TWSBR

Two Wheel Self Balancing Robot

Dynamical Model

Coordinate Axis of TWSBR System

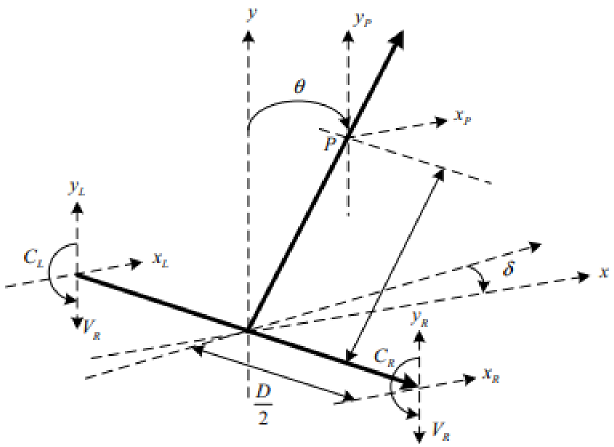


Fig.1 Coordinate Axis of TWSBR System

Equations

- The relationship between rotation angles of two wheels and location is,

$$\begin{cases} X_{RL} = \theta_{RL} R \\ X_{RR} = \theta_{RR} R \\ X_{RL} - X_{RR} = D \times \delta \end{cases}$$

- equations for left wheel

$$\begin{cases} \ddot{X}_{RL} \times M_{RL} = H_{TL} - H_L + f_{RL} \\ \ddot{Y}_{RL} \times M_{RL} = V_{TL} - M_{RL} g - V_L \\ J_{RL} \times \ddot{\theta}_{RL} = C_L - H_{TL} R \end{cases}$$

- Equations for body

$$\begin{cases} \ddot{X}_P \times M_P = H_R + H_L + f_P \\ \ddot{Y}_P \times M_P = V_R - M_P g + V_L \\ J_{RL} \times \ddot{\theta}_P = (V_R + V_L) L \sin \theta_p - (C_R + C_L) - (H_R + H_L) L \cos \theta_p \end{cases}$$

Linearization

Nonlinear Equations

$$\ddot{X}_{RM} = \frac{1}{J_P\left(2\left(J_W/R^2+M_W\right)+M_P\right)+2\left(J_W/R^2+M_W\right)M_PL^2+M_P^2L^2\sin^2\theta_P}$$
$$\cdot\Big[\left(J_P+M_PL^2\right)M_PL\sin\theta_P\dot{\theta}_P^2 - M_P^2L^2g\cos\theta_P\sin\theta_P$$
$$+\left(\frac{1}{R}\left(J_P+M_PL^2\right)+M_PL\cos\theta_P\right)\left(C_R+C_L\right)$$
$$+\left(J_P+M_PL^2\left(f_{RR}+f_{RL}\right)\right)\Big]$$

$$\ddot{\theta}_P = -\frac{1}{J_P\left(2\left(J_W/R^2+M_W\right)+M_P\right)+2\left(J_W/R^2+M_W\right)M_PL^2+M_P^2L^2\sin^2\theta_P}$$
$$\cdot\Big[M_P{}^2L^2\sin\theta_P\cos\theta_P\dot{\theta}_P^2+\left(2\left(J_W/R^2+M_W\right)+M_P\right)M_PgL\sin\theta_P$$
$$-\left(M_PL\cos\theta_P/R+2\left(J_W/R^2+M_W\right)+M_P\right)\left(C_R+C_L\right)$$
$$-M_PL\cos\theta_P\left(f_{RR}+f_{RL}\right)+2\left(J_W/R^2+M_W\right)L\cos\theta_Pf_P\Big]$$

$$\ddot{\delta} = [(C_L-C_R)/R+(f_L-f_R)]/(2J_PM_W+J_WD/R^2)$$

Quiescent Point

- In order to design controller by chips, the above equations is approximated by method of linearization. That is, near the equilibrium point and in a small angle, set

$$\sin\theta_P \approx \theta_P, \cos\theta_P \approx 1$$

State Space

First Equation

$$\begin{pmatrix} \dot{X}_{RM} \\ \dot{V}_{RM} \\ \dot{\theta}_P \\ \dot{\omega}_P \\ \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_P \\ \omega_P \\ \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ B_{21} & B_{22} \\ 0 & 0 \\ B_{41} & B_{42} \\ 0 & 0 \\ B_{61} & B_{62} \end{pmatrix} \begin{pmatrix} C_L \\ C_R \end{pmatrix}$$

Change of variables

- Set state variable x , input variable u and Output variable y be,

$$x = \left[X_{RM}, V_{RM}, \theta_p, \omega_P, \delta, \dot{\delta} \right]^T ,$$

$$y = \left[V_M, \theta_P, \dot{\delta} \right]^T ,$$

$$u = [C_L \quad C_R]^T$$

Standard State Space

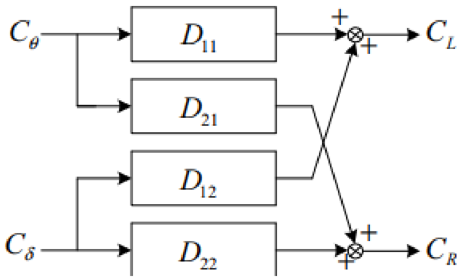
$$\dot{x} = Ax + Bu = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_P \\ \omega_P \\ \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ B_{21} & B_{22} \\ 0 & 0 \\ B_{41} & B_{42} \\ 0 & 0 \\ B_{61} & B_{62} \end{pmatrix} \begin{pmatrix} C_L \\ C_R \end{pmatrix}$$

$$y = Cx = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_P \\ \omega_P \\ \delta \\ \dot{\delta} \end{pmatrix}$$

where

$$\begin{cases} A_{23} = \dfrac{-M_P{}^2 L^2 g}{M_P J_P + 2\left(J_P + M_P L^2\right)\left(M_W + J_W/R^2\right)} \\ A_{43} = \dfrac{M_P{}^2 gL + 2M_P gL\left(M_W + J_W/R^2\right)}{M_P J_P + 2\left(J_P + M_P L^2\right)\left(M_W + J_W/R^2\right)} \\ B_{21} = B_{22} = \dfrac{\left(J_P + M_P L^2\right)/\left(R + M_P L\right)}{M_P J_P + 2\left(J_P + M_P L^2\right)\left(M_W + J_W/R^2\right)} \\ B_{41} = B_{42} = \dfrac{-\left(R + L\right) M_P/R - 2\left(M_W + J_W/R^2\right)}{M_P J_P + 2\left(J_P + M_P L^2\right)\left(M_W + J_W/R^2\right)} \\ B_{61} = B_{62} = \dfrac{D/2R}{J_P + \dfrac{D^2}{2R}\left(M_W R + \dfrac{J_W}{R}\right)} \end{cases}$$

Decoupling



$$\begin{pmatrix} \dot{X}_{RM} \\ \dot{V}_{RM} \\ \dot{\theta}_P \\ \dot{\omega}_P \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & A_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & A_{43} & 0 \end{pmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_P \\ \omega_P \end{pmatrix} + \begin{pmatrix} 0 \\ B_{21} \\ 0 \\ B_{41} \end{pmatrix} C_\theta$$

$$\begin{pmatrix} \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ B_{61} \end{pmatrix} C_\delta$$

Let $D_{11} = D_{21} = D_{12} = D_{22} = 0.5$, then

$$\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix} \begin{pmatrix} C_\theta \\ C_\delta \end{pmatrix} \qquad ($$

Controller Design

List. 1 Specification of TWSBR

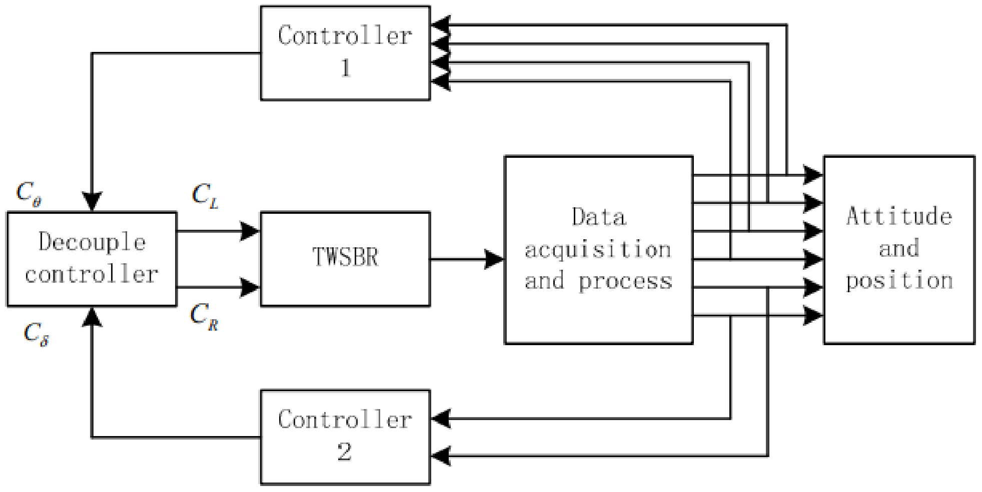
variable	value	variable	value
M_P	$20\,kg$	R	$0.2\,m$
M_W	$6\,kg$	D	$0.5\,m$
L	$0.2\,m$	J_P	$0.27\,kgm^2$
J_{Py}	$1.33\,kgm^2$	J_W	$0.12\,kgm^2$

$$\begin{pmatrix} \dot{X}_{RM} \\ \dot{V}_{RM} \\ \dot{\theta}_P \\ \dot{\omega}_P \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -6.388 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 60.6861 & 0 \end{pmatrix} \begin{pmatrix} X_{RM} \\ V_{RM} \\ \theta_P \\ \omega_P \end{pmatrix} + \begin{pmatrix} 0 \\ 0.3803 \\ 0 \\ -2.3629 \end{pmatrix} C_\theta$$

$$\begin{pmatrix} \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ 0.5085 \end{pmatrix} C_\delta$$

LINEAR QUADRATIC OPTIMAL CONTROL

Decoupling System



LQR Controller

With the decoupling system shown in Fig.4, two state-feedback controllers based on linear quadratic form are designed and the target function is set,

$$J = \int_0^\infty \left(X^T Q X + U^T R U \right) dt$$

and input is,

$$U = -KX$$

where $K = -R^{-1} B^T P$ and $R > 0, Q \geq 0$. P is the solution of Riccati differential equation,

$$\dot{P}(t) = -P(t)A(t) - A^T(t)P(t) + P(t)B(t)R^{-1}(t)B^T(t)P(t) - Q(t)$$

$$P(t_f) = F \quad t \in [t_0, t_f]$$

As a conclusion, if there exits a positive definite P to satisfy the above Riccati differential equation, TWSBR is stable. And then K, the optimal feedback gain, can be calculated by P.

Simulation

Balancing Robot

- Set the given velocity of wheel motors and inclination of body are 0 at the initial time. And then a certain initial angle is given to the body, such as 0.1rad.

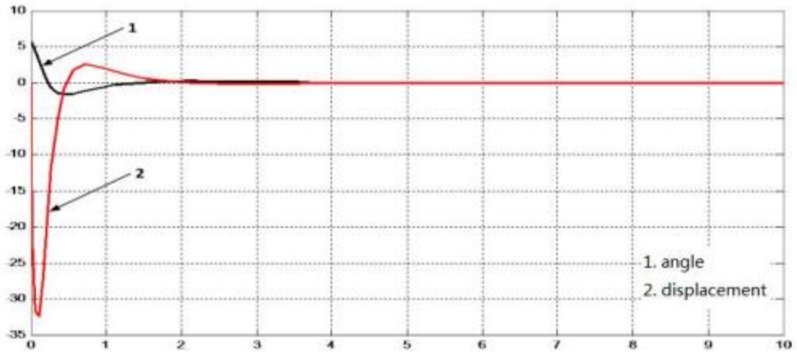
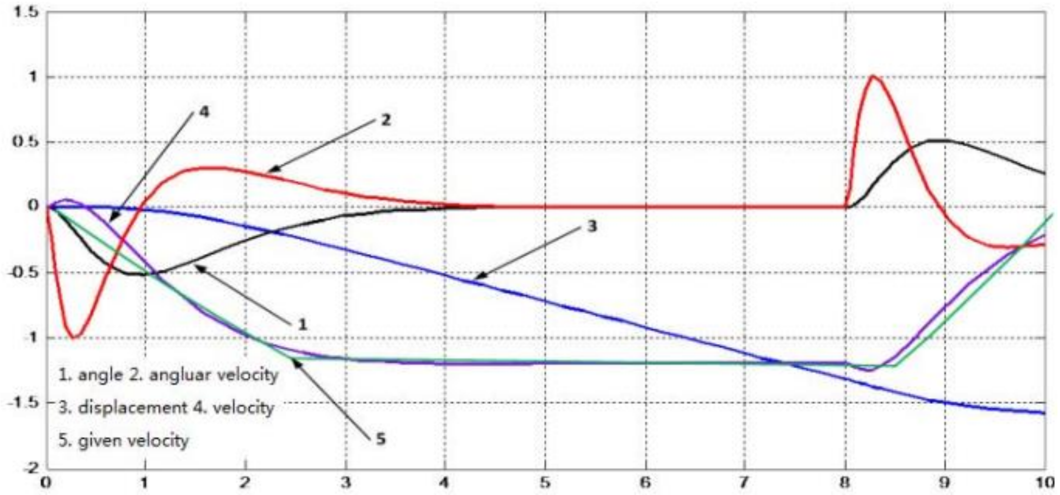


Fig7. Balance control experiment

Motion Control



- These above results show that the state feedback controller based on quadratic form can make TWSBR stable and has an effective control performance.