CENG 789 – Digital Geometry Processing

06- Surface Reconstruction,

Scientific Visualization and Information Visualization

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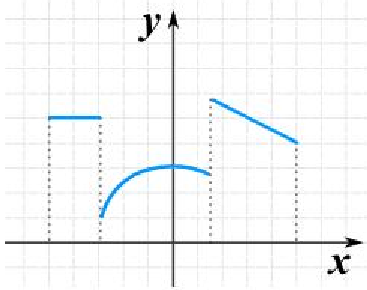
Explicit Representation

- ✓ RECALL: Polygon meshes are piecewise linear surface representatins.
- ✓ Analogous to piecewise functions:



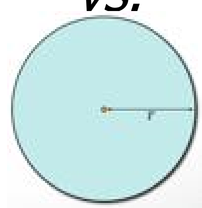
$$f(x) = \begin{cases} 6 & \text{if } x < -2 \\ x^2 & \text{if } x > -2 \text{ and } x \le 2 \\ 10 - x & \text{if } x > -2 \end{cases}$$

- ✓ Think surface as (the range of) a "shape" function.

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathrm{I\!R}^2$$

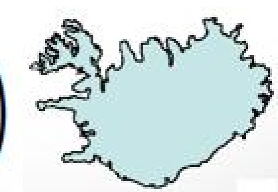
$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$

vs.



$$\mathbf{f} : [0, 2\pi] \rightarrow \mathrm{I\!R}^2$$

$$\mathbf{f}(t) = \begin{pmatrix} ? \\ ? \end{pmatrix}$$



Explicit Surface Representation

- ✓ For varying t, we get 2D shape (circle) points: t in [0, 2pi].
- ✓ Parameterization $\mathbf{f}$ maps a 1D parameter domain, e.g., [0, 2pi], to the curve (surface) embedded in R^2 (R^3).

$$\mathbf{f} : [0, 2\pi] \rightarrow \mathrm{I\!R}^2$$

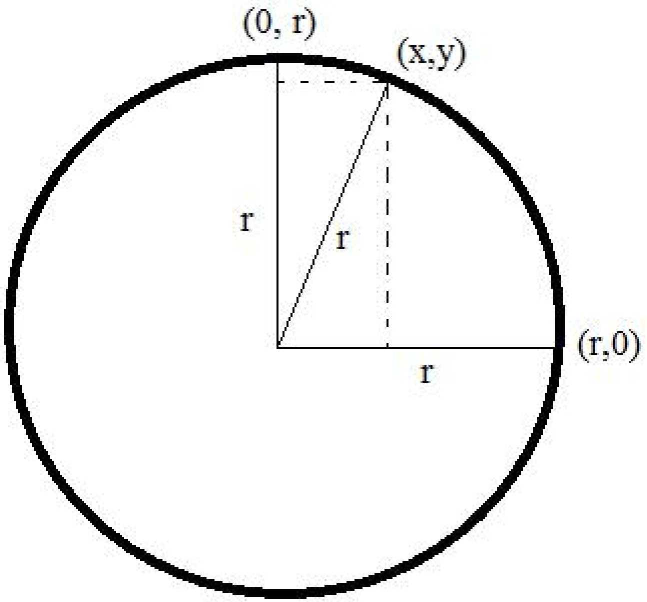
$$\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$$

Implicit Surface Representation

- ✓ Implicit or indirect surface representation.
- ✓ Surface is defined to be the zero-set of a scalar-valued function F.
  - ✓ That is, we have $F : \mathbb{R}^2 \to \mathbb{R}$, and then surface $\mathcal{S} = \{\mathrm{x} \in \mathbb{R}^2 \mid F(\mathrm{x}) = 0\}$.
  - ✓ Circle of radius r: $(x, y) \mapsto \sqrt{x^2 + y^2} - r$

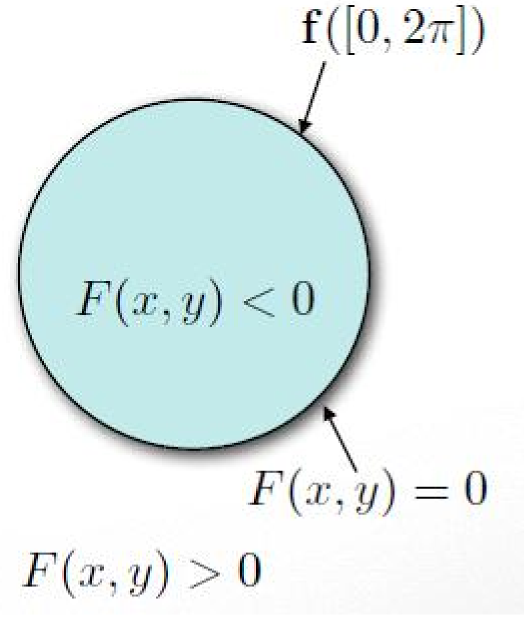
  is defined by (x,y) pairs

  that make the right-hand-side 0



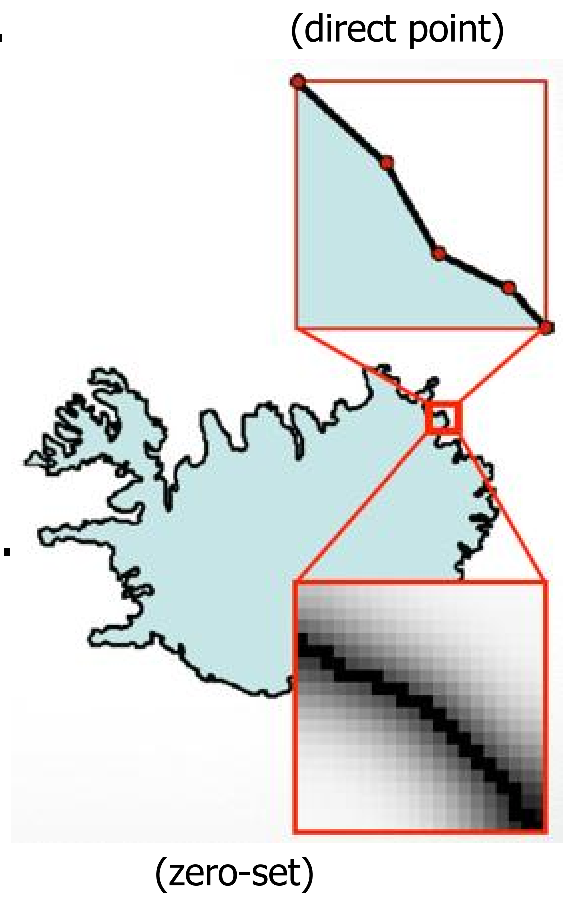
Explicit vs. Implicit

- Explicit: $\mathbf{f}(t) = \begin{pmatrix} r\cos(t) \\ r\sin(t) \end{pmatrix}$
  - Range of a parameterization function.
- Implicit: $F(x,y) = \sqrt{x^2+y^2} - r$
  - Kernel of an implicit function.



Explicit vs. Implicit

- ✓ Explicit: for non-mathematical/arbitrary shapes.
  - ✓ Range of a parameterization function.
  - ✓ Piecewise approximation.
- ✓ Implicit: for non-mathematical/arbitrary shapes.
  - ✓ Kernel of an implicit function.
  - ✓ Piecewise approximation.



Explicit vs. Implicit

- ✓ Explicit:
  - ✓ Range of a parameterization function.
  - ✓ Piecewise approximation.
  - ✓ Splines, triangle meshes, points.
  - ✓ Easy rendering.
  - ✓ Easy geom. modif. (deformation or new t).
- ✓ Implicit:
  - ✓ Kernel of an implicit function.
  - ✓ Piecewise approximation.
  - ✓ Scalar-valued 3D grid (mesh lies here).
  - ✓ Easy in/out test (just evaluate F).
  - ✓ Easy topology modification.

(a) The line in $\mathbb{R}^2$ through the points $(1, 2)$ and $(2, 3)$.

explicit: $(1, 2)$, $(2, 3)$
implicit: $x - y = -1$

(b) The point $(0, 1, 2)$ in $\mathbb{R}^3$.

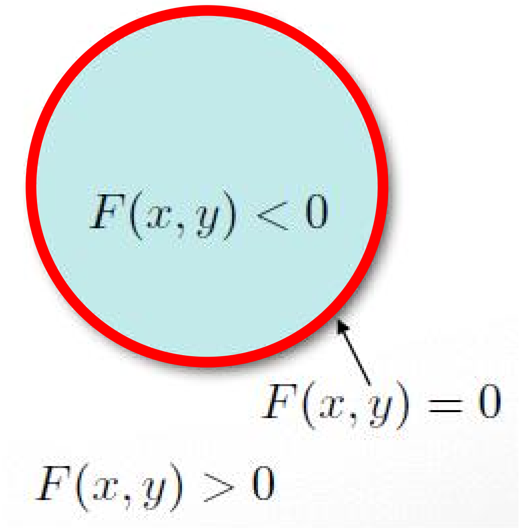
explicit: $(0, 1, 2)$
implicit: $x = 0 \wedge y = 1 \wedge z = 3$

(c) The plane in $\mathbb{R}^3$ through the point $(0, 1, 1)$ and pe
perpendicular to the vector $(2, 1, 0)$.

explicit: $(0, 1, 0)$, $(0, 1, 1)$, $(-1, 3, 0)$
implicit: $2x + y = 1$

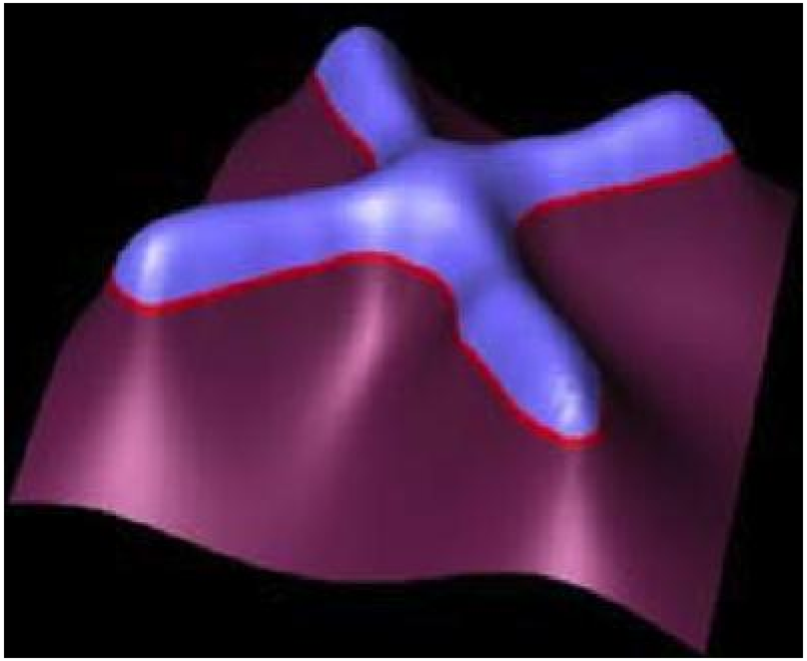
Implicit/Indirect Surfaces

- ✓ Level-set of a function defines the shape.
- ✓ Level means function values are the same = at the same level. Traditionally, level is 0, hence 0-set of the function F is sought.



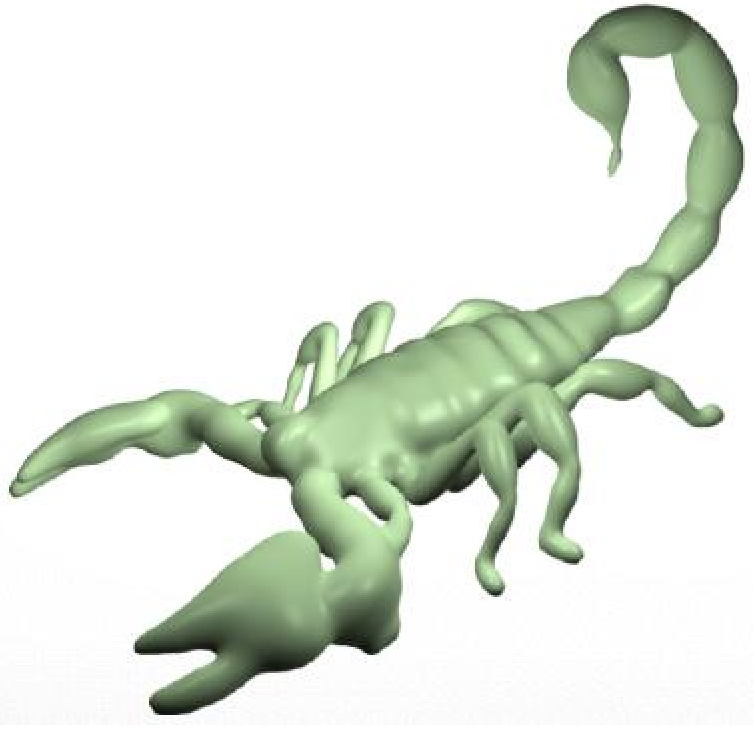
Implicit Surfaces

- ✓ Level-set of 2D function F(x, y) defines 1D curve (embedded in 2D).



Implicit Surfaces

- ✓ Level-set of 3D function F(x, y, z) defines 2D surface (embedded in 3D).

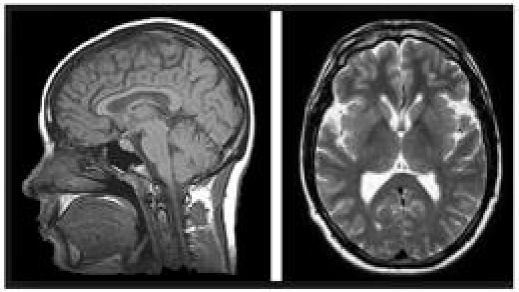


Implicit Function F

- ✓ Most common and natural representation is Signed Distance Function.
- ✓ Maps each 3D point x to its signed distance $F(\mathrm{x})$ from the surface.

$$F : \mathbb{R}^3 \to \mathbb{R}$$

- ✓ Sign indicates inside/outside. $F(\mathrm{x})=0$ means x is on the surface.
- ✓ Other than Signed Distance Function (SDF), continuous intensity function from CT/MRI scans are also common. Here, we will focus on the computation of SDF as it is a geometric problem. CT/MRI devices provide intensity info with their underlying radiologic technology.



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- ✓ Other than Signed Distance Function (SDF), continuous intensity function from CT/MRI scans are also common. Here, we will focus on the computation of SDF as it is a geometric problem. CT/MRI devices provide intensity info with their underlying radiologic technology.
  - ✓ Intensity: amount of vibration caused by magnets in MRI (no radiation).
  - ✓ Intensity: x-rays (x radiation) are beamed & received along the tunnel in CT.

Implicit Function F

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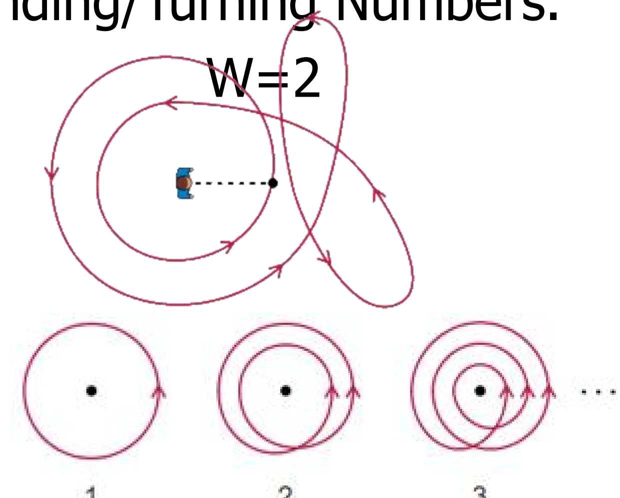
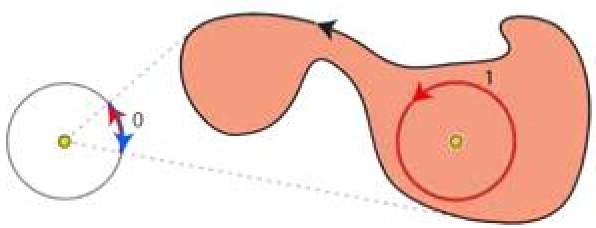
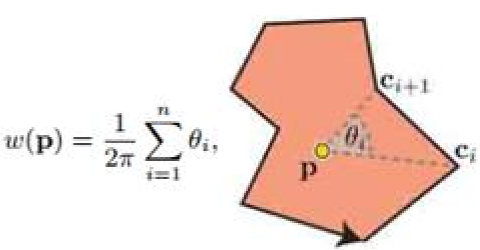
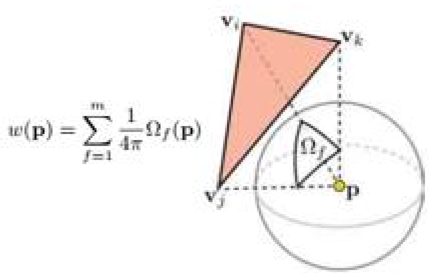
- ✓ Sign indicates inside/outside. $F(\mathrm{x})=0$ means x is on the surface.

- ✓ Another function for inside/outside test: Winding/Turning Numbers.









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Implicit Function F

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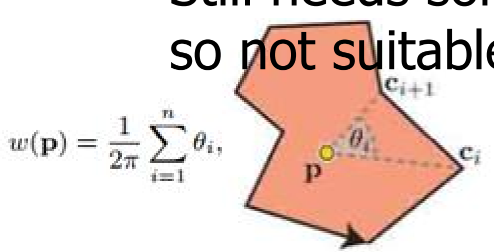
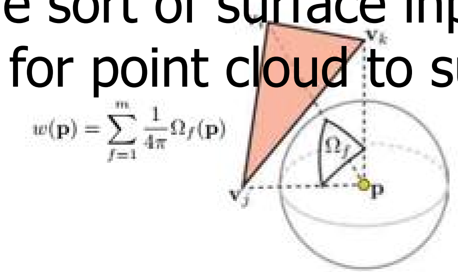
- ✓ Sign indicates inside/outside. $F(\mathrm{x})=0$ means x is on the surface.

- ✓ Another function for inside/outside test: Winding/Turning Numbers.
- ✓ Originally defined for closed curves (2D) and surfaces (3D).
- ✓ Made robust to self-intersections, open boundaries, and non-manifold pieces by Jacobson et al., 2013.
- ✓ Still needs some sort of surface input (tests point in/out of a surface), so not suitable for point cloud to surface conversion.





Implicit Function F

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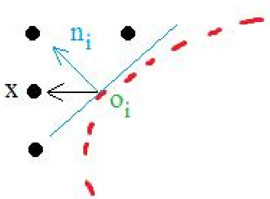
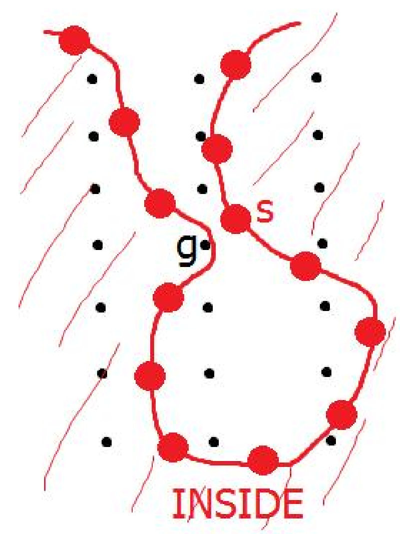
- ✓ Sign indicates inside/outside. $F(\mathrm{x})=0$ means x is on the surface.

- ✓ Another function for inside/outside test: Winding/Turning Numbers.
  - ✓ More robust than a signed distance function.
  - ✓ Although inside, grid point g will be wrongly treated as outside as it is coupled w/ the closest sample, which yields a positive signed distance (see slide 39).
  - ✓ No such problem w/ Winding Number test.



$$F(\mathrm{x}) = (\mathrm{x} - \mathrm{o}_i) \cdot \mathrm{n}_i$$



Implicit Function F

- ✓ Most common and natural representation is Signed Distance Function.
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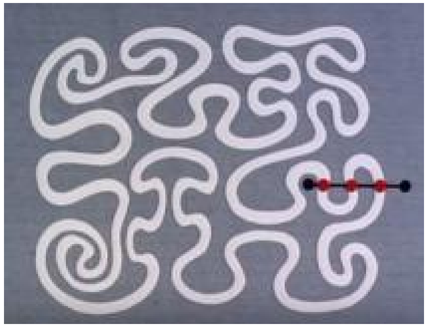
$$F : \mathbb{R}^3 \rightarrow \mathbb{R}$$

- ✓ Sign indicates inside/outside. $F(\mathrm{x})=0$ means x is on the surface.

- ✓ Another function for inside/outside test of a closed surface: ray count.
- ✓ Shooting a ray from the point and count # of times it intersects the surface: odd number means the point is inside, even means outside.



Implicit Function F

- ✓ Most common and natural representation is Signed Distance Function.
- ✓ Maps each 3D point x to its signed distance $F(\mathrm{x})$ from the surface.

$$F : \mathbb{R}^3 \to \mathbb{R}$$

- ✓ Sign indicates inside/outside. $F(\mathrm{x})=0$ means x is on the surface.

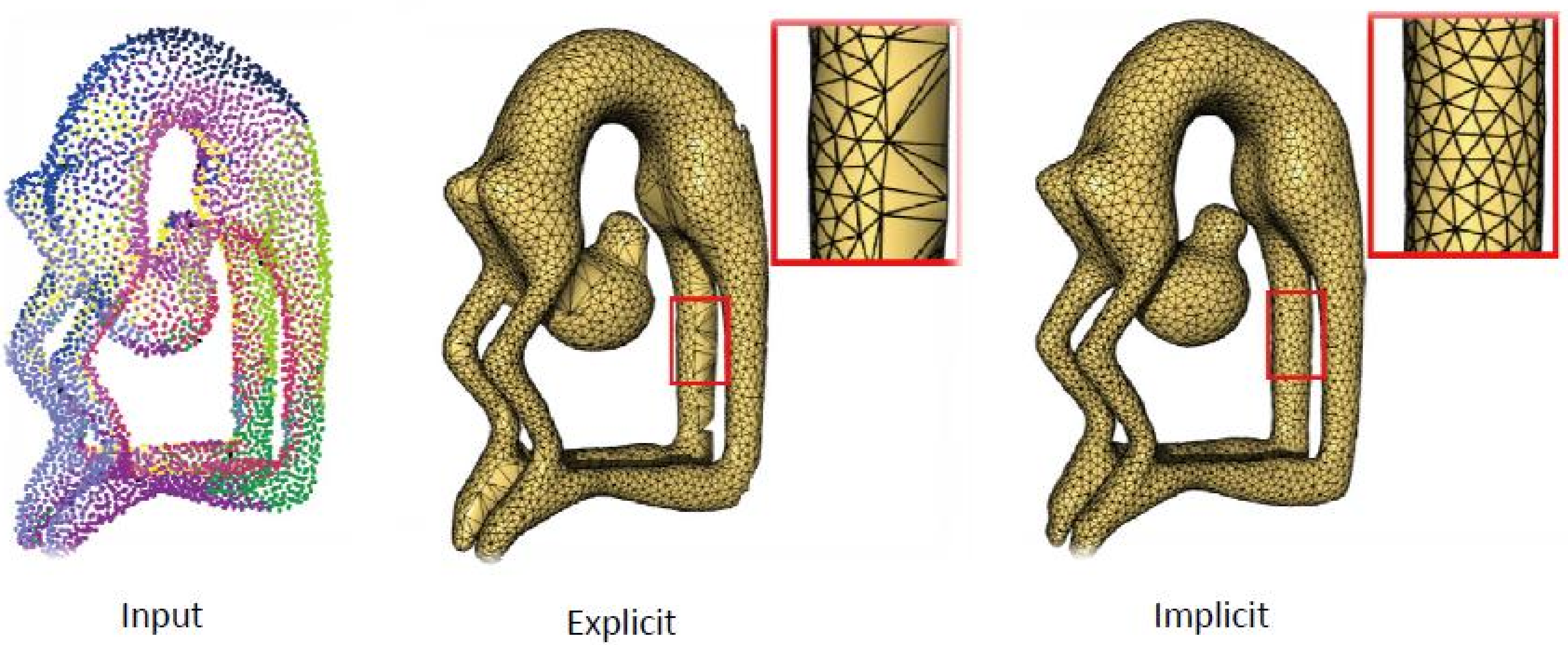
- ✓ A function for unsigned distance to a point cloud: UDF vs. SDF.
- ✓ Average the sum of squared distances to K-nearest neighbors.

$$d_U(x) = \sqrt{\frac{1}{K} \sum_{p \in N_K(x)} ||x - p||^2}$$

- ✓ Doesn't need a surface input and also pretty robust ☺.
- ✓ Can you sign this up to make it useful for Marching Cubes?
  - ✓ First do UDF (robust) and then sign it using the signs (not values) in SDF?

Surface Reconstruction

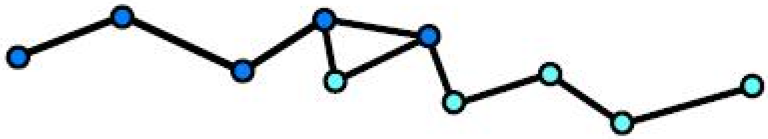
✓ Given a set of 3D points, compare Explicit vs. Implicit reconstruction.

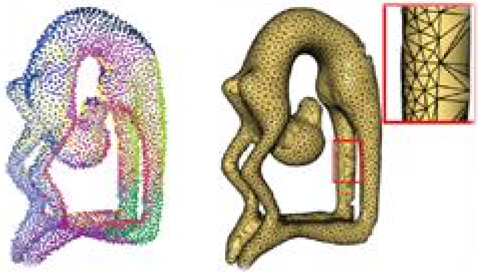


✓ Implicit algorithms *approximate*, explicit ones *interpolate* input data.

Surface Reconstruction

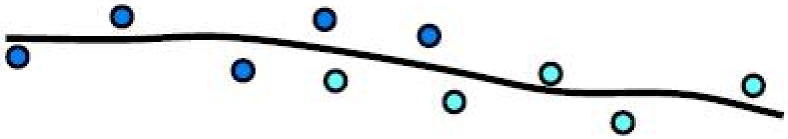
- ✓ Given a set of 3D points, compare Explicit vs. Implicit reconstruction.
- ✓ Explicit reconstruction method:
  - ✓ Connect sample points by triangles (connect the dots).
  - ✓ Exact interpolation of sample points.
  - ✓ Bad for noisy or misaligned data (common in scans).
  - ✓ May lead to holes or non-manifoldness.

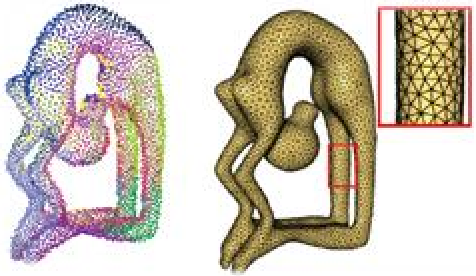




Surface Reconstruction

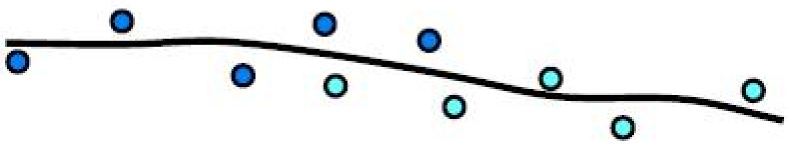
- ✓ Given a set of 3D points, compare Explicit vs. Implicit reconstruction.
- ✓ Implicit reconstruction method:
  - ✓ Estimate signed distance function (SDF) for each grid point (scalar-valued grid).
  - ✓ Extract level zero iso-surface from this grid (Marching Cubes).
  - ✓ Approximation of input points (better in noisy situations).
  - ✓ Manifoldness by construction.

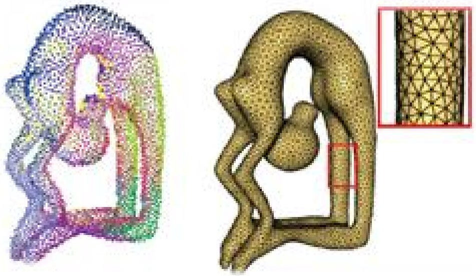




Surface Reconstruction

- ✓ Given a set of 3D points, compare Explicit vs. Implicit reconstruction.
- ✓ Implicit reconstruction method:
  - ✓ The distance function is interpolated and polygonalized by the marching squ/cubes algorithm.
  - ✓ The distance function is, however, based on an approximation of the input surface (via tangent planes), hence the whole process is an approximation.





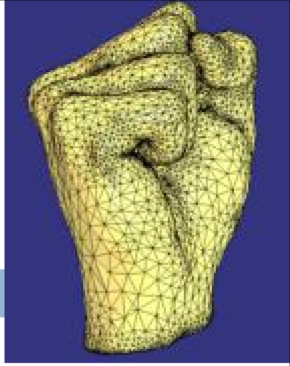
Surface Reconstruction

- ✓ Given a set of 3D points, compare Explicit vs. Implicit reconstruction.
- ✓ Implicit vs. Explicit:

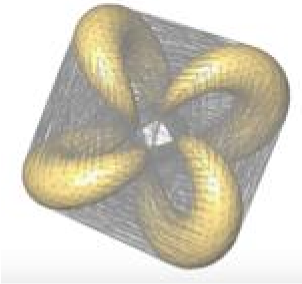
Smooth　　Piecewise Smooth　　Interpolant "Connect the Dots"

- ✓ Surface reconstruction is an ill-posed problem: want sharp corner?
- ✓ Solution: regularize via priors, e.g., smooth, piecewise smooth, simple.

Explicit Reconstruction

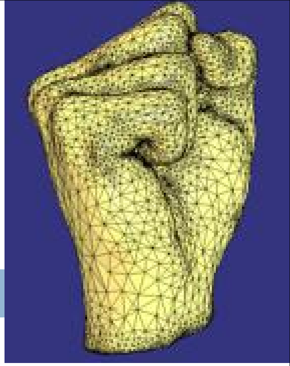


- ✓ Crust Algorithm: Since Delaunay triangulation encodes the proximity b/w the points, if points are dense enough, we should have the desired triangles within the Delaunay triangulation. Tri → Tetrahedron in 3D.



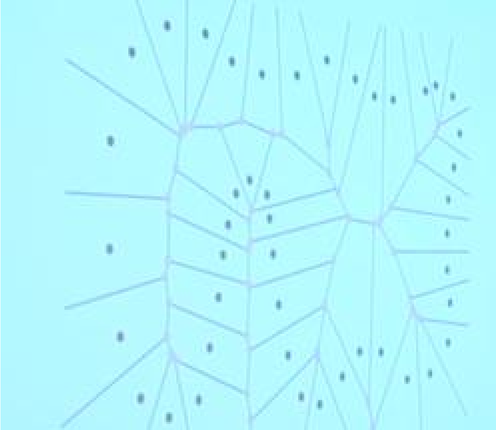
- ✓ Compute Delaunay triangulation of the input point set.
- ✓ So we have the edges we need, but also the extras.

Explicit Reconstruction

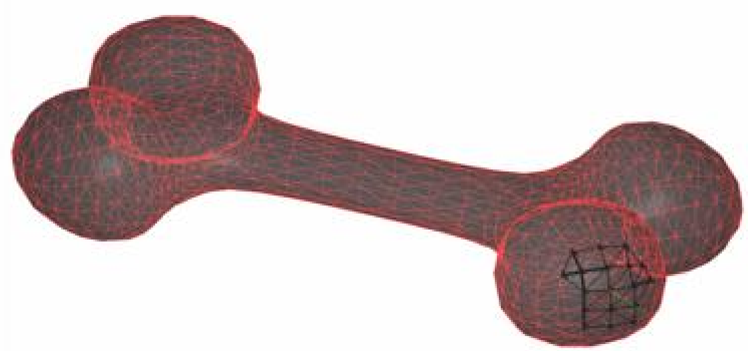


- ✓ Crust Algorithm: Since Delaunay triangulation encodes the proximity b/w the points, if points are dense enough, we should have the desired triangles within the Delaunay triangulation. Tri → Tetrahedron in 3D.

- ✓ Get rid of extras: Compute Voronoi diagram. Insert Voronoi vertices (pink) to the Delaunay triangulation. Keep edges b/w black dots only.



Explicit Reconstruction

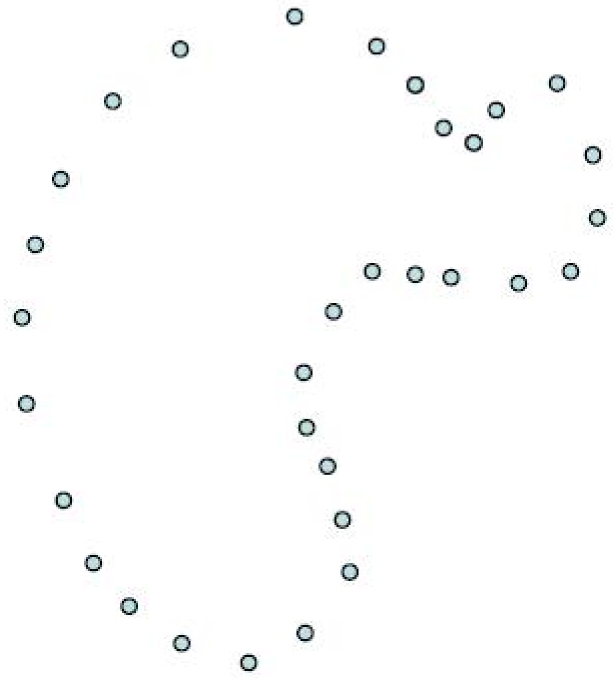


- ✓ Find local neighborhood L_i of each point p_i in the 3D point cloud input.
  - ✓ Closest k points (using a k-d tree).
- ✓ For each L_i compute tangent plane using PCA.
  - ✓ If the input is oriented, just take the average of all normals for the plane normal. Use the mean pnt as the plane pnt (for both oriented/unoriented).
- ✓ Project all points in L_i to the tangent plane and compute their 2D Delaunay triangulation D_i.
- ✓ Final triangulation is the composition of all these local triangulations.

- ✓ Yet another explicit algorithm is Poisson Reconstruction.

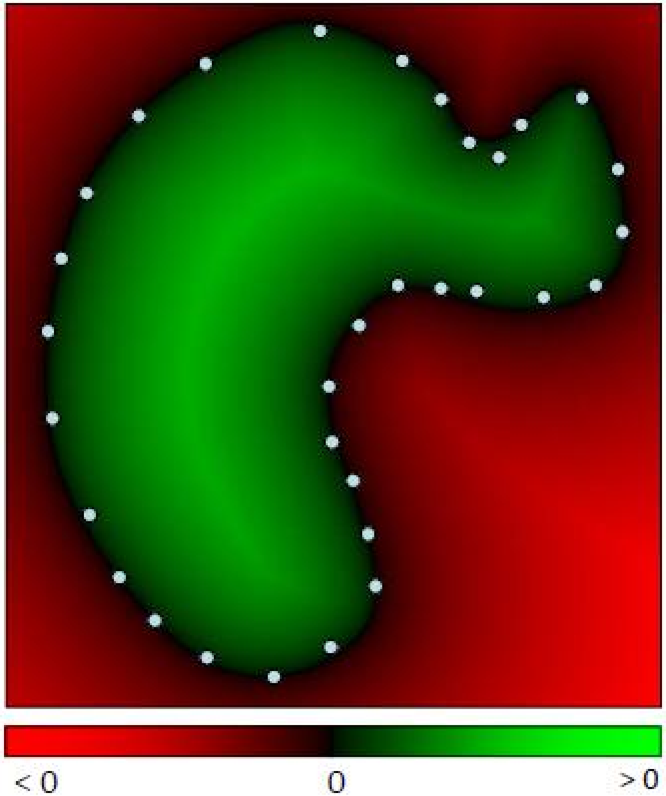
Implicit Reconstruction

- ✓ Implicit function approach.



Implicit Reconstruction

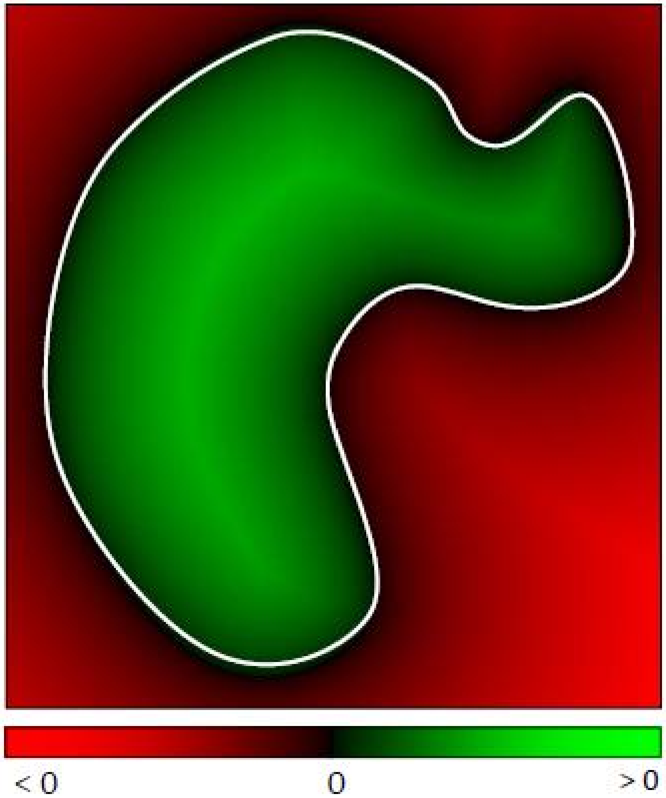
- ✓ Implicit function approach.



- ✓ Define a function $F : \mathrm{R}^3 \rightarrow \mathrm{R}$ with value <0 outside, >0 inside, =0 on.

Implicit Reconstruction

- ✓ Implicit function approach.

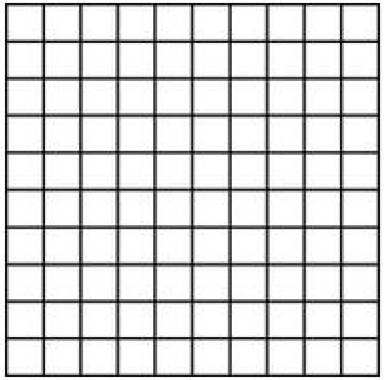


- ✓ Extract the zero-set isosurface, i.e., $S = \{x \text{ in } \mathbb{R}^3 \mid F(x) = 0\}$.

Implicit Reconstruction

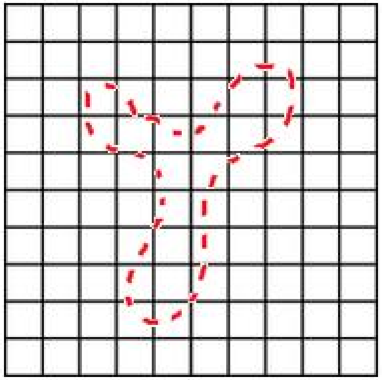
- ✓ Implicit function: Signed distance function $F : \mathrm{R}^3 \rightarrow \mathrm{R}$.
- ✓ Construction algorithm.
  - ✓ Input: point samples representing the object to be reconstructed.

  - ✓ Output: F value at each grid point, i.e., signed distance of that grid pnt to the sampled surface.

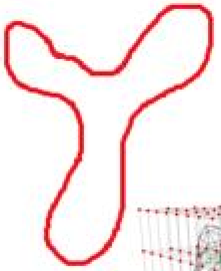


Implicit Reconstruction

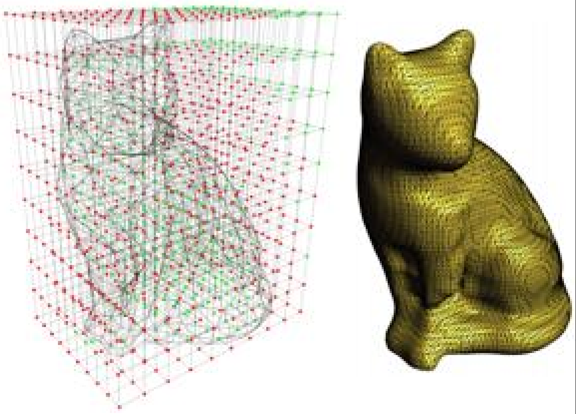
- ✓ So we need a grid that encapsulates the sampled surface.



- ✓ Uniform 2D grid.
  - ✓ Mesh to be extracted: 1D (closed) curve →

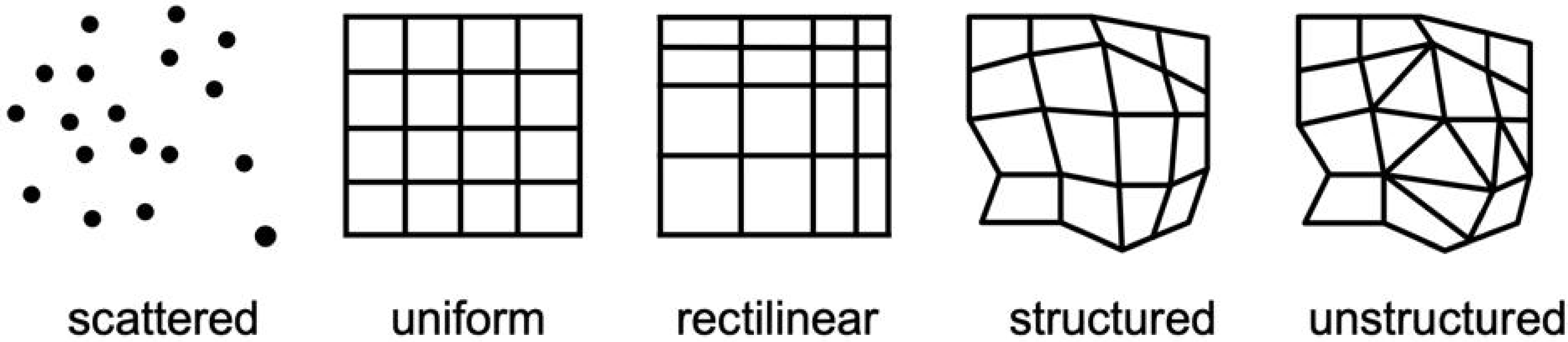


- ✓ Uniform 3D grid.
  - ✓ Mesh to be extracted: 2D (closed) surface →→→→



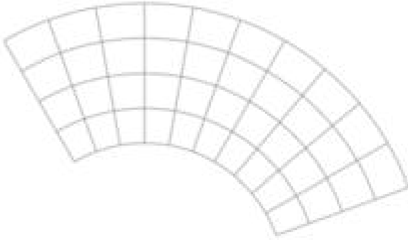
Implicit Reconstruction

✓ Other grid types.



Not a grid at all: no connectivity.

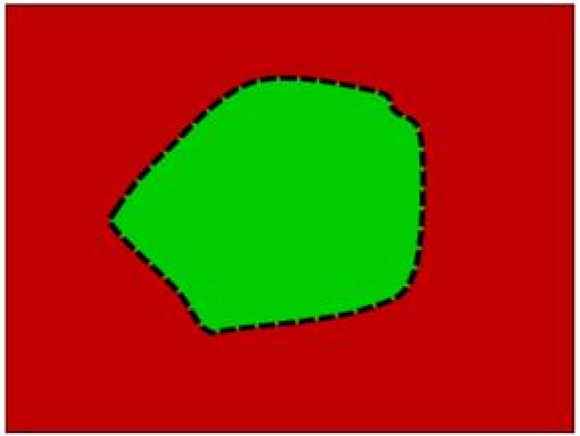
Uniform aka regular grid.

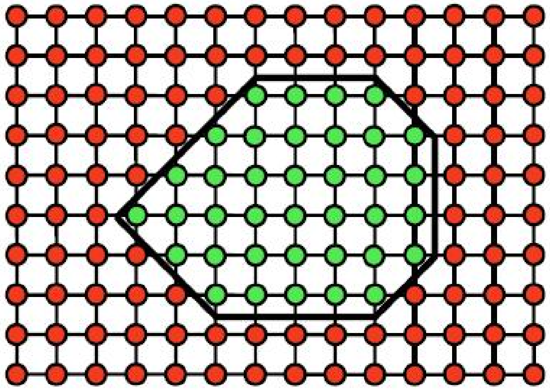


curvilinear

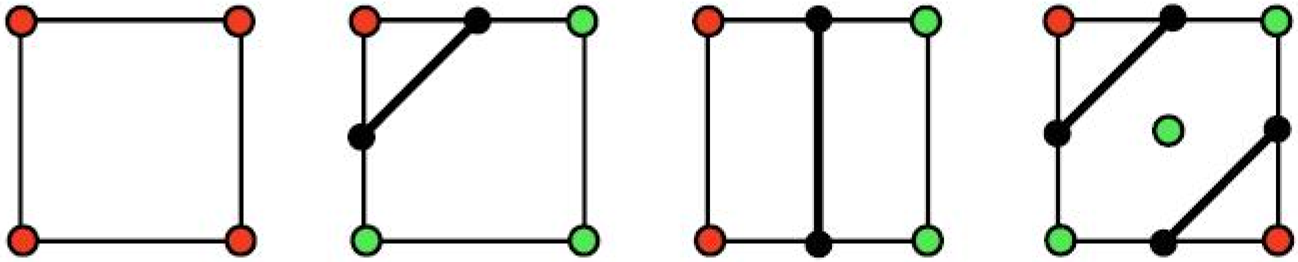
Marching X

- ✓ Mesh extraction idea.
- ✓ Marching Squares (2D), Marching Cubes (3D).



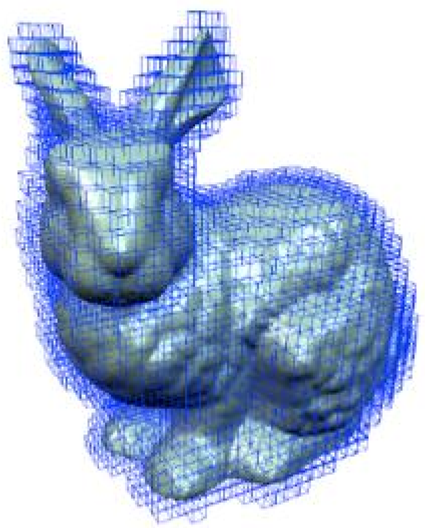


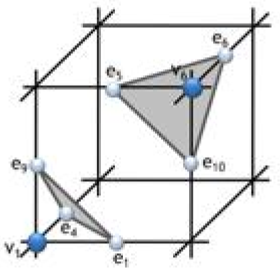
- ✓ 2^4 cases. May be reduced by symmetry, e.g., all red and all gre. equal.



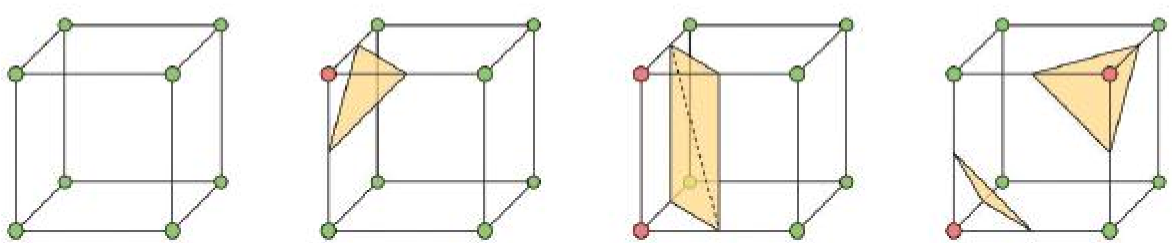
Marching X

- ✓ Mesh extraction idea.
- ✓ Marching Squares (2D), Marching Cubes (3D).



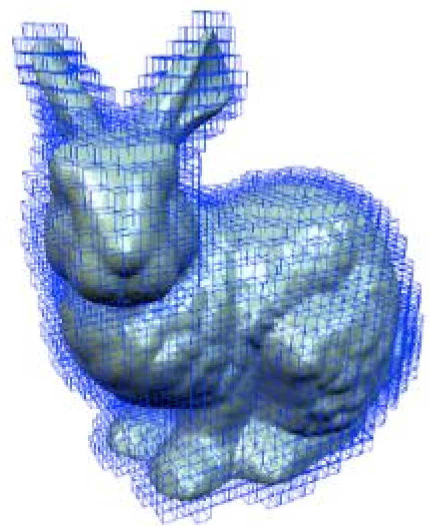


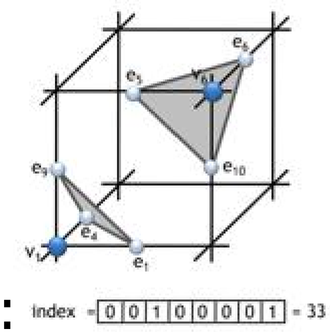
- ✓ 2^8 cases. Sym. applies. Fast look up: use 8-bit case index: index = 0 0 1 0 0 0 0 1 = 33



Marching X

- ✓ Mesh extraction idea.
- ✓ Marching Squares (2D), Marching Cubes (3D).

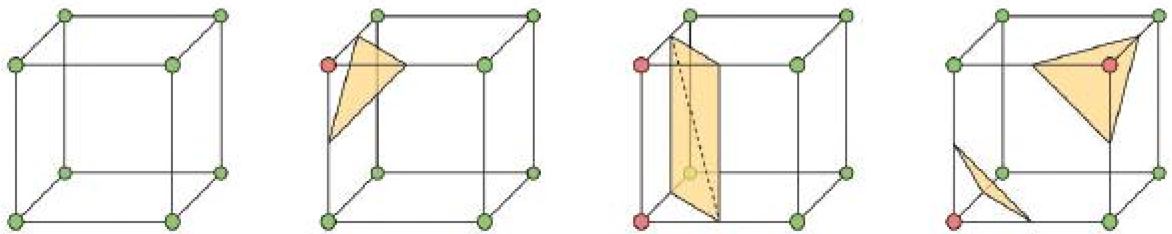


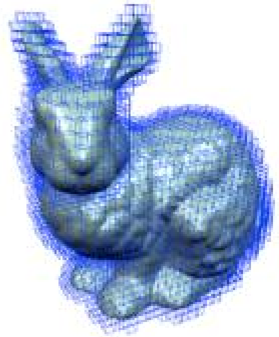


- ✓ 2^8 cases. Sym. applies. Fast look-up: use 8-bit case index:
- ✓ Case index brings connectivity from the look-up table.
  - ✓ Cut edges are e1, e4, e5, e6, e9, e10.
  - ✓ Output triangles are (e1, e9, e4), (e5, e10, e6).

Marching X

- ✓ Steps for Marching Squares/Cubes
  - ✓ Discretize space (use a regular grid enclosing the surface).
  - ✓ Evaluate signed distance function on grid.
  - ✓ Classify grid points (inside/outside w.r.t. surface).
  - ✓ Classify grid edges (one endpoint inside, one outside).
  - ✓ Compute intersections.
  - ✓ Connect intersections.





Marching X

- ✓ Steps for Marching Squares/Cubes
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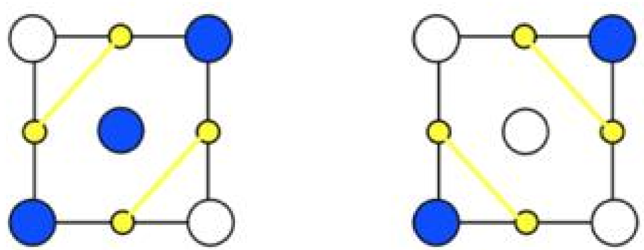
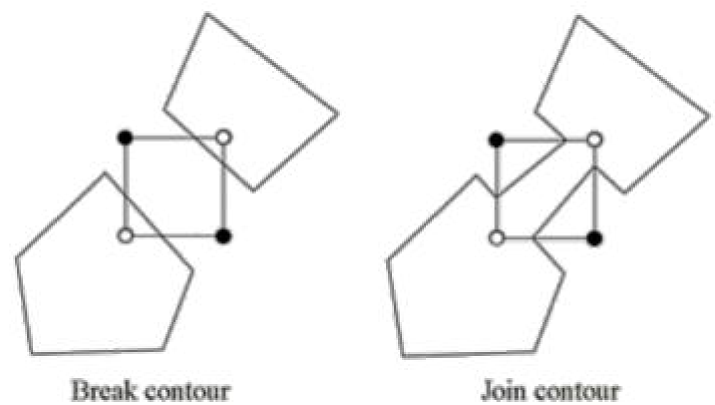
  - ✓ Connect intersections (ambiguities may arise).





  - ✓ Handle by subsampling inside a cell, or randomly picking 1 of the 2 possibilities.

Marching X

- ✓ Steps for Marching Squares/Cubes
  - ✓ Discretize space (use a regular grid enclosing the surface).
  - ✓ Evaluate signed distance function on grid.
  - ✓ Classify grid points (inside/outside w.r.t. surface).
  - ✓ Classify grid edges (one endpoint inside, one outside).
  - ✓ Compute intersections.
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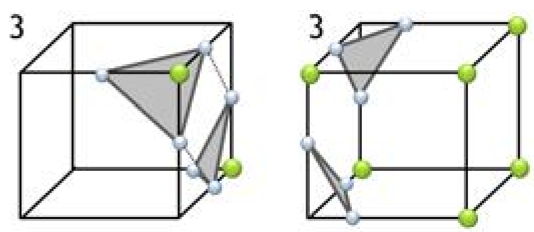
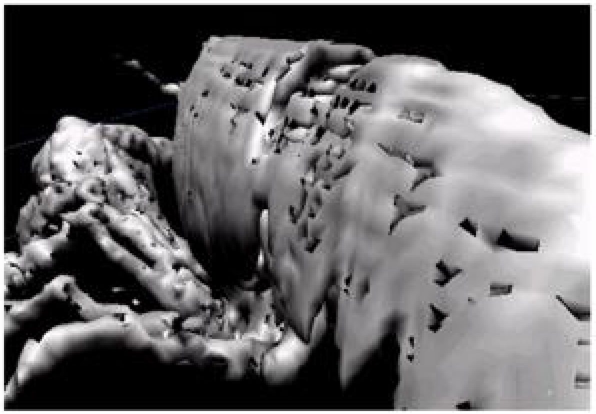
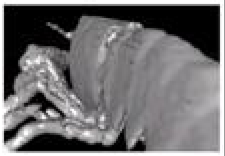


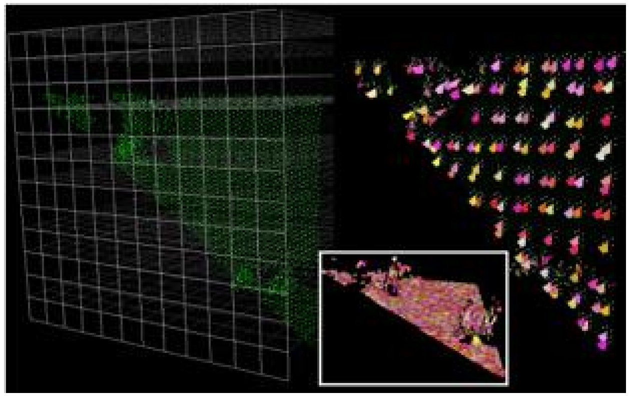




  - ✓ If random pick, then make choices consistently or this happens. Fixed:

Signed Distance Function (SDF)

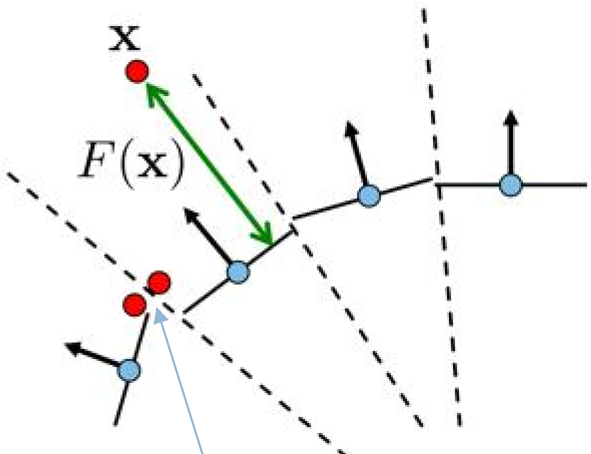
- ✓ Back to the construction of the Signed Distance Function $F : \mathrm{R}^3 \rightarrow \mathrm{R}$.
- ✓ Input: sample points.
- ✓ Output: F value at each grid point.
- ✓ Algo:
  - ✓ Associate a tangent plane with each sample point.
    - ✓ Why? Tangent planes are local linear approximations to the surface.

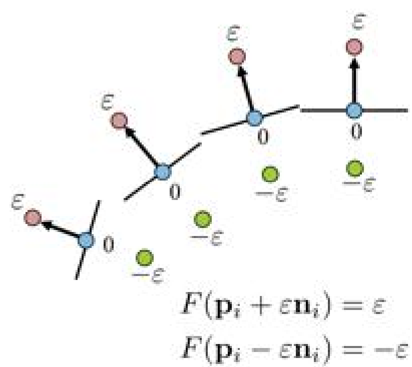


  - ✓ Compute $F(\mathrm{x}) = (\mathrm{x} - \mathrm{o}_i) \cdot \mathrm{n}_i$

SDF

- ✓ Back to the construction of the Signed Distance Function $F : R^3 \rightarrow R$.
- ✓ Input: sample points.
- ✓ Output: F value at each grid point.
- ✓



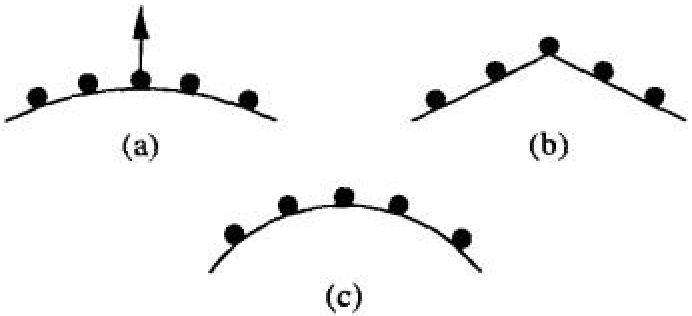


  - ✓ $F(x) = (x - o_i) \cdot n_i$
- ✓ $F(x)$ is discontinuous. How to fix? Hint: consider off-set constraints.
  - ✓ RBF, MLS, DeepSDF, <your idea here>.
    - ✓ Reconstruction and representation of 3D objects with radial basis functions, 2001.
    - ✓ Interpolating and Approximating Implicit Surfaces from Polygon Soup, 2004.
    - ✓ DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation, 2019.

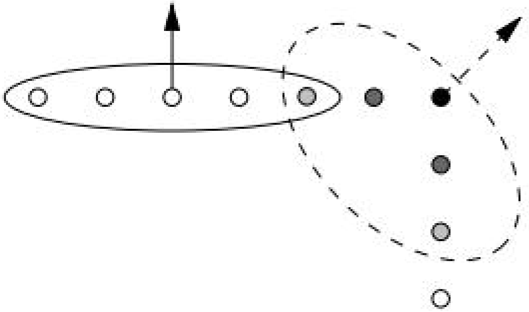
SDF

- ✓ Tangent plane for sample point s is found as follows:
  - ✓ Find k-nearest neighbors of s.
  - ✓ Apply principle component analysis on this set of close points.
    - ✓ Find the covariance matrix of the set.
    - ✓ Take the eigenvectors of this matrix as the principal axes.
    - ✓ Use the third best axis (w/ smallest eigenvalue) as the normal of the tangent plane, i.e. n_i.
    - ✓ Use the mean of this point set as the centering point of the plane, i.e., o_i.
    - ✓ The smaller the 3rd eigenvalue, the more confident the plane is, a is confident, b,c are not.

$$(\lambda_3/\lambda_1 < \Gamma)$$



Darker = more ambiguous



SDF

- ✓ Tangent planes have arbitrary normal signs; need a normal-orienter for smooth/consistent normals across the point set.
- ✓ MST-based: Surface Reconstruction from Unorganized Points, 1992.
- ✓ Ray cast-based: A Simple Method for Correcting Facet Orientations, ‘15.
- ✓ BFS-based: breadth-first search to enfrce a consistent facet orientation.
  - ✓ Last two are designed for surface/mesh inputs, thus not directly applicable.

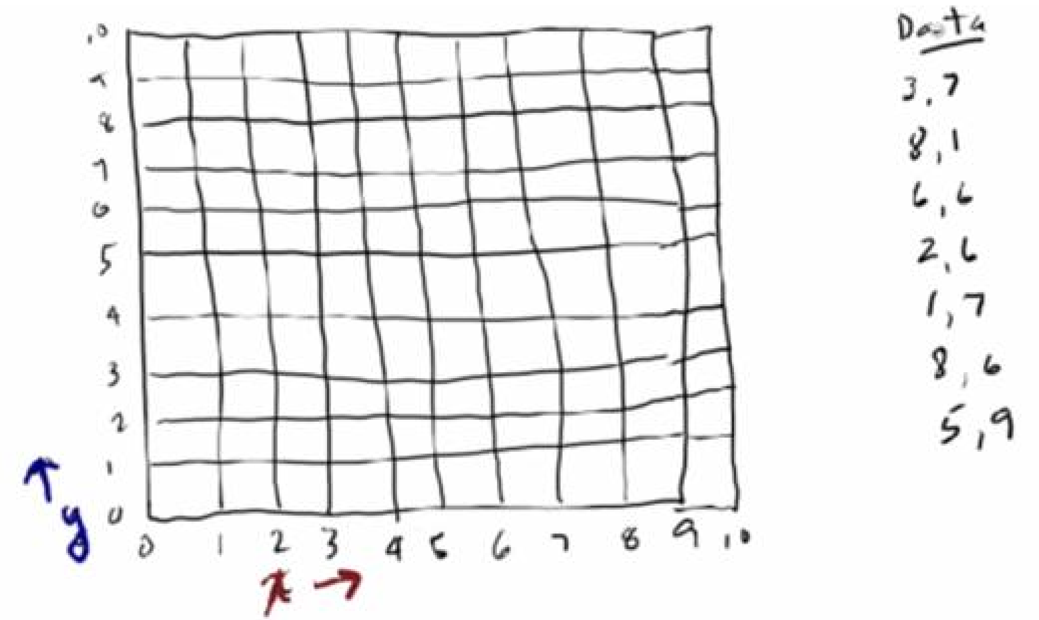
k-d Tree for Nearest Neighbors (NN)

- ✓ k-nearest points.
- ✓ Brute force: O(k * N) when we have N samples. //10^3 steps
- ✓ Can be done in O(k * lgN) using k-d trees. //10 steps.

- ✓ k-d tree: cells are axis-aligned bounding boxes.
  - ✓ k-d dimensional binary search tree.
  - ✓ Not to be confused w/ the k in k-nearest neighbors.

k-d Tree for NN

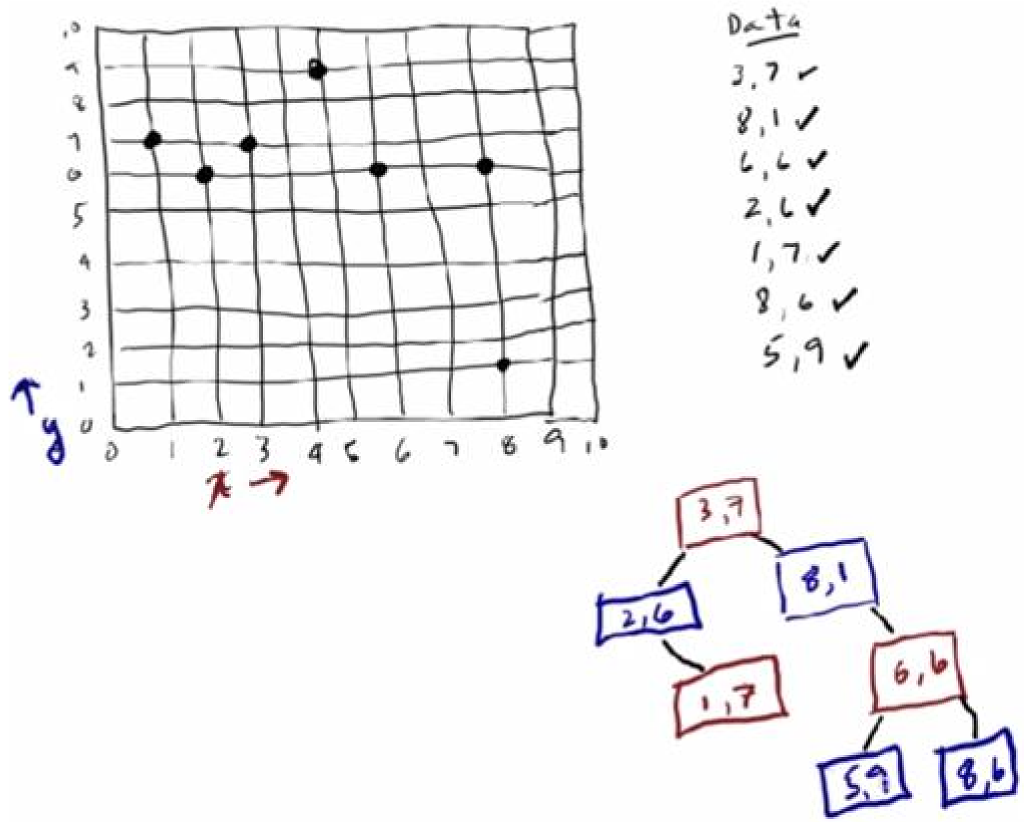
- ✓ k-d tree insertion. k = # of dimensions = 2 in example below.
- ✓ Idea: every time you go down in tree, you toggle decision-making dimension.



k-d Tree for NN

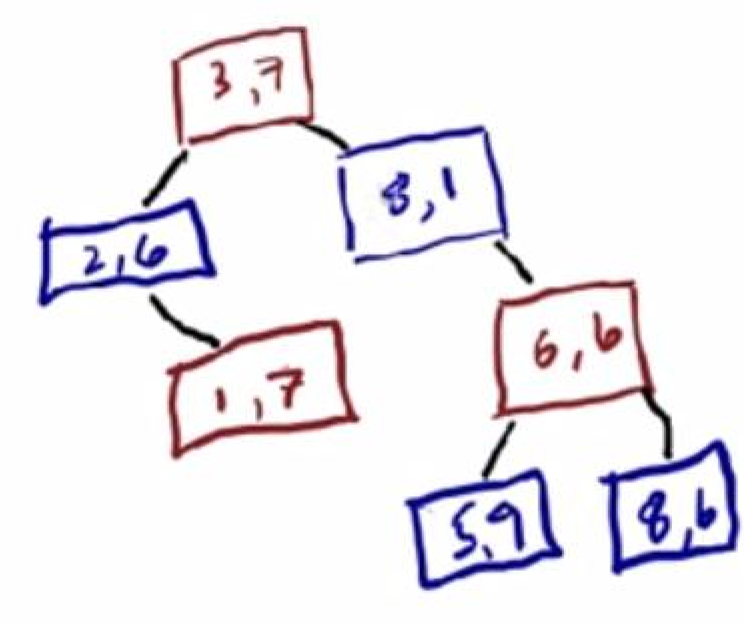
- ✓ k-d tree insertion. k = # of dimensions = 2 in example below.
- ✓ Idea: every time you go down in tree, you toggle decision-making dimension.

1) 3,7 is inserted
2) 8 > 3 (1st dimension) of 3,7, so put 8,1 to right; colored in blue 'cos if may have to make a decision at this point, i'll be deciding at the 2nd dim (red means 1st dim)
3) 6 > 3 go right, 6 > 1 go right
4) 2 < 3 go left
5) 1 < 3 go left, 7 > 6 go right
6) 8 > 3 go right, 6 > 1 go right, 8 > 6 go right
7) 5 > 3 go right, 9 > 1 go right, 5 < 6 go left



k-d Tree for NN

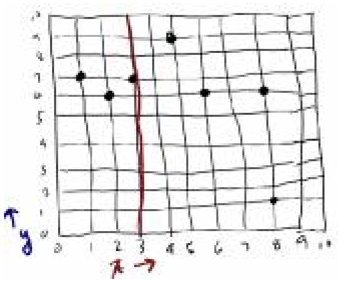
- ✓ k-d tree search operation. Search (5, 2) below.
- ✓ 5 > 3 go right, 2 > 1 go right, 5 < 6 go left, 2 < 9 look left, not found.



k-d Tree for NN

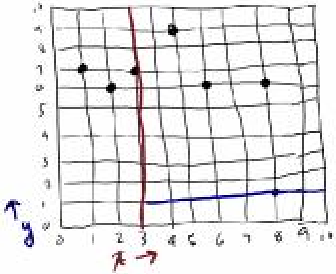
- ✓ k-d tree partitioning.
- ✓ Going left or right means partitioning the space.

(3,7) inserted



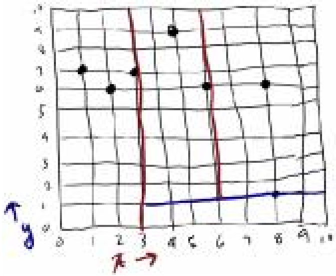
partition the space into 2 axis-aligned bounding boxes, i.e., everything <= 3 to left everything > 3 to right.

(8,1) inserted



everything <= 1
everything > 1
(w.r.t. y dimension)

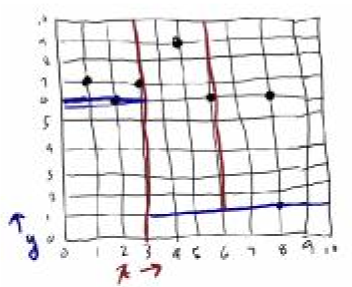
(6,6) right of the 3-line, top of the 1-line:



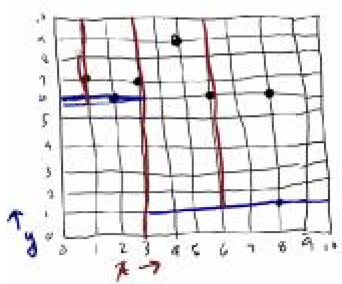
k-d Tree for NN

- ✓ k-d tree partitioning.
- ✓ Going left or right means partitioning the space.

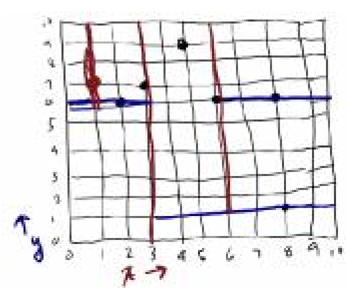
(2,6) inserted



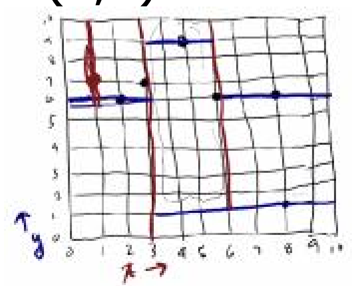
(1,7) inserted



(8,6) inserted

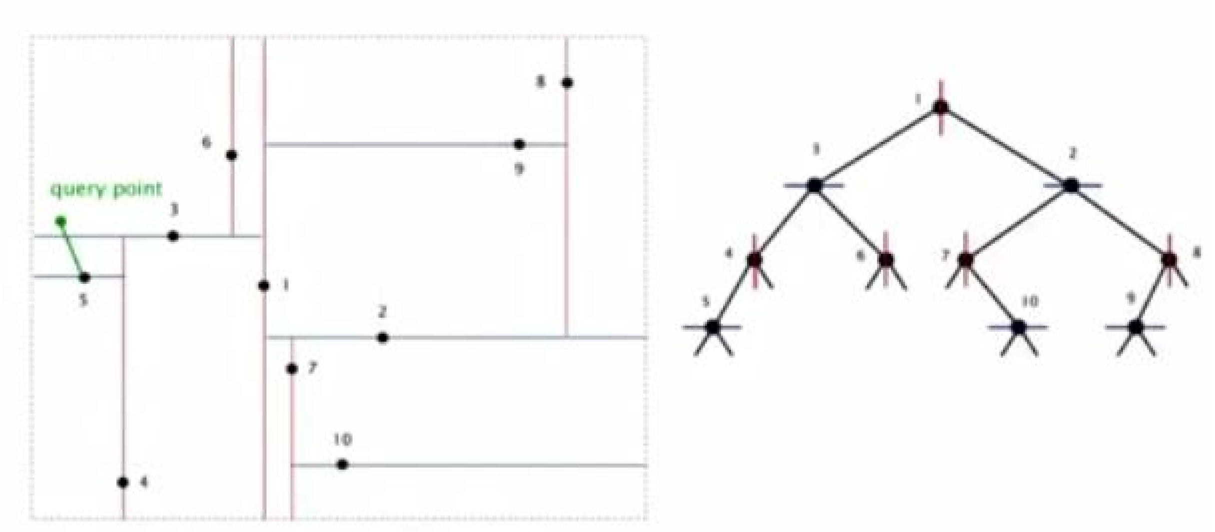


(5,9) inserted



k-d Tree for NN

- ✓ k-d tree nearest-neighbor to a query point (see Note below).
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Our algo should return point 5 (as the nearest one to the query).

k-d Tree for NN

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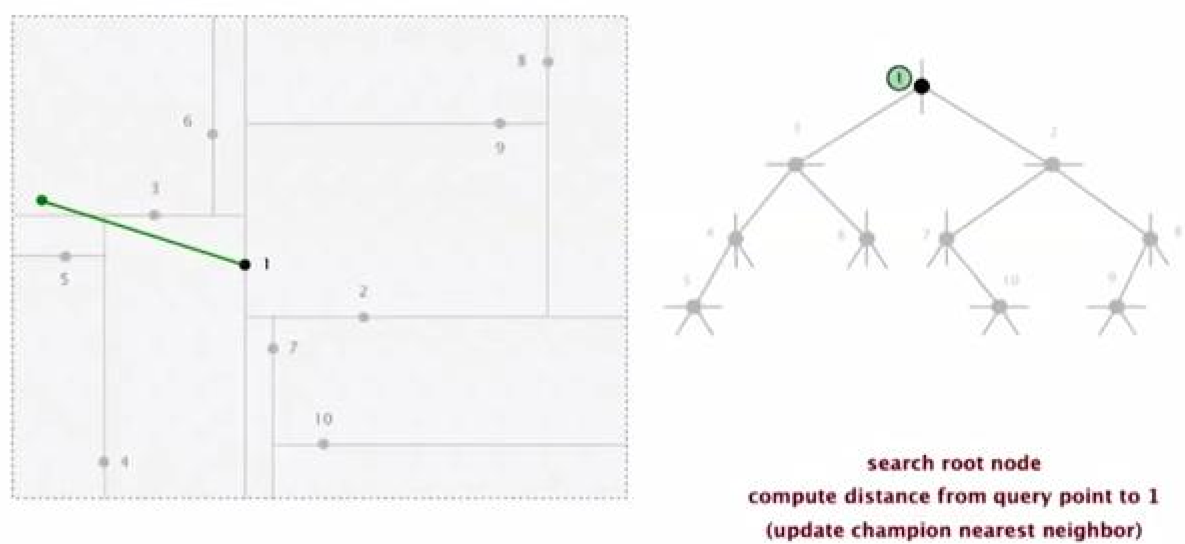
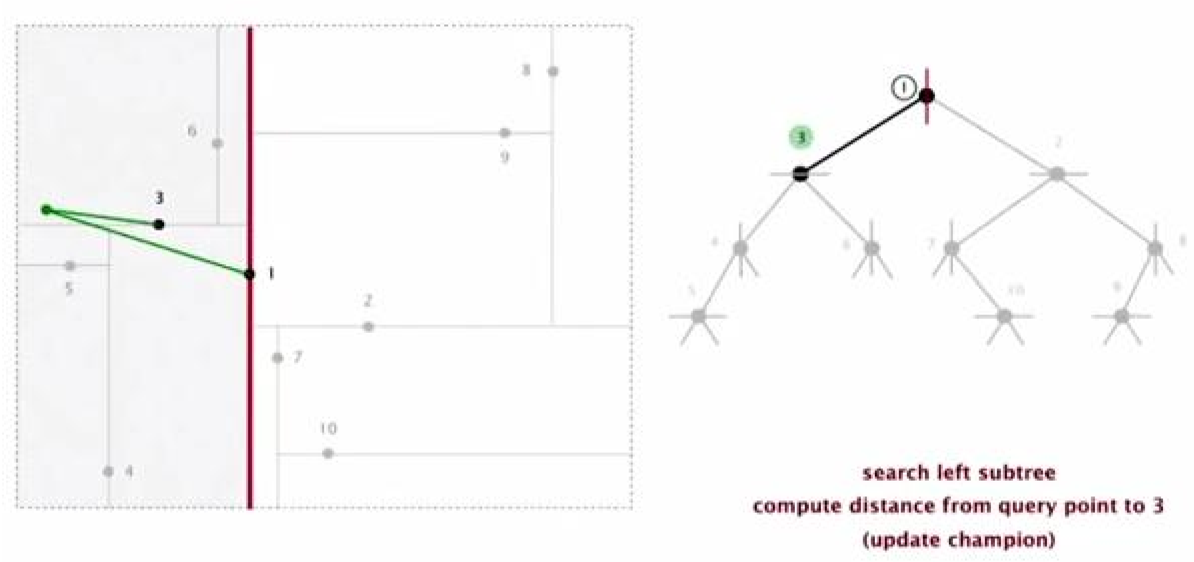
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Go towards query point, i.e., left. (there might be a closer pnt than 1 in the right; at least we think so now).

k-d Tree for NN

- k-d tree nearest-neighbor to a query point.
- Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- When recursion gets back to pnt 1, we won't search the right subtree 'cos there could be no point closer to query than 3. //cutting out ☺

k-d Tree for NN

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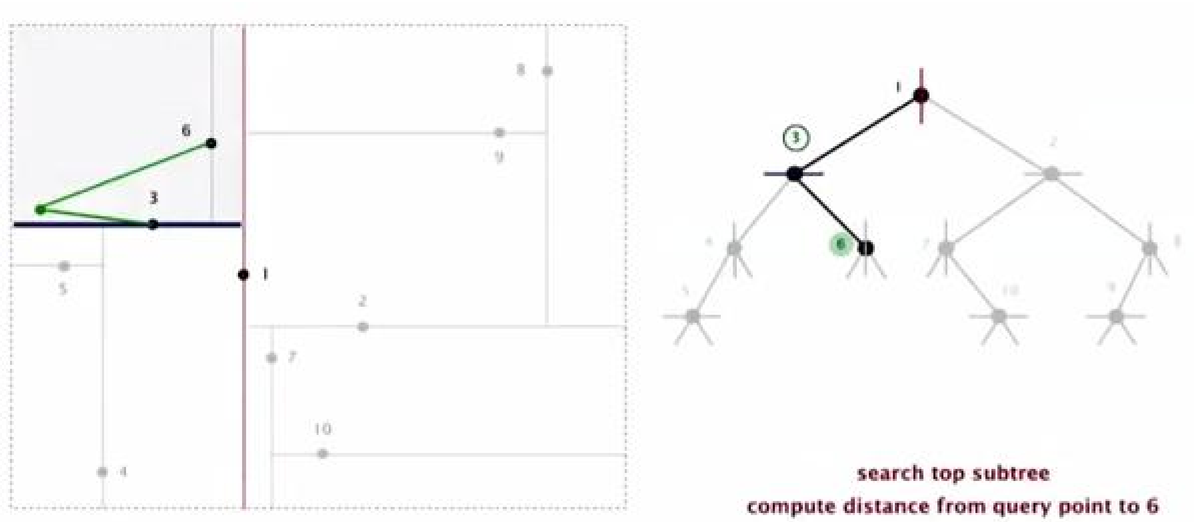

- ✓ Going towards the query point, so checking the top of 3. Pnt 6 is not better so do not update the champion.

k-d Tree for NN

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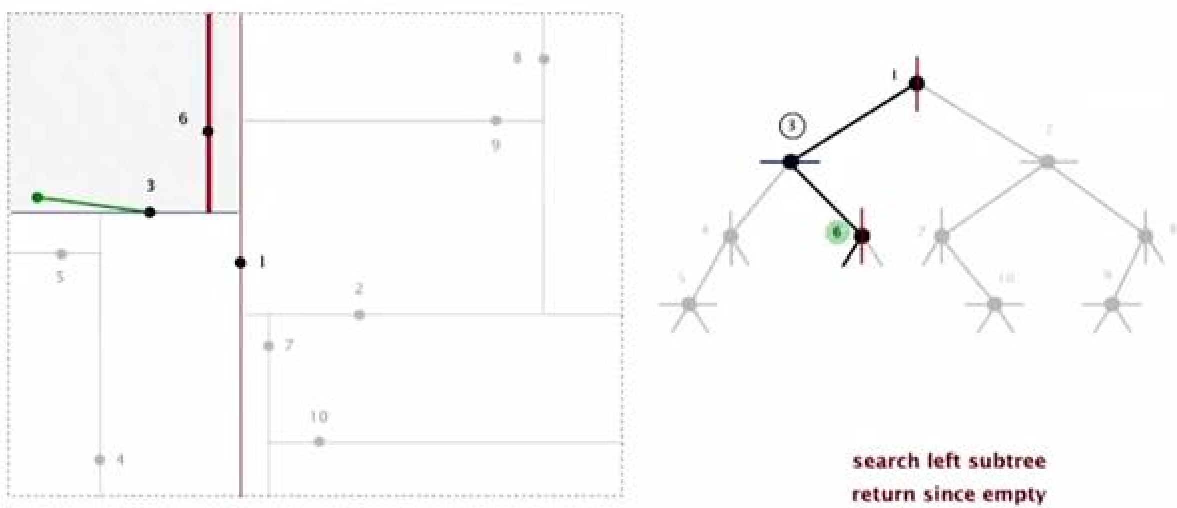
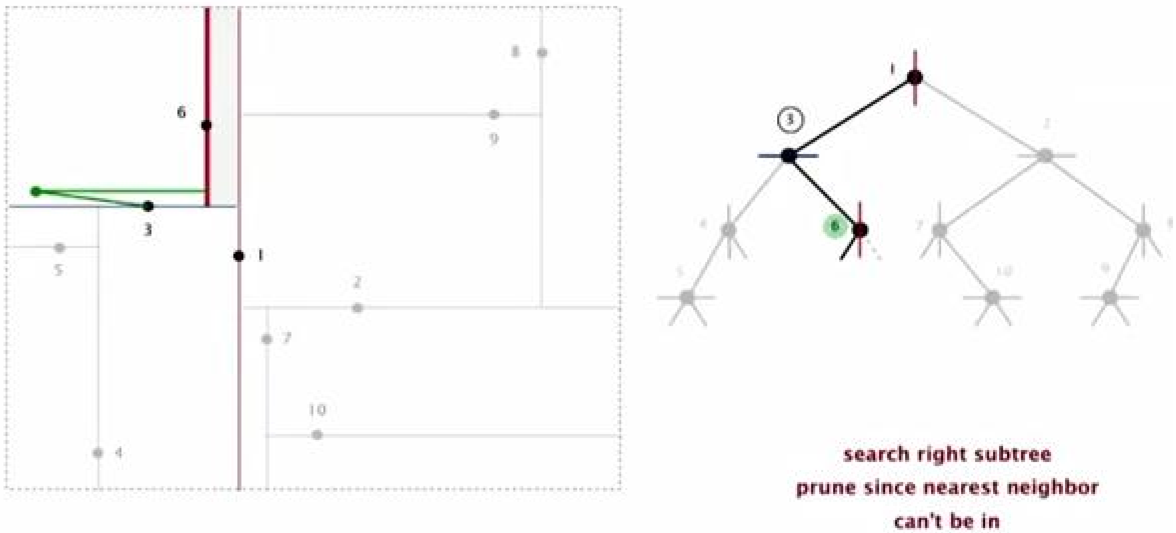
- Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- Check left subtree of pnt 6 (going towards qury), empty, so do nothing.

k-d Tree for NN

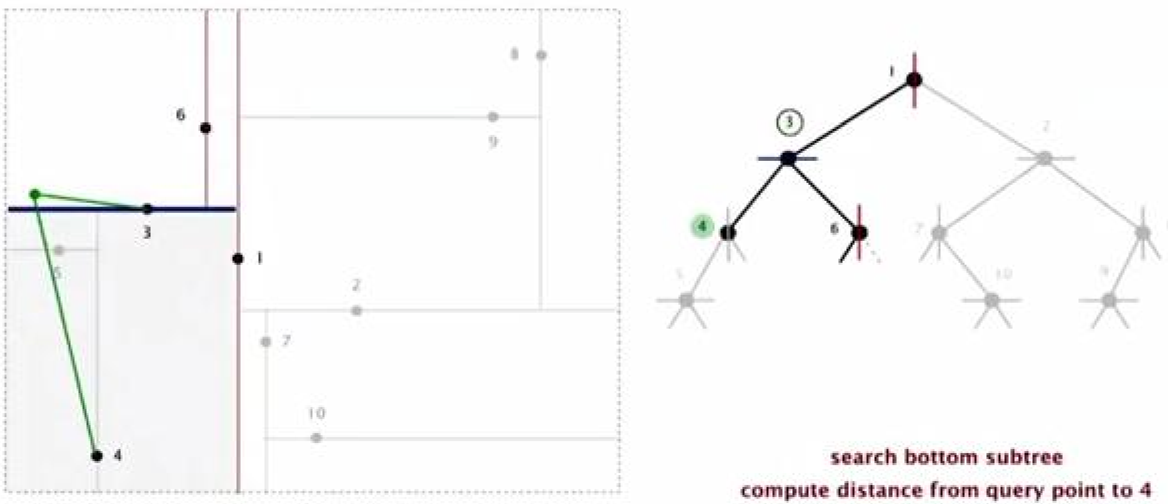
- ✓ k-d tree nearest-neighbor to a query point.
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Do not check the right subtree of pnt 6 ‘cos they can’t be closer than 3.

k-d Tree for NN

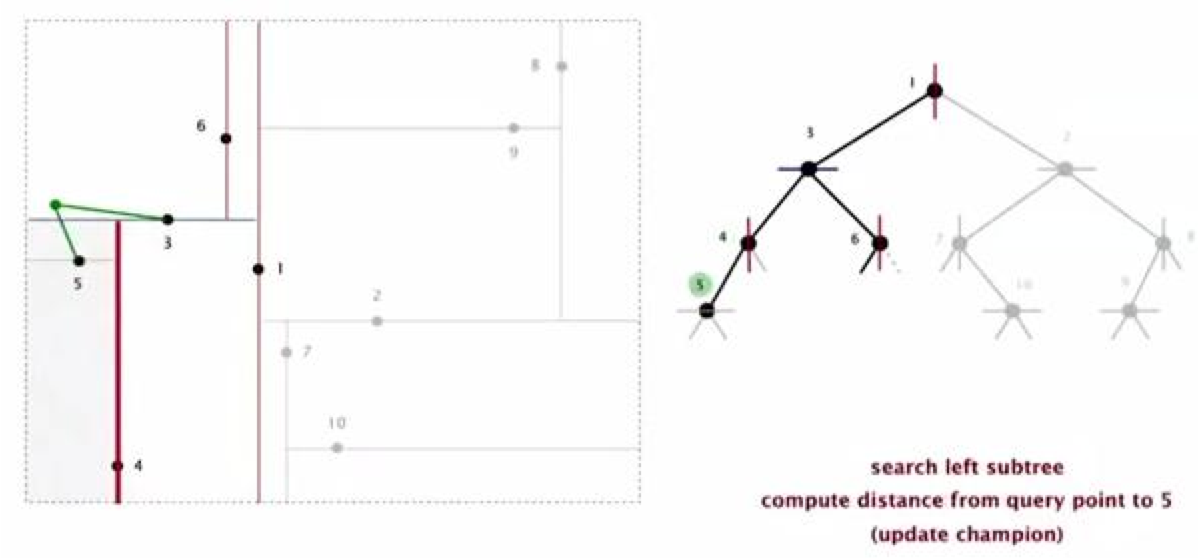
- ✓ k-d tree nearest-neighbor to a query point.
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Recursion checks the bottom subtree of pnt 3, which takes us to pnt 4, which is not closer; so 3 is still the champion.

k-d Tree for NN

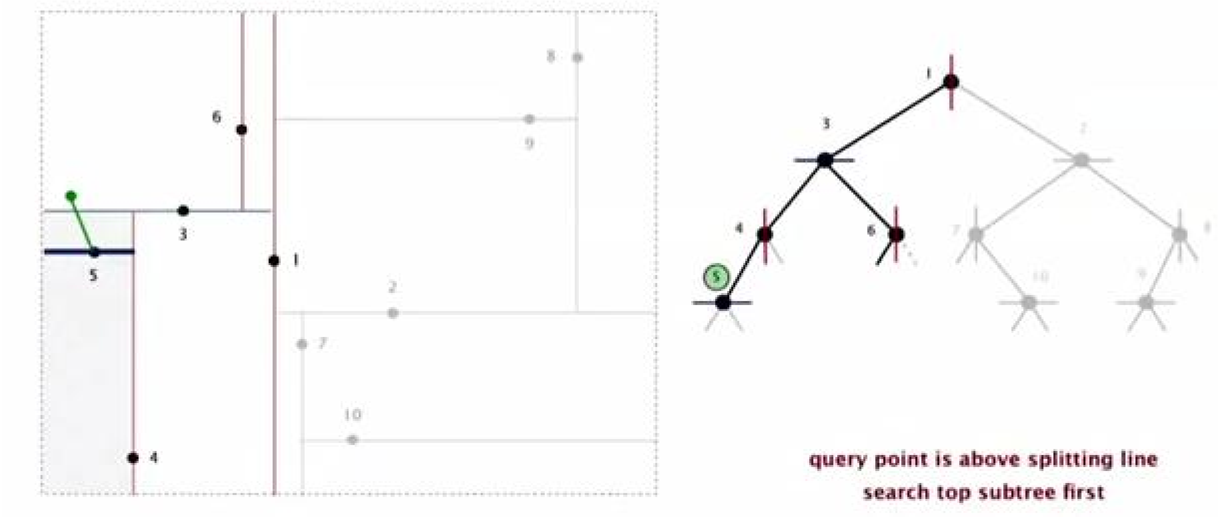
- k-d tree nearest-neighbor to a query point.
- Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- Check left subtree of pnt 4 (towards query), where we have pnt 5, the new champion.

k-d Tree for NN

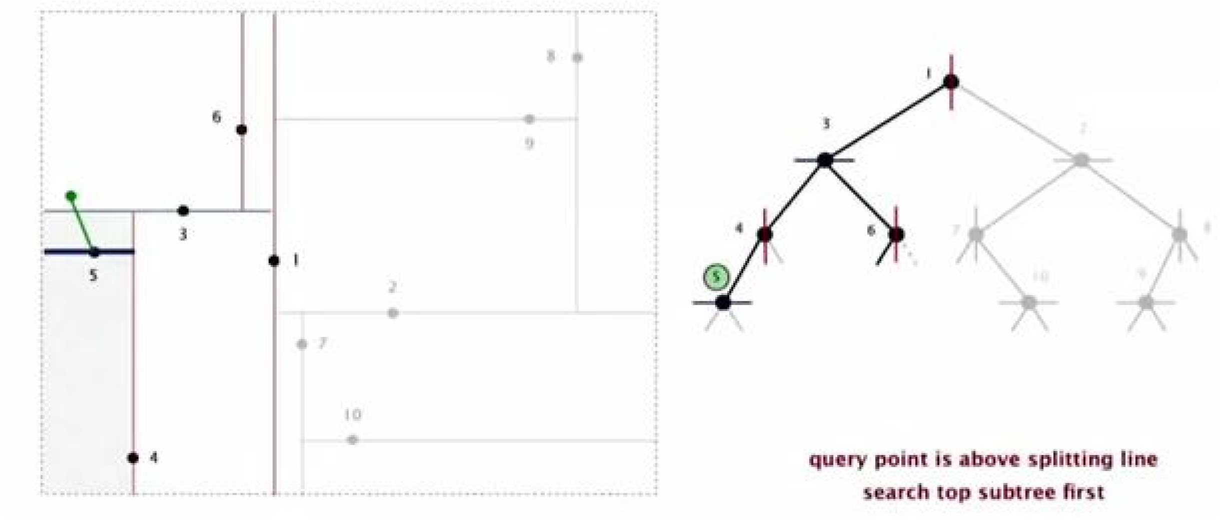
- k-d tree nearest-neighbor to a query point.
- Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- Check top of pnt 5 (no pnts there), do nothing.

k-d Tree for NN

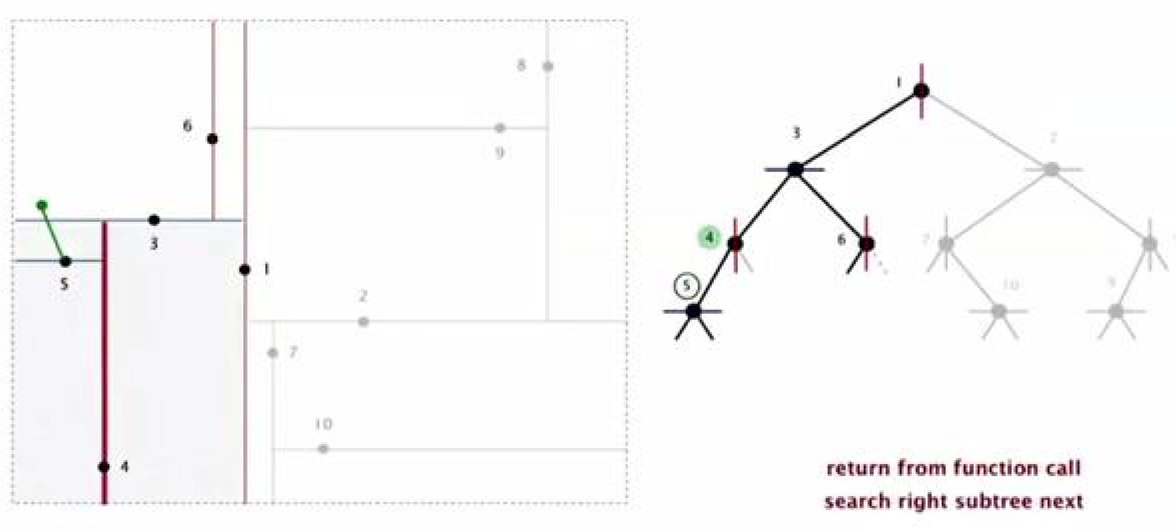
- ✓ k-d tree nearest-neighbor to a query point.
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Check bottom of pnt 5 (no pnts there), do nothing.

k-d Tree for NN

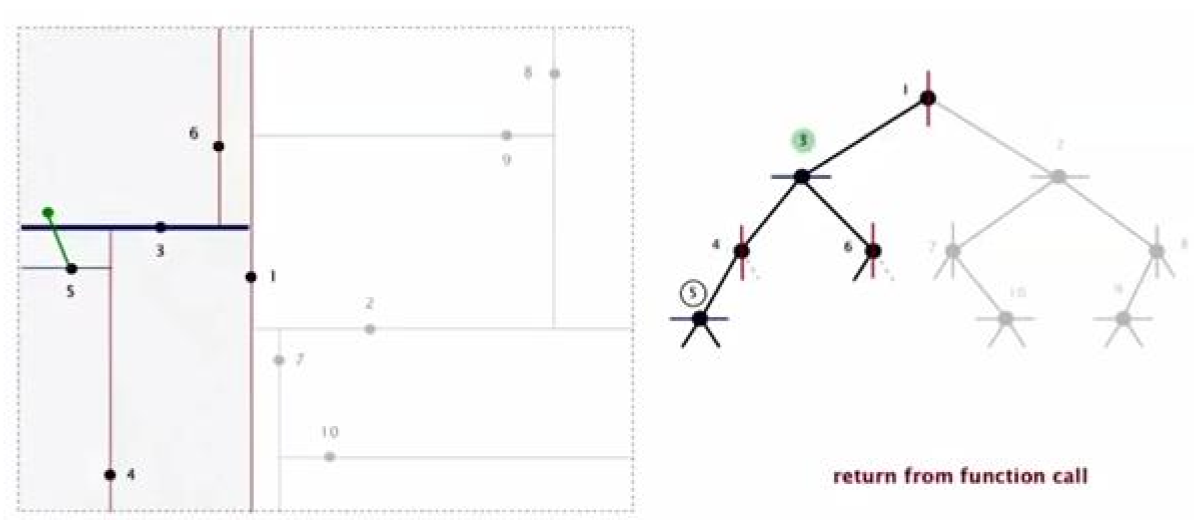
- ✓ k-d tree nearest-neighbor to a query point.
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Recursion comes back to the right subtree of pnt 4, but cut it out (no search) 'cos there can't be a closer pnt there than pnt 5.

k-d Tree for NN

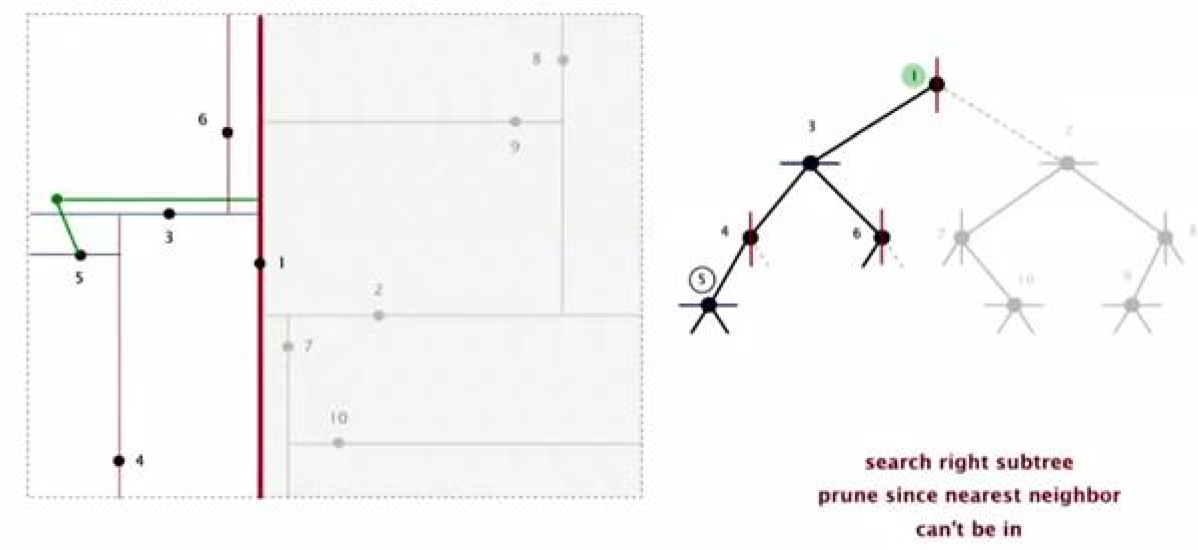
- ✓ k-d tree nearest-neighbor to a query point.
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Recursion comes back to pnt 3, whose both subtrees are handled.

k-d Tree for NN

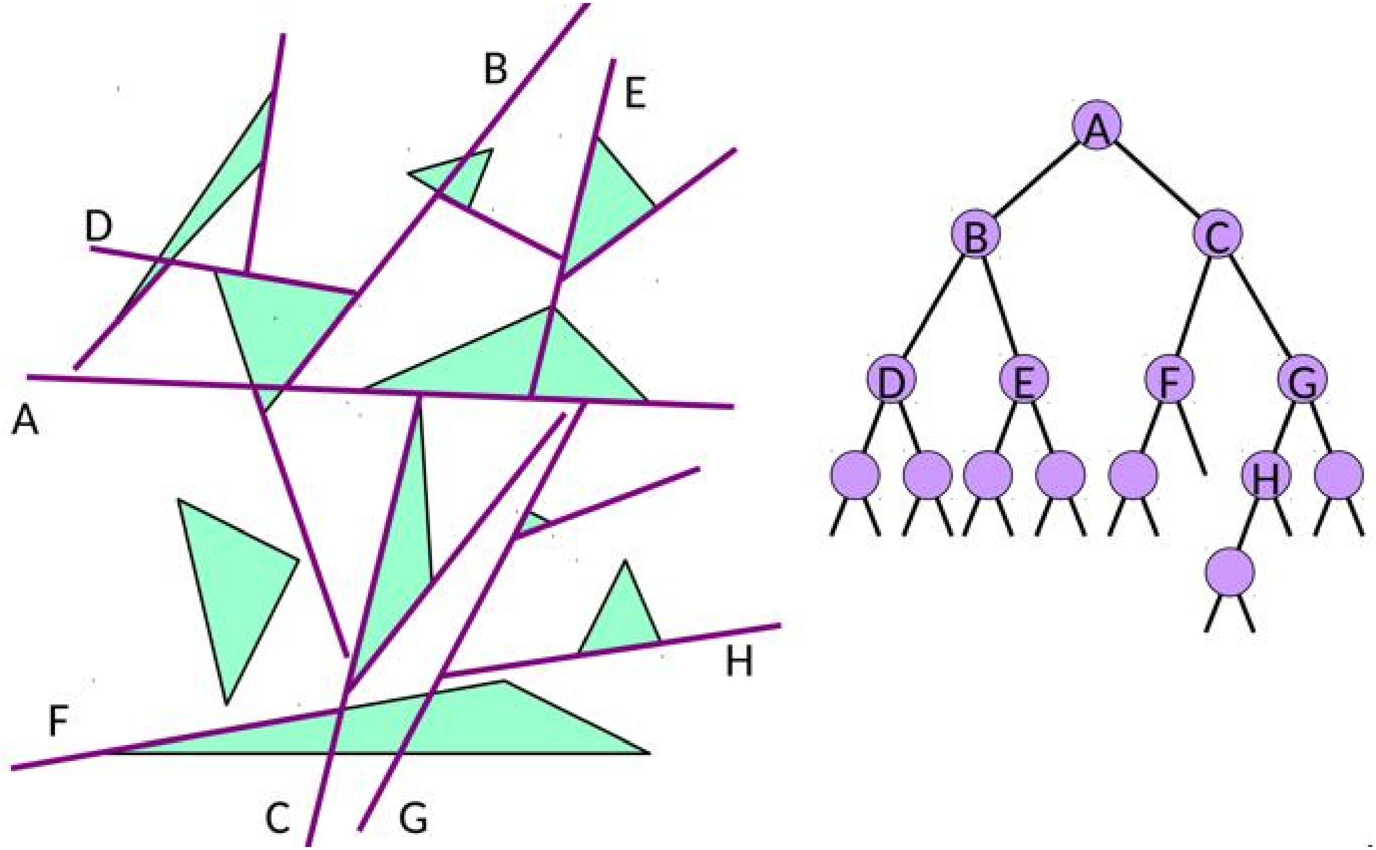
- ✓ k-d tree nearest-neighbor to a query point.
- ✓ Idea: As we get closer to the query point, we are cutting out all the subtrees that are away.



- ✓ Recursion comes back to pnt 1, right subtree is next, but cut it out (no search) ‘cos there can’t be a closer pnt there than pnt 5.

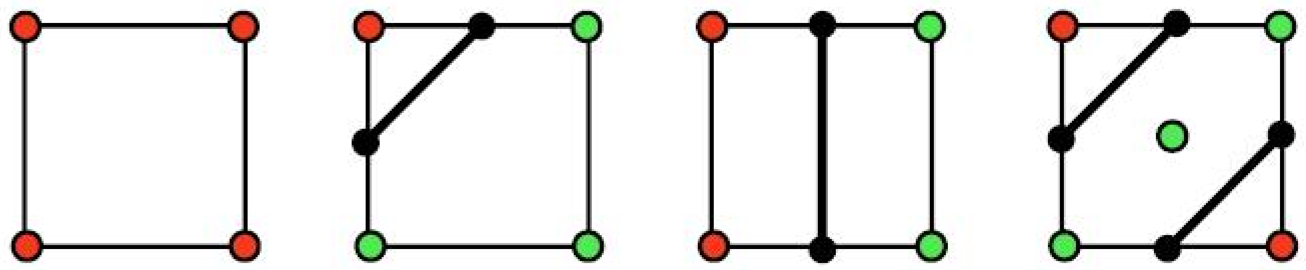
k-d Tree Alternative: BSP Tree

- ✓ k-d tree: cells are axis-aligned bounding boxes.
- ✓ BSP-tree: cells are arbitrarily shaped; splitting lines/planes through edge/triangles of the set to be stored. BSP: Binary Space Partitioning.

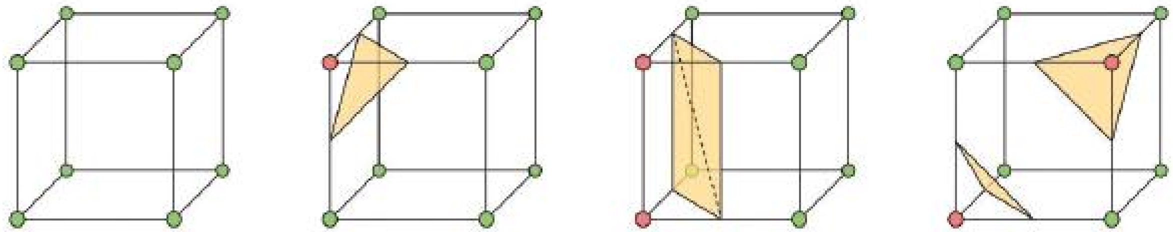


Marching X

- ✓ We efficiently (k-d tree) computed signed distance function F.
- ✓ The only thing left is to find the intersection point on the square/cube edge:
- ✓ Square:



- ✓ Cube:



Marching X

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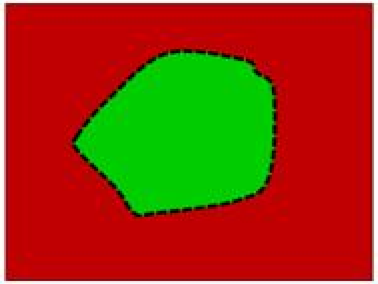
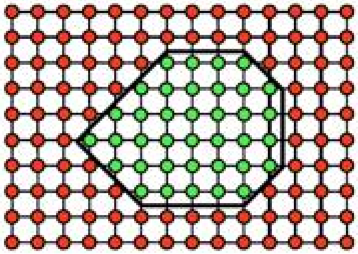
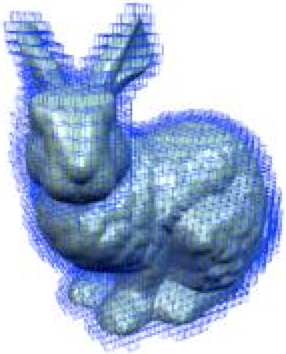
- ✓ The only thing left is to find the intersection point on the square/cube edge:

- ✓ Square:



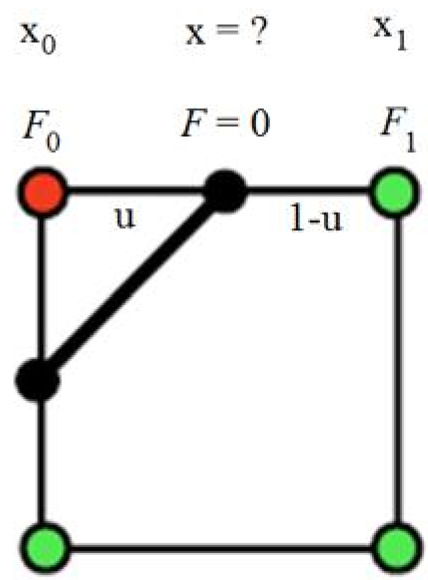


- ✓ Cube:



Marching X

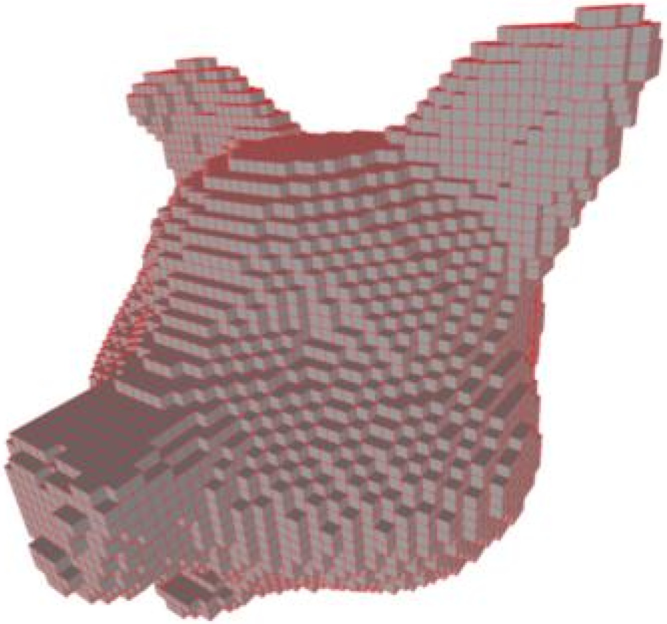
- ✓ We efficiently (k-d tree) computed signed distance function F.
- ✓ The only thing left is to find the intersection point x on the square/cube edge:

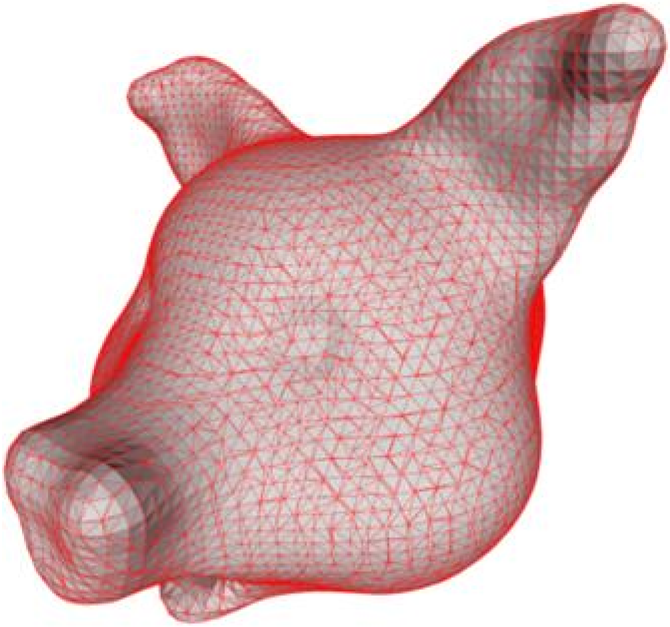


- ✓ $F = F_0 * (1\text{-u}) + F_1 * \mathrm{u}$ ➔ for F=0, $\mathrm{u} = F_0 / (F_0 - F_1)$ //F is linear on edges.
- ✓ $\mathrm{x} = \mathrm{x}_0 + \mathrm{u} * (\mathrm{x}_1 - \mathrm{x}_0)$ ➔ use u computed above and locate x ☺.
- ✓ Interpolation schemes discussed in Deformation lecture slides 26-31.

Marching X vs. Cuberille

- ✓ We efficiently (k-d tree) computed signed distance function F.
- ✓ Once we have this function, one can use the Cuberille method (1979) for a cruder approximation than Marching Cubes (1987).

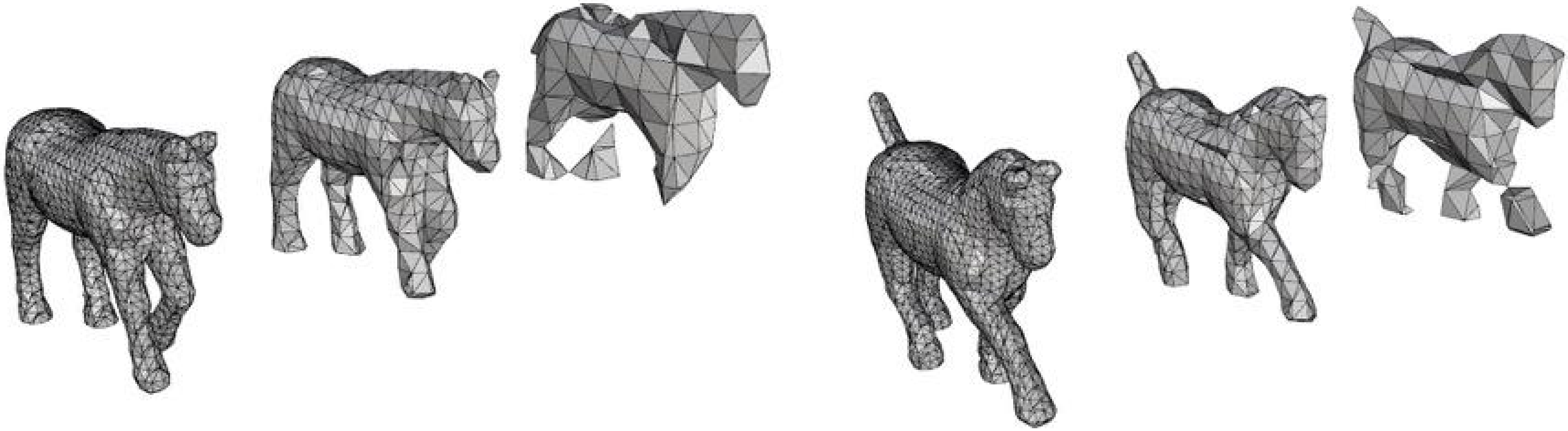




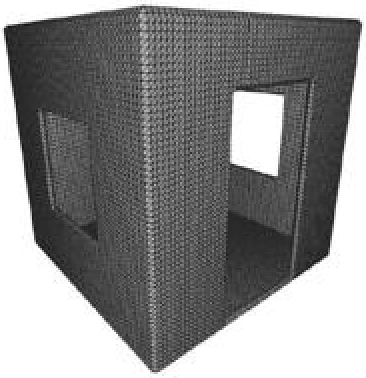
Check function at each cube center.

Marching Cubes Results

- ✓ Size of each voxel in the uniform grid determines level-of-detail.

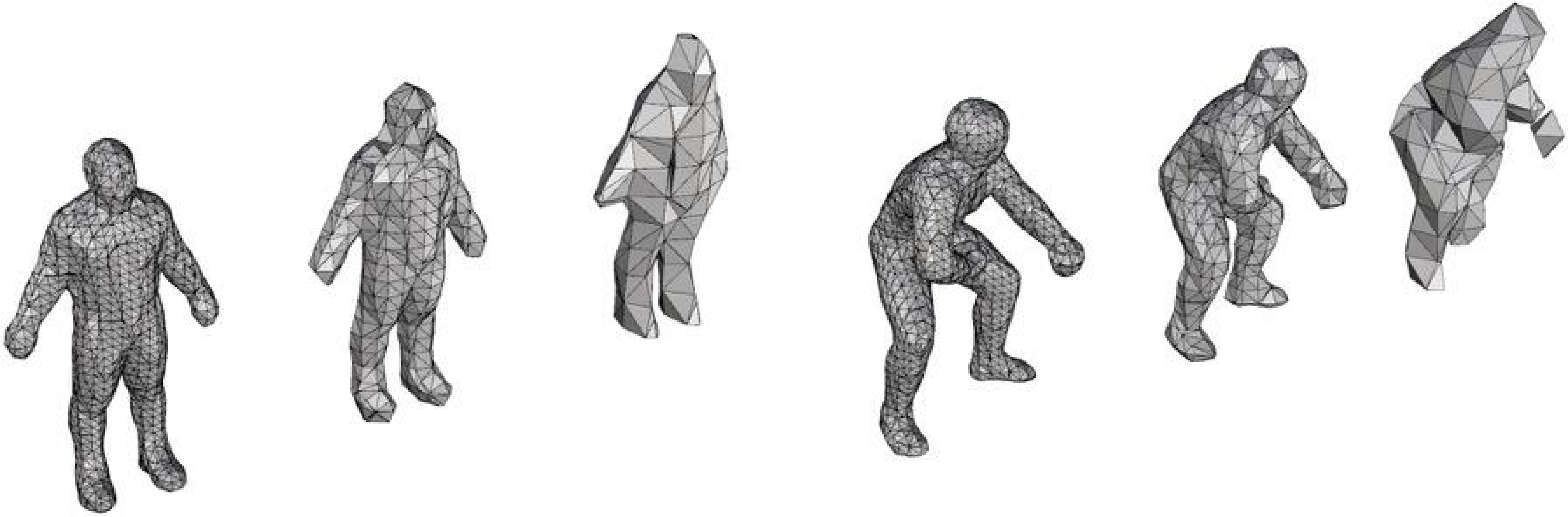


- ✓ Grid not adaptive ☹.

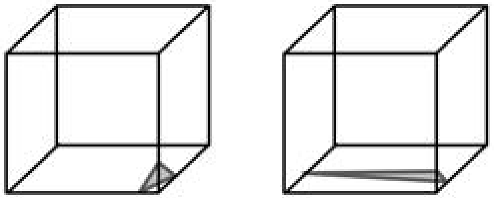


Marching Cubes Results

✓ Size of each voxel in the uniform grid determines level-of-detail.

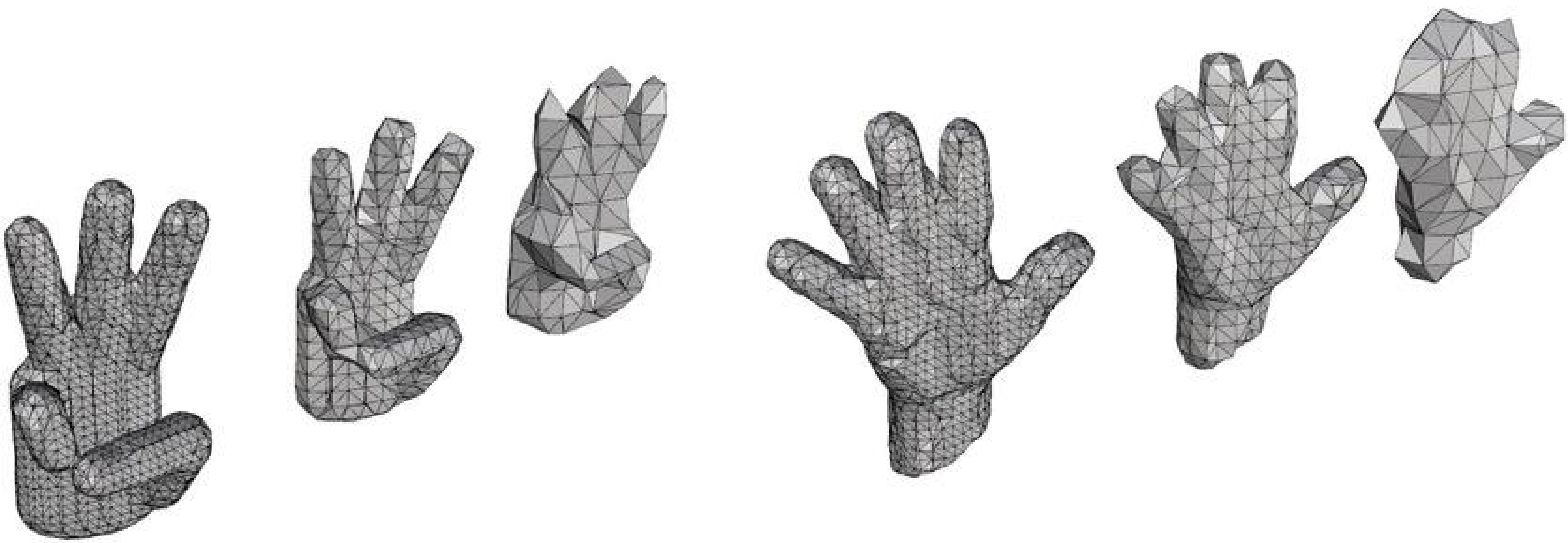


✓ Grid sampling can cause short triang edges, rendering tri uninformative.

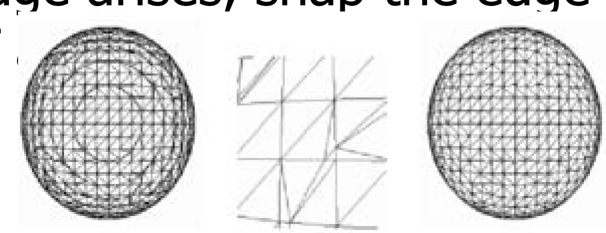


Marching Cubes Results

- ✓ Size of each voxel in the uniform grid determines level-of-detail.



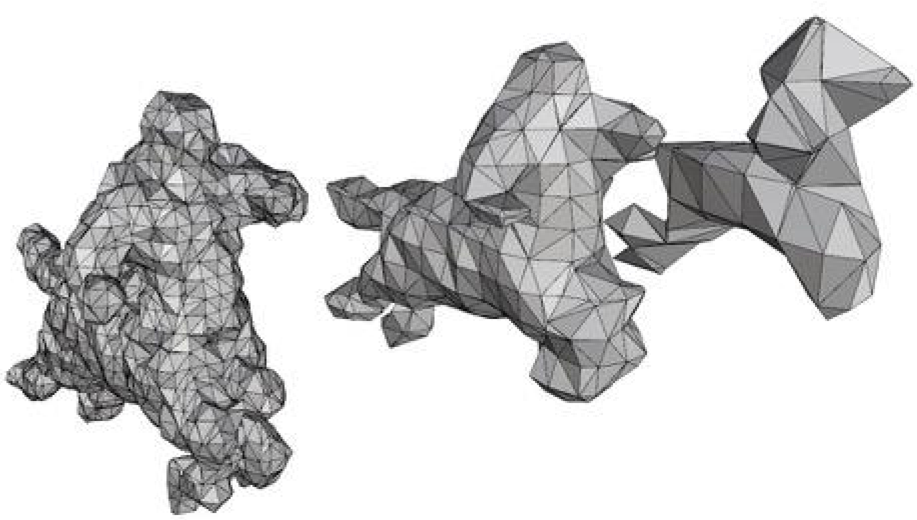
- ✓ Solution: if such an edge arises, snap the edge vertex to cube corner. If more than 1 vertex of

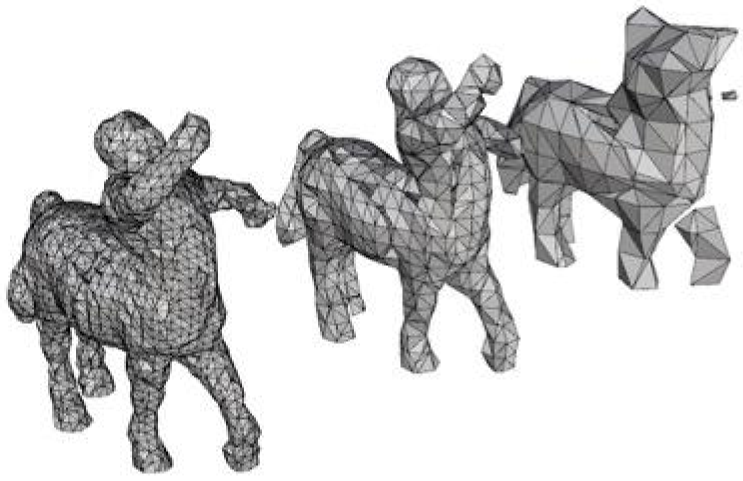


en discard triangle.

Marching Cubes Results

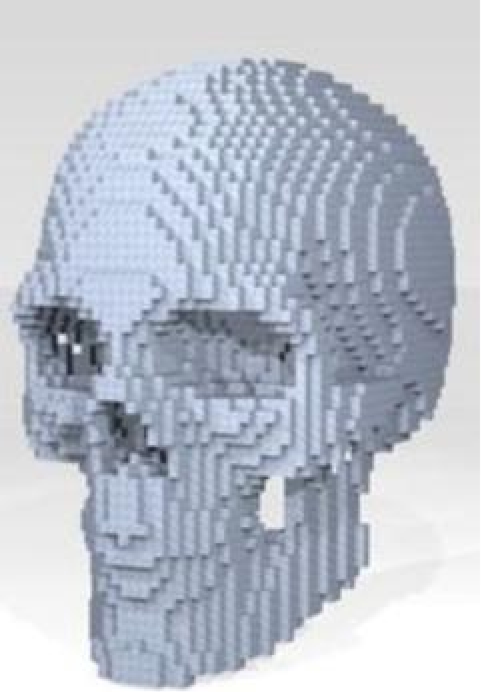
✓ Size of each voxel in the uniform grid determines level-of-detail.





Marching Cubes Case Study: CT Scan View

- ✓ In medical imaging, uniform grids are built based on CT/MRI scans.
- ✓ SDF is replaced by intensity values captured by radiology technology.
- ✓ CT scans generally come in DICOM format. Here is one way to convert them to NIfTI (nii) format, which can be parsed easily.
  - ✓ Use dcm2nii.exe software: https://people.cas.sc.edu/rorden/mricron/dcm2nii.html
  - ✓ E:\Postdoc\skull\mricron>dcm2nii.exe ..\CTsample\DIC
- ✓ Unzip output above somescan.nii.gz
- ✓ Learn target intensity using a software like ITK-SNAP, say it is e.
  - ✓ E.g., move cursor around skull and read intensities to decide on e.
- ✓ Marching cubes will extract the level-set corresponding to value e.



Marching Cubes Case Study: CT Scan View

- ✓ Read header and data.
  - ✓ #include "nifti1.h"

nifti1.h ×

```
/** \file nifti1.h
    \brief Official definition of the nifti1 header.  Written by Bob Cox, SSCC, NIMH.

    HISTORY:

       29 Nov 2007 [rickr]
          - added DT_RGBA32 and NIFTI_TYPE_RGBA32
          - added NIFTI_INTENT codes:
                TIME_SERIES, NODE_INDEX, RGB_VECTOR, RGBA_VECTOR, SHAPE
 */

#ifndef _NIFTI_HEADER_
#define _NIFTI_HEADER_

/*****************************************************************************
      ** This file defines the "NIFTI-1" header format.               **
      ** It is derived from 2 meetings at the NIH (31 Mar 2003 and    **
      ** 02 Sep 2003) of the Data Format Working Group (DFWG),        **
      ** chartered by the NIfTI (Neuroimaging Informatics Technology  **
      ** Initiative) at the National Institutes of Health (NIH).      **
      **--------------------------------------------------------------**
      ** Neither the National Institutes of Health (NIH), the DFWG,   **
      ** nor any of the members or employees of these institutions   **
      ** imply any warranty of usefulness of this material for any    **
      ** purpose, and do not assume any liability for damages,        **
      ** incidental or otherwise, caused by any use of this document. **
      ** If these conditions are not acceptable, do not use this!     **
      **--------------------------------------------------------------**
      ** Author:   Robert W Cox (NIMH, Bethesda)                      **
      ** Advisors: John Ashburner (FIL, London),                      **
      **           Stephen Smith (FMRIB, Oxford),                     **
      **           Mark Jenkinson (FMRIB, Oxford)                     **
******************************************************************************/
```

Marching Cubes

- ✓ Read header and data.
  - ✓ #include "nifti1.h"

```
void VoxelSet::loadNii(char* niiFile)
{
    //binary nii format reader adapted from http://sourceforge.net/projects/niftilib/

    cout << "Voxel grid initializing (to " << niiFile << ")..\n";

    nifti_1_header hdr;
    FILE *fp;
    int ret,i;
    double totalIntensity;
    MY_DATATYPE *data = NULL;

    /********** open and read header */
    fp = fopen(niiFile,"rb"); //seeing this took my 2 hours :(
    if (fp == NULL) {
            fprintf(stderr, "\nError opening header file %s\n",niiFile);
            exit(1);
    }
    ret = fread(&hdr, MIN_HEADER_SIZE, 1, fp);
    if (ret != 1) {
            fprintf(stderr, "\nError reading header file %s\n",niiFile);
            exit(1);
    }

    fclose(fp); //close here 'cos i'll reopen it below with a jump to the start of the image data

    /********** print a little header information */
    fprintf(stderr, "\nnii file header information:");
    fprintf(stderr, "\nXYZT dimensions: %d %d %d %d",hdr.dim[1],hdr.dim[2],hdr.dim[3],hdr.dim[4]);
    fprintf(stderr, "\nDatatype code and bits/pixel: %d %d",hdr.datatype,hdr.bitpix);
    fprintf(stderr, "\nScaling slope and intercept: %.6f %.6f",hdr.scl_slope,hdr.scl_inter);
    fprintf(stderr, "\nByte offset to data in datafile: %ld",(long)(hdr.vox_offset));
    fprintf(stderr, "\nVoxel spacing: %f, %f, %f\n", hdr.pixdim[1], hdr.pixdim[2], hdr.pixdim[3]);

    /********** open the datafile, jump to data offset */
    fp = fopen(niiFile,"rb");
    if (fp == NULL) {
            fprintf(stderr, "\nError opening data file %s\n",niiFile);
            exit(1);
    }

    ret = fseek(fp, (long)(hdr.vox_offset), SEEK_SET); //jump
    if (ret != 0) {
            fprintf(stderr, "\nError doing fseek() to %ld in data file %s\n",
                (long)(hdr.vox_offset), niiFile);
            exit(1);
    }

    /********** allocate buffer and read first 3D volume from data file */
    data = (MY_DATATYPE *) malloc(sizeof(MY_DATATYPE) * hdr.dim[1]*hdr.dim[2]*hdr.dim[3]);
    if (data == NULL) {
            fprintf(stderr, "\nError allocating data buffer for %s\n",niiFile);
            exit(1);
    }
    fread(data, sizeof(MY_DATATYPE), hdr.dim[1]*hdr.dim[2]*hdr.dim[3], fp);
    fclose(fp);
```

Marching Cubes Case Study: CT Scan View

✓ Create voxels useful to Marching Cubes, e.g., the ones that contain skull intensities.

```
//learn width, height, depth of each voxel and then initialize each one (new Voxel)
width = hdr.pixdim[1];
height = hdr.pixdim[2];
depth = hdr.pixdim[3]; //of each cell to be added to voxels[] below
int toRight = 0, toUp = 0, toFront = 0;
float x, y, z;
//all voxels that includes bones, brains, eyes, .. everything; just need skull bones here
for (i = 0; i < hdr.dim[1]*hdr.dim[2]*hdr.dim[3]; i++)
{
    //id, intensity, and coord of this current voxel
    x = toRight * hdr.pixdim[1]; //voxel spacing, space b/w x coords, i.e. pixdim[1]
    toRight++;
    y = toUp * hdr.pixdim[2]; //voxel spacing, space b/w y coords, i.e. pixdim[2]
    z = toFront * hdr.pixdim[3]; //voxel spacing, space b/w z coords, i.e. pixdim[3]
    if ( isSkullVoxel(data[i], x, y, z) ) //returns true if (intensity > 200 && intensity < 1000) where intensity is data[i]
    {
        if (toRight % 5 == 0 && toUp % 5 == 0 && toFront % 5 == 0) //extra thresholding to downsample the bone voxels by picking
                                                                  //every fifth in x-direction and in y-direction and in z-direction
        {
            float* coords = new float[3];
            coords[0] = x;
            coords[1] = y;
            coords[2] = z;
            voxels.push_back( new Voxel(i, data[i], coords) ); //coords is the center of this voxel
        }
    }

    //direction/pointer adjustments
    if (toRight >= hdr.dim[1])
    {
        toRight = 0; //reset to the left end
        toUp++; //1 level up
        if (toUp >= hdr.dim[2])
        {
            toFront++; //1 slice towards me, that is towards front
            toUp = 0; //start from the bottom level
        }
    }
} //end of i

//you may then create coarse voxels by merging every non-overlapping 3x3x3 box (of 27 voxels)
//in data[] to 1 big coarseVoxel if 1+ constituent element is a segmented skull/bone voxel
//..
```

Scientific Visualization

- ✓ Scalar field: a scalar value is associated with each point in n-d space.
  - ✓ Can be obtained by associating SDF or intensity values to grid points.
- ✓ Vector field: an n-d vector is associated with each point in n-d space.
  - ✓ Can be obtained by simulations.

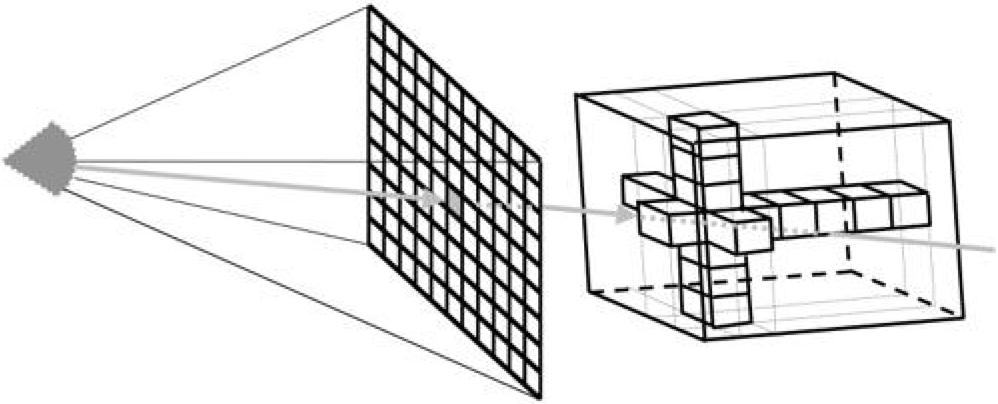
- ✓ Scalar fields can be visualized via
  - ✓ indirect (volume) rendering: isosurface/mesh extraction and rendering (corresponding to a certain scalar value – Marching Cubes in 3D).
    - ✓ *We handled this already.*
  - ✓ direct (volume) rendering – ray casting.
    - ✓ *Upcoming.*
- ✓ Vector fields can be visualized via
  - ✓ vector arrows located at each point.
    - ✓ *Upcoming.*

Direct Scalar Field Visualization

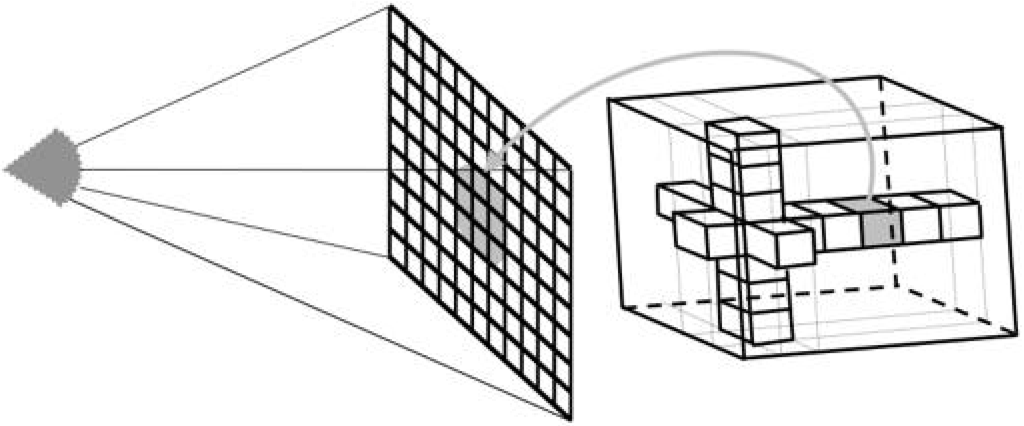
- ✓ So far we've dealt with visualizing a scalar field indirectly, via isosurface extraction and rendering.
- ✓ Let's now do it directly via ray casting.
- ✓ Aka direct volume rendering (DVR) if the field is defined in 3D.

Direct Scalar Field Visualization

✓ Ray casting is a backward method where we send rays from the eye.



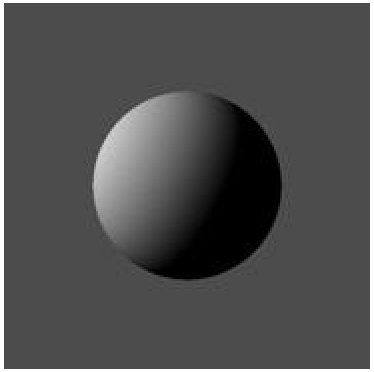
✓ Opposite of the forward method where we use projections to the eye.

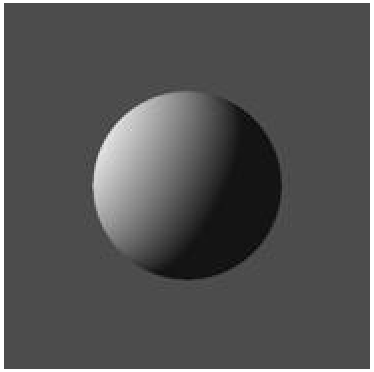


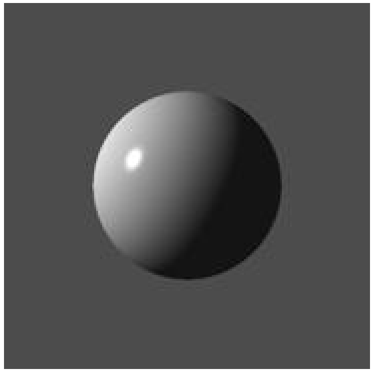
Direct Scalar Field Visualization

- ✓ Ray casting is similar to ray tracing in surface-based computer graphics.
  - ✓ Ray tracing deals with primary rays (first rays cast into the scene from the camera/eye) and secondary rays (recursive rays to generate shadows, reflections, refractions, etc.)
    - ✓ 1979, Turner Whitted.
  - ✓ Ray casting deals only with primary rays.
    - ✓ 1968, Arthur Appel.

- ✓ Check out CENG477 Computer Graphics course for details.
  - ✓ http://user.ceng.metu.edu.tr/~ys/ceng477-gfx/
  - ✓ Look at Ray Tracing parts 1 and 2, up to the Recursive Ray Tracing section.





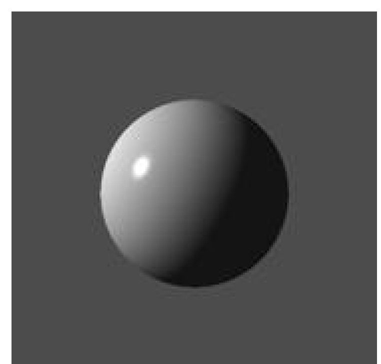
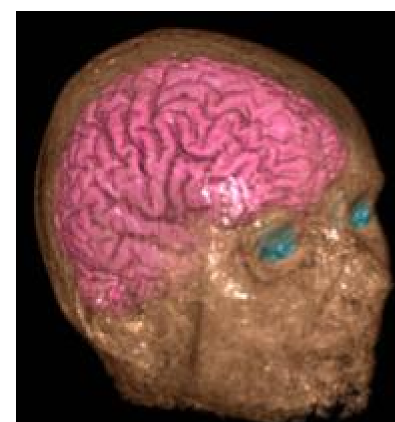


Additional color due to lighting.

Direct Scalar Field Visualization

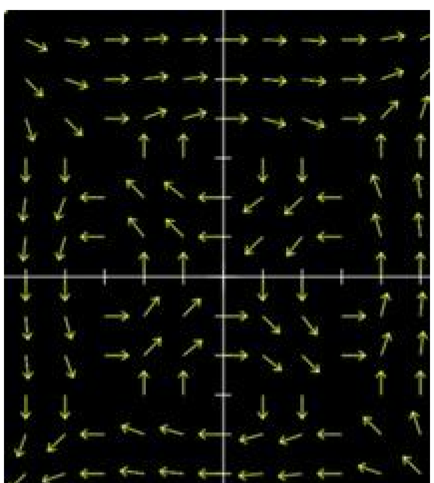
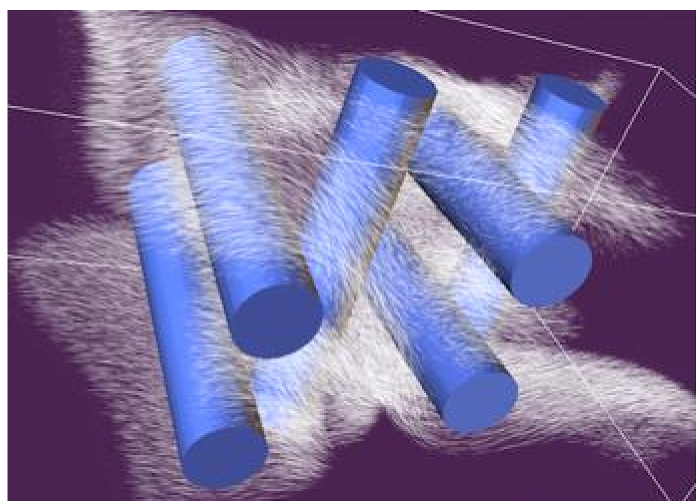
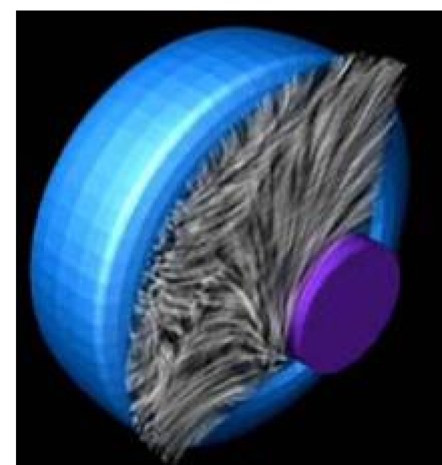
- ✓ How to calculate ray-surface intersection since we've no surfaces here.
- ✓ Step through the volume.
- ✓ A ray is cast into the volume, sampling the volume at certain intervals.
- ✓ Sampling intervals are usually equidistant, but don't have to be (e.g. importance sampling).
- ✓ At each sampling location, a sample is interpolated / reconstructed from the voxel grid.
- ✓ Opacity and color in each cell is predefined according to classification.



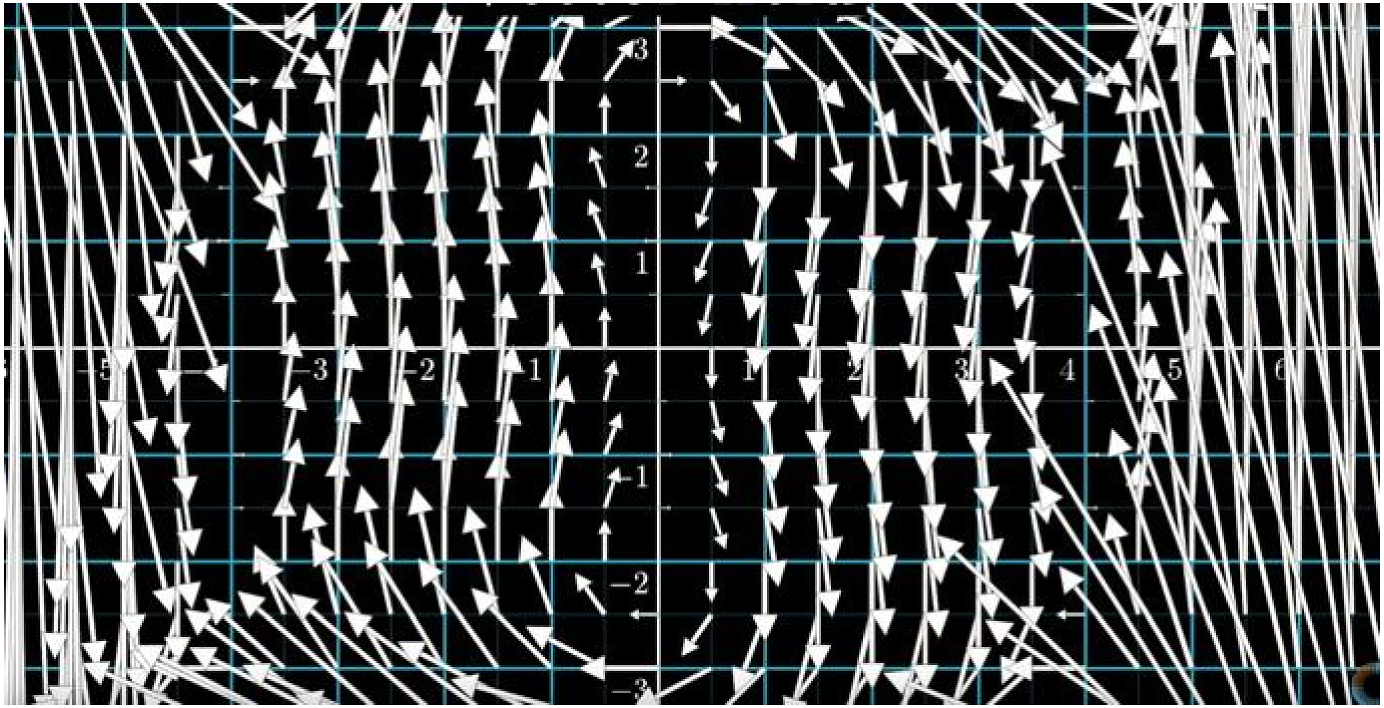
Vector Field Visualization

- ✓ So far we've dealt with visualizing a scalar field, i.e., a grid with function values at each grid point, for a given scalar value.
  - ✓ Arises frequently in medical imaging, 3D point cloud to surface conversions.
- ✓ Let's now extend this to vector field visualization, where we have an n-dimensional space with n-dimensional vectors attached at each point.
  - ✓ Arises frequently in fluid simulation, air flow simulation, or in general visualizing functions that have the same # of dimensions in their input as in their output.
    - ✓ $f(x, y) = [x^2 + 5y \quad xy]^T$, $f(x, y, z) = [4x \quad x^3(y^2 - 5) \quad 33]^T$, and so on.

Vector Field Visualization

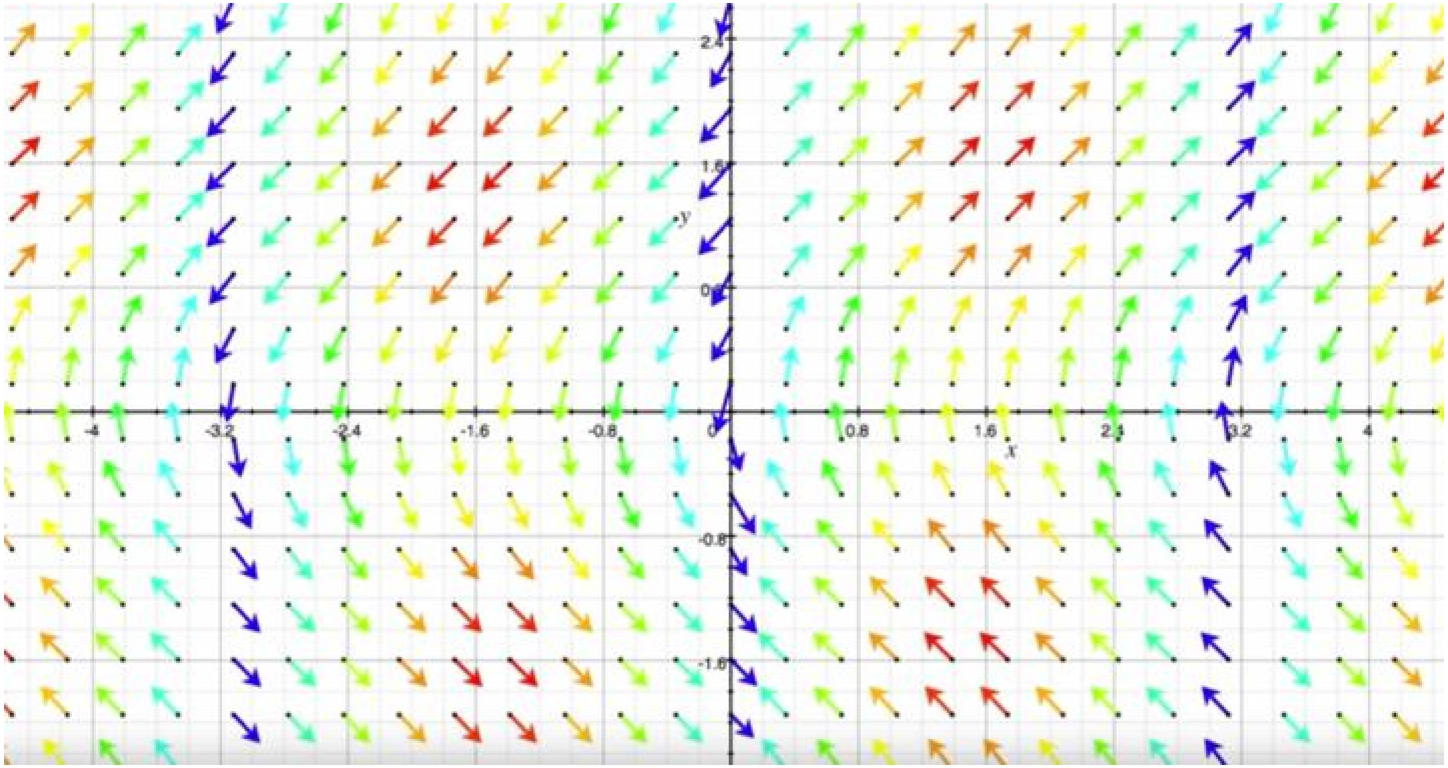
- ✓ If all vectors are drawn proportional to their magnitudes, the longer ones mess up the screen.



- ✓ This one and the subsequent visuals're collected from the fluid flow youtube series of Khan Academy and 3Blue1Brown.

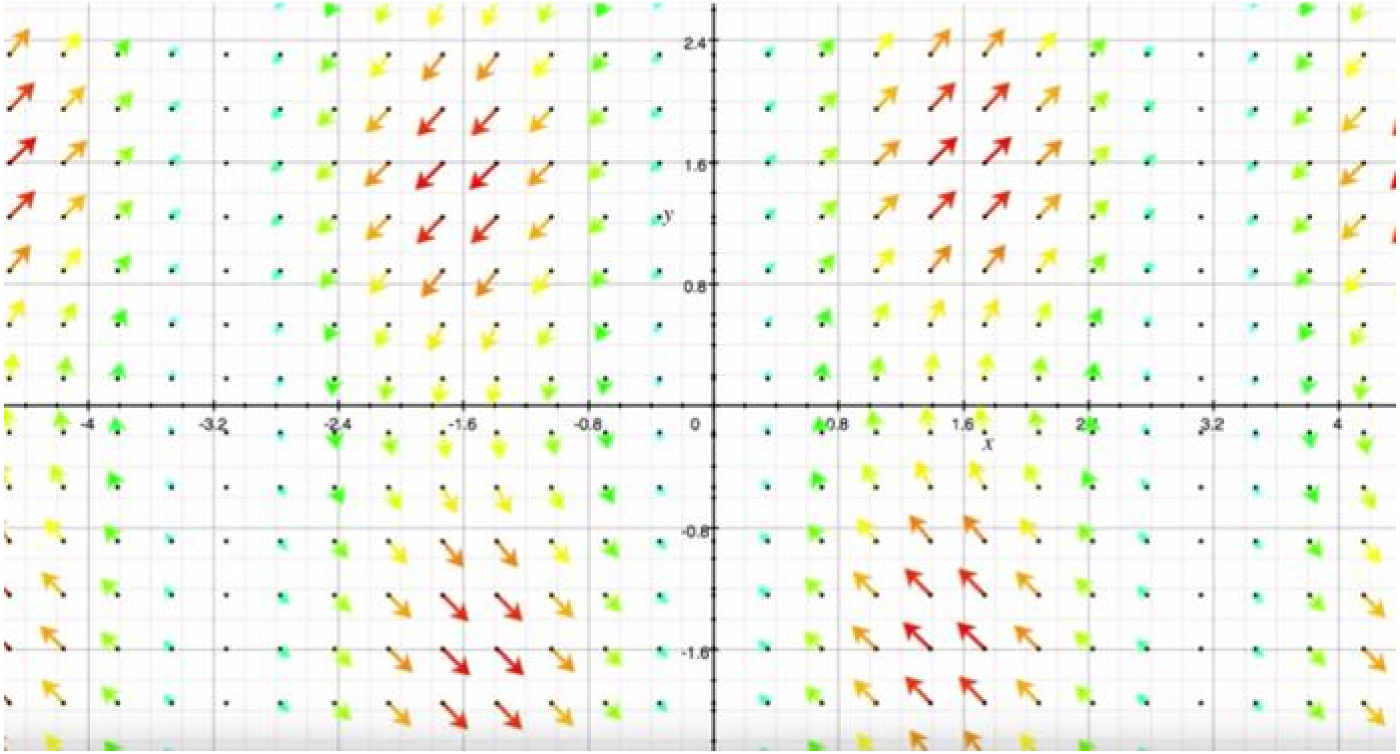
Vector Field Visualization

- ✓ All vectors are made unit (length 1) for visual clarity.
- ✓ We can still infer their magnitudes via color coding: red long blue short.



Vector Field Visualization

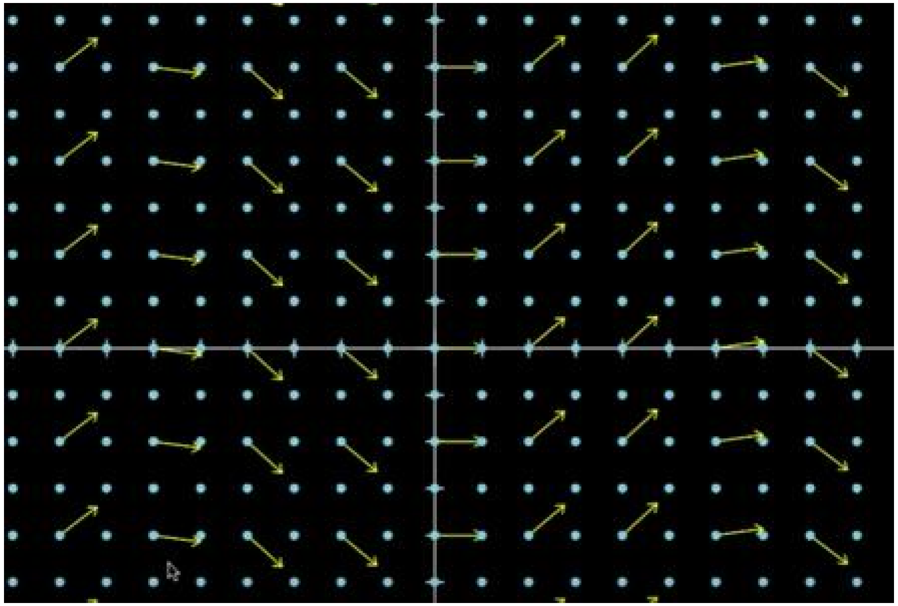
- ✓ All vectors are made unit (length 1) for visual clarity.
- ✓ Combined w/ proper scaling where max length is 1 (prevent mess).



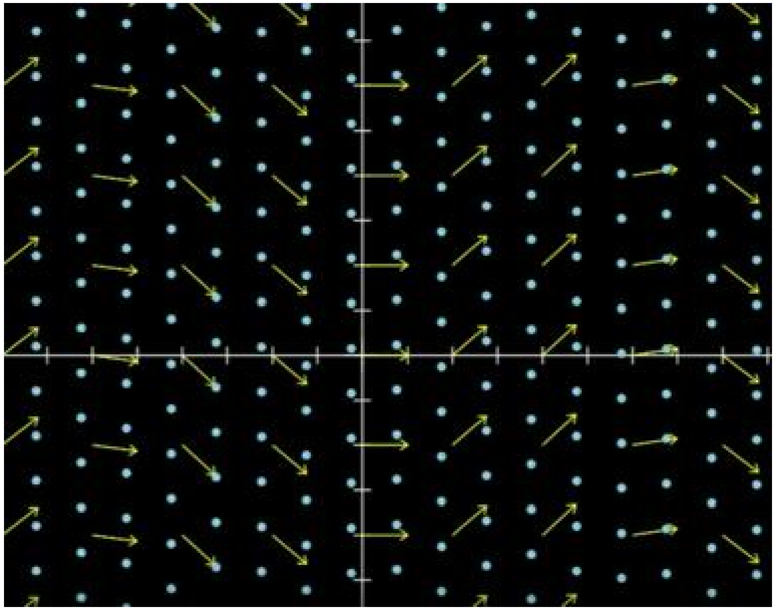
Vector Field Visualization

- ✓ Case study: fluid flow.
- ✓ Droplets of water are moving according to the vectors visualized.

initial state



t + 10

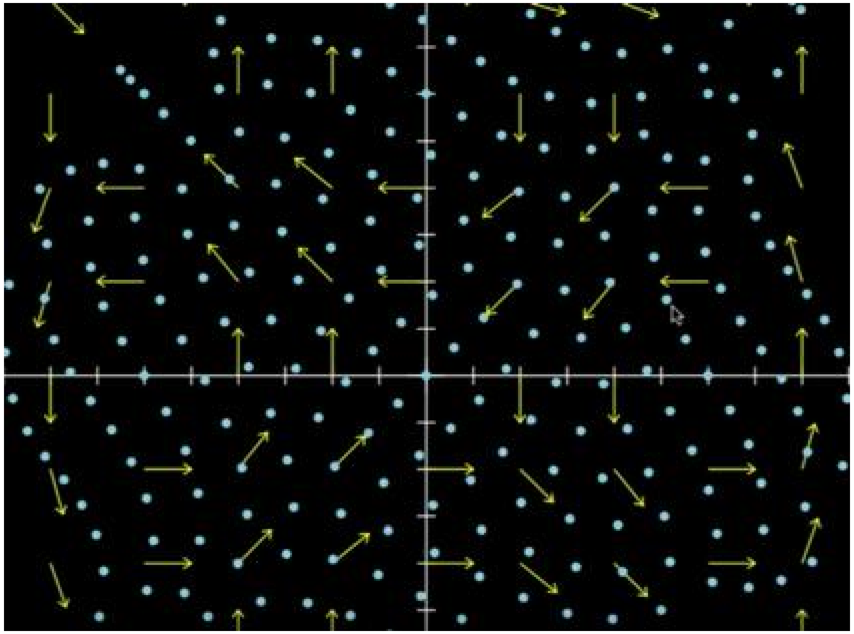


- ✓ Vectors are representing the velocities of particles of fluids here.
  - ✓ Alternatives: force of gravity, magnetic field, etc.

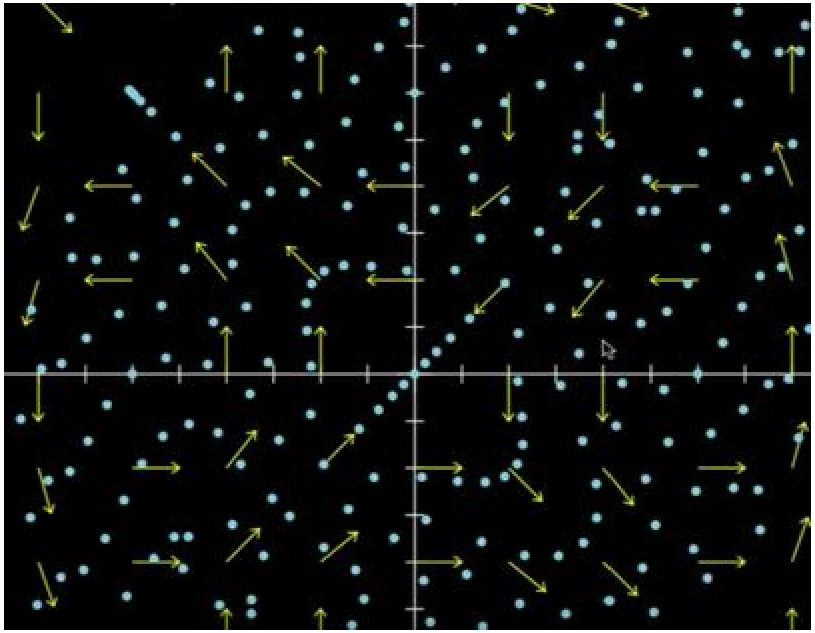
Vector Field Visualization

- ✓ Case study: fluid flow.
- ✓ Droplets of water are moving according to the vectors visualized.

t



t + 10

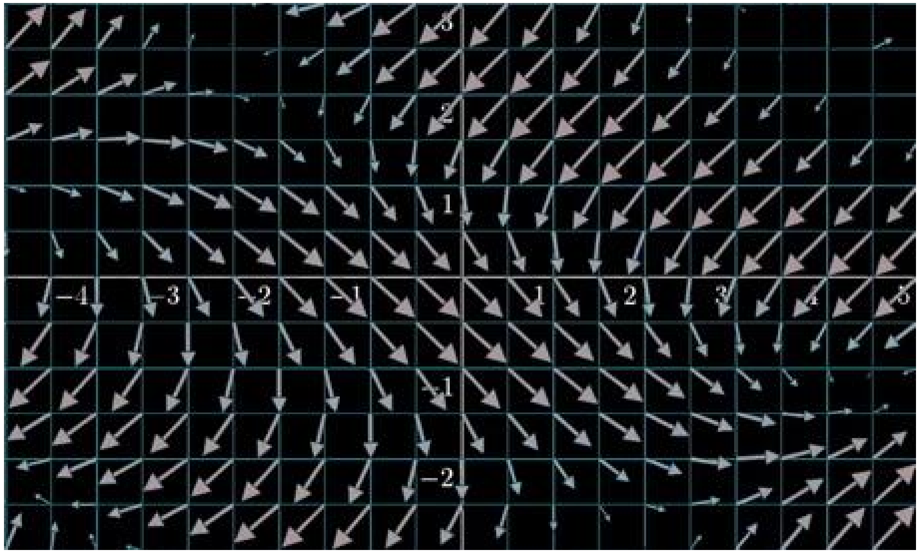


- ✓ This is a static vector field depicting a steady state system.

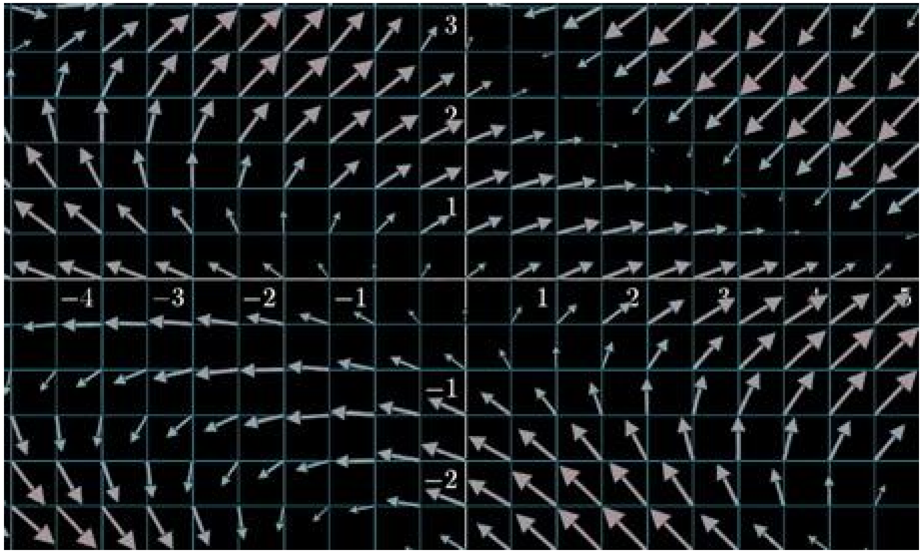
Vector Field Visualization

- ✓ Case study: fluid flow.
- ✓ Vector fields may change over time, e.g., in real-world fluid flow, the velocities of particles change in response to the surrounding context.

t



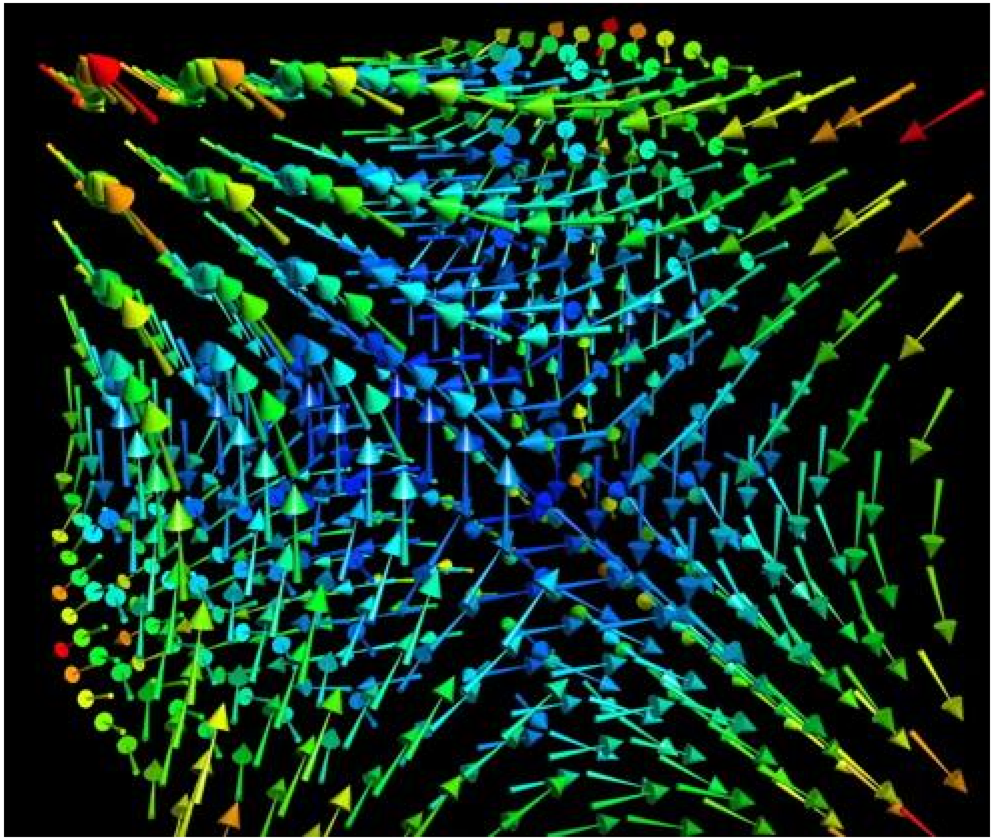
t + 10



- ✓ This is a dynamic vector field depicting a more realistic fluid flow.
  - ✓ Wind is not constant; electric field not constant (charged particles move).

Vector Field Visualization

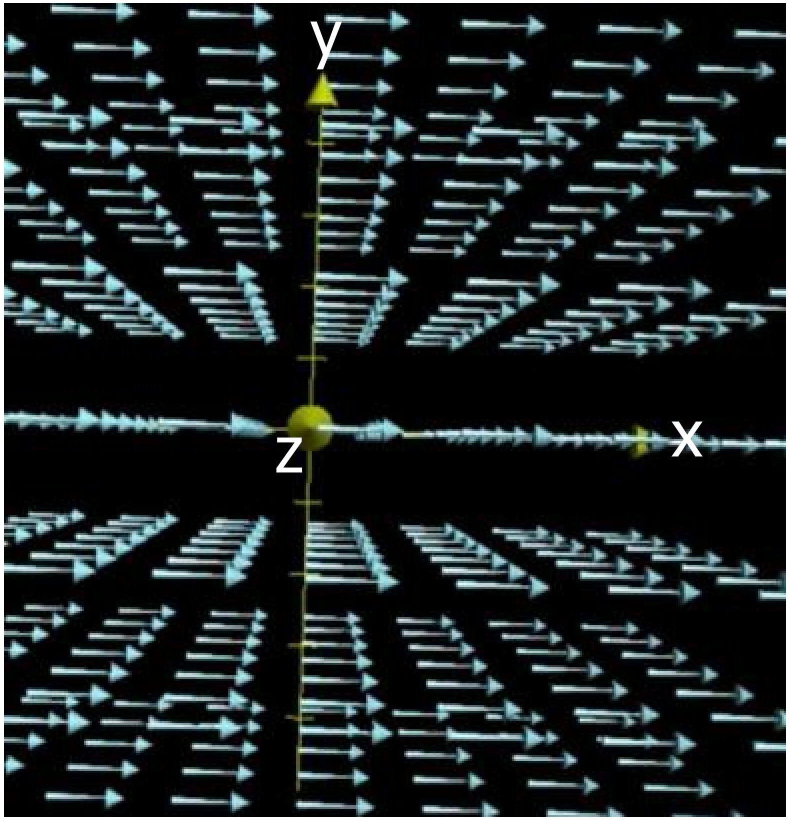
- ✓ n-d spaces are equipped with n-d vectors.
- ✓ So far we studied n=2. Here n=3:



- ✓ Vectors at each point are defined as $f(x, y, z) = [yz \quad xz \quad xy]^T$.

Vector Field Visualization

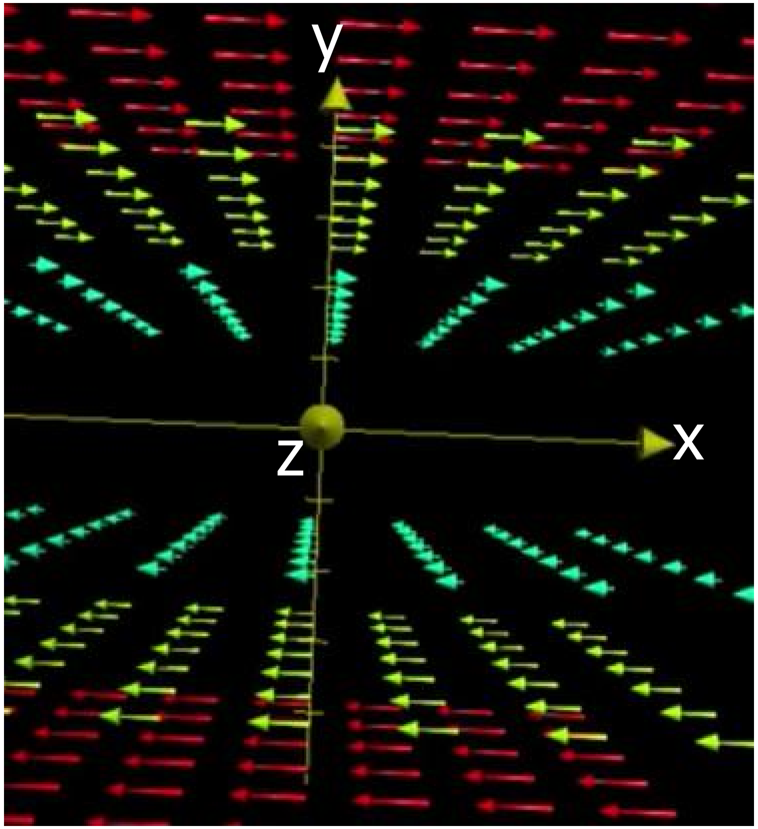
- ✓ n-d spaces are equipped with n-d vectors.
- ✓ So far we studied n=2. Here n=3:



- ✓ Vectors at each point are defined as $f(x, y, z) = [1 \quad 0 \quad 0]^T$.

Vector Field Visualization

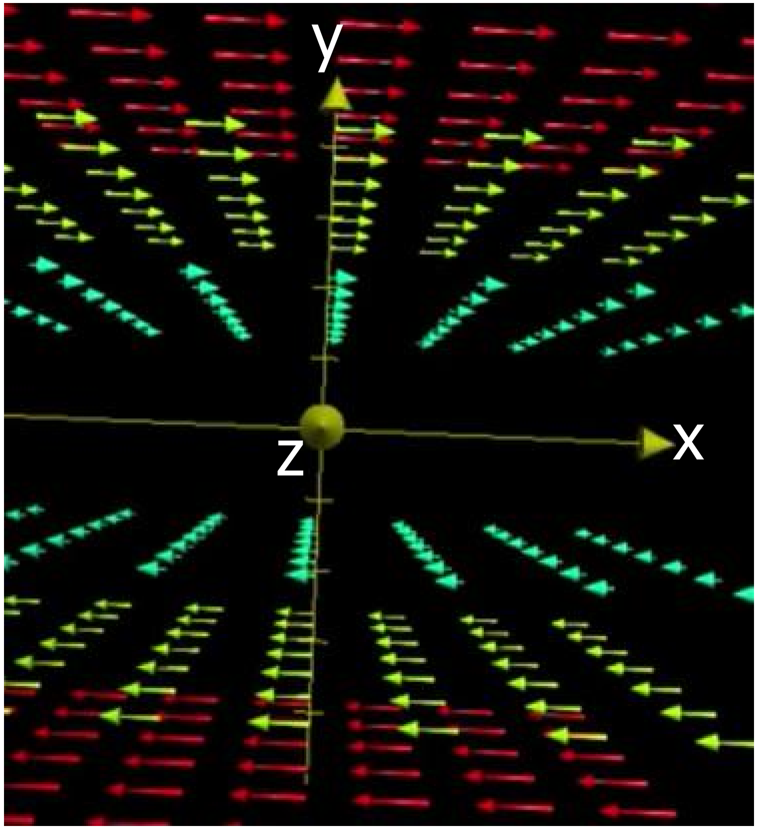
- ✓ n-d spaces are equipped with n-d vectors.
- ✓ So far we studied n=2. Here n=3:



- ✓ Vectors at each point are defined as f(x, y, z) = ?. Color ~ magnitude.

Vector Field Visualization

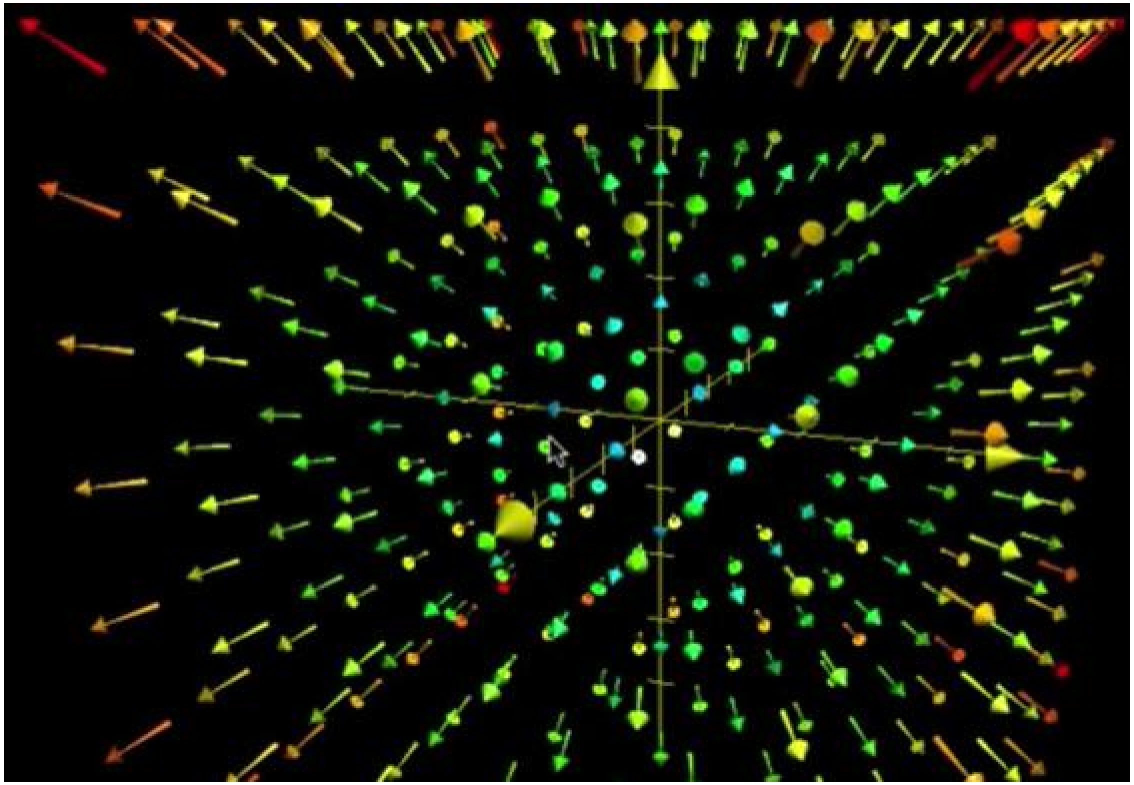
- ✓ n-d spaces are equipped with n-d vectors.
- ✓ So far we studied n=2. Here n=3:



- ✓ Vectors at each point are defined as $f(x, y, z) = [y \quad 0 \quad 0]^T$.

Vector Field Visualization

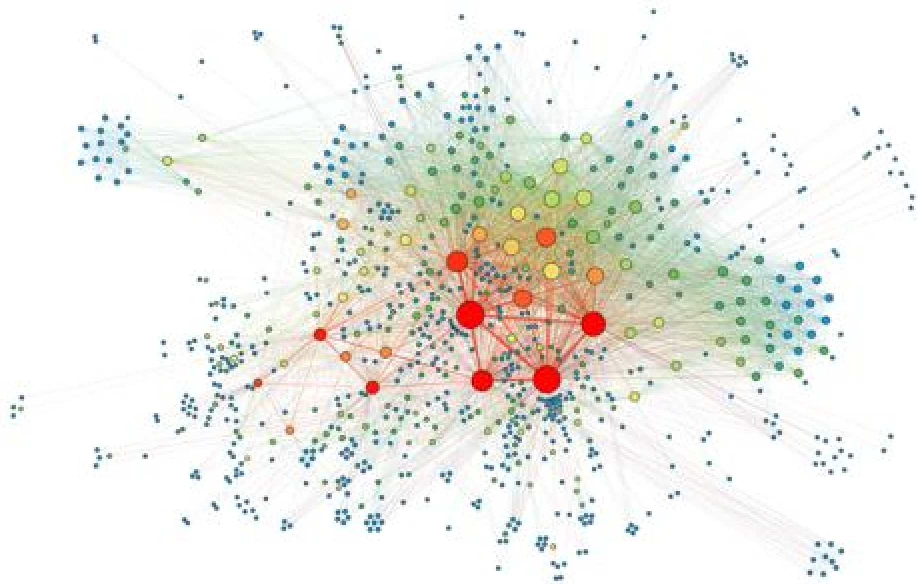
- ✓ n-d spaces are equipped with n-d vectors.
- ✓ So far we studied n=2. Here n=3:

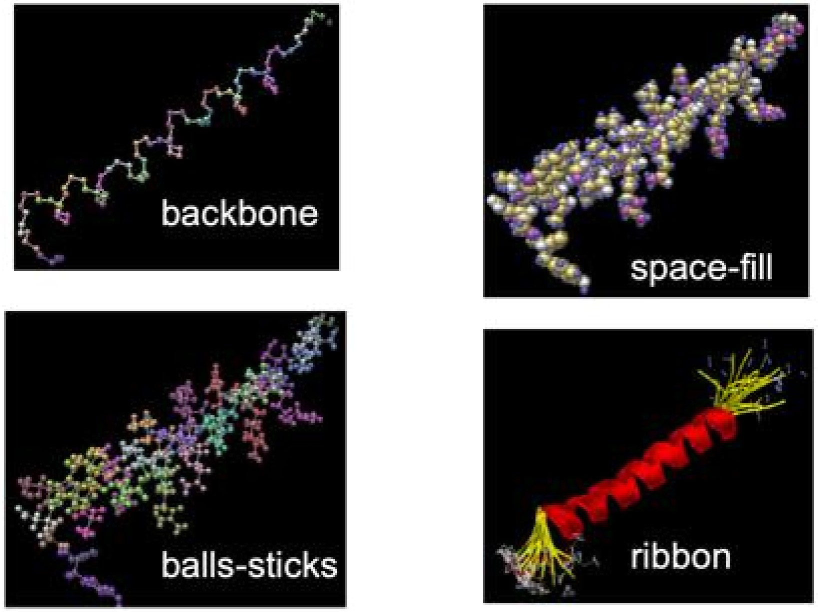


- ✓ Vectors at each point are defined as $f(x, y, z) = [x \quad y \quad z]^T$.

Information Visualization

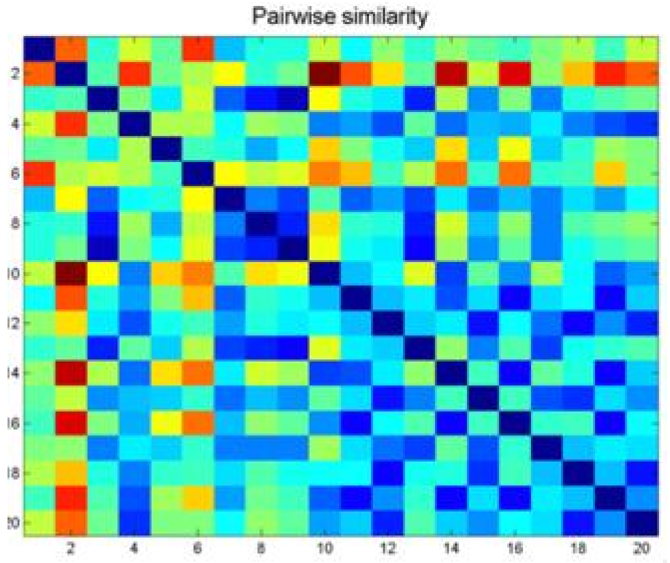
- ✓ Study of visual representations of abstract data to reinforce human cognition.
- ✓ Differs from Scientific Visualization: it is infovis (information visualization) when the spatial representation is chosen, and it is scivis (scientific visualization) when the spatial representation is given.
- ✓ Some examples include social network analysis, protein visualization.



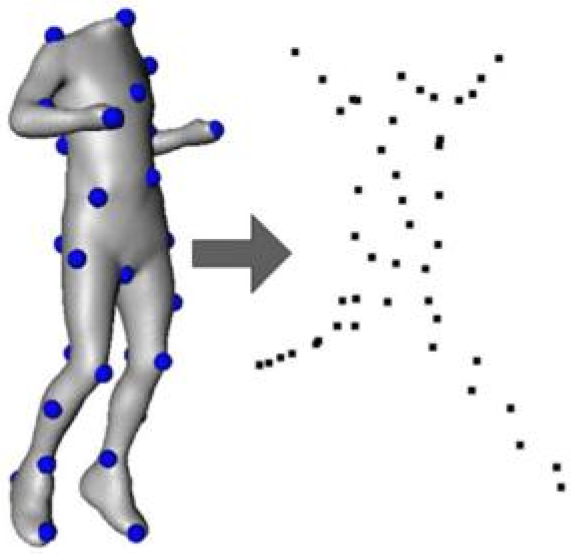


Information Visualization - MDS

- ✓ Multidimension Scaling (MDS) is a powerful tool to visualize pairwise dissimilarity data.
- ✓ Translates “info about the pairwise 'distances' among a set of n objects” into a configuration of n points mapped into the Euclidean space.
  - ✓ A form of non-linear dimensionality reduction: see more in Shape Gen. slides.



Geodesic distances b/w sample pairs.



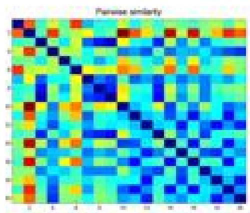
MDS mapping of samples.

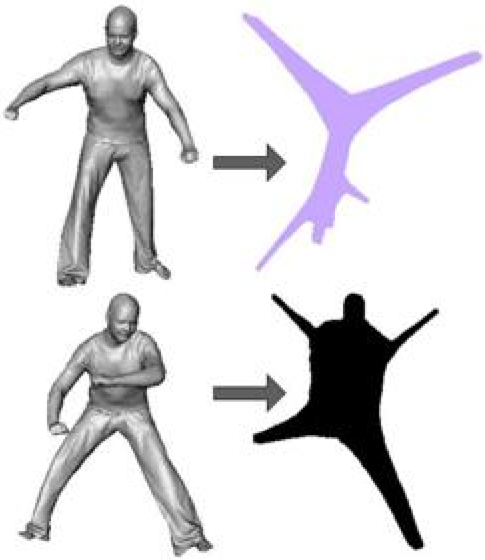
Information Visualization - MDS

- ✓ Given a distance matrix with the distances $\delta_{i,j}$ between each pair of objects, and a chosen number of dimensions *k*, an MDS algorithm places each object into *k* – dimensional Euclidean space such that the between-object distances $\|x_i - x_j\|$ are preserved as well as possible. For *k* =1, 2, and 3, the resulting points can be visualized on a scatter plot.

$$\min_{x_1,\ldots,x_I} \sum_{i<j} (\|x_i - x_j\| - \delta_{i,j})^2$$

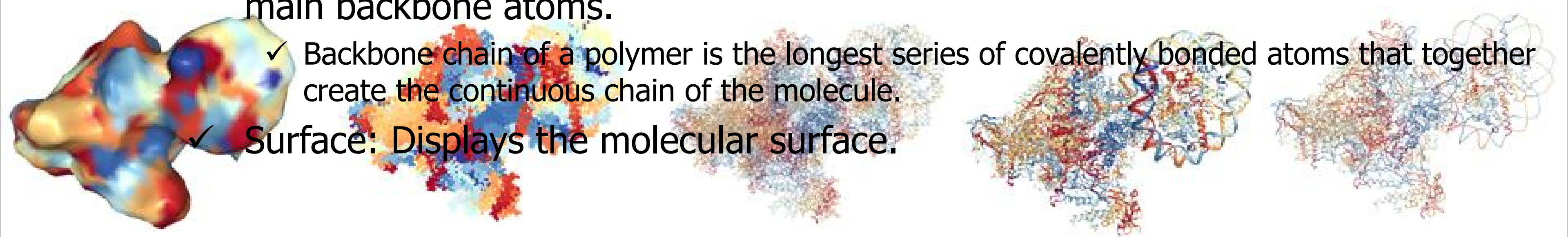
Information Visualization - MDS

- ✓ Main MDS techniques are as follows.
- ✓ Least-squares MDS: minimizes the energy: $$\min_{x_1,\ldots,x_I} \sum_{i<j} (\|x_i - x_j\| - \delta_{i,j})^2$$
- ✓ Classical MDS: uses k leading eigenvectors of the associated affinity matrix () to obtain the k-d configuration.
- ✓ Landmark MDS: embeds a small number of landmark points via least-squares or classical method, and then computes the embedding coordinates of the remaining data points based on their distances from the landmark points.



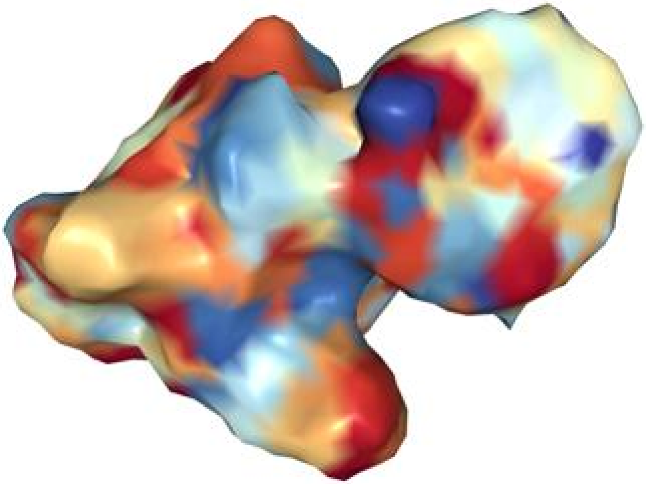
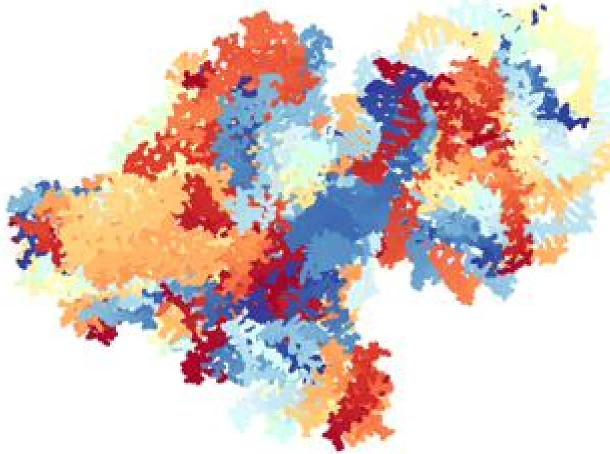
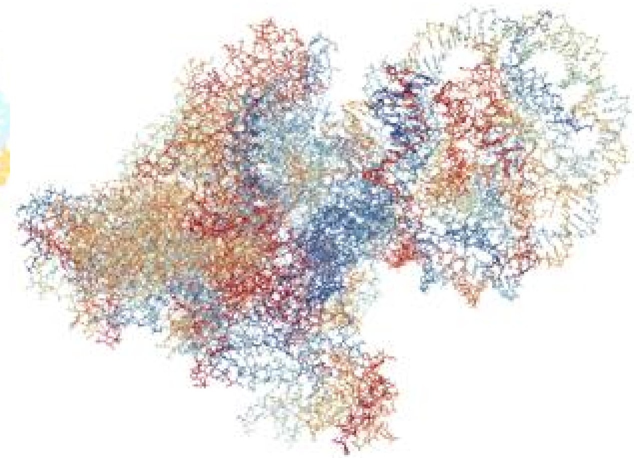
Information Visualization – Protein Vis.

- ✓ Some popular visualization modes:
  - ✓ Ball+stick: Atoms are displayed as spheres (balls) and bonds as cylinders (sticks). Aspect ratio defines how much bigger sph radius compred to cyl radius.
  - ✓ Licorice: Ball+stick variant where balls and sticks have the same radius.
  - ✓ Line: Bonds are displayed by a flat, unshaded line.
  - ✓ Point: Atoms are displayed by textured points.
  - ✓ Spacefill: Atoms are displayed as a set of space-filling spheres.
  - ✓ Trace: A flat, unshaded line is displayed along the main backbone trace.
  - ✓ Ribbon: A thin ribbon is displayed along the main backbone trace.
  - ✓ Cartoon: A smooth trace connect successive residues of unbroken chains by their main backbone atoms.
  - ✓ Backbone: Cylinders connect successive residues of unbroken chains by their main backbone atoms.
    - ✓ Backbone chain of a polymer is the longest series of covalently bonded atoms that together create the continuous chain of the molecule.
  - ✓ Surface: Displays the molecular surface.

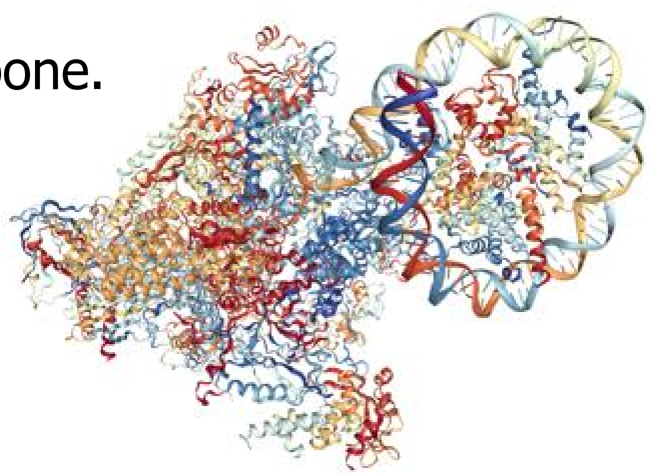
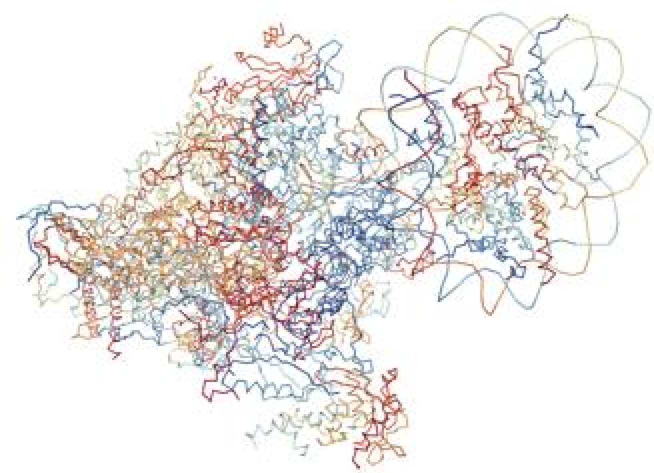


Information Visualization – Protein Vis.

- ✓ Other modes from left to right: surface, line, licorice ..

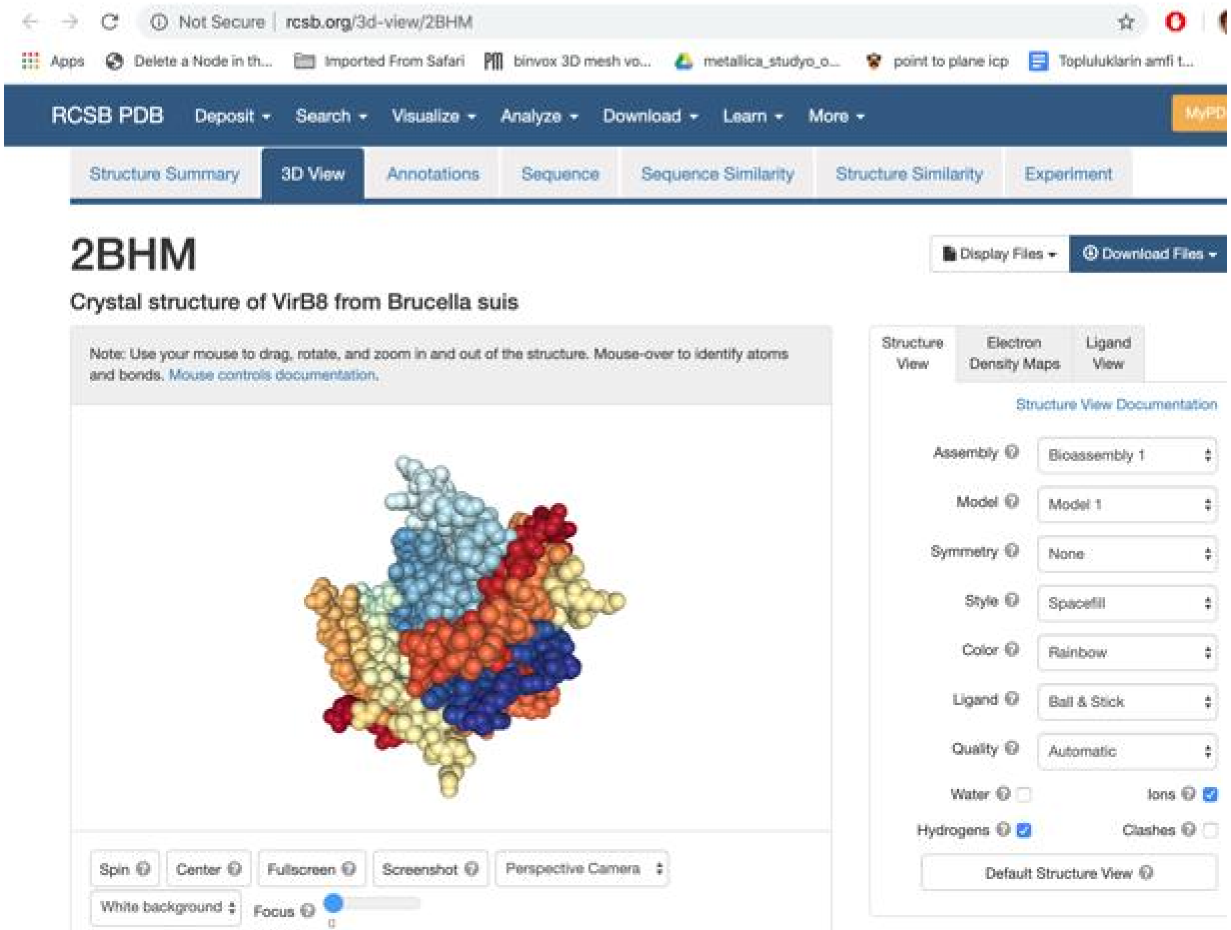




- ✓ .. cartoon, backbone.

Information Visualization – Protein Vis.

✓ https://www.rcsb.org/ provides state-of-the-art visualizations of protein structures in different intuitive ways, e.g., http://www.rcsb.org/3d-view/2BHM



Spacefill mode.

Potential Project Topics

- ✓ Implement Marching Cubes algorithm for implicit surface reconstructin.
  - ✓ Address the related research questions posed in Slide 17* and Slide 39**.

  * Rather than contouring the surface with 0 isovalue, take the isovalue as the median signed distance value over the input points, suggested by the paper Signing the Unsigned: Robust Surface Reconstruction from Raw Pointsets.

  ** Fixing the discontinuity issue in the original SDF.
- ✓ Implement Crust algorithm for explicit surface reconstruction.
- ✓ Implement MDS by casting it as a mass-spring problem.