Pulsar Noise Analysis using ENTERPRISE

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"Frequency"

1. Pulsar frequency == pulse frequency == rotational frequency F

$\neq$

2. Radio frequency == observation frequency ν

$\neq$

3. Fourier frequency == conjugate frequency f

$\neq$

4. Gravitational wave frequency == source frequency

Hobbs+ 2006
Lentati+ 2014

Pulsar timing likelihood (ideal)

Number of TOAs – N
TOA – t_i
TOA uncertainty – σ_i
Timing model
Estimated pulse phase – $\Phi_i = \Phi(t_i)$
Estimated pulse number – $n_i = \frac{\Phi_i}{2\pi}$
Pulse frequency – $F_i = \frac{dn}{dt}(t_i)$
Timing residual – $R_i = \frac{n_i - \lfloor n_i \rfloor}{F_i}$
Weighted least squares metric – $\chi^2 = \sum_{i=1}^{N} \left(\frac{R_i}{\sigma_i}\right)^2$

$$\Lambda = \left[\prod_{i=1} \frac{1}{\sqrt{2\pi\sigma_i^2}}\right] \exp\left[-\frac{\chi^2}{2}\right]$$

$$\ln\Lambda \sim -\frac{\chi^2}{2} - \sum_{i=1}^{N} \ln\sigma_i$$

Lentati+ 2014

Pulsar timing likelihood (extra white noise - EFAC & EQUAD)

Scaled TOA uncertainty – $\varsigma_i^2 = E_f^2\sigma_i^2 + E_q^2$

Weighted least squares metric – $\chi^2 = \sum_{i=1}^{N}\left(\frac{R_i}{\varsigma_i}\right)^2$

$$\Lambda = \left[\prod_{i=1} \frac{1}{\sqrt{2\pi\varsigma_i^2}}\right]\exp\left[-\frac{\chi^2}{2}\right]$$

$$\ln\Lambda \sim -\frac{\chi^2}{2} - \sum_{i=1}^{N}\ln\varsigma_i$$

- EFAC $\boldsymbol{E_f}$ – Underestimated radiometer noise
- EQUAD $\boldsymbol{E_q}$ – Pulse jitter

Lentati+ 2014

Pulsar timing likelihood (single-pulsar correlated noise)

Covariance matrix (white noise) – $C^w_{ij} = \begin{cases} \varsigma_i^2 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$

Covariance matrix (red noise) – C^r_{ij}

Covariance matrix (total) – $C_{ij} = C^w_{ij} + C^r_{ij}$

Generalized least squares metric – $\chi^2 = C^{-1}_{ij} R_i R_j$

$$\Lambda = \left[\frac{1}{\sqrt{2\pi \det[C]}}\right] \exp\left[-\frac{\chi^2}{2}\right]$$

$$\ln \Lambda \sim -\frac{\chi^2}{2} - \frac{1}{2} \ln \det C$$

- ECORR – Narrowband TOAs from the same observation are correlated
- Achromatic noise – Radio frequency-independent noise
  - Spin noise – Stochastic pulsar frequency variations
- Chromatic Noise – Frequency-dependent noise (includes DM noise, scattering noise etc)
  - DM Noise – DM variations
  - Scattering noise – Variations in the scattering timescale, scattering index

Red noise

$$P(f) = A f^{-\gamma}\,\Theta[f - f_{\min}]$$

Power spectrum

$$C(\tau) = \int_0^\infty df\; P(f)\cos(2\pi f \tau)$$

Autocorrelation function

$$C_{ij} = (\nu_i \nu_j)^{-\alpha}\, C\left(|t_i - t_j|\right)$$

Correlation matrix

α = Chromatic index
γ = Power law index / spectral index

α = 0 —- Achromatic noise
α = 2 —- DM noise

SPNA vs SPNTA

1. SPNTA – Estimate timing model parameters together with noise parameters (eg: TEMPONEST, PINT (under development))

$$\ln \Lambda \sim -\frac{\chi^2}{2} - \frac{1}{2} \ln \det C$$

2. SPNA – Estimate noise parameters assuming that the best-fit timing model parameters are known (eg: ENTERPRISE)
    a. Timing model parameters are analytically marginalized

(See van Haasteren+ 2009 Eq 18 for the analytically marginalized likelihood function)

Fourier representation of red noise

- C_{ij} is an N x N matrix
- Computing its inverse and determinant are $O(N^3)$ operations ⇒ *slow*.
- We can speed this up by approximating C_{ij} using a truncated Fourier representation.

$$C^r_{ij} \approx F_{im} S_{mn} F_{jn}$$

$$F_{jn} = \begin{cases} \sin\left[2\pi n F_{\min} \left|t_j - t_0\right|\right] & \text{even } n \\ \cos\left[2\pi n F_{\min} \left|t_j - t_0\right|\right] & \text{odd } n \end{cases} \qquad \text{for } n = 1...2N_{\text{harm}}$$

$$S_{mn} = \begin{cases} P(2\pi n F_{\min}) & \text{if } m = n \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } m, n = 1...2N_{\text{harm}}$$