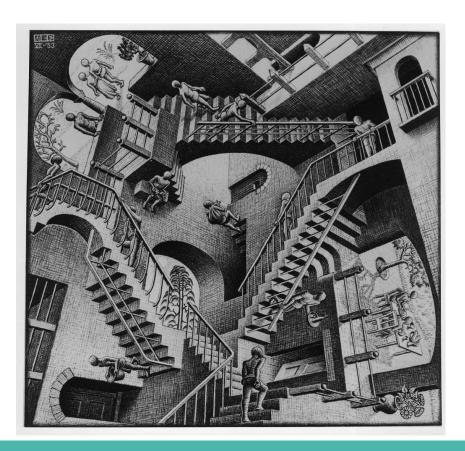
ME 536

Week 9-10: Images as Tensors

Pixels: oh pixels



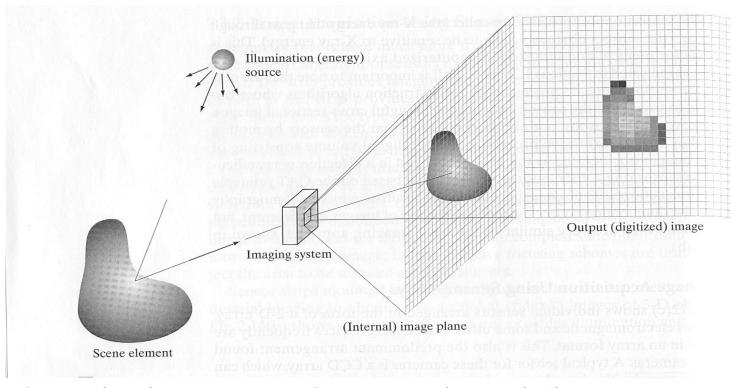
Neighbor pixels are mostly related

Far away pixels are sometimes / somehow related

• Physically related: a path

 Contextually related: common meaning

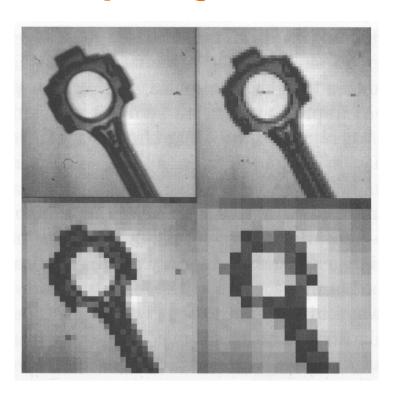
Digital Image Acquisition Process



Quantization → Sampling →

Gray-Level Resolution
Spatial Resolution

Example: Quantization vs Sampling





Quantization → from 32 to 4 gray levels all with 256x256 sampling

Sampling \rightarrow 256x256 to 16x16 all with 128 gray levels

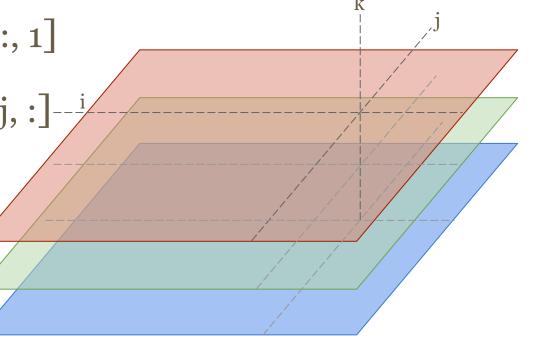
Pixels in Images: RGB implied...

Pixels in tensor I:

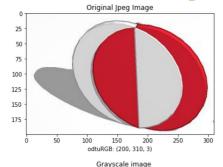
All pixels on Green plane:

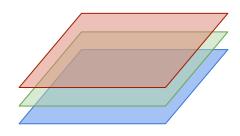
RGV values of pixel (i,j):

Any plane -> grayscale



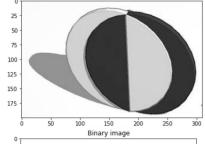
Color Depth in Images: Color, Gray, Binary

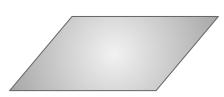




Color images:

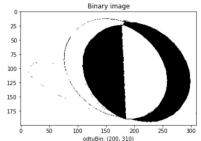
- has layers such as RGB
- Typical values: [0-255] or [0.0 1.0]





Gray-scale images:

- o has 1 layer
- Typical values: [0-255] or [0.0 1.0]





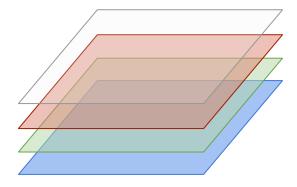
Binary images

- o just like Gray scale has 1 layer
- Values are 0 or 1

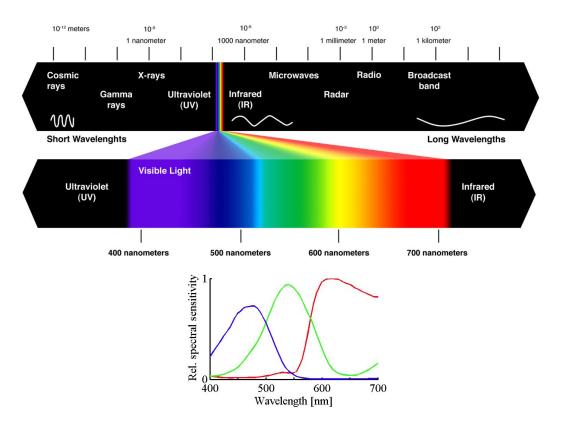
https://www.tinkercad.com/things/htir6tWlgGA-odtu-metu-logo

Color Depth in Images: Transparency, Hyperspectral

- Color images with transparency:
 - o Typically has an additional layer

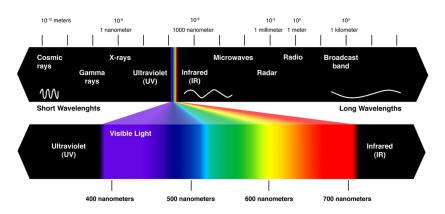


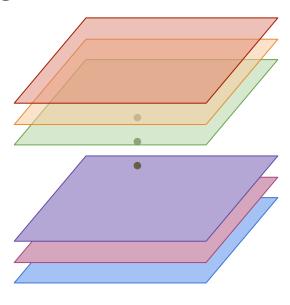
RGB response: of some/typical daylight camera*



Color Depth in Images: Transparency, Hyperspectral

- Multispectral / Hyperspectral images:
 - has several layer corresponding to different wavelengths
 - Near IR (~700nm 900nm)
 - Short-Wave IR (SWIR: ~1.7um-2.5um)
 - Mid-Wave IR (MWIR: ~2.7um-5.3um)
 - Long-Wave IR (LWIR: ~ 8um-12um)



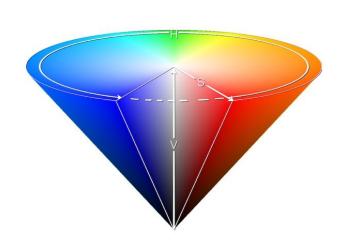


Color Spaces: RGB → **Most famous but ?**

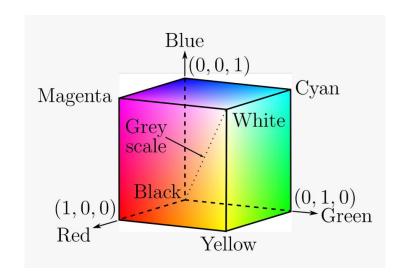
- a.k.a. Color Formats
- RGB is one of many: CMYK, CIE, HSV, ...
- Different uses, different formats
- HSV / (a.k.a. *HSB*) is more ?
- How about you come up with one? Other do so*

Color Spaces: RGB vs HSV

When lighting conditions change less play on HSV



HSB Cone. Image courtesy of Wikimedia.



Python libraries available

Scikit-Image

OpenCV

PIL

• • •

Scikit-Image

Check out:

Examples page

for more...

Manipulating exposure and color channels



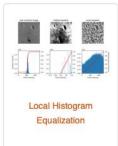




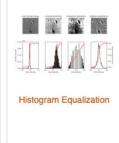










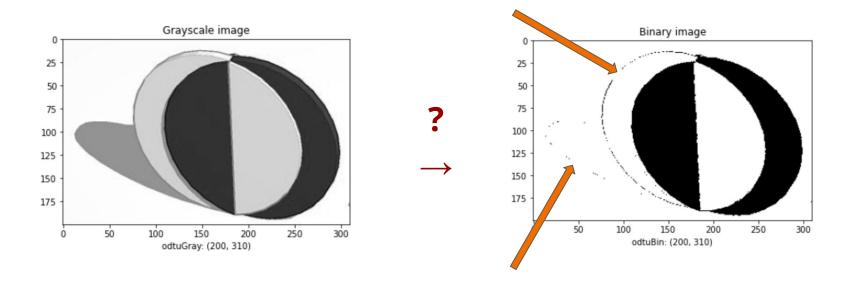


Format change:

- RGB → HSV
 - Linear transformation
 - not unique different transformation in literature
- RGB \rightarrow Gray
 - Generally a linear transformation
 - not unique different transformation in literature
 - GrayValue = 0.2125 R + 0.7154 G + 0.0721 B ¹
 - Used by CRT phosphors as they better represent human perception of red, green and blue than equal weights
- Gray → Binary
 - Tricky
 - Depends on the objective
 - Manual inspection is useful
 - Automatic thresholding methods exist such as OTSU² and many more³
 - 1- https://scikit-image.org/docs/stable/auto_examples/color_exposure/plot_rgb_to_gray.html#id2
 - 2- https://en.wikipedia.org/wiki/Otsu%27s method
 - 3- https://scikit-image.org/docs/dev/api/skimage.filters.html#skimage.filters.threshold_isodata

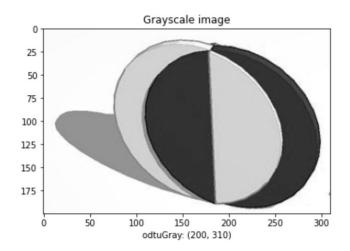
Binary images: Thresholding

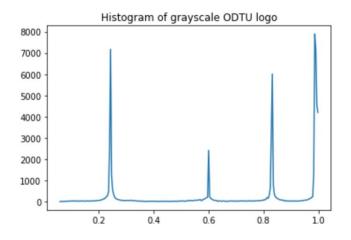
Proper thresholding improves detection performance



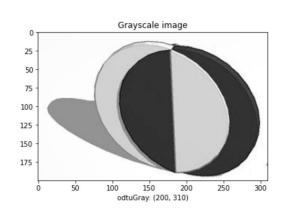
Histograms:

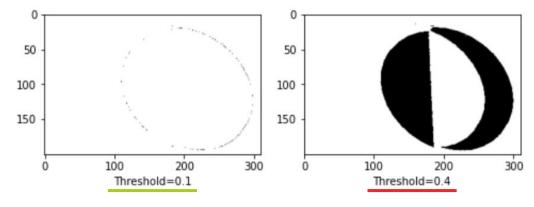
A way of analyzing distribution of data

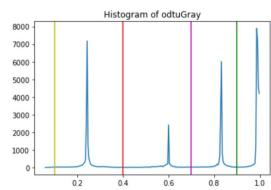


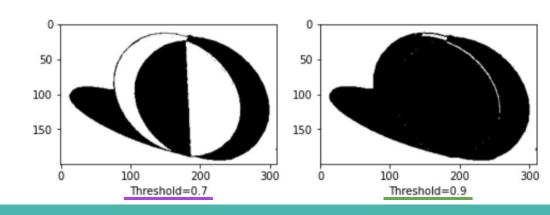


Histograms: Manual Thresholding a delicate balance

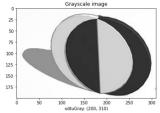


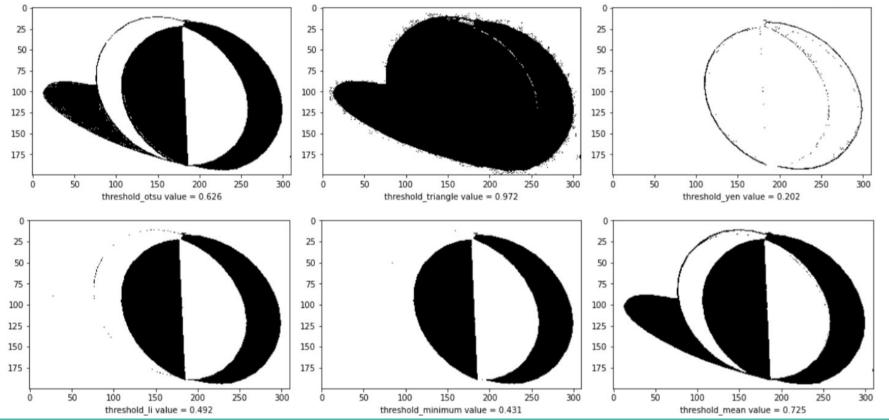






Histograms: Automatic Thresholding





Histograms: Proper Choice

Doesn't it sound like a clustering problem in a way? Where there are 2 clusters...



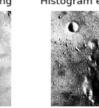
How about a **thresholding method** that **understands what's in the image**?

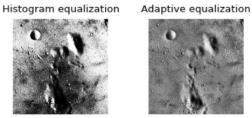
Histogram: Beyond Thresholding

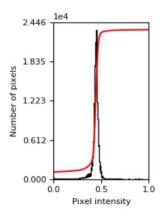
Histogram enhancement

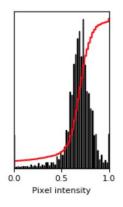
Low contrast image

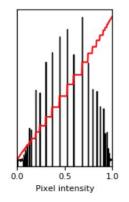


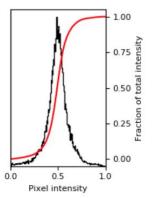












Histogram: Beyond Thresholding

Histogram matching:

- when groups of images are analyzed
- extends to histogram equalization

Source



Reference



Matched

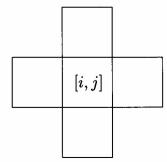


Basic Definitions: used in dealing with images

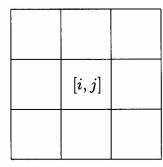
- Neighbors
- Path
- Foreground, Background
- Connectivity
- Connected Components
- Boundary, Interior, Surrounds

Neighbors: 4 or 8

4-neighbors [i+1, j], [i-1, j], [i, j+1], [i, j-1]



8-neighbors [i+1,j+1], [i+1,j-1], [i-1,j+1], [i-1,j-1] plus all of the 4-neighbors



Path:

• A path from the pixel P_0 to pixel P_n is a sequence of pixel indices $(i_0,j_0),(i_1,j_1),(i_2,j_2),...,(i_n,j_n)$ such that the pixel at P_k is a neighbor of the pixel at P_{k+1} for all k with $0 \le k \le n-1$

• If a 4-connection is used the path is a 4-path, and it is an 8-path in case 8-connection is used

Foreground vs Background:

• Set of all 1 pixels in an image is called the foreground and denoted by S.

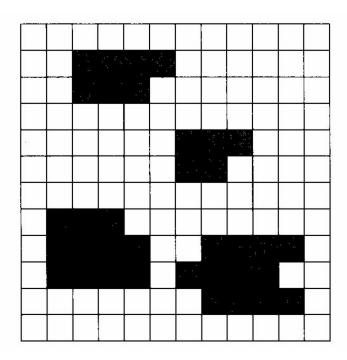
• Set of all 0 pixels in an image is called the background and denoted by \overline{S} .

Connectivity:

• A pixel p in S is said to be connected to pixel q in S if there is a path from p to q.

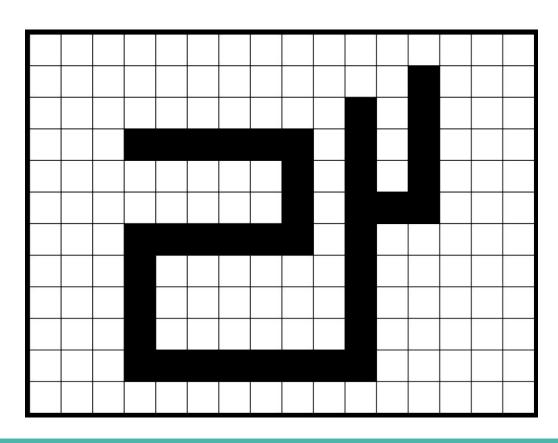
- Connectivity is:
 - Reflexive: p is connected to p
 - Commutative: if p is connected to q, then q is connected to p
 - Transitive: If p is connected to q and q is connected to r, then p is connected to r

Connected Component Labeling



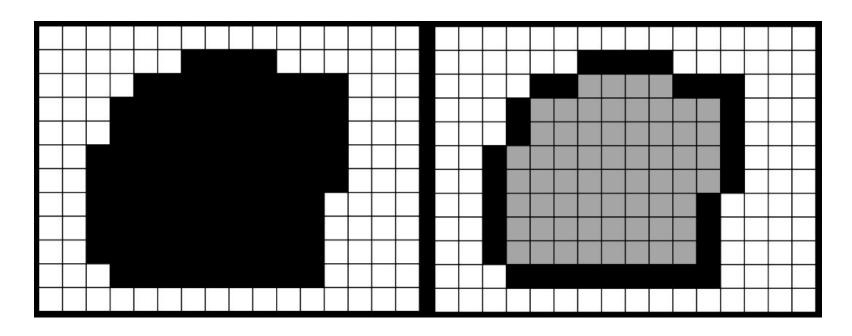
		1	1	1	1						
		1	1	1							
						2	2	2			
						2	2				
	3	3	3								
	3	3	3	3			4	4	4	4	
	3	3	3	3		4	4	4	4		
10.			7 mg 12 to 4 2 to				4	4	4	4	

Connected Component Labeling



Boundary, Interior, Surrounds

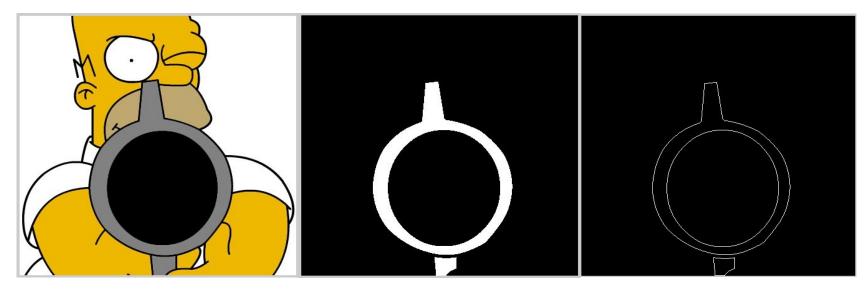
Neighborhood?



Properties

- Size → Area, perimeter
- Position
- Orientation
- Compactness
- Euler number
- Distance measures
- Medial Axis

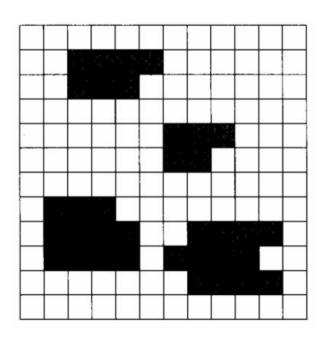
Size: Area, Perimeter,...



Area = 15132 Perimeter = 1414

Position: Center of mass of each component

Orientation: Axis that minimized second moment

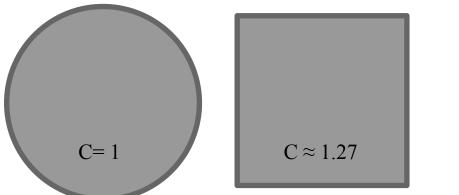


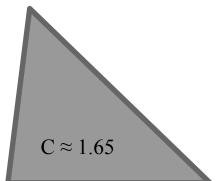
Compactness:

 Compactness of a continuous geometric figure is measured by the isoperimetric inequality:

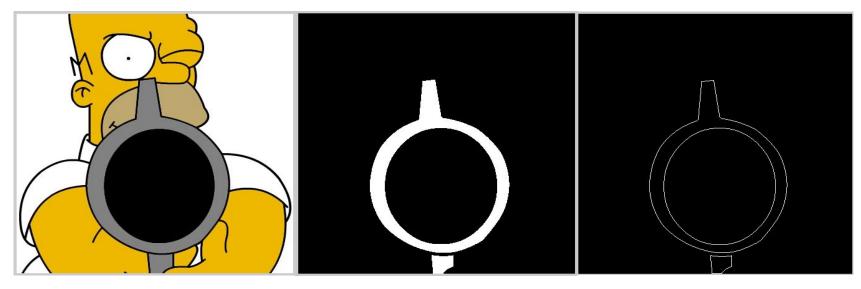
$$C = (P^2/A)/4\pi \ge 1$$

where P is the perimeter and A is the area of the object





Compactness: C≈ 132



Area = 15132 Perimeter = 1414

Euler Number:

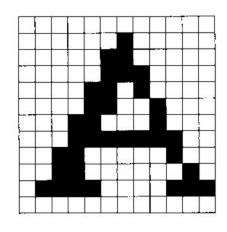
 Euler number is the number of components minus the number of holes on the image:

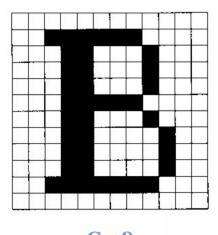
$$E = C - H$$

- Neighborhood definition is important for both the back and the foreground
- Invariant to translation, rotation, and scaling!

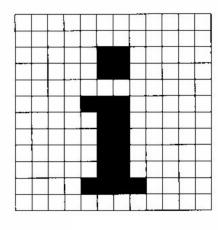
Euler Number: $\mathbf{E} = \mathbf{C} - \mathbf{H}$

Background N4, foreground N8



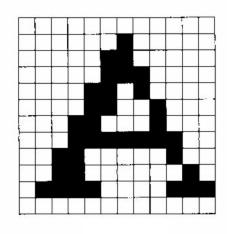


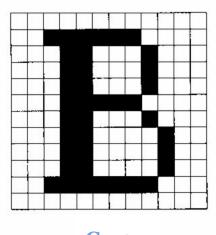


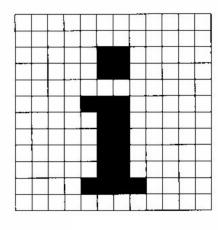


Euler Number: $\mathbf{E} = \mathbf{C} - \mathbf{H}$

Background N4, foreground N8







$$C = 2$$
 $H = 0$

Distance:

Euclidean

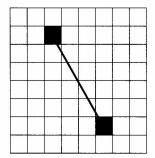
$$d_{ ext{Euclidean}}([i_1,j_1],[i_2,j_2]) = \sqrt{(i_1-i_2)^2+(j_1-j_2)^2}$$

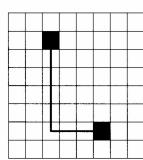
City-block

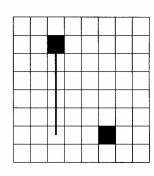
$$d_{\text{citv}} = |i_1 - i_2| + |j_1 - j_2|$$

Chessboard

$$d_{\text{chess}} = \max(|i_1 - i_2|, |j_1 - j_2|)$$







Euclidean distance:

City-block distance:

Chessboard distance:

Medial Axis

• If the distance $d((i,j), \overline{S})$ for the pixel (i,j) in S to \overline{S} is locally maximum i.e.

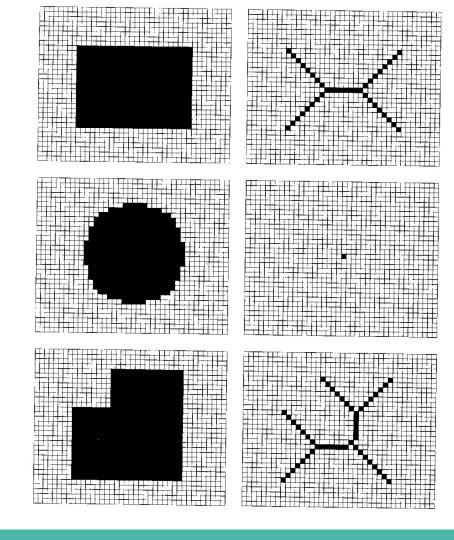
$$d((i,j), \overline{S}) \ge d((u,v), \overline{S})$$

for all pixels (u,v) in the neighborhood of (i,j), then pixel (i,j) is on the medial axis.

(Recall that: \overline{S} is the background and S is the foreground)

- Medial axis has been used for compact representation of objects.
- Also check out scikit-image: skeletonize

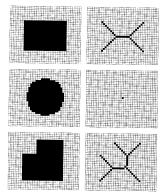
Medial Axis: Examples



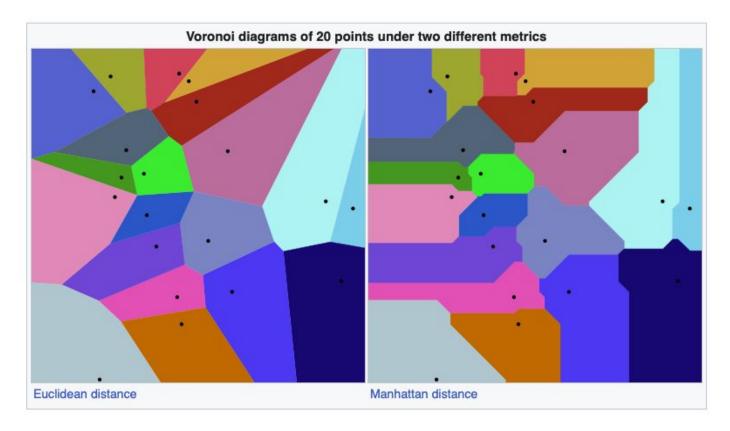
Recalls or extends to: Voronoi graphs

Observe that:

Voronoi graphshas a **k-means**flavor



Voronoi: Distance Metrics



Dilation & Erosion

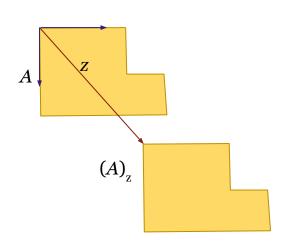
 Fundamental to morphological processing of binary images

 Many other algorithms are based on these two primitives

Dilation & Erosion: Translation and Reflection

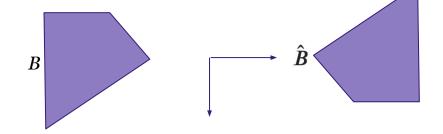
On image I, A & B are foreground objects:

• Translation $(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$



Reflection

$$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$$



Erosion:

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

A: foreground on the image to be eroded

B: structuring element (i.e. selem in skimage)

Erosion \rightarrow contains all points where the translated selem is fully contained by the image foreground (i.e. $(B)_{\tau}$ is completely inside A)

Analogy → using an eraser going over the perimeter of the foreground components

erosion noun

Save Word

ero·sion | \ i-ˈrō-zhən 🚳 \

Definition of erosion

1 a : the action or process of eroding

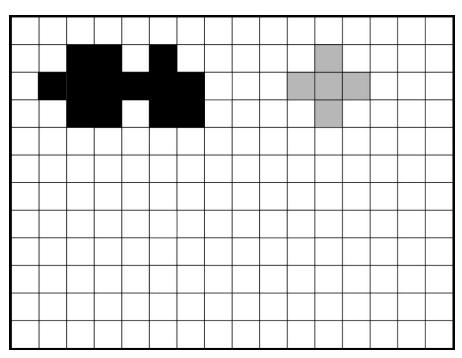
b : the state of being eroded

2 : an instance or product of erosion

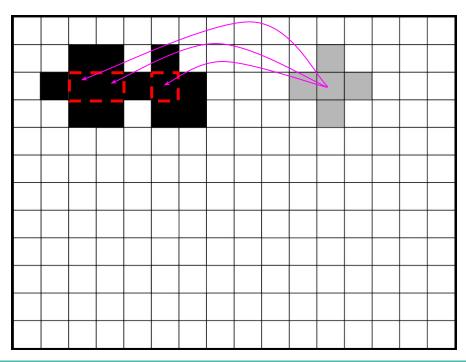
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

- 1. Hold selem at its center
- 2. Put it on the top left of the image
- 3. If selem is *fully* contained by the foreground (1s of selem match the 1s that of the image), mark the pixel that coincides with the center of the selem to belong to the eroded image
- 4. Systematically move selem pixel by pixel all over the *foreground*

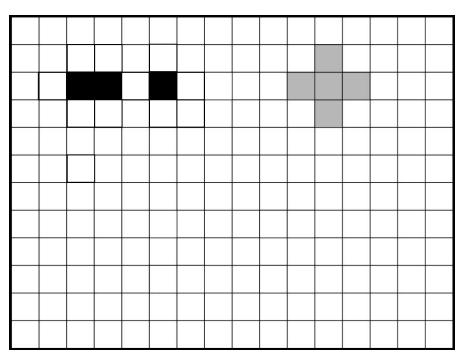
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



Dilation:

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$

A: foreground on the image to be dilated

B: structuring element



Save Word

di·la·tion | \ dī-ˈlā-shən 💿 \

Definition of dilation

- 1 : the act or action of <u>dilating</u>: the state of being <u>dilated</u>: <u>EXPANSION</u>, <u>DILATATION</u>
- 2 : the action of stretching or enlarging an organ or part of the body

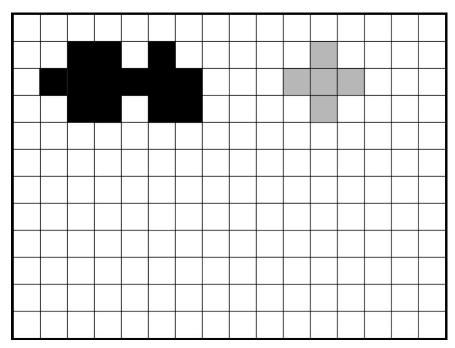
Dilation \rightarrow contains all points where the reflection of the translated selem has any intersection (even 1 pixel) with the image foreground

Analogy \rightarrow painting over the perimeter of the foreground components with a Brush (i.e. the structuring element B)

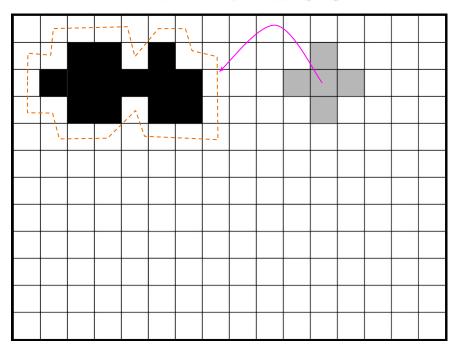
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$

- 1. Hold the reflection of selem at its center
- 2. Put it on the top left of the image
- 3. If selem has any overlap with the foreground, mark the pixel that coincides with the center of the selem to belong to the dilated image
- 4. Systematically move selem pixel by pixel all over the image

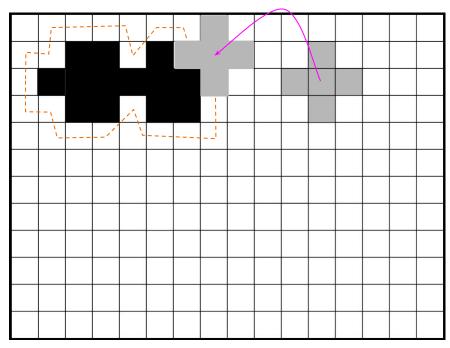
$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$



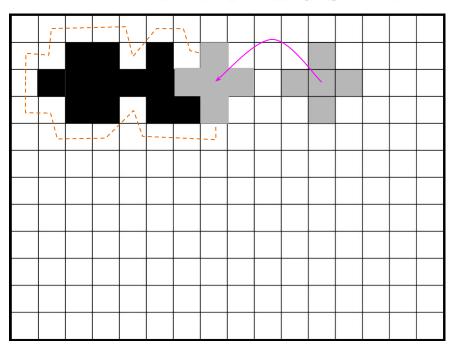
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$



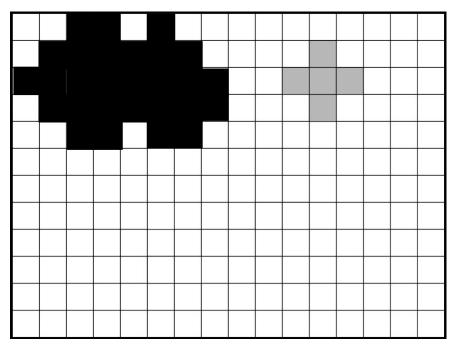
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$



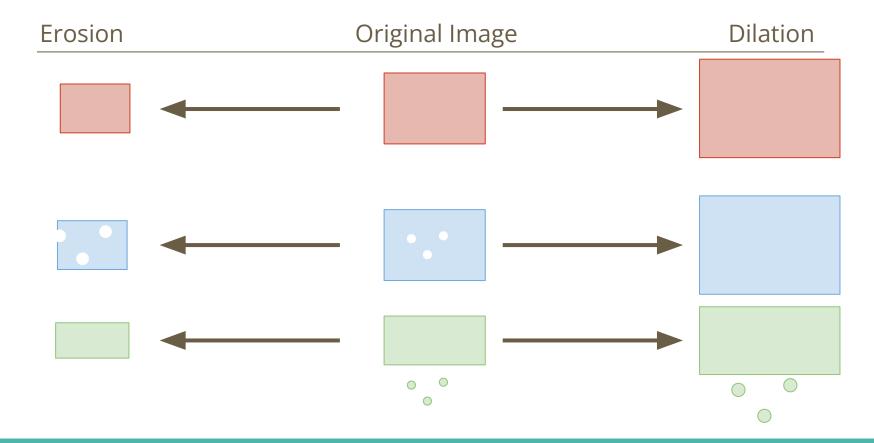
$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$



$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$



Erosion & Dilation



Opening & Closing an Image

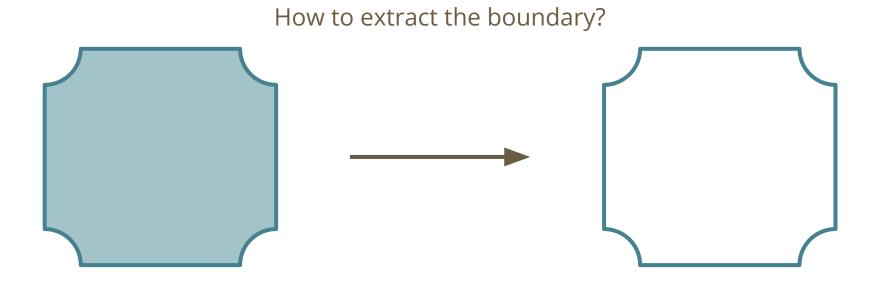
Opening A:
$$A \circ B = (A \ominus B) \oplus B$$

- Erode first, then dilate with B
- Get rid of tiny foreground components
- Separate weakly connected components

Closing A: $A \cdot B = (A \oplus B) \ominus B$

- Dilate first, then erode with B
- Get rid of holes and small irregularities on the perimeter
- close gaps between components

Boundary Extraction:



Images multiplied: say what?

Hadamard (a.k.a Schur) product: A o B

Yes even the simplest things/ ideas have names as usual

Note that this is the default multiplication for numpy, pytorch etc for matrices of equal size

$$egin{bmatrix} a_{11} & a_{12} & a_{13} \ a_{21} & a_{22} & a_{23} \ a_{31} & a_{32} & a_{33} \end{bmatrix} \circ egin{bmatrix} b_{11} & b_{12} & b_{13} \ b_{21} & b_{22} & b_{23} \ b_{31} & b_{32} & b_{33} \end{bmatrix} = egin{bmatrix} a_{11} b_{11} & a_{12} b_{12} & a_{13} b_{13} \ a_{21} b_{21} & a_{22} b_{22} & a_{23} b_{23} \ a_{31} b_{31} & a_{32} b_{32} & a_{33} b_{33} \end{bmatrix}$$

Images multiplied: come again Hadamard product

A

B

 $\mathbf{A} \circ \mathbf{B}$

 $\Sigma(\mathbf{A} \circ \mathbf{B})$

1	5	10
1	2	0
3	0	5

1	1	1
1	1	1
1	1	1

27

What is the meaning of $\Sigma(\mathbf{A} \circ \mathbf{B})$?

Images multiplied: sum of Hadamard product

A

B

 $\mathbf{A} \circ \mathbf{B}$

 $\Sigma(\mathbf{A} \circ \mathbf{B})$

1	5	10
1	2	0
3	0	5

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

1/9	5/9	10/9
1/9	2/9	0/9
3/9	0/9	5/9

3

What is the **meaning of** $\Sigma(A \circ B)$?

Why are all neighbors equally important?

Images multiplied: come again Hadamard product

A

B

 $\mathbf{A} \circ \mathbf{B}$

 $\sum (\mathbf{A} \circ \mathbf{B})$

1	5	10
1	2	0
3	0	5

1/9	5/9	1/9
5/9	10/9	5/9
1/9	5/9	1/9

1/9	25/9	10/9
5/9	20/9	0/9
3/9	0/9	5/9

7.66

What is the meaning of $\Sigma(\mathbf{A} \circ \mathbf{B})$?

Note that
$$\rightarrow \sum \mathbf{B} = 34 / 9$$

Images multiplied: come again Hadamard product

A

B

 $\mathbf{A} \circ \mathbf{B}$

 $\sum (\mathbf{A} \circ \mathbf{B})$

1	5	10
1	2	0
3	0	5

1/34	5/34	1/34
5/34	10/34	5/34
1/34	5/34	1/34

1/34	25/34	10/34
5/34	20/34	0/34
3/34	0/34	5/34

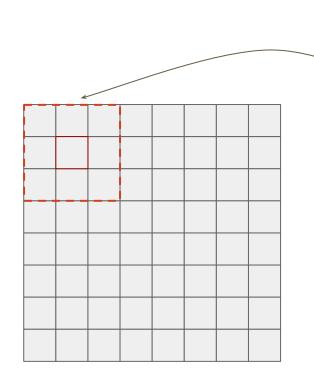
2.03

What is the meaning of $\Sigma(\mathbf{A} \circ \mathbf{B})$?

$$\sum \mathbf{B} = 1$$

Convolution: very similar to cross correlation

Recall: reflection from morphological operators $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$



I: image

B: kernel

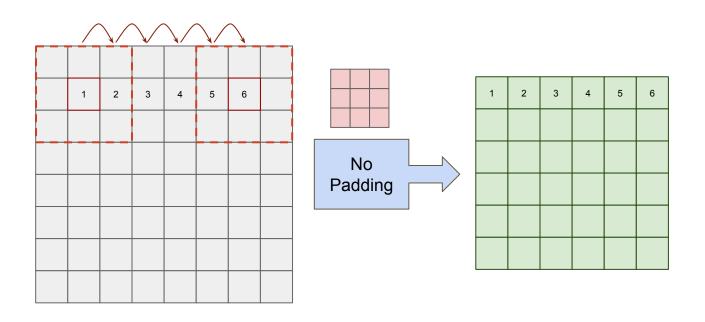
Convolution: Reflection of the kernel

B travels around the image I

For simplicity, we will **not** incorporate **reflection** in the following examples, but sample codes demonstrate the actual usage

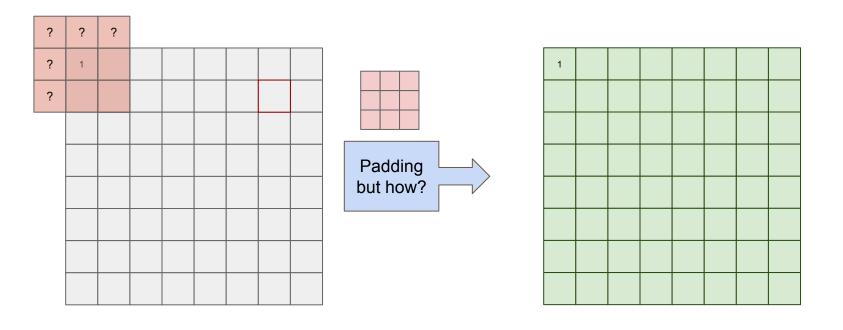
Convolution dilemma: pad or shrink

What happens when center pixel of B is on the edge of the I?



Convolution dilemma: pad or shrink

What happens when center pixel of B is on the edge of the I?

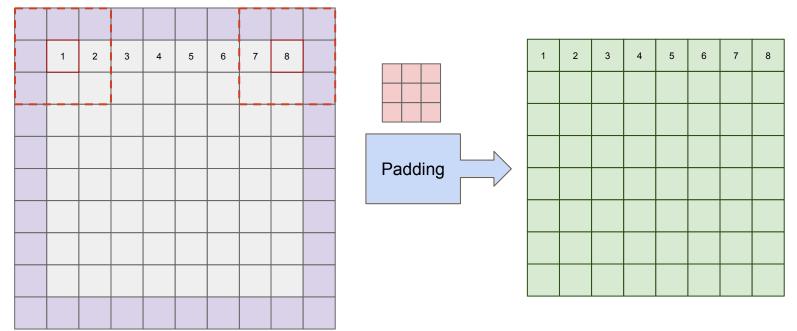


Convolution dilemma: pad not to shrink

Note that padding depends on the size of the kernel

Kernel size: in general a center pixel (i.e. kernel fully contained by the image)

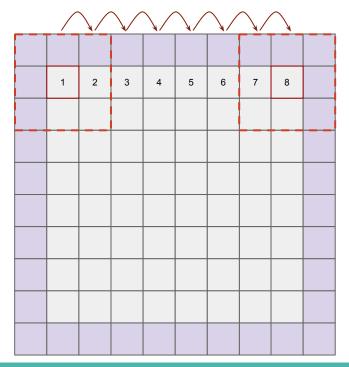
can be found without ambiguity



Convolution: pad but how?

Note that **number** of padded pixels **depends** on the **size of the kernel**

Kernel size: in general a center pixel can be found without ambiguity



Constant: zero, one padding commonly used

Reflect: dcba|abcd|dcba

Nearest: aaaa|abcd|ddd

Mirror: dcb|abcd|cba

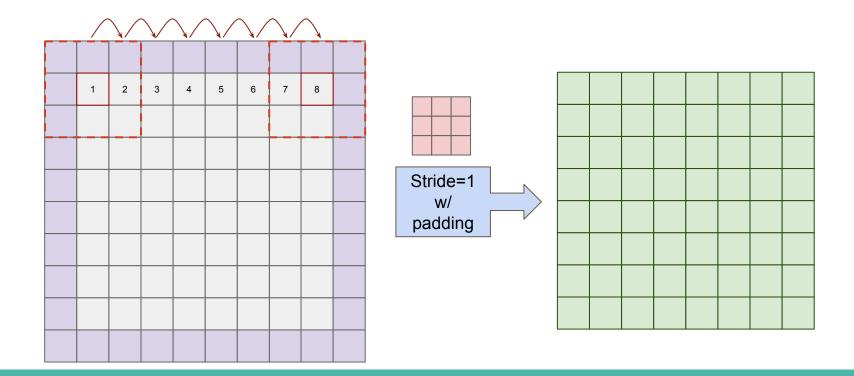
Wrap: abcd|abcd|abcd

and more...

scipy.ndimage.convolve()

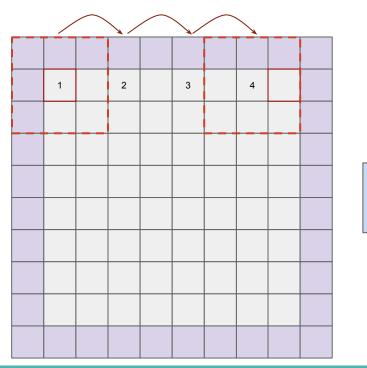
Convolution: stride - 1

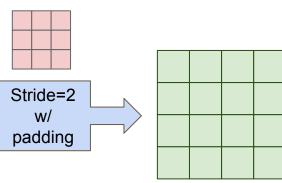
Does the kernel have to move 1 step at a time?



Convolution: stride - 2

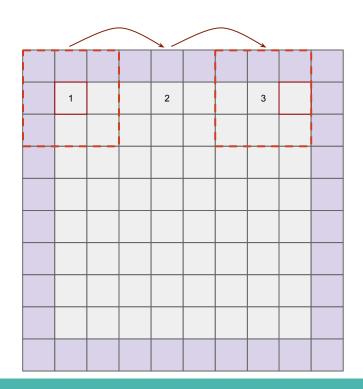
Does the kernel have to move 1 step at a time?

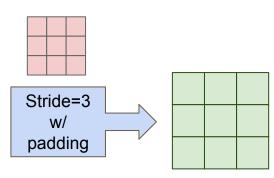




Convolution: stride - 3

With a proper kernel, result is like a summary of the bigger picture





Convolution: popular kernels

$$\left[egin{array}{ccc} 1 & 0 & -1 \ 0 & 0 & 0 \ -1 & 0 & 1 \end{array}
ight]$$

Blur: $\frac{1}{9} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$

Edge Detection:
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$
$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Gaussian Blur 3x3: $\frac{1}{16} \begin{vmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{vmatrix}$

Sharpen: $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

Gaussian Blur 5x5: $\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \end{bmatrix}$

Different Kernels in action

Operation	Kernel ω	Image result g(x,y)
Identity	$\left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$	
	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Ridge or edge detection	$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$	
Sharpen	$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$	
Box blur (normalized)	$\frac{1}{9} \left[\begin{array}{rrr} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right]$	

Operation	Kernel ω	Image result g(x,y)		
Gaussian blur 3 x 3 (approximation)	$\frac{1}{16} \left[\begin{array}{ccc} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{array} \right]$			
Gaussian blur 5 x 5 (approximation)	$\frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}$			
Unsharp masking 5 × 5 Based on Gaussian blur with amount as 1 and threshold as 0 (with no image mask)	$ \frac{-1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & -476 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix} $			

Kernels: how to choose the best?

Choice depends on the need

Tailored based on *experience* or *trial* & *error*

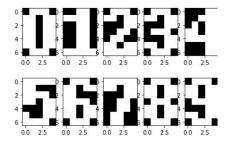
trial & *error* → *neural network training* ?

Using a set of specialized kernels to detect objects

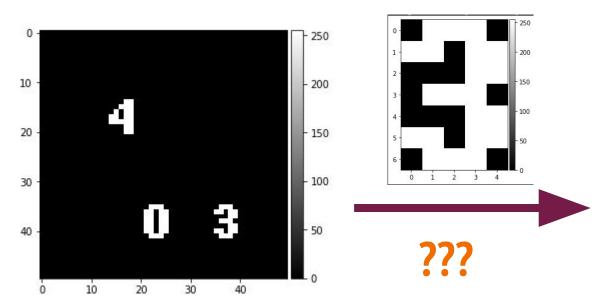
Hands on with this notebook

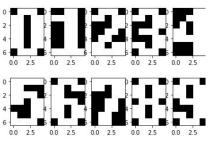
This is where convolutional networks shine!!!

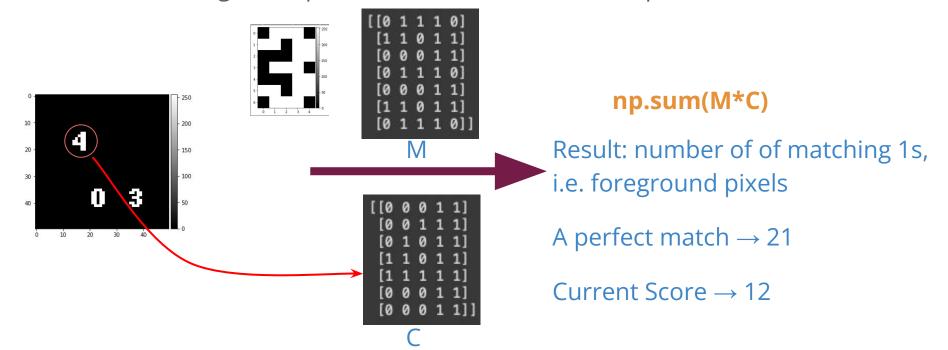
Talking about detection:

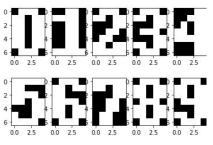


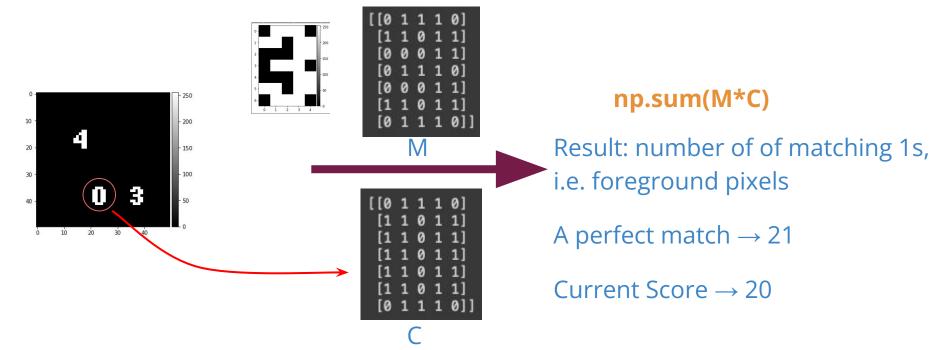
Check this page out for matching patterns using the simple convolution idea

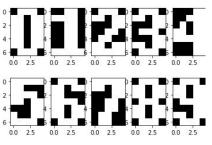


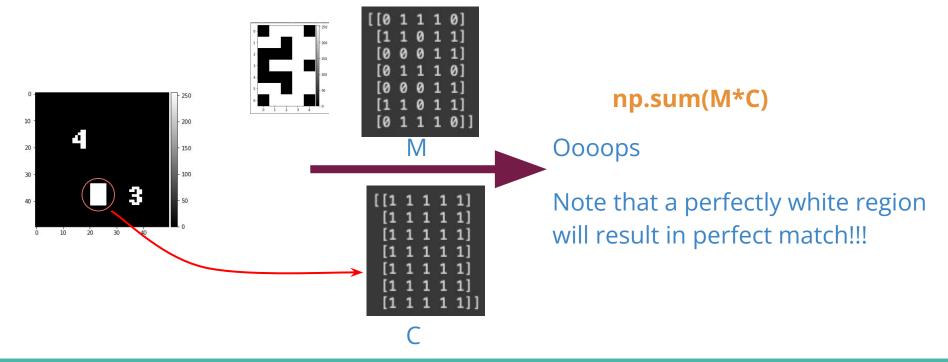


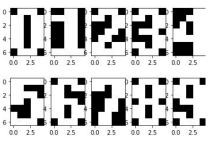


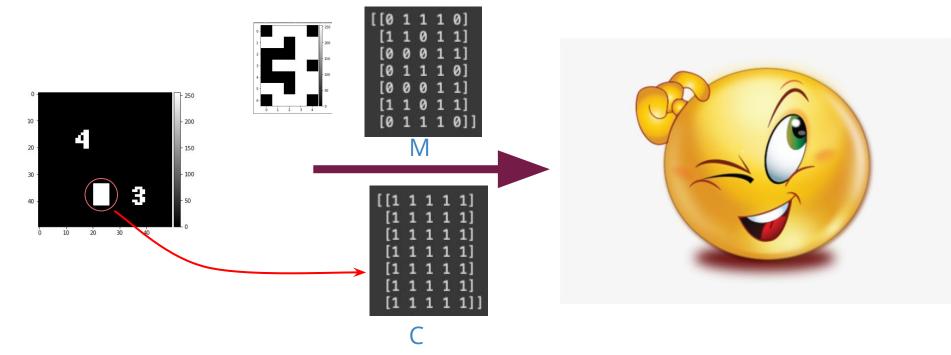


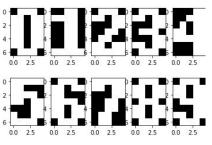




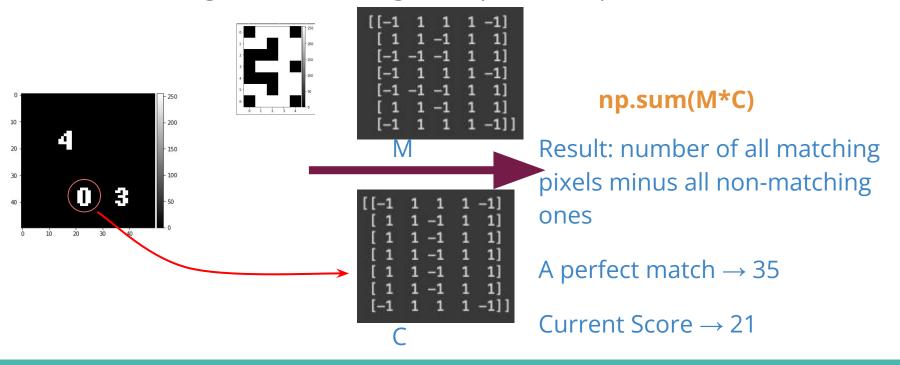


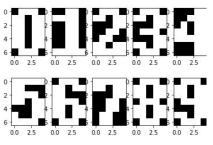




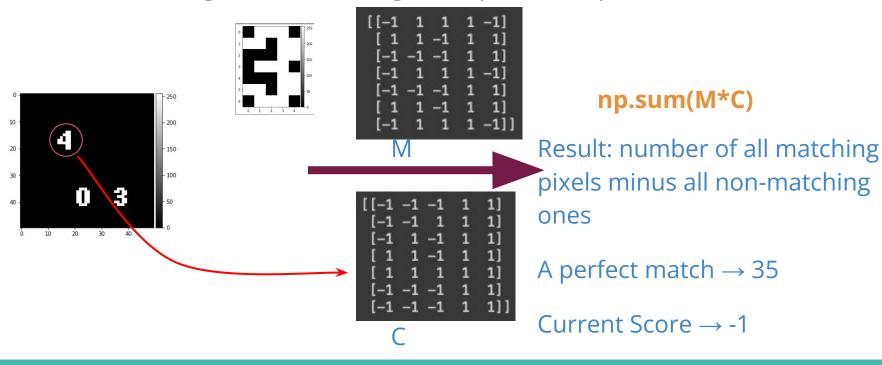


Match same foreground and background pixels and punish for mismatches





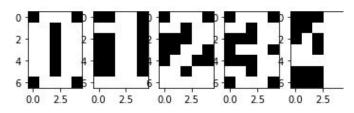
Match same foreground and background pixels and punish for mismatches



Matching results for all the digits

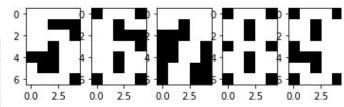
Method 1

26.00	12.00	18.00	20.00	17.00	20.00	23.00	14.00	24.00	23.00
12.00	19.00	15.00	13.00	10.00	13.00	12.00	13.00	13.00	13.00
18.00	15.00	22.00	18.00	12.00	14.00	17.00	15.00	19.00	18.00
20.00	13.00	18.00	21.00	12.00	16.00	19.00	15.00	21.00	21.00
17.00	10.00	12.00	12.00	21.00	13.00	14.00	9.00	15.00	14.00
20.00	13.00	14.00	16.00	13.00	24.00	18.00	12.00	18.00	19.00
23.00	12.00	17.00	19.00	14.00	18.00	24.00	13.00	23.00	21.00
14.00	13.00	15.00	15.00	9.00	12.00	13.00	19.00	15.00	15.00
24.00	13.00	19.00	21.00	15.00	18.00	23.00	15.00	25.00	23.00
23.00	13.00	18.00	21.00	14.00	19.00	21.00	15.00	23.00	24.00

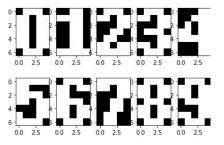


Method 2

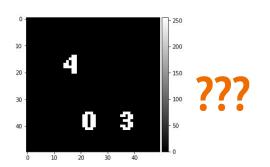
1	35.00	-7.00	11.00	21.00	9.00	15.00	27.00	1.00	29.00	27.00	
Ĺ	-7.00	35.00	13.00	7.00	-5.00	1.00	-3.00	11.00	-1.00	1.00	
Ì	11.00	13.00	35.00	21.00	-3.00	-1.00	11.00	13.00	17.00	15.00 j	
İ	21.00	7.00	21.00	35.00	-1.00	9.00	21.00	15.00	27.00	29.00	
Ī	9.00	-5.00	-3.00	-1.00	35.00	-3.00	1.00	-9.00	3.00	1.00	
ĺ	15.00	1.00	-1.00	9.00	-3.00	35.00	11.00	-3.00	9.00	15.00	
Ì	27.00	-3.00	11.00	21.00	1.00	11.00	35.00	1.00	29.00	23.00	
Ì	1.00	11.00	13.00	15.00	-9.00	-3.00	1.00	35.00	7.00	9.00	
Ì	29.00	-1.00	17.00	27.00	3.00	9.00	29.00	7.00	35.00	29.00	
Ì	27.00	1.00	15.00	29.00	1.00	15.00	23.00	9.00	29.00	35.00 j	

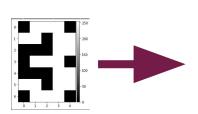


My way or highway:



You can come up with your own method...





What if:

- Digits are scaled?
- Digits are rotated?
- There is noise in the image?