CENG 789 – Digital Geometry Processing

01- Introduction

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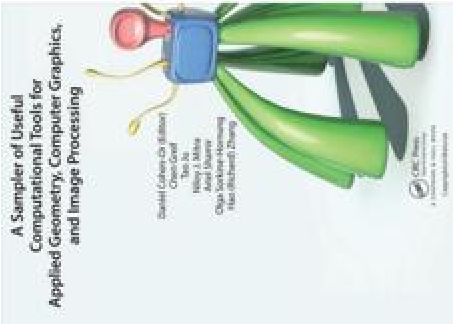
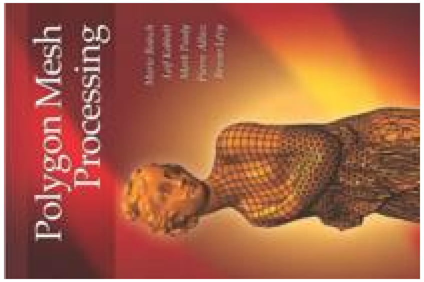
Computer Eng. Dept, , Turkey

Administrative

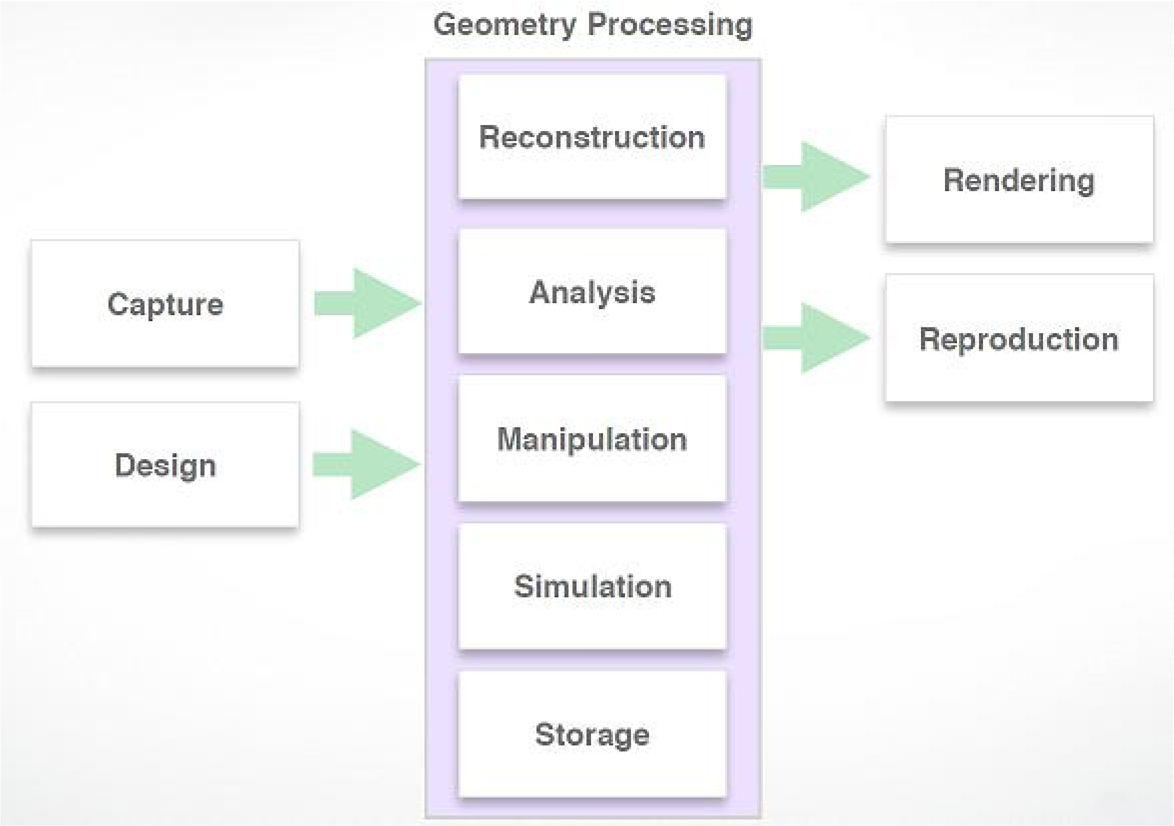
- ✓ Tue 14:40-17:30 @ G102.
- ✓ www.ceng.metu.edu.tr/~ys/ceng789-dgp/
- ✓ Grading
  - ✓ 40%: 2 programming assignments.
  - ✓ 40%: Term Project.
  - ✓ 20%: Final Exam.
- ✓ Reference book: Polygon Mesh Processing:
- ✓ C++ programming required.
  - ✓ Code framework including 3D UI will be provided.
- ✓ Similar courses.
  - ✓ EPFL, M. Pauly.
  - ✓ Stanford, V. Kim.
  - ✓ Technion, M. Ben-Chen.
- ✓ Instructor: Yusuf Sahillioğlu (ys@ceng.metu.edu.tr)
  (office: B107)





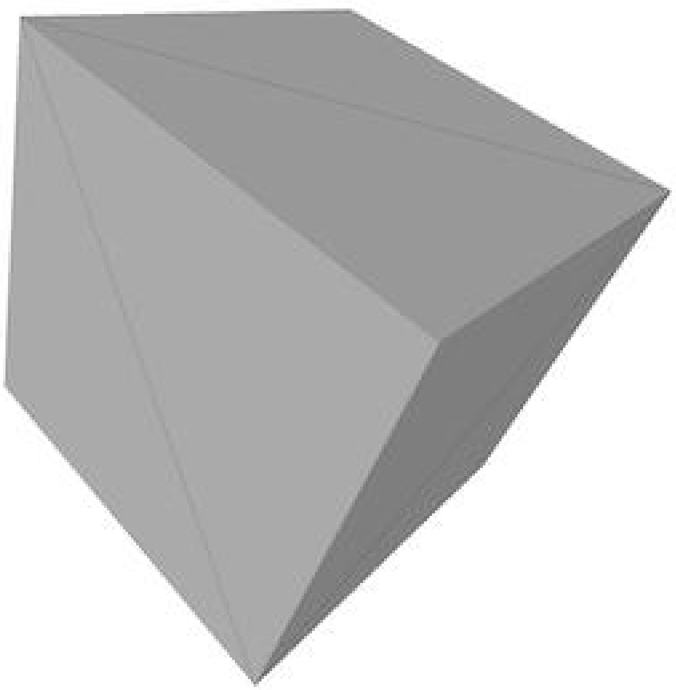
Objective

✓ Learn concepts and algorithms for a complete geometry processing system.



Objective

✓ Typically geometry comes through a mesh structure.



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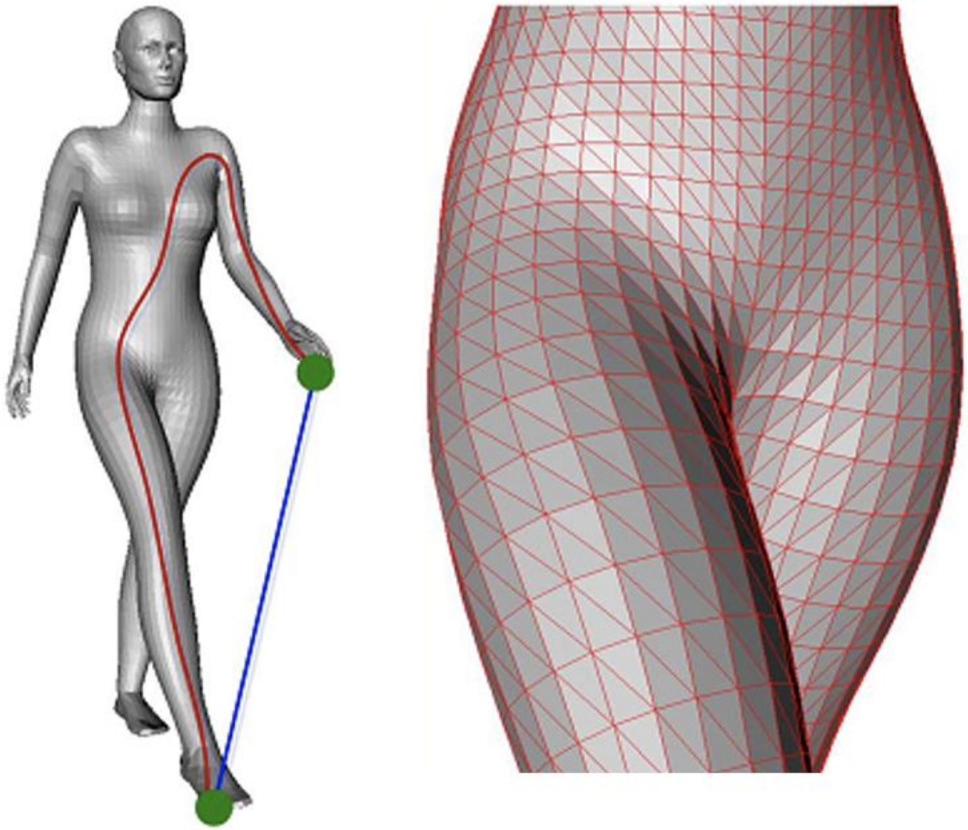
v 1.000000 -1.000000 1.000000
v -1.000000 -1.000000 1.000000
v -1.000000 -1.000000 -1.000000
v 1.000000 1.000000 -0.999999
v 0.999999 1.000000 1.000001
v -1.000000 1.000000 1.000000
v -1.000000 1.000000 -1.000000
f 2 3 4
f 8 7 6
f 5 6 2
f 6 7 3
f 3 7 8
f 1 4 8
f 1 2 4
f 5 8 6
f 1 5 2
f 2 6 3
f 4 3 8
f 5 1 8
```

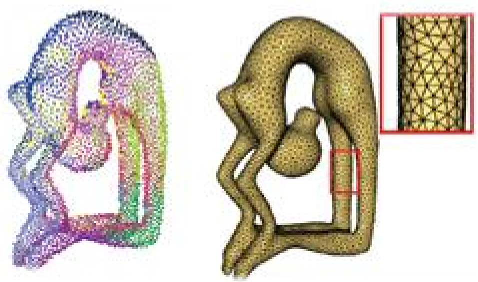
Objective

✓ Typically geometry comes through a mesh structure.

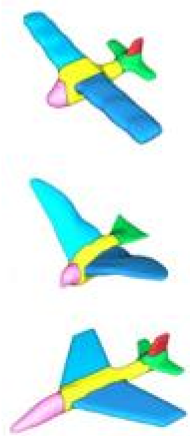


Objective

✓ We mostly deal with



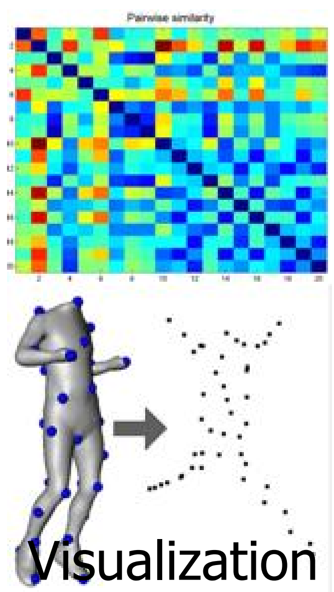
Reconstruction



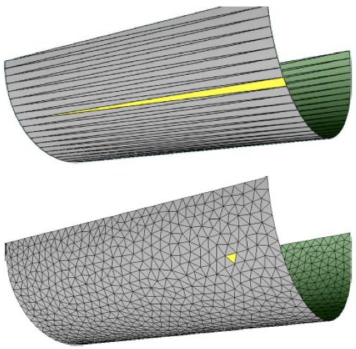
Analysis

Manipulation/Editing

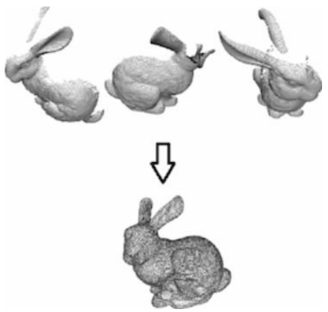
Parameterization



Visualization



Remeshing

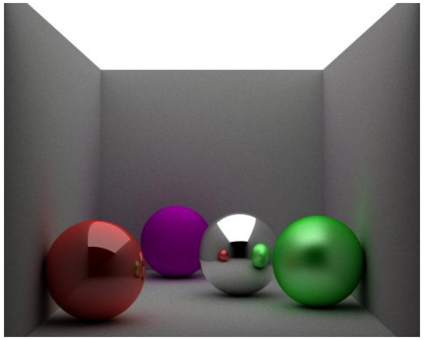


Registration

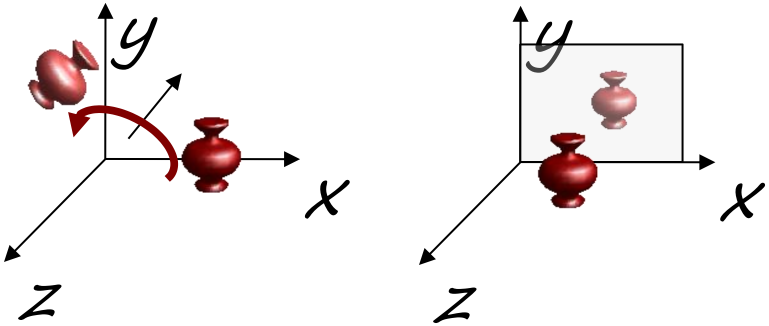
3D Printing

Objective

- ✓ We don't deal with Computer Graphics material (take CENG 477)
  - ✓ Rendering
    - ✓ Ray tracing
    - ✓ Rasterization

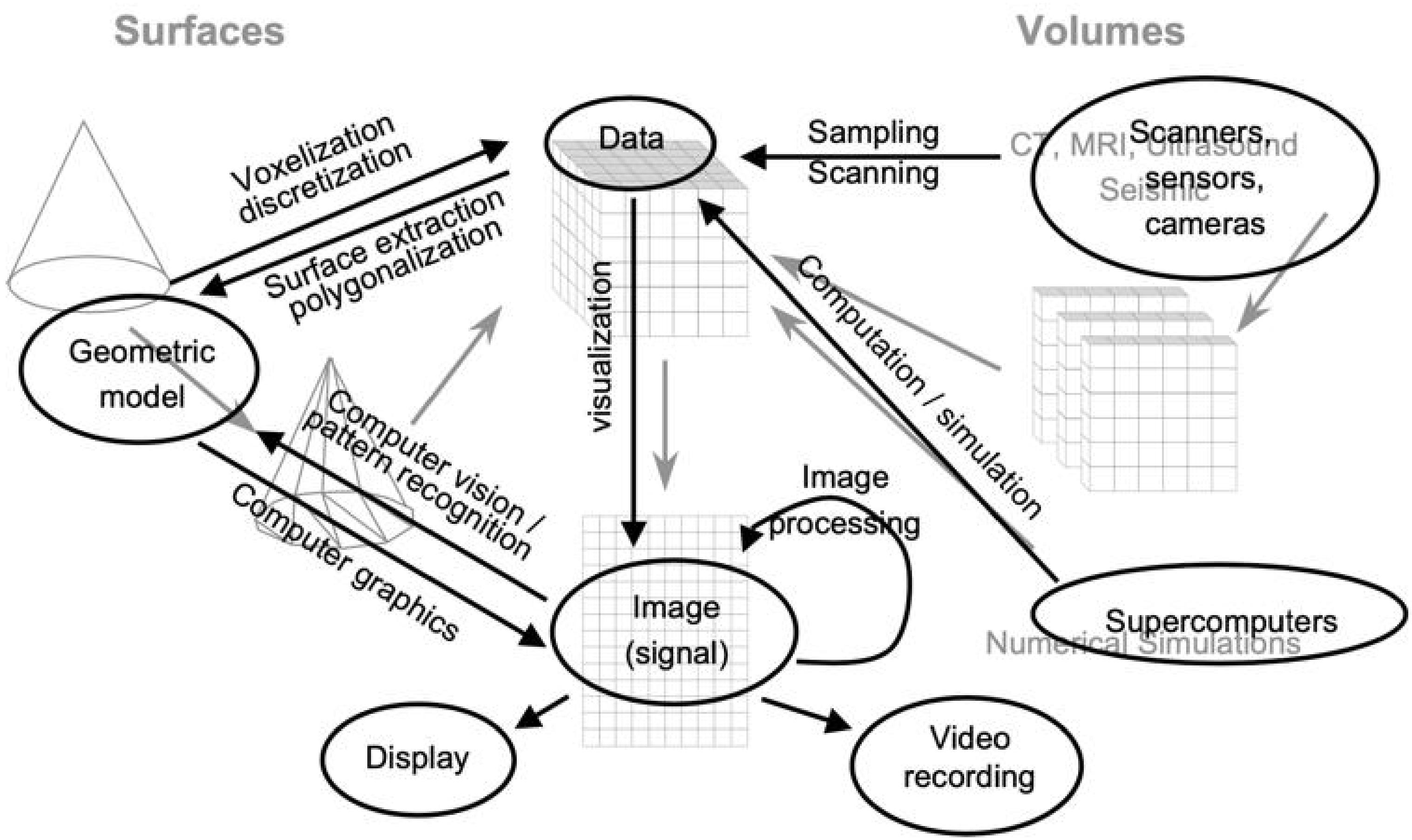


  - ✓ Transformations



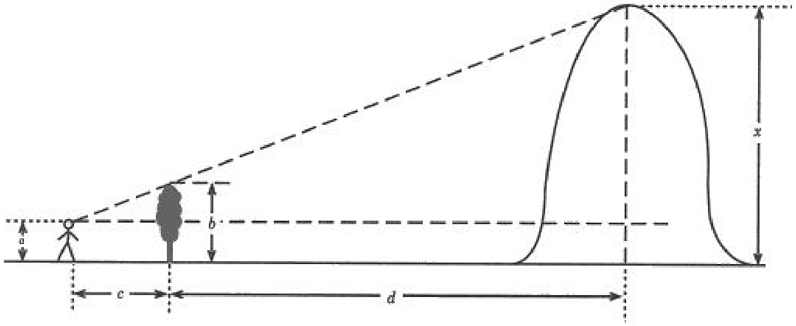
Objective

✓ Connecting terms: geometric modeling, graphics, vision, visualization, ..



Why do we care?

- ✓ Geometry: geo (earth) + metron (measurement)
- ✓ From ancient times:



- ✓ To modern times:

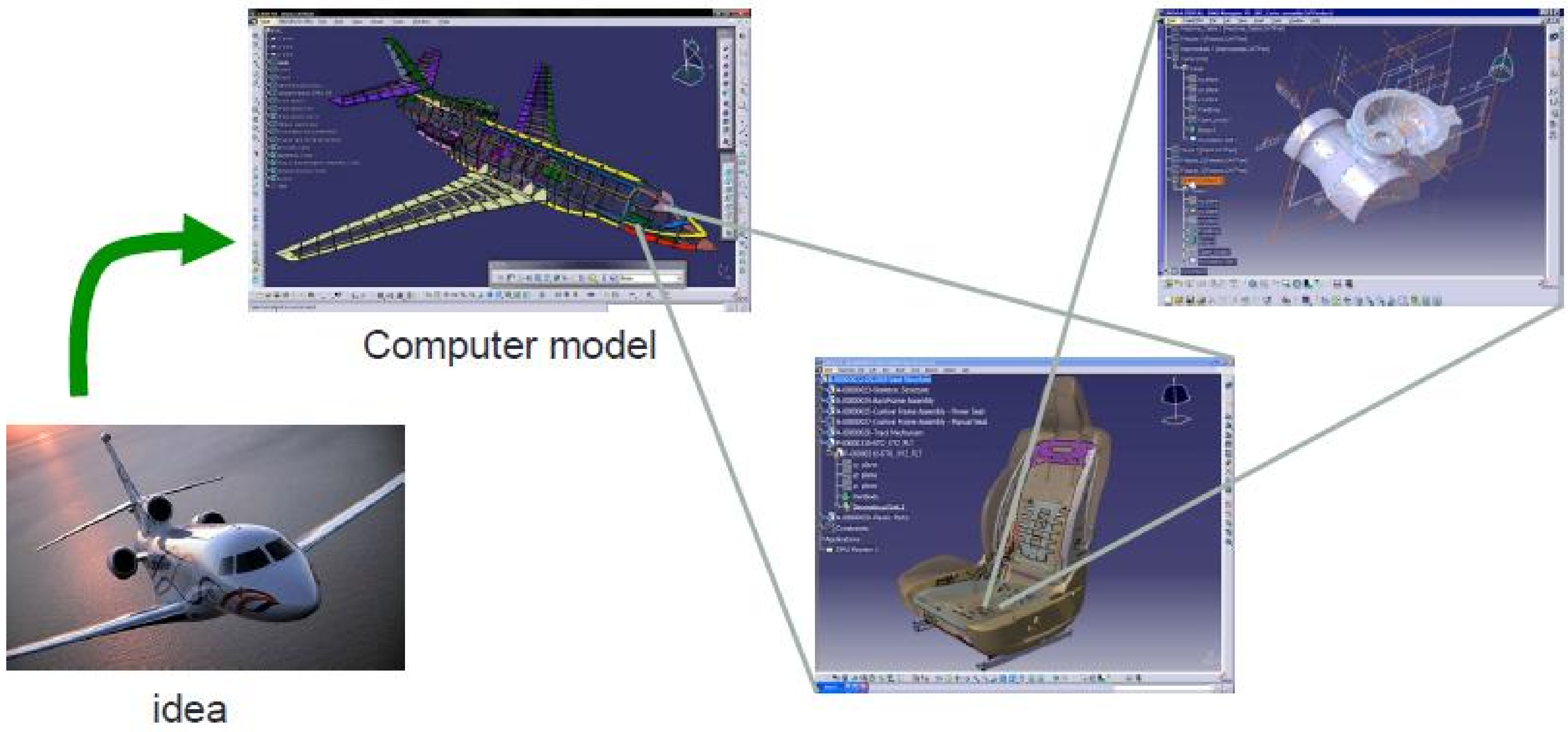
Geometric Digital Modeling

- ✓ Geometric objects in the world are digitally modeled (representation) for
  - ✓ easy manipulation
  - ✓ easy repairing
  - ✓ easy comparison
  - ✓ easy synthesis
  - ✓ cheaper simulation

Digitially modeled
(designed on a computer)

Digital Models

✓ Geometric objects in the world are digitally modeled (representation).



Digital Models

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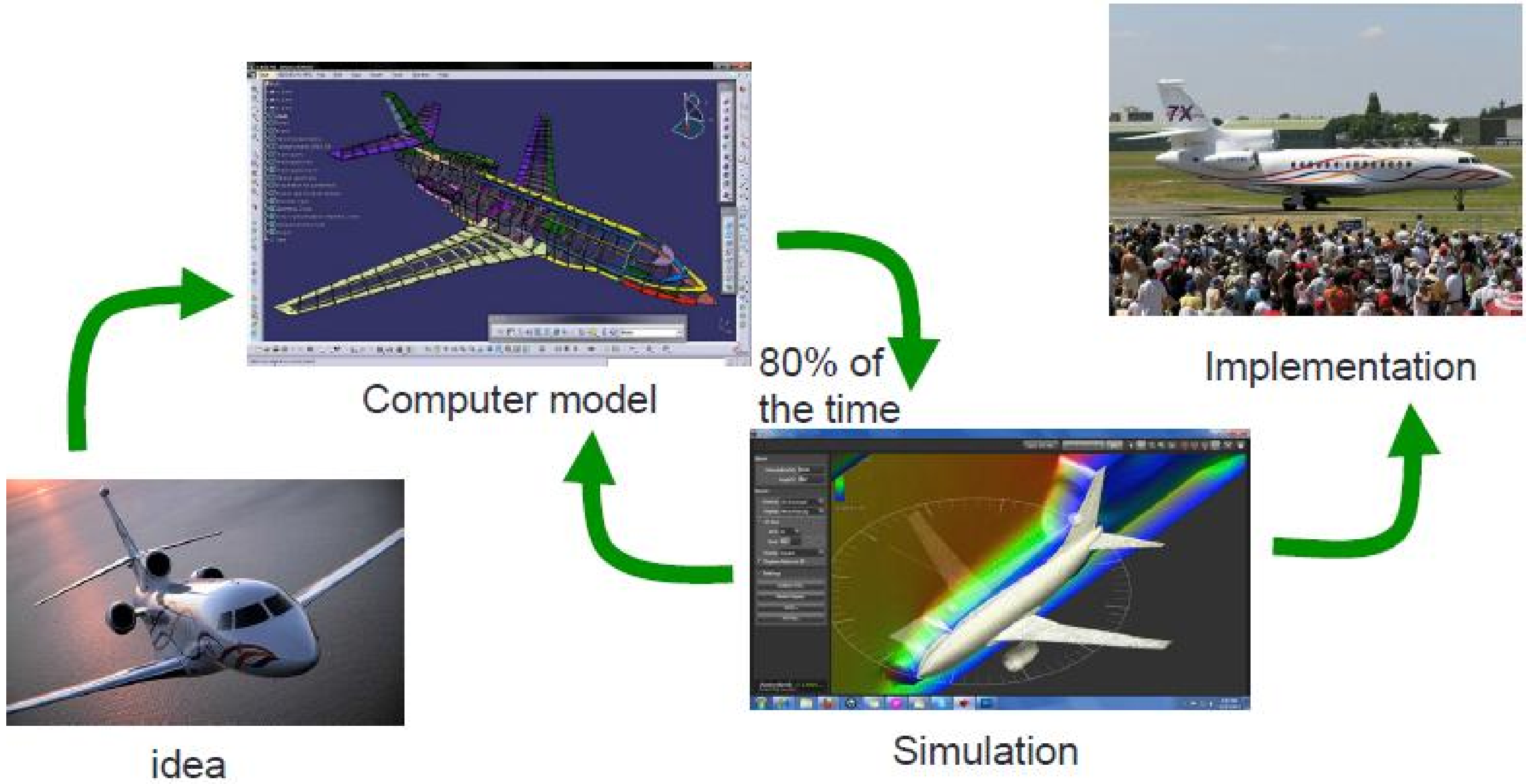
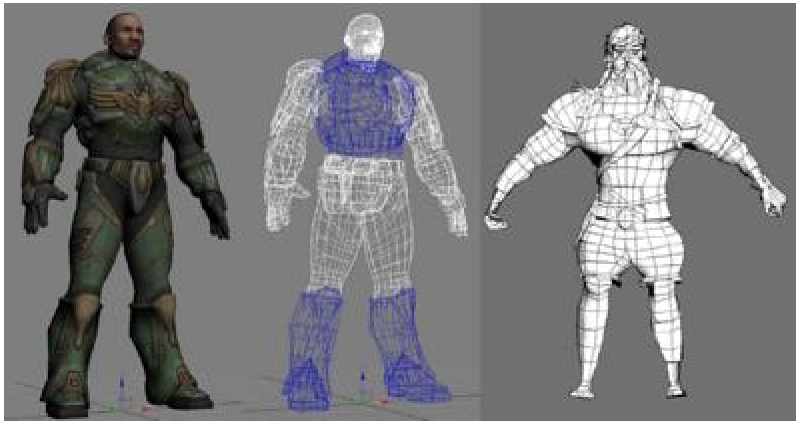


Digital Models

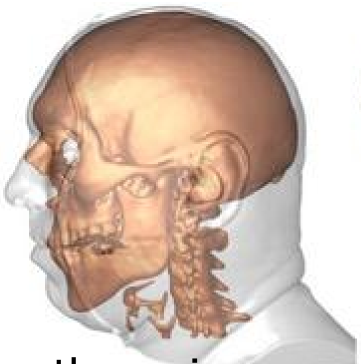
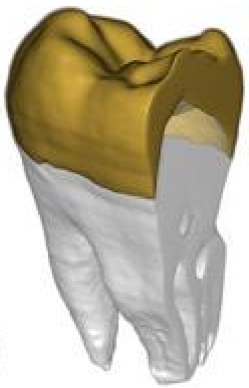
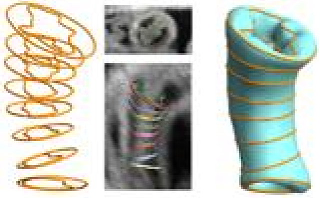
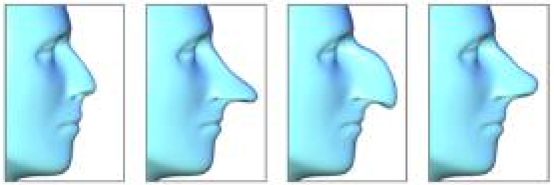
- ✓ Digital models are used in
  - ✓ video games



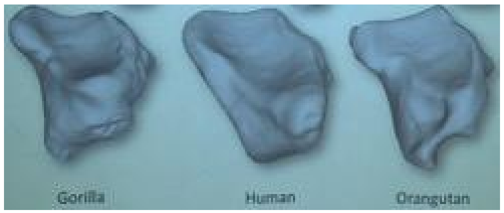
  - ✓ 3d cinema

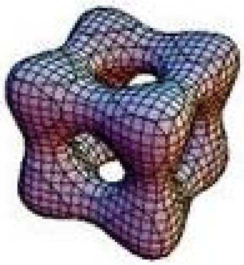
Digital Models

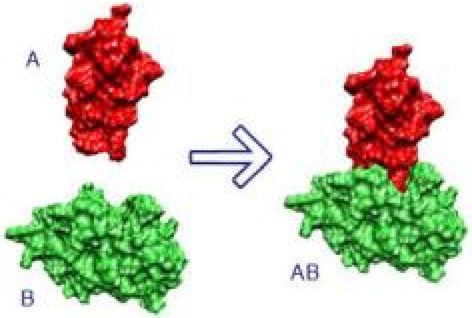
- ✓ Digital models are used in
  - ✓ medicine, esthetics

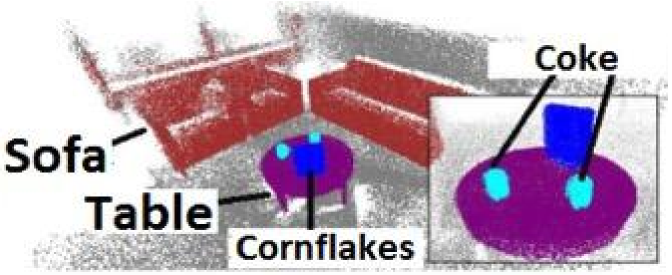
  - ✓ paleontology., math, other sciences



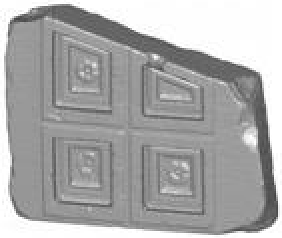


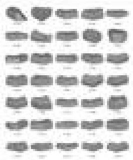
  - ✓ robotics, autonomous driving

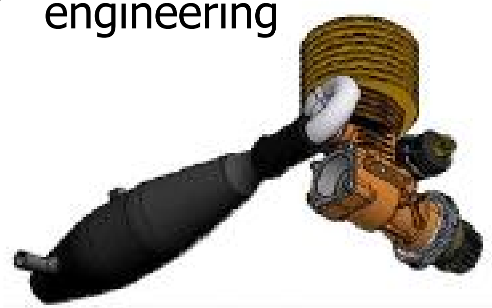
Digital Models

- ✓ Digital models are used in
  - ✓ cultural heritage (reconstruction, matching)

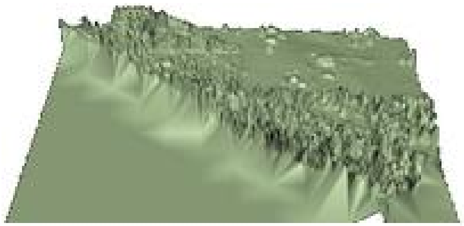




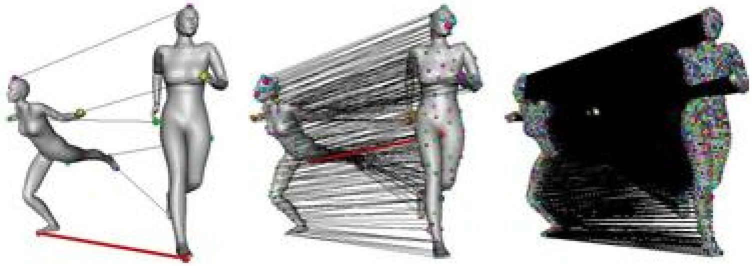
  - ✓ engineering



  - ✓ geological data



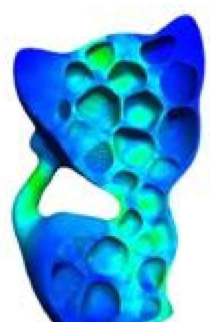
  - ✓ new digital model creation



Digital Models

- ✓ Digital models are used in
  - ✓ virtual shopping (AR)

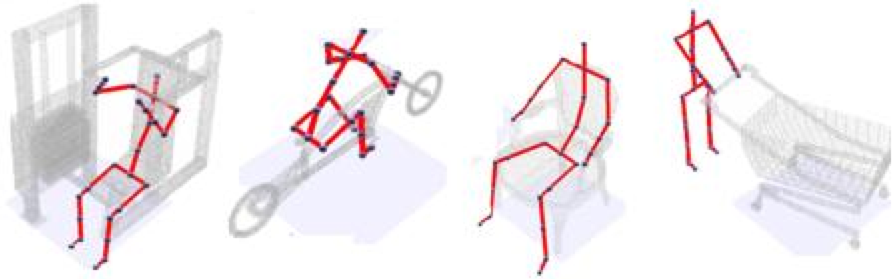
  - ✓ fabrication (3d printing)



Digital Models

- ✓ Digital models are used in
  - ✓ virtual reality (VR)

  - ✓ simulation (ergonomy etc.)

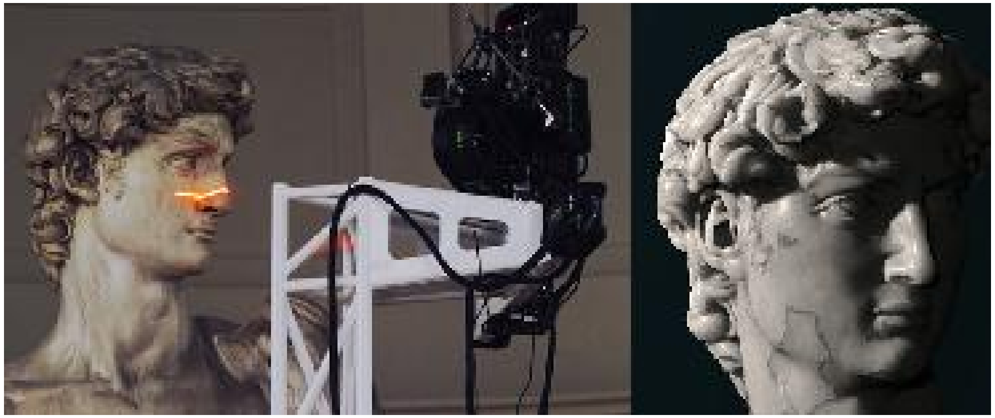


Digital Models

- ✓ Digital models are useful because
  - ✓ A digital model allows easy manipulation.
  - ✓ Digital simulation is much cheaper.
  - ✓ Model optimization and repair is possible.
  - ✓ Comparison across models is easy.
  - ✓ Creation of new models from other ones is easy.

Geometry Capture

- ✓ Static.



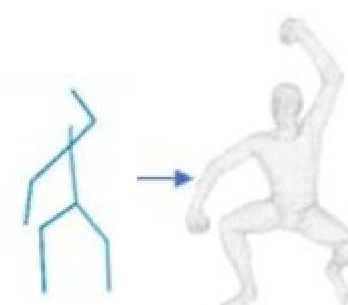
Geometry Capture

- ✓ Dynamic (performance capture).

Geometry Creation

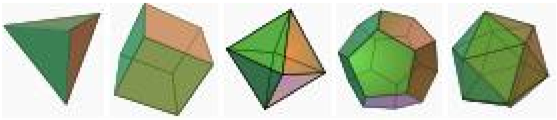
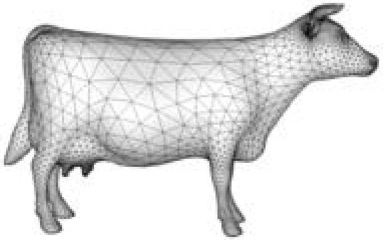
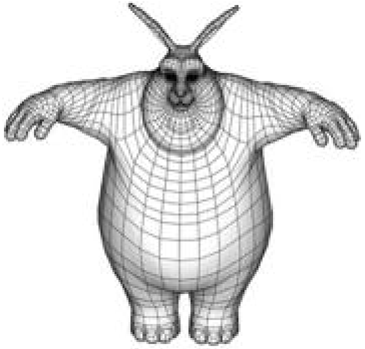
✓ Artists/interactive modeling.

✓ Automated tools.

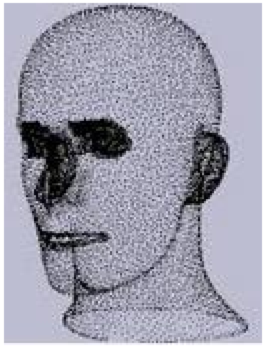
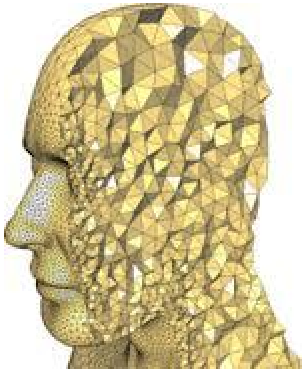
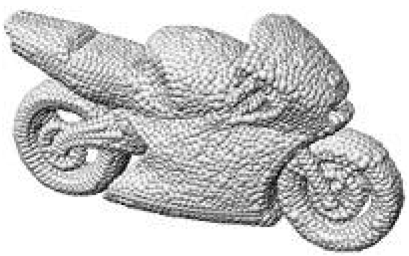


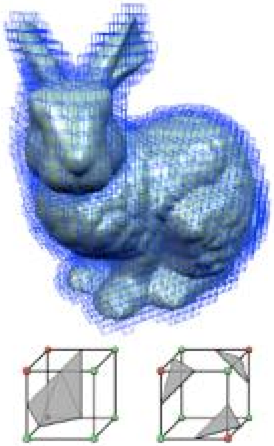
Geometry Representation

- ✓ We are interested in thin-shell surfaces, represented by polygon meshes: set of polygons representing a 2D *surface* embedded in 3D.

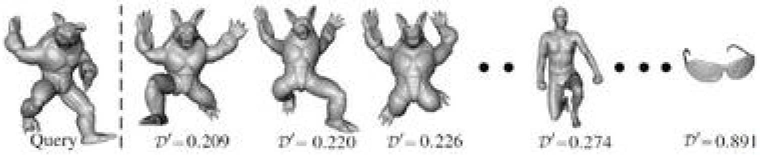
- ✓ Other representations.

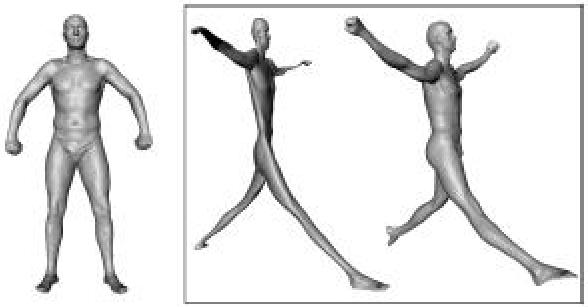


Polygon Mesh

- ✓ Why polygon meshes as discretization of surfaces?
  - ✓ Surface of a solid shape is sufficient for
    - ✓ Rendering
    - ✓ Adaptive refinement
    - ✓ Similarity comparison
    - ✓ New surface generation
    - ✓ Segmentation
    - ✓ Many other analyses

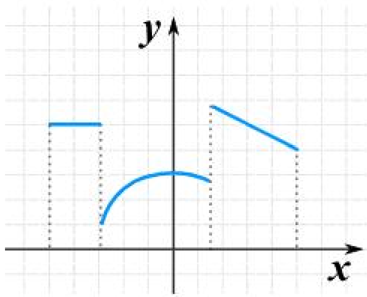


  - ✓ Not realistic for deformations though (use tetmeshes here).



Polygon Mesh

- ✓ Polygon meshes are piecewise linear surface representations.
- ✓ Analogous to piecewise functions:



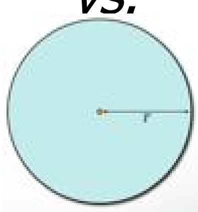
$$f(x) = \begin{cases} 6 & \text{if } x < -2 \\ x^2 & \text{if } x > -2 \text{ and } x \leq 2 \\ 10 - x & \text{if } x > -2 \end{cases}$$

- ✓ Think surface as (the range of) a “shape” function.

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathrm{I\!R}^2$$

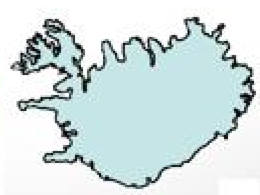
$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$

vs.



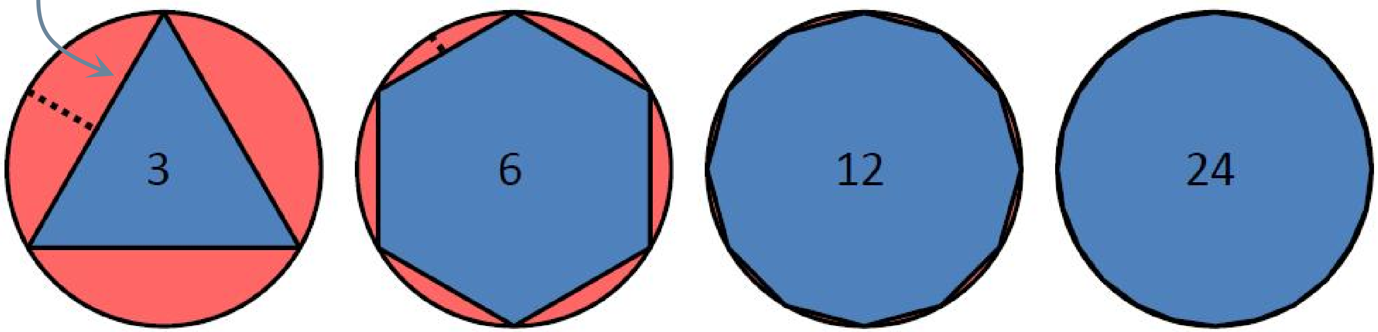
$$\mathbf{f} : [0, 2\pi] \rightarrow \mathrm{I\!R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} ? \\ ? \end{pmatrix}$$

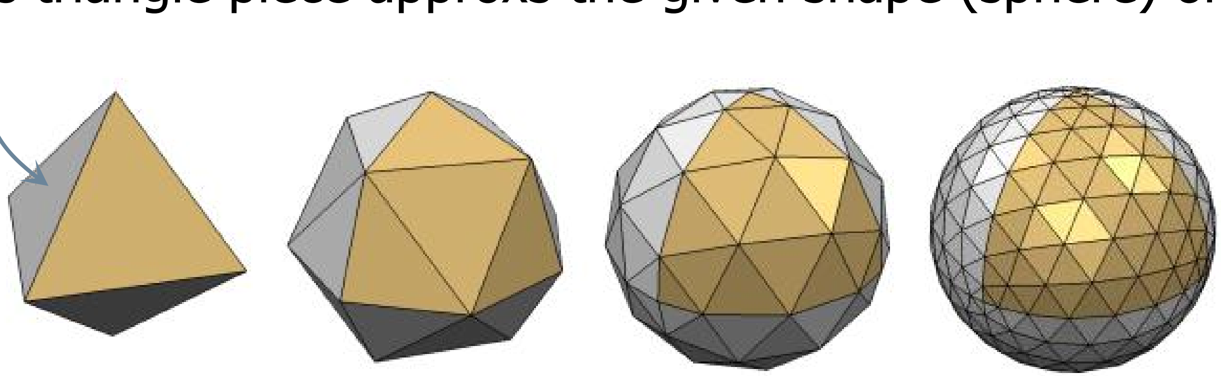


Polygon Mesh

- ✓ 1D: This line piece approximates the given shape (circle) only locally.

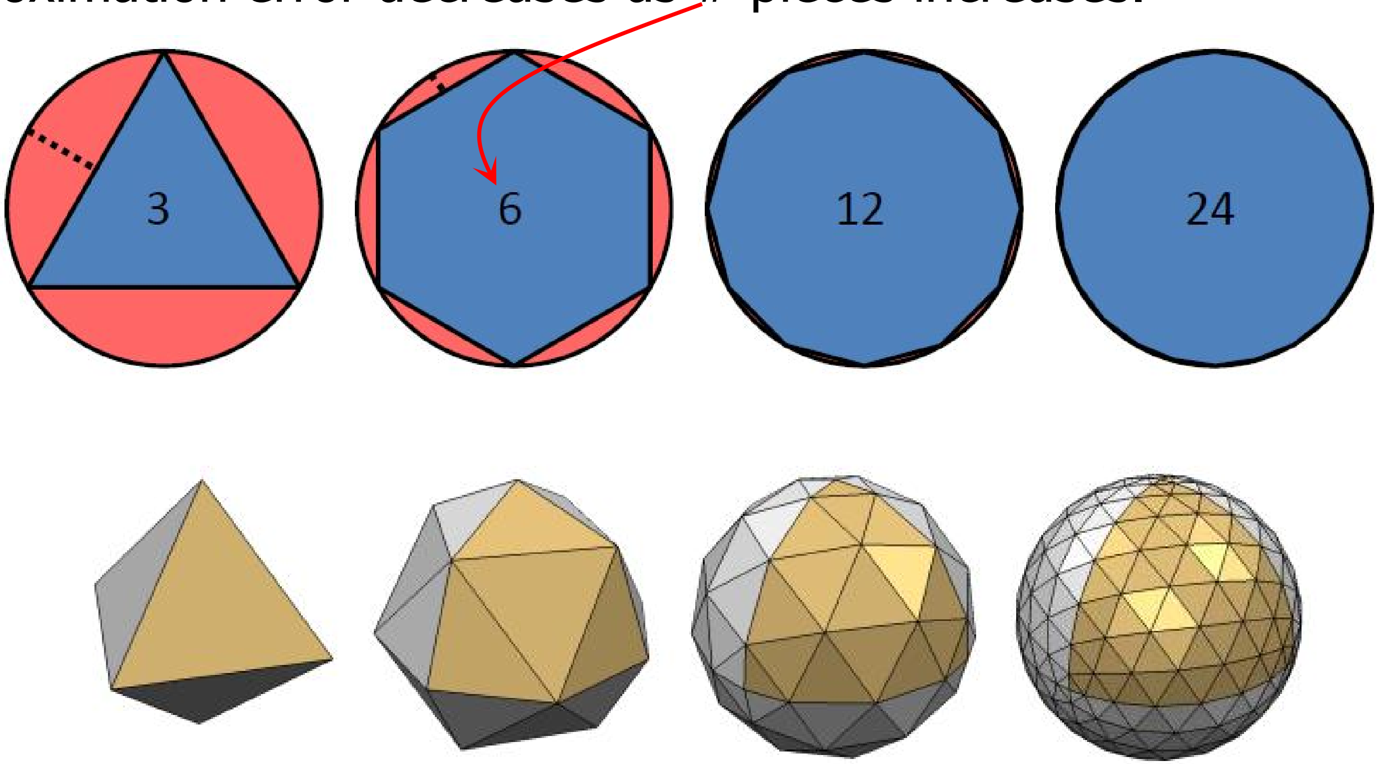


- ✓ 2D: This triangle piece approxs the given shape (sphere) only locally.



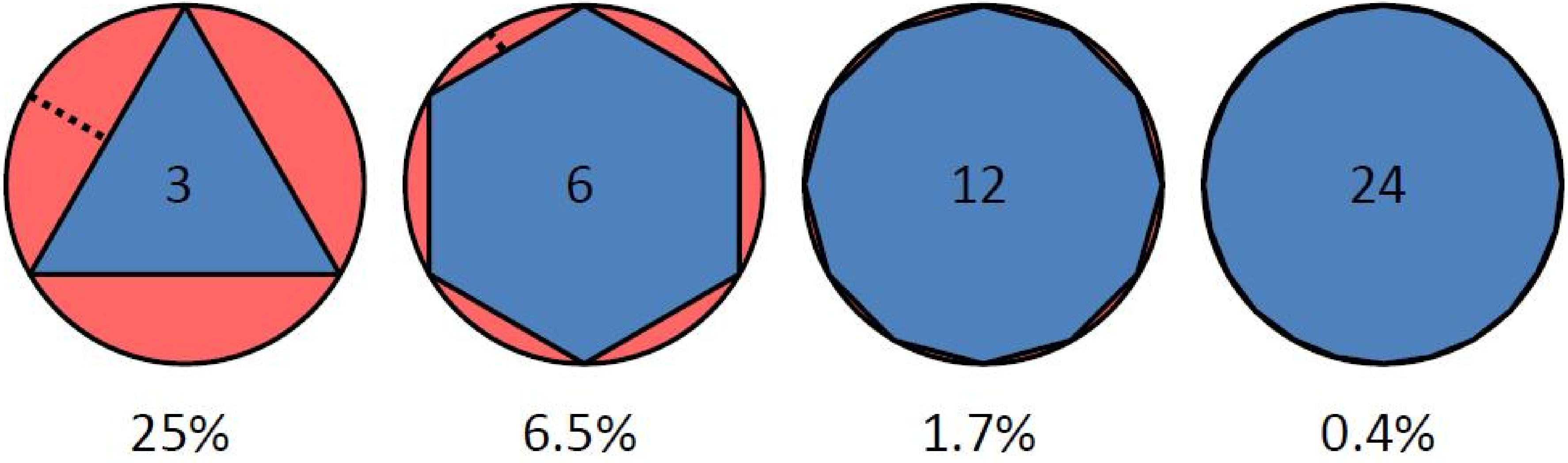
Polygon Mesh

✓ Approximation error decreases as # pieces increases.



Polygon Mesh

- ✓ Approximation error is quadratic.
  - ✓ As # pieces doubled, error decreases one forth.

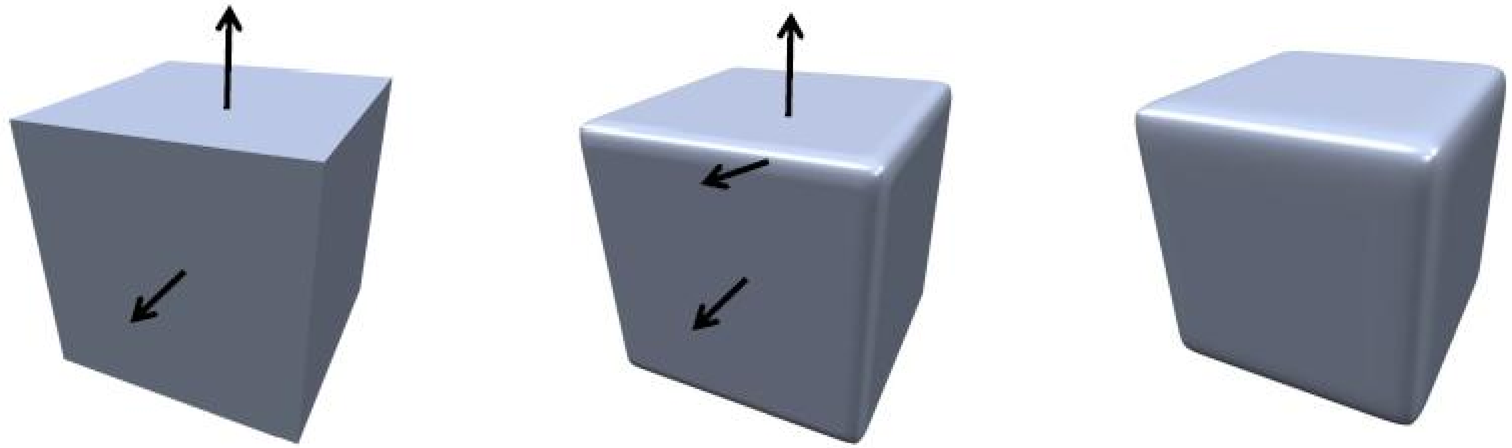


Polygon Mesh

- ✓ Polygon meshes are C^0 piecewise linear surface representations.

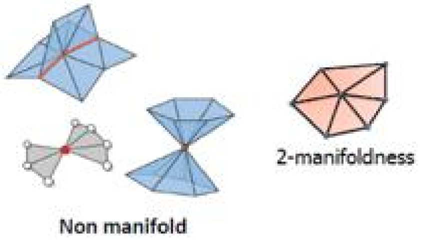
- ✓ Smoothness levels:

C^0: Position continuity C^1: Tangent continuity C^2: Curvature continuity



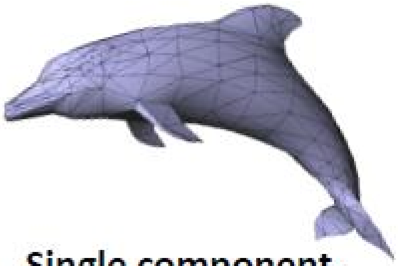
Polygon Mesh Types

- ✓ Manifold meshes: keep things simple.
  - ✓ Images: assume every pixel has 4 neighbors. Likewise, assume meshes are manifold. It keeps formulas simple and leads to fewer special cases in code.
    - ✓ Edges are contained in at most 2 polygonal faces.
    - ✓ Vertices are contained in disk of triangles.

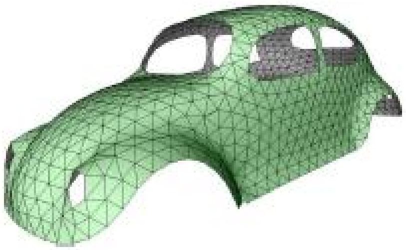


- ✓ Watertight meshes are 2-manifold meshes w/o boundary edges.
  - ✓ Suitable for simulations and 3D printing.
- ✓ No holes or non-manifold structures.
- ✓ Closed mesh (no boundary edges).
- ✓ Euler's formula (slide 36) applies.
- ✓ Imagine filling the inside of the mesh w/ water, would anything leak out? If not, then chances are the mesh is watertight.

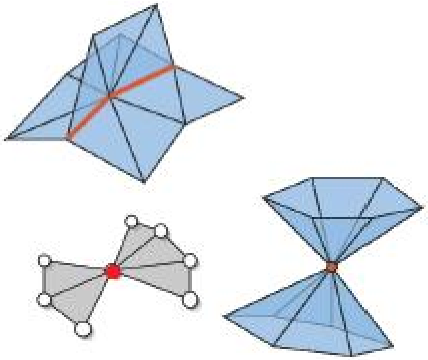
Polygon Mesh Types



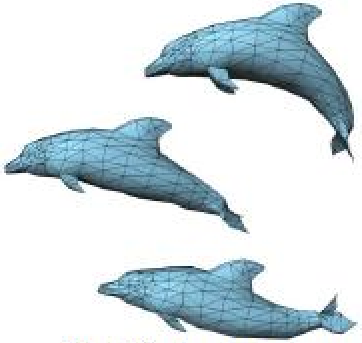
Single component, closed, triangular, 2-manifold



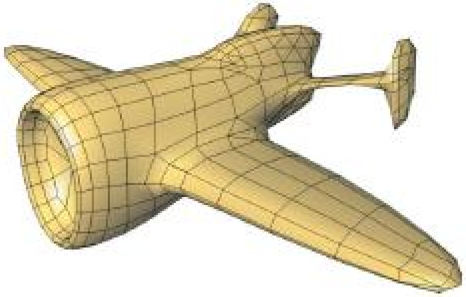
With boundaries 2-manifold



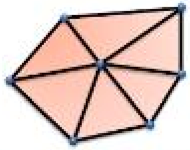
Non manifold



Multiple components 2-manifold



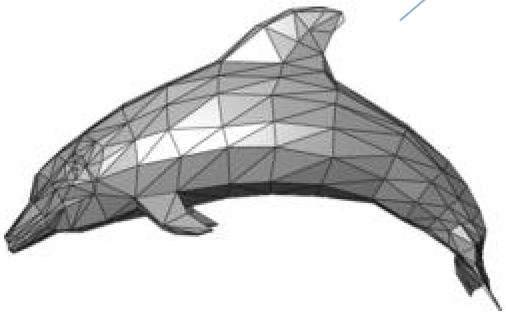
Not only triangles 2-manifold

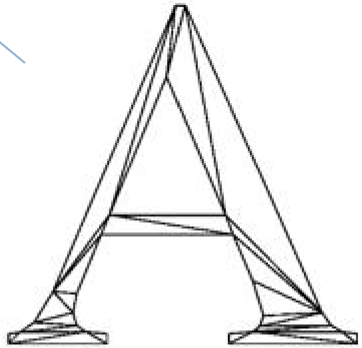


2-manifoldness

Triangle Meshes

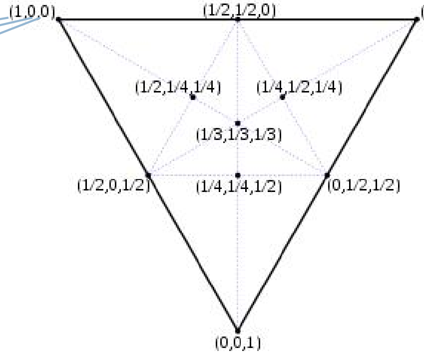
- ✓ Most common *piece* is triangles (quads come second).
- ✓ A set of triangles (embedded in 3D or 2D) that are connected by their common edges or corners.





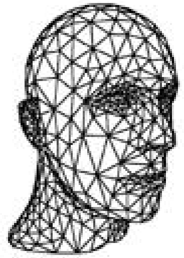
- ✓ Each triangle defines, via its barycentric parameterization, a segment of a piecewise linear surface representation, so that you can, e.g., interpolate function values at triangle vertices a, b, and c.

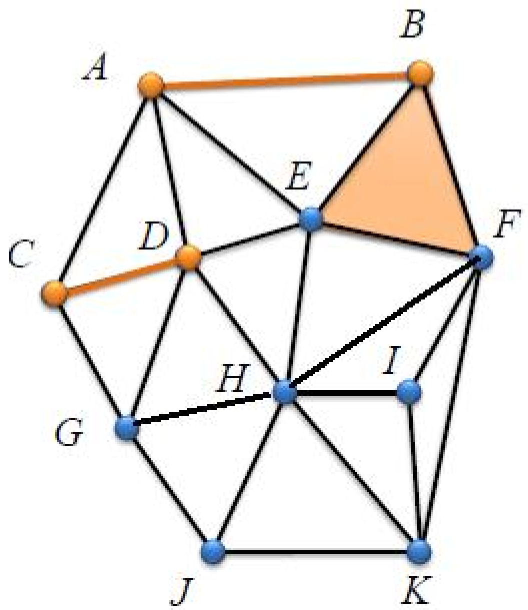
$$\mathbf{p} = \alpha\,\mathbf{a} + \beta\,\mathbf{b} + \gamma\,\mathbf{c},$$
$$\alpha + \beta + \gamma = 1, \quad \alpha, \beta, \gamma \geq 0.$$



Triangle Meshes

✓ An undirected graph, with triangle faces.





G = graph = $<V, E>$

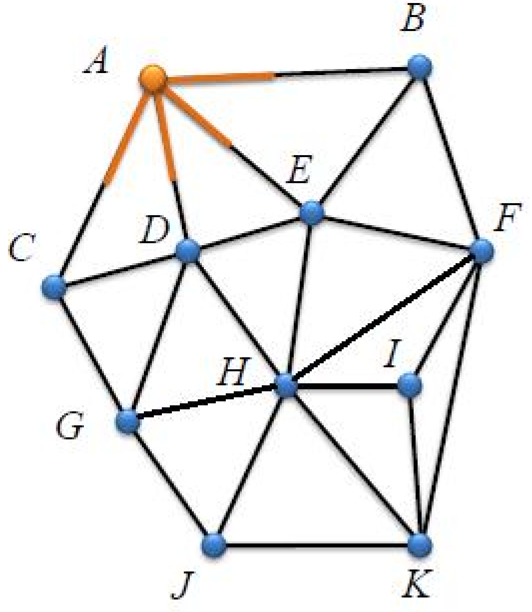
V = vertices = $\{A, B, C, \ldots, K\}$

E = edges = $\{(AB), (AE), (CD), \ldots\}$

F = faces = $\{(ABE), \quad \ldots\}$

Triangle Meshes

✓ An undirected graph, with triangle faces.

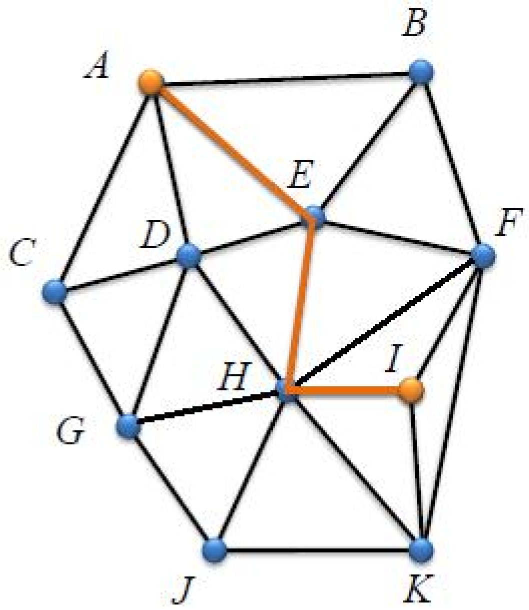


Vertex degree or valence = # incident edges
deg(A) = 4   deg(B) = 3

k-regular mesh if all vertex degrees are equal to k.

Triangle Meshes

✓ An undirected graph, with triangle faces.



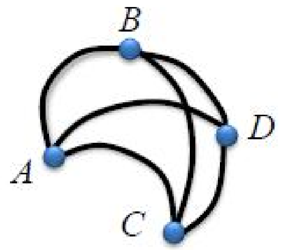
connected if every pair of vertices are connected by a path (of edges).

Triangle Meshes

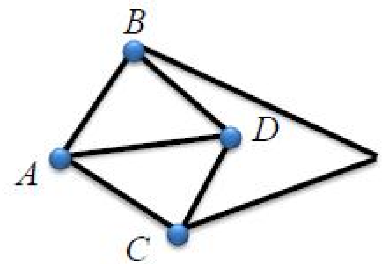
- ✓ A specific undirected graph: straight-line plane graph (embedded in 2/3D) where every face is a triangle, a.k.a. triangulation.

- ✓ Planar graph: graph whose vertices and edges can be embedded in 2D without intersecting edges.

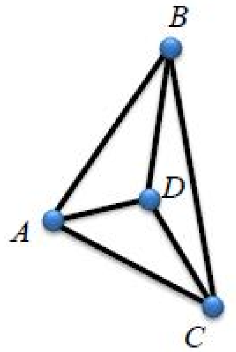
Planar graph



Plane graph

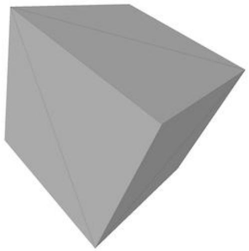


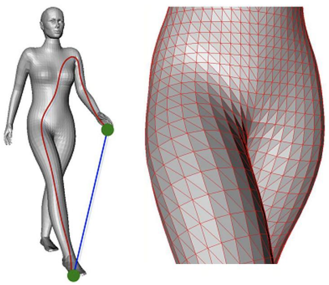
Straight-line plane graph



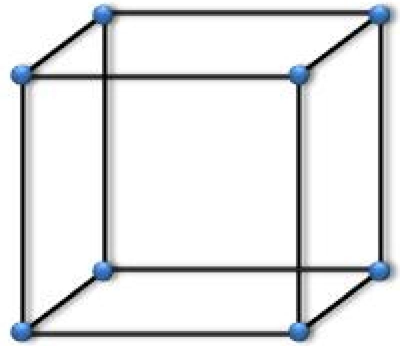
Mesh Statistics

- ✓ Euler formula for connected planar graphs help us derive mesh stats.
- ✓ Holds for triangle, quad, pentagon, .. faces, i.e., polygon faces.

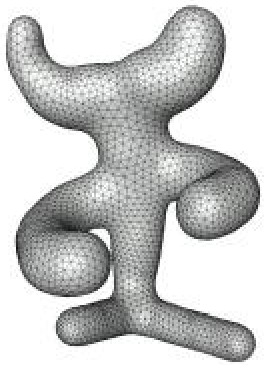




$$V - E + F = 2$$



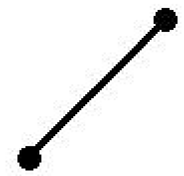
$$V = 8$$
$$E = 12$$
$$F = 6$$
$$\chi = 8 + 6 - 12 = \mathbf{2}$$

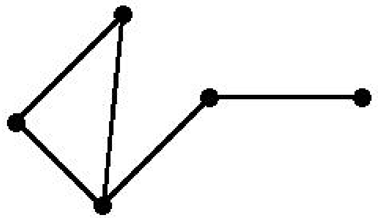


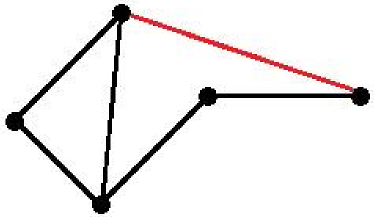
Triangle mesh is a planar triangulation embedded in R^3.

$$V = 3890$$
$$E = 11664$$
$$F = 7776$$
$$\chi = \mathbf{2}$$

Mesh Statistics

- ✓ Proof of Euler's formula: V – E + F = 2
- ✓ Proof: Induct on E, # edges.
- ✓ Base Case:  2 – 1 + 1 = 2 //holds ☺
- ✓ Inductive Step: Assume formula is True for planar subgraph with E edges. Show that it must also be T for planar graph with E+1 edges.





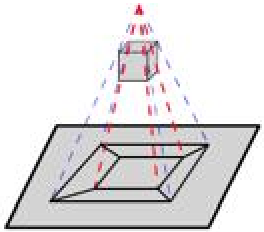
Add new (red) edge → V – (E + 1) + (F + 1) = 2 ☺ 'cos

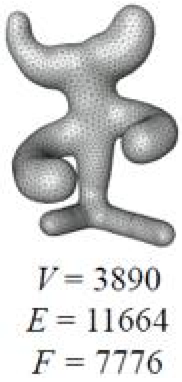
V – E + F = 2 by inductive hypothesis.

Mesh Statistics

- ✓ Based on Euler's formula, we can derive:
  - ✓ F ~ 2V
  - ✓ E ~ 3V
  - ✓ Average vertex degree is 6.

  for a closed triangular mesh.





- ✓ Proofs:
- ✓ For E ~ 3V: count 3 edges for each face → 3F = E

  this way each edge counted twice → 3F = 2E

  V – E + F = 2 → V – E + 2E/3 = 2 negligible →E ~ 3V

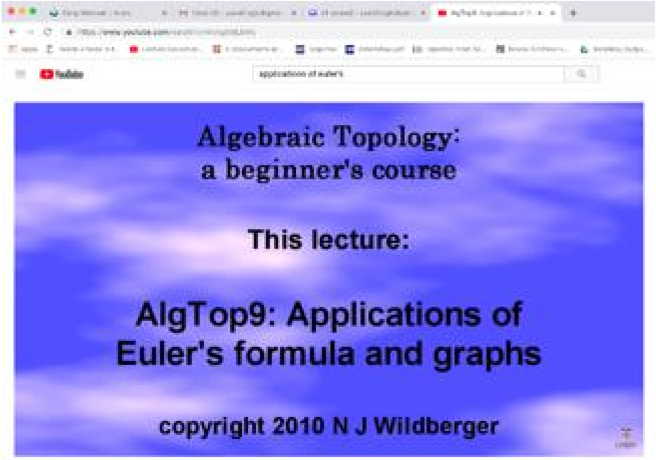
- ✓ For F ~ 2V: V – E + F = 2 → V – 3F/2 + F = 2 negligible → F ~ 2V

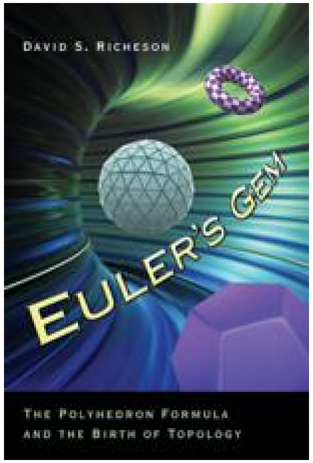
- ✓ For avg degree = sum_v deg(v) / V = 2E / V //by handshaking lemma

  = 6V / V = 6.

More on Euler's Formula

- ✓ There are lectures and books about this Euler's formula: V-E+F=2

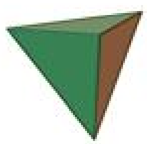
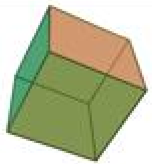
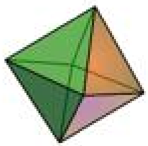
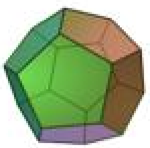
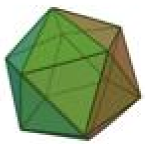




- ✓ Although it's about Topology, it still relates to our Geometry Processing class, e.g., for topology-invariant shape analysis.
- ✓ As applications, let's see how it helps to prove some cool theorems.

More on Euler’s Formula

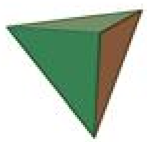
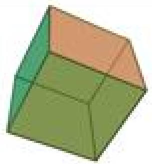
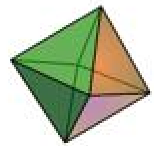
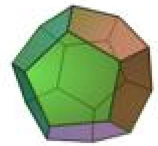
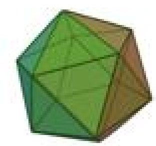
- ✓ Theorem: there are at most 5 Platonic Solids.

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
				

- ✓ Named after Greek philosopher Plato.
- ✓ *Congruent* (identical in shape and size), *regular* (all angles/sides equal) polygonal faces with the same number of faces meeting at each vertx.

More on Euler's Formula

- ✓ Theorem: there are at most 5 Platonic Solids.

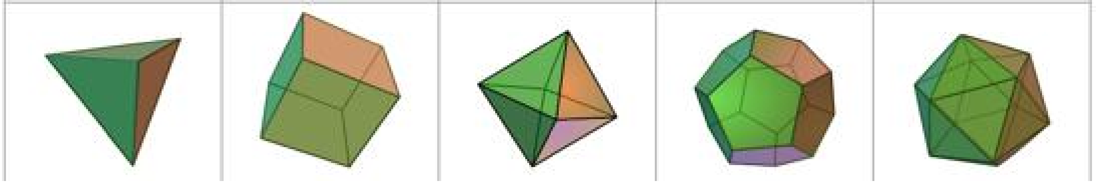
Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces
				

- ✓ Proof: n = # of edges on each face, m = # of edges meeting at a vert.
- ✓ F x n counts each edge twice ➔ E = F x n / 2
- ✓ F x n counts each vertex too many times ➔ V = F x n / m
- ✓ V – E + F = 2 ➔ Fn/m – Fn/2 + F = 2 ➔ F = 4m / (2n – mn + 2m)
- ✓ 2n – mn + 2m > 0 'cos we want a +ve # of faces ➔ 2n > m (n - 2)
- ✓ n, m >= 3 'cos every face of a polyhedron has at least 3 edges (triangular) and each vertex appears in at least 3 edges (closed).
- ✓ 2n / (n-2) > m >= 3 ➔ 2n > 3n – 6 ➔ n < 6, by symmetry m < 6
- ✓ (n,m) = {(3,3), (3,4), (3,5), (4,3), (5,3)}, e.g., first one is tetrahedron.

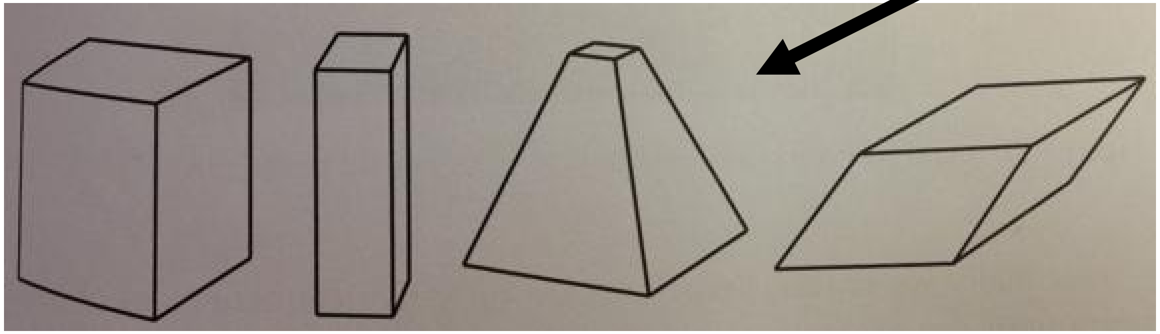
More on Euler's Formula

- ✓ Theorem: there are at most 5 Platonic Solids.

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
Four faces	Six faces	Eight faces	Twelve faces	Twenty faces



- ✓ We gave a topological/combinatorial proof that did not take geometry into account: side lengths and angle measures (regularity) ignored.
- ✓ We proved that our Platonic Solids *resemble* Tet, Cube, Oct, Dod, Ico.



- ✓ See Euclid's geometrical proof forthe existence of *exact* Platonic Solids.

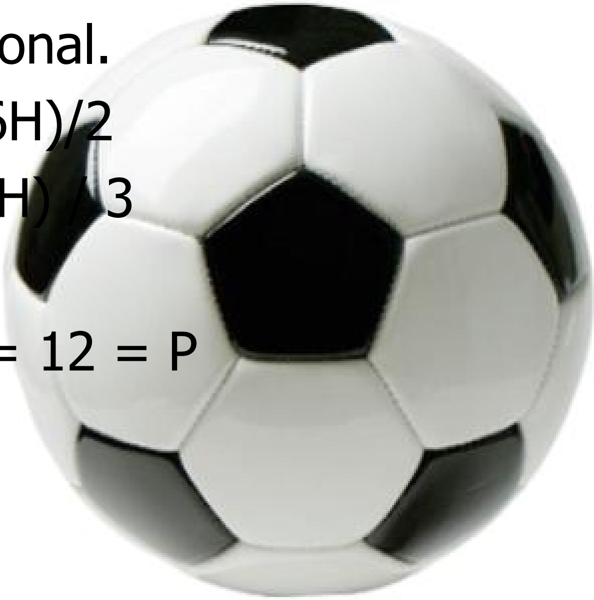
More on Euler's Formula

- ✓ Theorem: if every face of a polyhedron is a pentagon or a hexagon and if the degree of every vertex is 3 (soccer ball), then the polyhedron has exactly 12 pentagonal faces.

More on Euler's Formula

- ✓ Theorem: if every face of a polyhedron is a pentagon or a hexagon and if the degree of every vertex is 3 (soccer ball), then the polyhedron has exactly 12 pentagonal faces.
- ✓ Proof: P = # of pentagonal faces, H = hexagonal.
- ✓ 5P + 6H counts each edge twice ➔ E=(5P+6H)/2
- ✓ 5P + 6H counts each vert thrice ➔ V=(5P+6H) / 3
- ✓ V – E + F = 2 ➔ =(5P+6H) / 3 - (5P+6H)/2

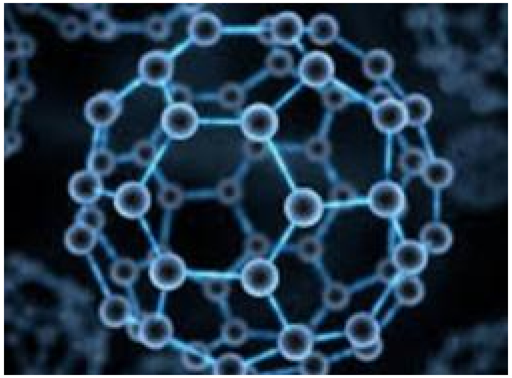
  + (P+H) = 2 ➔ 10P+12H-15P-18H+6P+6H = 12 = P



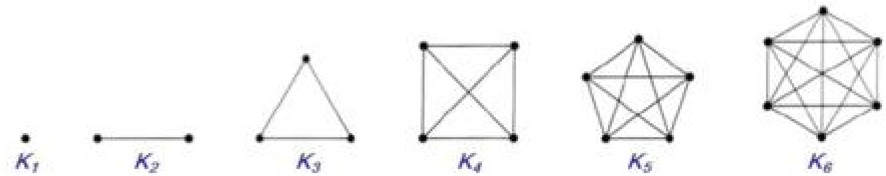
More on Euler's Formula

- ✓ Theorem: if every face of a polyhedron is a pentagon or a hexagon and if the degree of every vertex is 3 (soccer ball), then the polyhedron has exactly 12 pentagonal faces.

- ✓ Rest assured, a golf ball, fullerene (carbon), or soccer ball is not a sixth Platonic Solid: close inspection reveals pentagon/hexagon hybrid.



More on Euler’s Formula



- ✓ Theorem: K_5 is not planar.
- ✓ Proof: suppose it is. Then,

  V – E + F = 2 ➔ 5 – 10 + F = 2 ➔ F = 7 (including exterior).
- ✓ Closed triangle mesh: Each edge borders 2 faces, every face has *exactly* 3 edges ➔ 3F counts each edge twice ➔ 3F = 2E (slide 38).
- ✓ K_5: Each edge borders 2 faces, every face has *at least* 3 edges ➔ 3F <= 2E ➔ 21 <= 20 is a contradiction so K_5 is not planar.

  3F = 2E //each face has 3 edges

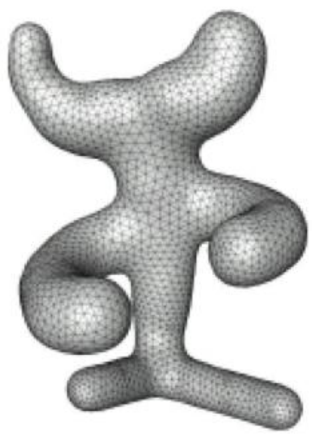
  4F = 2E //each face has 4 edges

  5F = 2E //each face has 5 edges

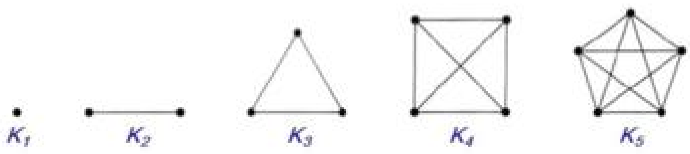
  .

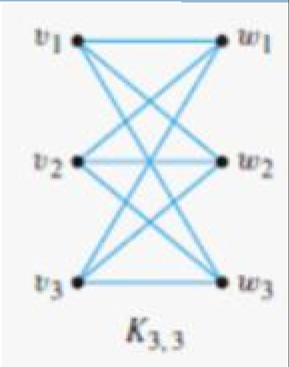
  .

  So, 2E is at least 3F ➔ 3F <= 2E



More on Euler's Formula

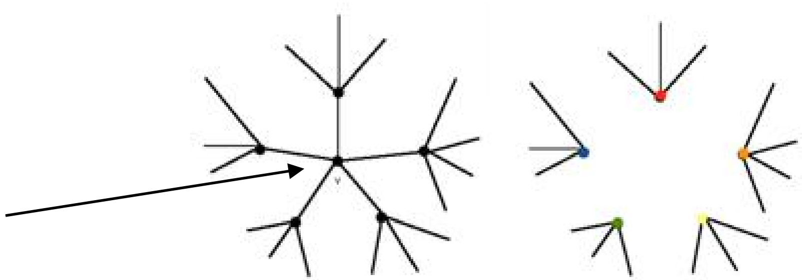


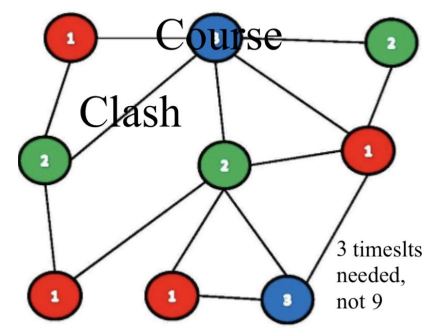


- ✓ Theorem: K_5 is not planar.
- ✓ Theorem: $K_{3,3}$ is not planar.
- ✓ Why do we care?
- ✓ Theorem (Kuratowski): Graph G is planar iff it does not have K_5 or $K_{3,3}$ as a subgraph.
- ✓ So regardless how large G is, you can reason about its planarity fastly.

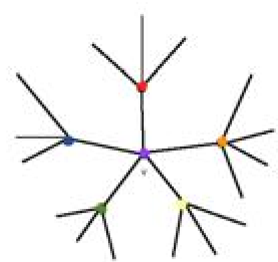
More on Euler's Formula

- ✓ Theorem (6-color): Every connected planar G can be properly colored w/ 6 or less colors.
  - ✓ 5-color proved (harder). 4-color also proved via computers (controversial).
- ✓ Proof: Induct on V, # vertices.
- ✓ Base: 1 <= V <= 6 colored by assigning a different color to each vert.
- ✓ Inductive Step: Every connected planar G contains a vertex v of degree 5 or less (proof next slide via Euler). Remove v. Resulting sub-graph can be colored w/ 6 or less colors (induction), so color it.





- ✓ Now add v back using the 6th color:



More on Euler's Formula

- ✓ Theorem: Every connctd planar graph has a vertex of degree 5 or less.
- ✓ Proof: enlarge graph by adding edges so all faces are triangles and still planar.

Exterior face has 3 curvy edges.

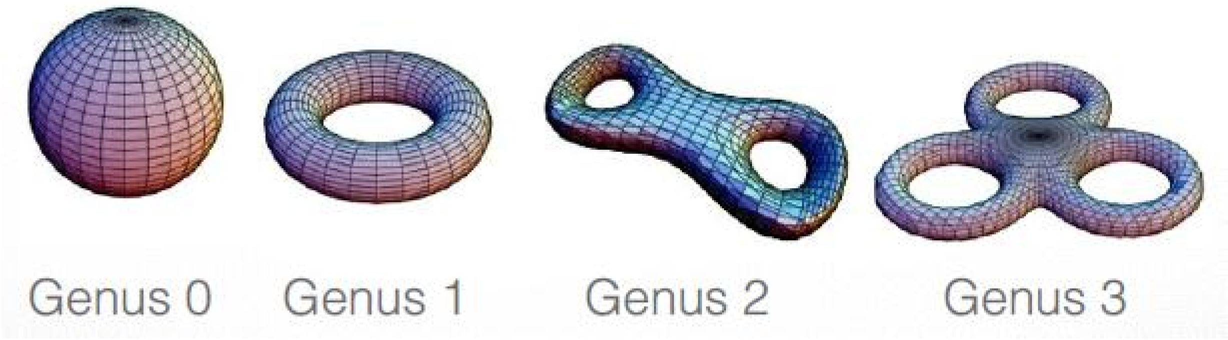
- ✓ Proving that enlarged G has a vert v with degree 5- does the job 'cos we just added edges, meaning that v must exist in the original G (with a less than or equal to degree value compared to the v in enlarged G.)
- ✓ $3F = 2E$ (slide 38) ➔ $V - E + F = 2$ ➔ $V - 2 = E - F$ ➔ $6V - 12 = 6E - 6F$ ➔ $6V - 12 = 2E$.
- ✓ Sum of all degrees is 2E (handshaking). So, average degree is $2E / V = (6V - 12) / V = 6 - 12/V < 6$.
- ✓ Average degree is less than 6 ➔ there must be 1+ vert w/ degree 5-.

More on Euler's Formula

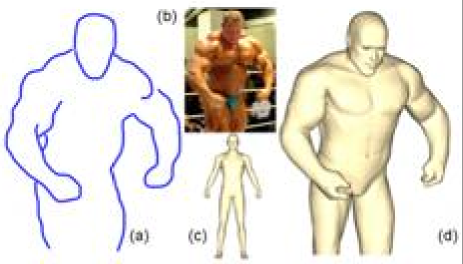
✓ Generalized Euler Formula:

$$V - E + F = 2(1-g)$$

where g is the genus of the surface, i.e., # handles of an object.



Some Cool DGP videos



- ✓ Modeling.
  - ✓ Zoomorphic: https://youtu.be/0gWomNI9CuI
  - ✓ Blending: https://youtu.be/U_XfYzy2c9w or https://youtu.be/WWLHPKsExaI
  - ✓ PCA: https://youtu.be/nice6NYb_WA
  - ✓ Printing: https://youtu.be/_drZksLRx94 or https://youtu.be/kFkD45NUIzQ
  - ✓ Segmenting: https://youtu.be/KTBdmY4U5Yw or https://youtu.be/aahC7i7FpTc
  - ✓ Multi-resolution: https://youtu.be/tTEGDPHv_AI
  - ✓ Image-guided: https://youtu.be/bmMV9aJKa-c
- ✓ 3D-supported Image Editing.
  - ✓ Sweeping & PCA: https://youtu.be/Oie1ZXWceqM & /9LJ-Gn5BM7A
- ✓ Deformation.
  - ✓ Energy-based: https://youtu.be/dERjpAaoNjk or youtu.be/F0ijAzOOWR4 https://youtu.be/QgrQuBwlbSE or /8C3uZOXLBIA or /bmMV9aJKa-c
  - ✓ Physically-based: https://youtu.be/CCIwiC37kks
- ✓ Exploration and organization of large 3D collections.
  - ✓ https://youtu.be/cmmCVrbgpnU