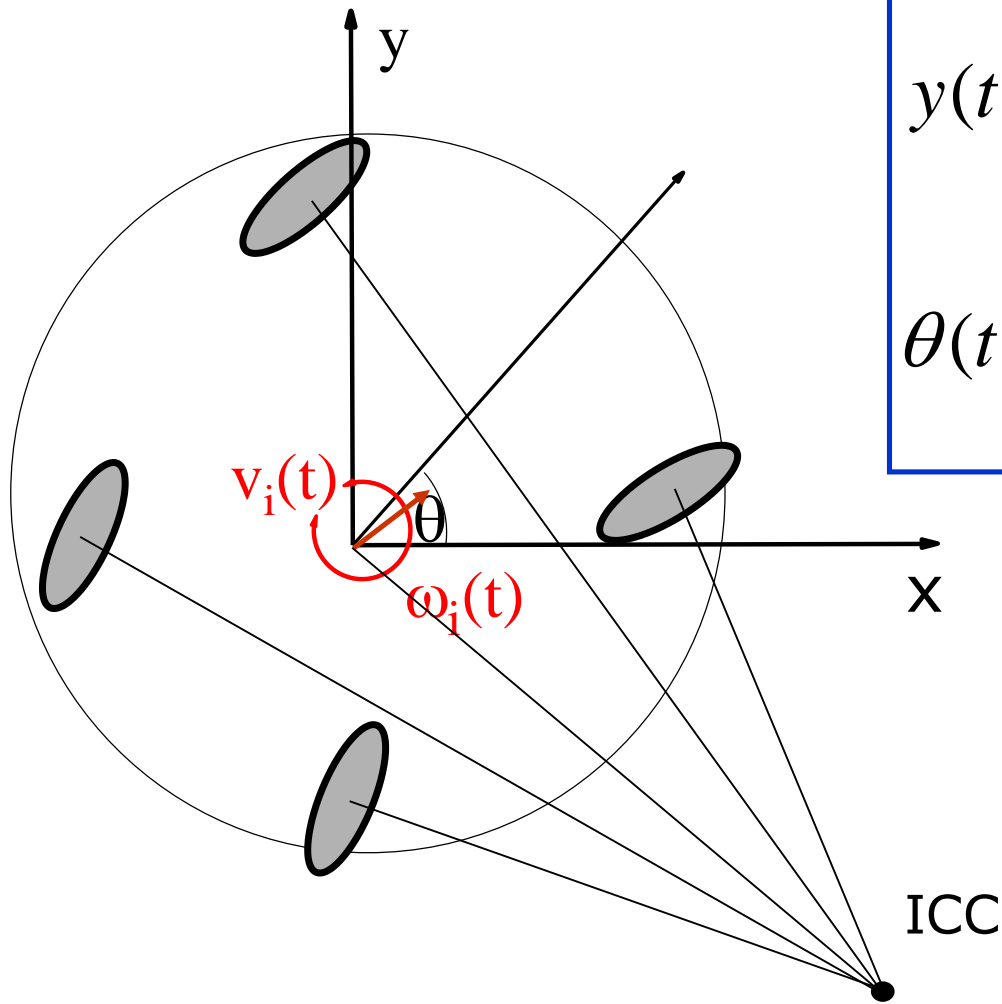
$$\theta(t) = \int_0^t \omega(t') dt'$$



# Synchro-Drive Robot



# XR4000 Drive



$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$
$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$
$$\theta(t) = \int_0^t \omega(t') dt'$$

# XR4000



[courtesy by Oliver Brock & Oussama Khatib]

# Mecanum Wheels



$$v_y = (v_0 + v_1 + v_2 + v_3) / 4$$

$$v_x = (v_0 - v_1 + v_2 - v_3) / 4$$

$$v_\theta = (v_0 + v_1 - v_2 - v_3) / 4$$

$$v_{error} = (v_0 - v_1 - v_2 + v_3) / 4$$

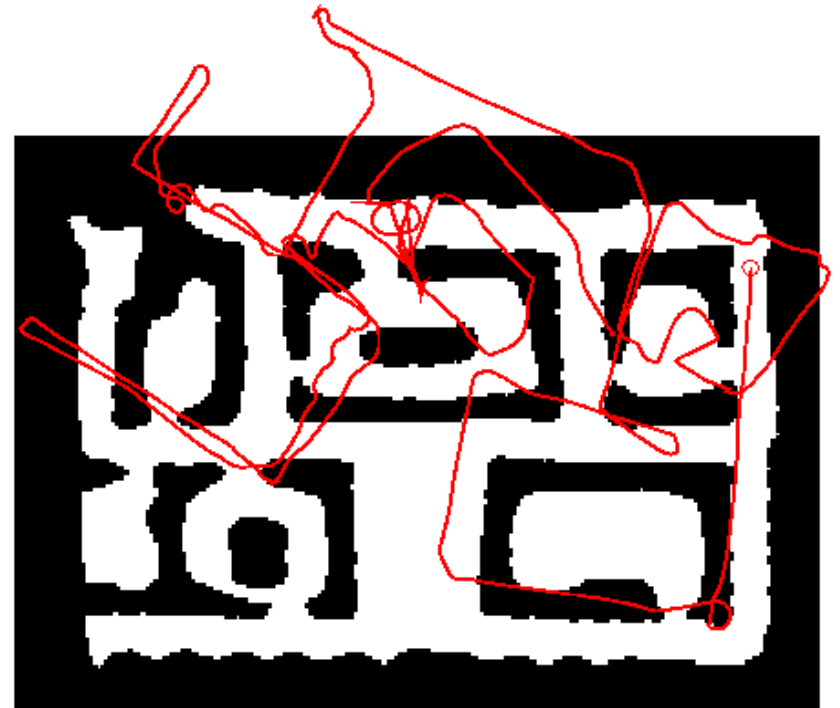
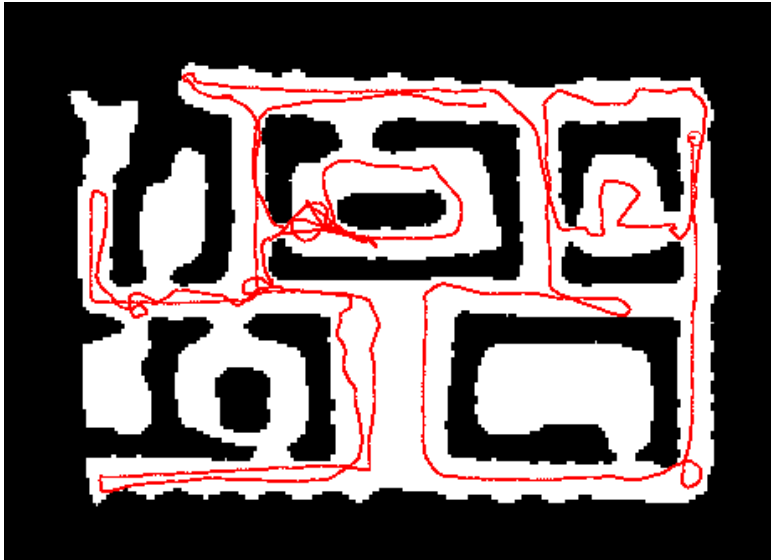
# Example: Priamos (Karlsruhe)



# Example



# Odometry



# Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- XR-4000 or Mecanum-wheeled robots can move sideways.



# Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the control space with respect to the current configuration (e.g., moving sideways is impossible).
- Holonomic constraints reduce the configuration space.