AMATH 483 / 583 - HW3

Exam 1

Please create a directory (folder) called `<uwnetid>_exam1` and put a single `<uwnetid>_exam1.pdf` file for your handwritten solutions, and your code work there. Create a `<uwnetid>_exam1.tar` archive file, and submit this by midnight Friday April 28 PT.

1 code

Please use the source code file `exam1-starter-code.cpp` provided with the exam to complete the following methods for the row-major matrix class I have defined. You will simply submit this file after editing it with your solutions. Don't modify my code in `main()`, but you may add to it to check your work. You will receive (+10) points if the code you submit compiles.

1. (+10) Matrix transpose for $A \in \mathbb{R}^{m\times n}$ is defined $A^T_{i,j} = A_{j,i}$ and so $A^T \in \mathbb{R}^{n\times m}$. This method returns a matrix as defined by the class.

   - ```
     Matrix<T> transpose() const{}
     ```

2. (+10) Matrix infinity norm for $A \in \mathbb{R}^{m\times n}$ is defined $\|A\|_\infty = \max\limits_{0\leq i\leq m} \sum_{j=1}^{j=n} |A_{i,j}|$. This method returns a number.

   - ```
     T infinityNorm() const{}
     ```

3. (+10) Write the method to operator overload multiplication $*$ for the matrix class I provided. This method returns a matrix as defined by the class.

   - ```
     Matrix<T> operator*(const Matrix<T> &other) const{}
     ```

4. (+10) Write the method to operator overload addition $+$ for the matrix class I provided. This method returns a matrix as defined by the class.

   - ```
     template <typename T>
     Matrix<T> Matrix<T>::operator+(const Matrix<T> &other) const{}
     ```

2 hand-written. show your work.

1. (+10) Linear independence. Determine if the given functions ($x \in \mathbb{R}$) are linearly dependent or not:

   a) $f_1(x) = x^2 - 3,\ f_2(x) = 2 - x,\ f_3(x) = (x-1)^2$
   b) $f_1(x) = e^x,\ f_2(x) = e^{x+1}$
   c) $f_1(x) = e^x,\ f_2(x) = e^{2x},\ f_3(x) = e^{3x}$

2. (+10) Basis. Consider the family of degree-2 polynomials $\mathbb{P}_2$, and the set of polynomials $\{1, (1-x), (1-x)^2\}$.

   a) Prove that the given set of polynomials form a basis for this space of quadratic polynomials. (hint: span and linear independence)
   b) Find the coefficients a, b, c such that $p(x) = 1 + x^2 = a(1) + b(1-x) + c(1-x)^2$.

3. (+10) Norm. Consider the set of all continuously differentiable functions with domain $[0,1]$, $X = C^1([0,1],\mathbb{C})$. Define quantity $N(f) = |f(1)| + \max\limits_{0\leq x\leq 1} |f'(x)|$ for $f \in X$.

a) Prove that N is a norm on X. (hint: show it satisfies all the requirements of a norm)

4. (+10) Gram Schmidt. Consider the inner product $(x, y) = \int_{-1}^{1} x(t)y(t)dt$ and set $x_1(t) = t^2, x_2(t) = t, x_3(t) = 1$.

   a) Orthonormalize the set.

   b) Check normality and orthogonality.

5. (+10) LU factorization. Find the LU factorization of $A = \begin{pmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{pmatrix}$.