

CS410
Advanced Functional Programming
2015/16 Session

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Chapter 1

Introduction

1.1 Language and Tools

For the most part, we'll be using the experimental language, Agda Norell [2008], which is a bit like Haskell (and implemented in Haskell), but has a more expressive type system and a rather fabulous environment for typed programming. Much of what we learn here can be ported back to Haskell with a bit of bodging and fudging (and perhaps some stylish twists), but it's the programming environment that makes it worth exploring the ideas in this class via Agda.

The bad news, for some of you at any rate, is that the Agda programming environment is tightly coupled to the Emacs editor. If you don't like Emacs, tough luck. You may have a job getting all this stuff to work on whatever machines you use outside the department, but the toolchain all works fine on departmental machines.

Teaching materials, exercise files, lecture scripts, and so on, will all pile up in the repository <https://github.com/pigworker/CS410-15>, so you'll need to get with the git programme. You should each keep your own version of the repository, with your solutions, in a *private* repository (e.g., at <https://bitbucket.org/>) and invite the class staff ([pigworker](#) and [jmchapman](#)) as collaborators. Your repository is your portfolio of coursework: we will add our written feedback to it directly, in your presence. The class is small enough that we can see you each individually for a bit of a chat and some marking relatively frequently.

These notes are a work in progress. Perhaps, one day, they will be a textbook.

1.2 Lectures, Labs, Office Hours

Monday: Lecture, 11am–12pm, JA326; Office Hour, 2–3pm, LT1317; Lab, 3–5pm LT1301

Tuesday: Office Hour, 2–3pm, LT1317; Lecture, 4–5pm, LT209

Friday: Lab, 1–2pm, LT1301

Lectures are where we introduce new ideas, usually livecoding in Agda, but although we'll be as interactive as possible, lectures can be a bit of a passive experience, like television. The real learning happens in labs, where you have to turn your

understanding into stuff, so you find the gaps in your understanding and ask the questions which get you to make progress from where you are. That is why we have lots of scheduled lab time. Office hours, divided into ten minute slots with signup sheets on the door of LT1317, are your opportunity to get some one-to-one supervision, as well as the marking of your assignments. Our feedback policy is ‘Come and get it!’.

We should add that the existence of designated office hours does not mean that you are otherwise unwelcome in LT1317 (or LT1310, where you can also find many Agda experts). If you turn up on spec at LT1317, the worst that can happen is that you find nobody home and then sign up for a slot.

1.3 Twitter @CS410afp

This class has a twitter feed. Largely, this is so that I can post pictures of the whiteboard. I don’t use it for essential communications about class business, so you need neither join twitter nor follow this user. You can access all the relevant stuff just by surfing into <http://twitter.com/CS410afp>. This user, unlike my personal account, will follow back all class members who follow it, unless you ask it not to.

1.4 Hoop Jumping

CS410 Advanced Functional Programming is a level 4 class worth 20 credits. It is assessed *entirely* by coursework. Departmental policy requires class convenors to avoid deadline collisions by polite negotiation, so I’ve agreed the following dates for handins, as visible on the 4th year noticeboard.

- Friday¹ week 2
- Friday week 5
- Friday week 9
- Friday week 12
- Friday week 15
- Semester 2 assignment, issued immediately after fourth year project deadline, to be submitted as late as I consider practicable before the exam board

As mentioned above, marking will happen ‘live’ and one-to-one, in office hours on a sign-up basis.

In the 2015/16 session, final year project submission is in week 11 of semester 2, which is the last week before the Spring break. We shall be sure to arrange an orientation session for the final assignment after the project deadline but before the break. You are not expected to work in your break, but want to make sure that it can at least be useful if you do.

Semester 1 assignments are each worth 15% and marked out of 15. The semester 2 assignment is worth 25% and marked out of 25. CS410 students can thus observe

¹A deadline officially on Friday means work submitted before we get to it on Monday is acceptable.

directly their accumulation of marks towards their degree. It is possible to bag the 20 credits by doing the first three assignments very well, and it is not unusual to finish semester 1 in a very healthy position.

1.5 Getting Agda Going on Departmental Machines

Note that these instructions are brittle and easily broken by software ‘upgrades’. We’ll keep them under review and try to make them as accurate as possible.

Step 1. Use Linux. Get yourself a shell. (It’s going to be that sort of a deal, all the way along. Welcome back to the 1970s.)

Step 2 for *bash* users. Ensure that your `PATH` environment variable includes the directory where Haskell’s `cabal` build manager puts executables. Under normal circumstances, this is readily achieved by ensuring that your `.profile` file contains the line:

```
export PATH=$HOME/.cabal/bin:$PATH
```

After you’ve edited `.profile`, grab a fresh shell window before continuing.

Step 2 for *tcsh* users. Ensure that your `path` environment variable includes the directory where Haskell’s `cabal` build manager puts executables. Under normal circumstances, this is readily achieved by ensuring that your `.cshrc` file contains the line:

```
set path = ($home/.cabal/bin $path)
```

After you’ve edited `.cshrc`, grab a fresh shell window before continuing.

Step 3. Ensure that you are in sync with the Haskell package database by issuing the command:

```
cabal update
```

Step 4. Install Agda by issuing the command:

```
cabal install agda
```

Yes, that’s a lower case ‘a’ in ‘agda’. In some situations, it may not manage the full installation in one go, delivering an error message about which package or version it has failed to install. We’ve found that it’s sometimes necessary to do `cabal install cpps`, `cabal install happy`, and `cabal install alex` separately.

Step 5. Wait.

Step 6. Wait some more.

Step 7. Assuming all of that worked just fine, set up the Emacs interactive environment with the command:

```
agda-mode setup; agda-mode compile
```

Step 8. Get this repository. Navigate to where in your file system you want to keep it and do

```
git clone https://github.com/pigworker/CS410-15.git
```

Step 9. Navigate into the repo.

```
cd CS410-15
```

Step 10. Start an emacs session involving an Agda file, e.g., by the command:

```
emacs Hello.agda &
```

The file should appear highlighted, and the mode line should say that the buffer is in Agda mode. In at least one case, this has proven problematic. To check what is going on, load the configuration file `~/.emacs` and find the LISP command which refers to `agda-mode locate`. Try executing that command: select it with the mouse, then type `ESC x`, which should get you a prompt at which you can type `eval-region`, which will execute the selected command. If you get a message about not being able to find `agda-mode`, then edit the LISP command to give `agda-mode` the full path returned by asking `which agda-mode` in a shell. And if you get a bad response to `which agda-mode`, go back to step 2.

Step 11. When you're done, please confirm by posting a message on the class discussion forum.

1.6 Making These Notes

The sources for these notes are included in the repo (in the subdirectory imaginatively called `notes`) along with everything else. They're built using the excellent `lhs2TeX` tool, developed by Andres Löh and Ralf Hinze. This, also, can be summoned via the Haskell package manager.

```
cabal install lhs2tex
```

With that done, the default action of `make` is to build these notes as `CS410-notes.pdf` at the top level. Naughtily, we also keep the notes in the repository, so they have a steady url: github.com/pigworker/CS410-15/raw/master/CS410-notes.pdf.

1.7 What's in `Hello.agda`?

It starts with a `module` declaration, which should and does match the filename.

```
module Hello where
```

Then, as in Haskell, we have comments-to-end-of-line, as signalled by `--` with a space.

```
-- Oh, you made it! Well done! This line is a comment.

-- In the beginning, Agda knows nothing, but we can teach it about numbers.
```

Indeed, this module has not `imported` any others, and unlike in Haskell, there is no implicit 'Prelude', so at this stage, the only thing we have is the notion of a `Set`. The following `data` declaration creates three new things—a new `Set`, populated with just the values generated by its constructors.

```
data Nat : Set where
  zero  : Nat
  suc   : Nat -> Nat
```

We see some key differences with Haskell. Firstly, *one* colon means ‘has type’, rather than ‘list cons’. Secondly, rather than writing ‘templates’ for data, we just state directly the types of the constructors. Thirdly, there’s a lot of space: Agda has very simple rules for splitting text into tokens, so space is often necessary, e.g., around `:` or `->`. It is my habit to use even more space than is necessary for disambiguation, because I like to keep things in alignment.

Speaking of alignment, we do have the similarity with Haskell that indentation after **where** indicates subordination, showing that the declarations of the **zero** and **suc** value constructors belong to the declaration of the **Nat** type constructor.

Another difference is that I have chosen to begin the names of **zero** and **suc** in *lower* case. Agda enforces no typographical convention to distinguish constructors from other things, so we can choose whatever names we like. It is conventional in Agda to name data-like things in lower case and type-like things in upper case. Crucially, **zero**, **suc**, **Nat** and **Set** all live in the *same* namespace. The distinction between different kinds of things is achieved by referring back to their declaration, which is the basis for the colour scheme in the emacs interface.

The declaration of **Nat** tells us exactly which values the new set has. When we declare a function, we create new *expressions* in a type, but *no new values*. Rather, we explain which value should be returned for every possible combination of inputs.

```
-- Now we can say how to add numbers.
```

```
_+_ : Nat -> Nat -> Nat
zero  +  n  = n
suc m  +  n  = suc (m + n)
```

What’s in a name? When a name includes *underscores*, they stand for places you can put arguments in an application. The unspaced `_+_` is the name of the function, and can be used as an ordinary identifier in prefix notation, e.g. `_+_ m n` for `m + n`. When we use `+` as an infix operator (with arguments in the places suggested by the underscores), the spaces around it are necessary. If we wrote `m+n` by accident, we would find that it is treated as a whole other symbol.

Meanwhile, because there are no values in **Nat** other than those built by **zero** and **suc**, we can be sure that the definition of `+` covers all the possibilities for the inputs. Moreover, or rather, lessunder, the recursive call in the **suc** case has as its first argument a smaller number than in the pattern on the left hand side, so the recursive call is strictly simpler. Assuming (rightly, in Agda), that *values* are not recursive structures, we must eventually reach **zero**, so that every addition of values is bound to yield a value.

```
-- Now we can try adding some numbers.
```

```
four : Nat
four = (suc (suc zero)) + (suc (suc zero))
```

```
-- To make it go, select "Evaluate term to normal form" from the
-- Agda menu, then type "four", without the quotes, and press return.
```

```
-- Hopefully, you should get a response
--   suc (suc (suc (suc zero)))
```

Evaluation shows us that although we have enriched our expression language with things like $2 + 2$, the values in `Nat` are exactly what we said they were: there are no new numbers, no error cases, no ‘undefined’s, no recursive black holes, just the values we declared.

That is to say, Agda is a language of *total* programs. You can approach it on the basis that things mean what they say, and—unusually for programming languages—you will usually be right.

1.8 Where are we going?

Agda is a language honest, expressive and precise. We shall use it to explore and model fundamental concepts in computation, working from concrete examples to the general structures that show up time and time again. We’ll look at examples like parsers, interpreters, editors, and servers. We’ll implement algorithms like arithmetic, sorting, search and unification. We’ll see structures like monoids, functors, algebras and monads. The purpose is not just to teach a new language for instructing computers to do things, but to equip you with a deeper perception of structure and the articulacy to exploit that structure.

Agda is a dependently typed language, meaning that types can mention values and thus describe their intended properties directly. If we are to be honest and ensure that we mean what we say, we had better be able to say more precisely what we do mean. This is not intended to be a course in dependently typed programming, although precision is habit-forming, so a certain amount of the serious business is inevitable. We’ll also be in a position to state and prove that the programs we write are in various ways sensible. What would it take to convince you that the `+` operator we constructed above really does addition?

I’m using Agda rather than Haskell for four reasons, two selfish, two less so.

- I am curious to see what happens.
- Using Agda brings my teaching a lot closer to my research and obliges me to generate introductory material which will help make this area more accessible. (The benefit for you is that I have lots of motivation to write thorough notes.)
- Agda’s honesty will help us see things as they really are: we cannot push trouble under the rug without saying what sort of rug it is. Other languages are much more casual about run time failure or other forms of external interaction.
- Agda’s editing environment gives strong and useful feedback during the programming process, encouraging a type-centred method of development, hopefully providing the cues to build good mental models of data and computation. We do write programs with computers: we don’t just type them in.

Bibliography

Ulf Norell. Dependently typed programming in agda. In Pieter W. M. Koopman, Rinus Plasmeijer, and S. Doaitse Swierstra, editors, *Advanced Functional Programming*, volume 5832 of *LNCS*, pages 230–266. Springer, 2008.

Appendix A

Agda Mode Cheat Sheet

I use standard emacs keystroke descriptions. E.g., ‘C-c’ means control-c. I delimit keystrokes with square brackets, but don’t type the brackets or the spaces between the individual key descriptions.

A.1 Managing the buffer

[C-c C-l] load buffer

This keystroke tells Agda to resynchronize with the buffer contents, typechecking everything. It will also make sure everything is displayed in the correct colour.

[C-c C-x C-d] deactivate goals

This keystroke deactivates Agda’s goal machinery.

[C-c C-x C-r] restart Agda

This keystroke restarts Agda.

A.2 Working in a goal

The following apply only when the cursor is sitting inside the braces of a goal.

[C-c C-,] what’s going on?

If you select a goal and type this keystroke, the information buffer will tell you the type of the goal and the types of everything in the context. Some things in the context are not in scope, because you haven’t bound them with a name anywhere. These show up with names Agda chooses, beginning with a dot: you cannot refer to these things, but they do exist.

[C-c C-.] more on what's going on?

This is a variant of the above which in addition also shows you the type of the expression currently typed into the hole. This is useful for trying different constructions out before giving/refining them!

[C-c C-spc] give expression

If you think you know which expression belongs in a goal, type the expression between its braces, then use this keystroke. The expression can include `?` symbols, which become subgoals.

[C-c C-c] case split

If your goal is immediately to the right of `=`, then you're still building your program's decision tree, so you can ask for a case analysis. Type the name of a variable in the goal, then make this keystroke. Agda will try to split that variable into its possible constructor patterns. Amusingly, if you type several variables names and ask for a case analysis, you will get all the possible combinations from splitting each of the variables.

[C-c C-r] refine

If there's only one constructor which fits in the hole, Agda deploys it. If there's a choice, Agda tells you the options.

[C-c C-a] ask Agsy (a.k.a. I feel lucky)

If you make this keystroke, Agda will use a search mechanism called 'Agsy' to try and guess something with the right type. Agsy may not succeed. Even if it does, the guess may not be the right answer. Sometimes, however, there's obviously only one sensible thing to do, and then Agsy is your bezzy mate! It can be an incentive to make your types precise!

A.3 Checking and Testing things**[C-c C-d] deduce type of expression**

If you type this keystroke, you will be prompted for an expression. If the expression you supply makes sense, you will be told its type.

If you are working in a goal and have typed an expression already, Agda will assume that you want the type of that expression.

[C-c C-n] normalize expression

If you type this keystroke, you will be prompted for an expression. If the expression you supply makes sense, you will be told its value.

If you are working in a goal and have typed an expression already, Agda will assume that you want to normalize (i.e. compute as far as possible) that expression. The normal form might not be a value, because there might be some variables in your expression, getting in the way of computation. When there are no free variables present, the normal form is sure to be a value.

A.4 Moving around

[C-c C-f]/[C-c C-b] move to next/previous goal

A quick way to get to where the action is to use these two keystrokes, which takes you to the next and previous goal respectively.

[M-.] go to definition

If you find yourself wondering what the definition of some identifier is, then you can put the cursor at it and use this keystroke – it will make Agda take you there.