Causal parameter estimat
using the state learner

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Abstract

The super learner is a machine learning algorithm which combines a librar
learner based on cross-validated loss. Unfortunately, the commonly used parti
super learning, and inverse probability of censoring weighted loss functions req
censoring distribution. To relax this, we introduce the state learner, a new super
evaluates the loss based on the observed data simultaneously using libraries of
of interest and the censoring distribution. We establish an oracle inequality for
performance through numerical experiments. We illustrate how the state learner
in a competing risks setting without having to prespecify models for neither th
nor the censoring distribution.

Motivation

Prespecification of estimators for survival and censoring probabilit

vational data. Super learning uses the data to decides which estima
learning can correct the bias stemming from this data-adaptive mo
valid post-selection inference for causal parameters.

Cox model?
Proportional hazard?
Random forest?
Penalization?
Interactions?
Independent censoring?
Complex censoring?

$\xrightarrow{\text{Super learning}}$ Nuisance parameter estimation

Targe

Problem statement

Ideal data: $(W, T^0, D^0, T^1, D^1) \sim Q \in \mathcal{Q}$

$W \in \mathcal{W} \subset \mathbb{R}^d$ is a vector of covariates, $T^a \in [0, \tau]$ is a counterfactual time to event variable under treatment a, and $D^a \times \{1, 2\}$ denotes the cause of the event. The maximal

Observed da

$A \in \{0, 1\}$ is
tered in observe
censored outco
$T^a \wedge C^a$ and

denotes the cause of the event. The maximal length of followup is $\tau < \infty$.

$A = a$, for some

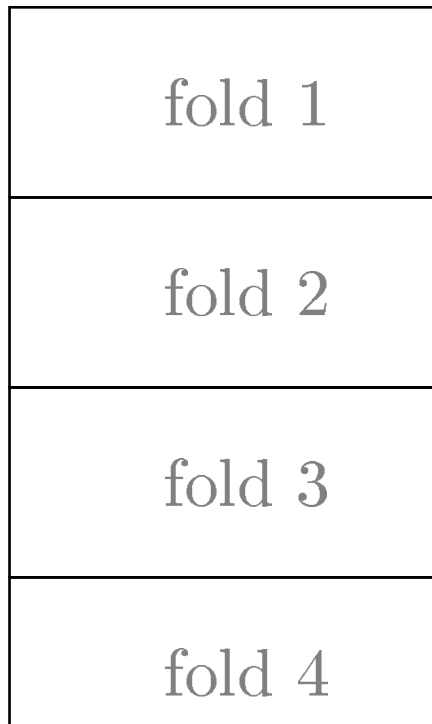
Targeted estimation of causal effects such as the average treatment cause-specific cumulative hazard functions,

$$\Lambda_j(t \mid w, a) = \int_0^t \frac{Q(T^a \in \mathrm{d}s, D^a = j \mid W = w)}{Q(T^a \geq s \mid W = w)},$$

and the censoring probability. Assuming coarsening at random [2, 9], the cause-specific hazard functions and the censoring probability can be estimated based on samples from P. Many existing methods are available for this task, including the Nelson-Aalen estimator, Cox models, random survival forests, Poisson regression, neural networks, and many others.

A super learner can be used to data-adaptively select a method from a library of candidates. Super learning uses cross-validation to estimate and

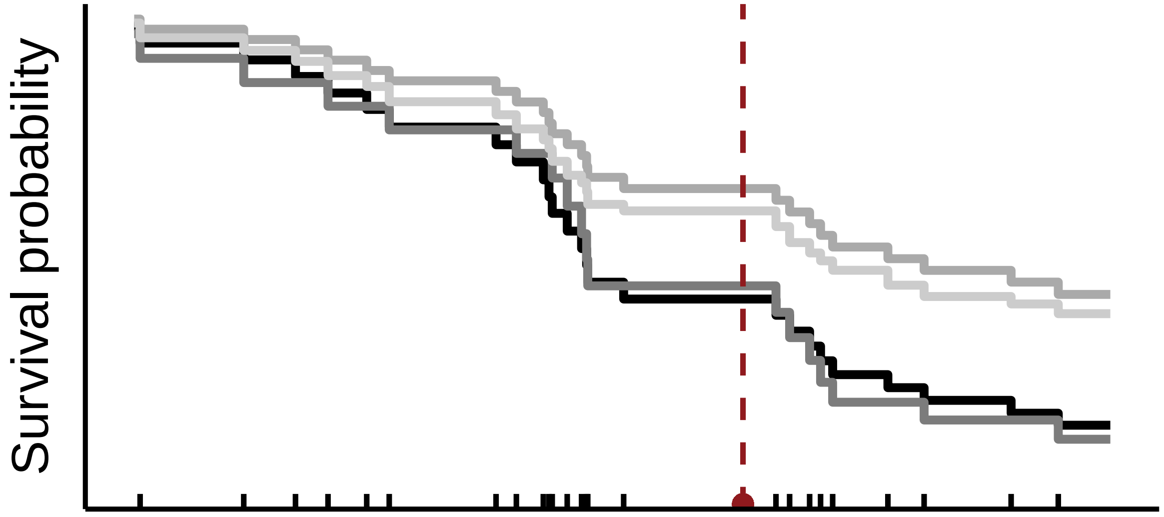
Train learner



per learning uses cross-validation to estimate and evaluate the performance of each learner in the library using a given loss function. Our focus in this work is on how a super learner can assess the performance of a learner in a right-censored validation set.

Existing methods

A commonly used loss function in survival analysis is the negative
loss is unsuited for super learner because many common survival es
in any hold out sample as illustrated in the figure below to the righ
inverse probability of censoring weighting [4, 3], pseudo-observation
transformations [8] rely on a pre-specified estimator of the censoring
circular reasoning as illustrated in the figure below to the right. A rece
competing events is based on iterative estimation of the survival and t
hope that this is not a vicious circle [10].



Estimate outcome

↓

Prespecify outcome

[1] Per Kragh Andersen, John P Klein, and Susanne Rosthøj. Generalised linear models for correlated pseudo-observations, with applications to multi-state models. *Biometrika*, 2003.

[2] Richard D Gill, Mark J van der Laan, and James M Robins. Coarsening at random: Characterizations, conjectures, counter-examples. In *Proceedings of the First Seattle Symposium in Biostatistics*, 1997.

[3] Pablo Gonzalez Ginestet, Ales Kotalik, David M Vock, Julian Wolfson, and Erin E Gabriel. Stacked inverse probability of censoring weighted baggin
of the Royal Stati

[4] Erika Graf, Clau
macher. Assess
schemes for survi

[5] Michael W Katta
Scardino, Zvi Fu
for predicting th
therapy in prosta

ion with right-censored data

gen

y of prediction models into a meta ial likelihood loss is not suited for uire a prespecified estimator of the learner for survival analysis, which predictions models for the event(s) the state learner and investigate its allows us to estimate causal effects e cause-specific hazards of interest

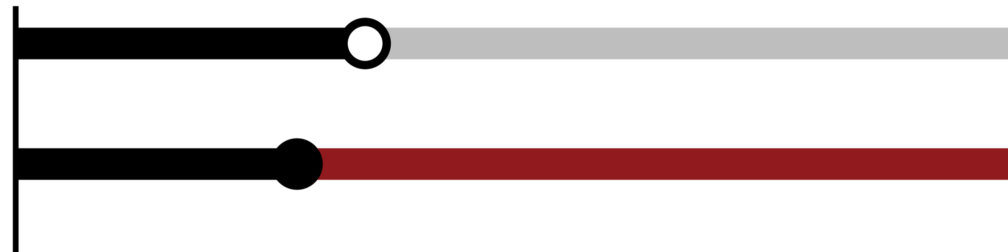
ies can be difficult with obser-

The state learner

The state learner models the observed syst considered a state. After baseline the observ defined as

$$\eta(t) = \mathbb{1}\{\tilde{T} \leq t, \tilde{D} = 1\} + 2\,\mathbb{1}\{\tilde{T} \leq$$

The process η

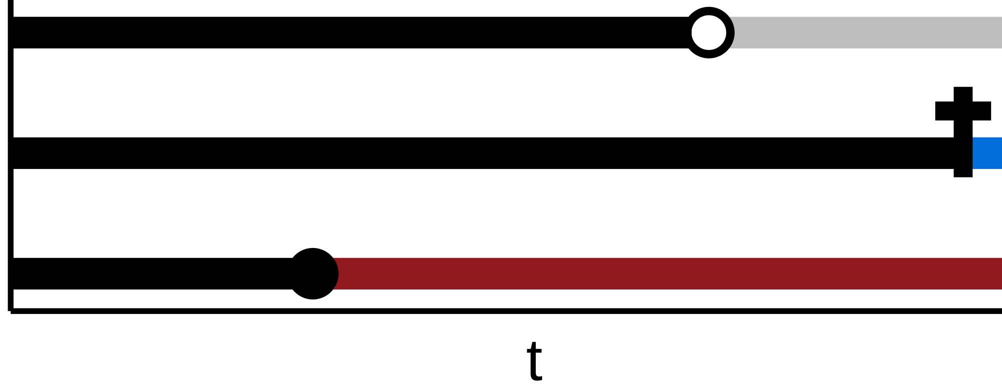


ators best fit the data. Targeted
del selection step, and provide

eted learning $\longrightarrow$ Valid inference for causal parameters

ata: $(W, A, \tilde{T}, \tilde{D}) \sim P \in \mathcal{P}$

a binary treatment adminis-
ed data. The pair $(\tilde{T}, \tilde{D})$ is the
me variable, defined as $\tilde{T} =$
$\tilde{D}^a = \mathbb{1}\{T^a \leq C^a\}D^a$, when



The state learner is a super learner for the c

$$F(t, k, x) = P(\eta(t) = k \mid X = x)$$

Performance of a model for F can be estima
Brier score,

$$\bar{B}_\tau(F, O) = \int_0^\tau B_t(F, O)\,\mathrm{d}t, \quad \text{wher}$$

Building a library for state l

Learners of F are not directly available. F e censoring time C^a.

effect rely on estimators of the

$j \in \{1, 2\}$,

rs Predict in test data

Evaluate performance using censored outcomes

mulative hazard functions Λ_1 and Λ_2, and a

denoted by Γ, we can obtain a library of lea

$$F(t, 0, x) = P\big(\tilde{T} > t \;\big|\; X = x\big) =$$

$$F(t, j, x) = P\big(\tilde{T} \leq t, \Delta = j \;\big|\; X =$$

$$F(t, -1, x) = P\big(\tilde{T} \leq t, \Delta = 0 \;\big|\; X =$$

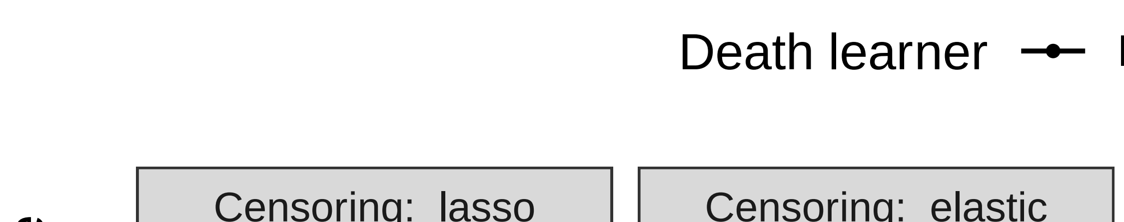
Theoretical and empirical re

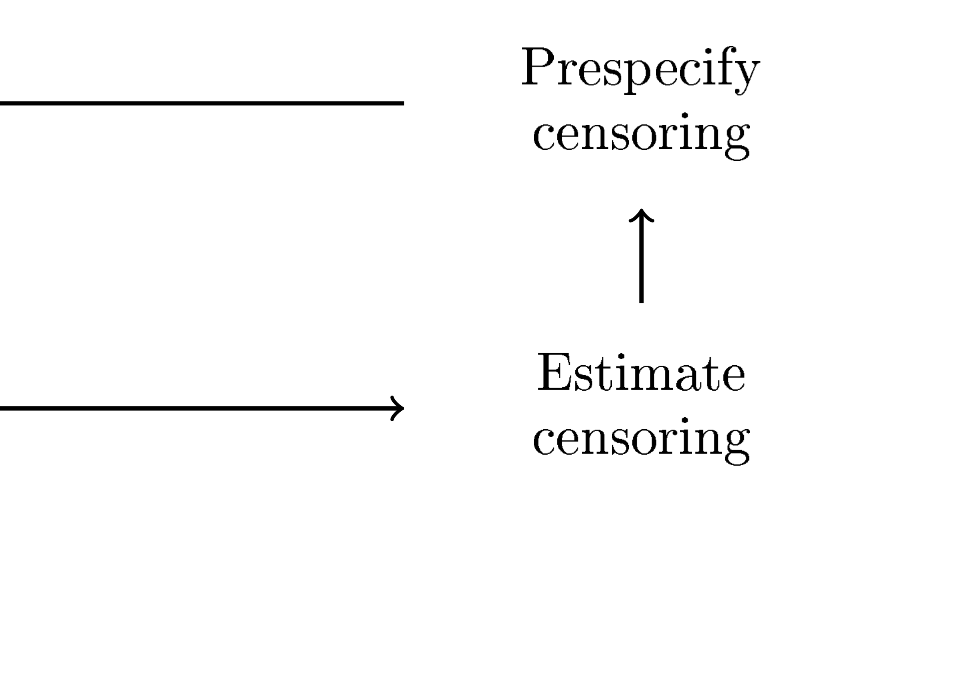
In [6] we provide a finite sample oracle in e
ity for the state learner along with other the
ical results. We confirm these results in a
lation study, where the state learner perfor
well as a super learner that uses a correctly s
fied model to estimate inverse probability o

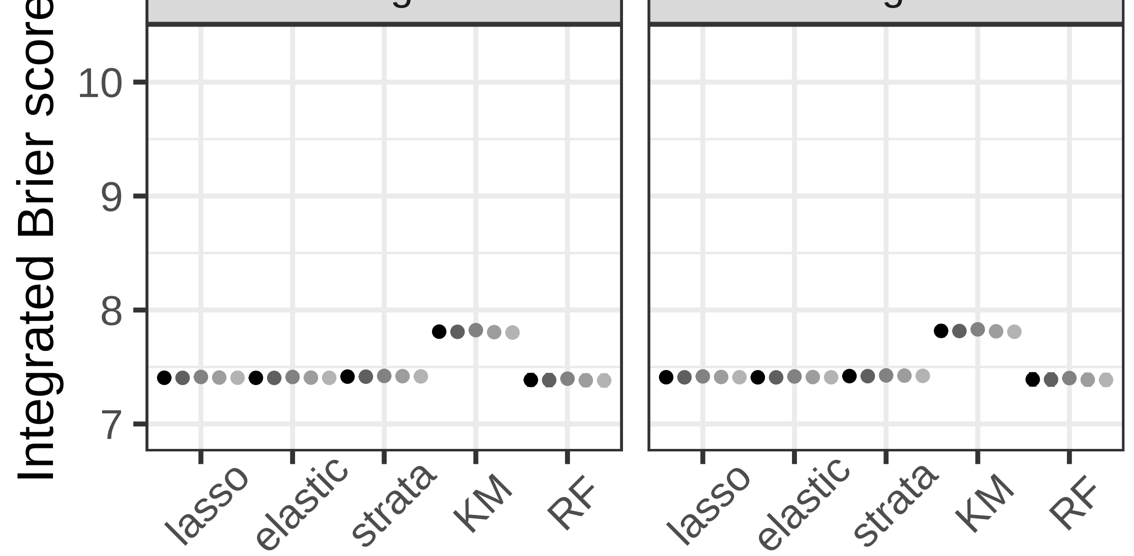
partial log-likelihood loss. This
stimators will have infinite loss
t. Alternative methods such as
s [1, 7], and censoring unbiased
g probability. This can lead to a
ent proposal in a context without
he censoring probabilities in the

soring weights. The state learner learns the
rect censoring model from the data at the
times as learning the outcome model. This
lustrated in the figure to the right.

We apply the state learner to data from a
servational prostate cancer study [5]. The
learner's ranking of all triples of learners
the provided libraries are visualized in the
below. In [6] we show how estimators of th
erage treatment effect of hormone therapy
mor recurrence and death can be obtained
the output of the state learner. The resul
shown in the figure to the right.







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Erin E Gabriel. Ensemble prediction of time-to-event outcomes
competing risks: A case-study of surgical complications in Cr
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Censoring unbiased regression trees and ensembles. *Journal*

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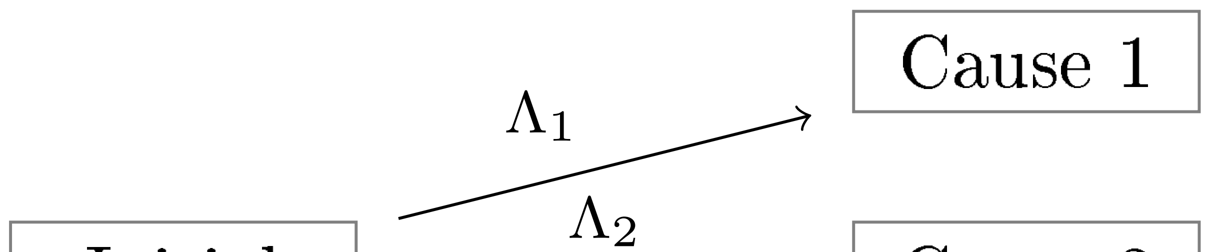
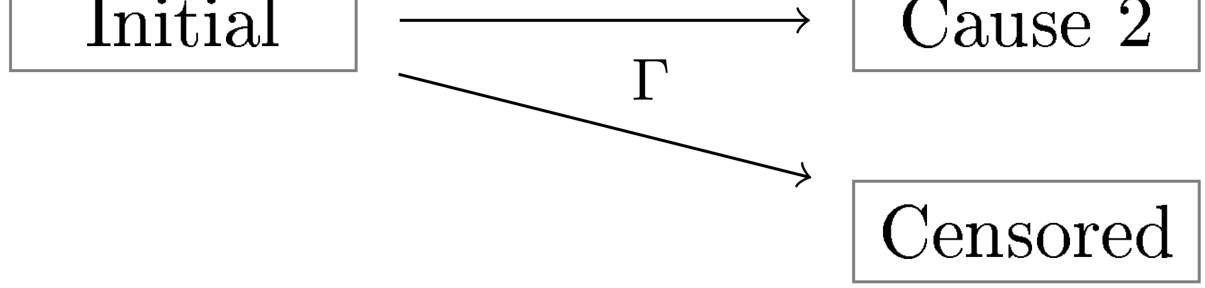
em as an *artificial* multi-state system where censoring is
ved data can be encoded by the process $\eta(t) \in \{-1, 0, 1, 2\}$

$$\leq t, \tilde{D} = 2\big\} - \mathbb{1}\big\{\tilde{T} \leq t, \tilde{D} = 0\big\}, \quad \text{for} \quad t \in [0, \tau].$$

The states of the observed system





onditional state-occupation probability function,

$$), \quad \text{for all} \quad t \in [0, \tau], k \in \{-1, 0, 1, 2\}, x \in \mathcal{X}.$$

ated directly in the observed data, e.g., using the integrated

$$\text{re} \quad B_t(F, O) = \sum_{j=-1}^{2} \left(F(t, j, X) - \mathbb{1}\{\eta(t) = j\}\right)^2 .$$

earning

However, given libraries of learners of cause-specific cu-

a library for learning the cumulative hazard of censoring,
rners for F using the following relations.

$$\prod_{0}^{t} \left(1 - [\Lambda_1 + \Lambda_2 + \Gamma](\mathrm{d}s \mid x)\right),$$

$$x) = \int_0^t F(t-, 0, x)\Lambda_j(\mathrm{d}s \mid x), \quad \text{for} \quad j \in \{1, 2\},$$

$$x) = \int_0^t F(t-, 0, x)\Gamma(\mathrm{d}s \mid x),$$

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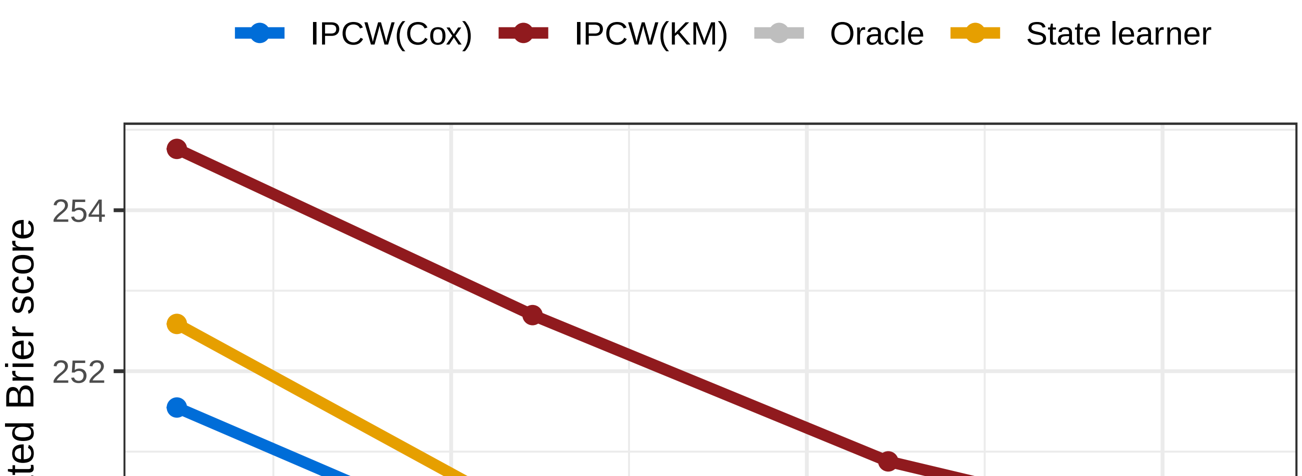

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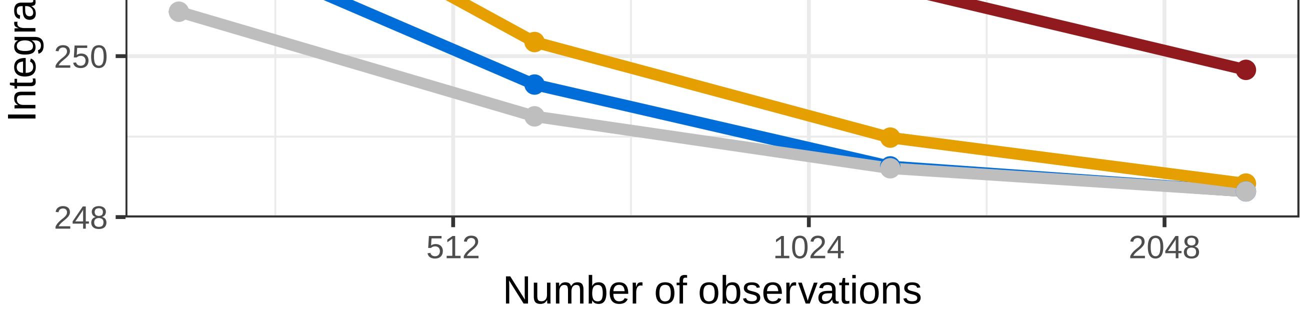

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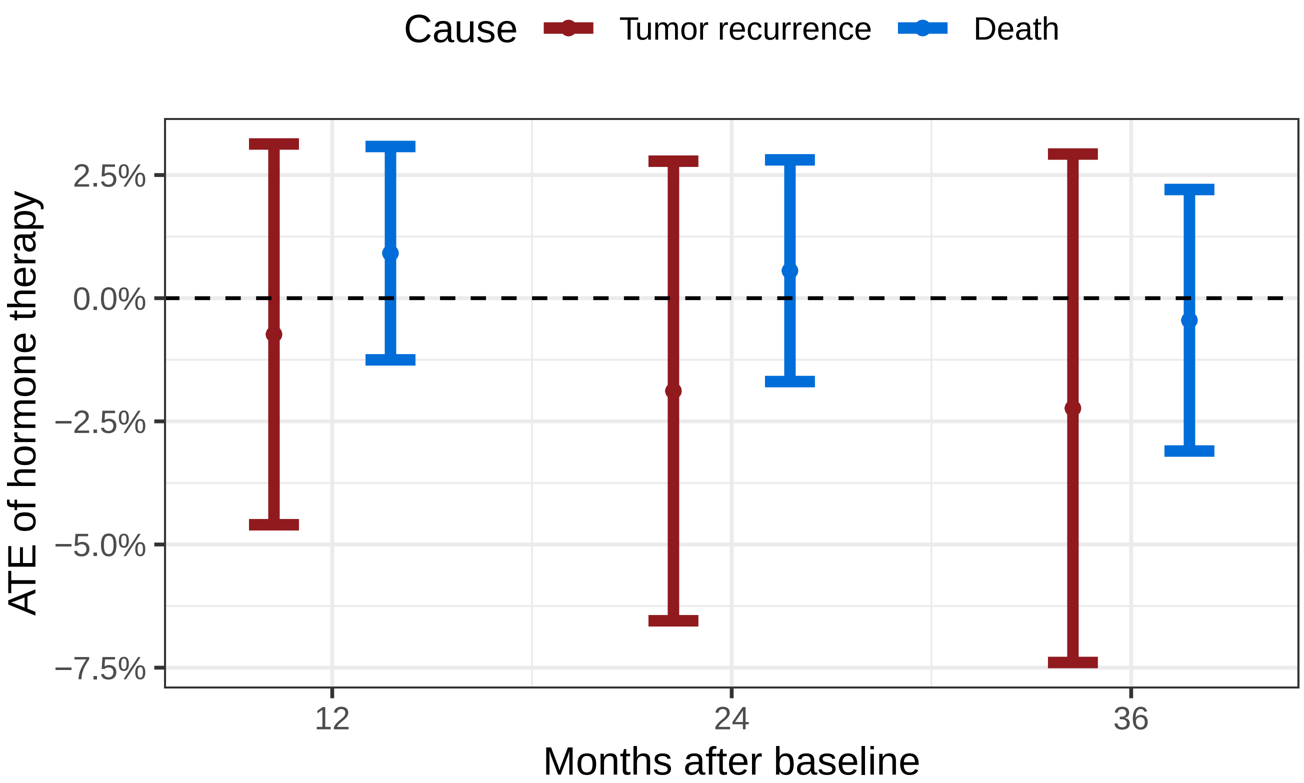

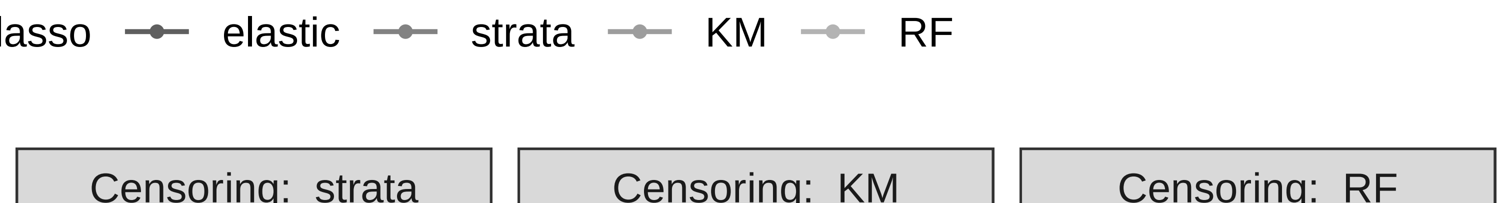

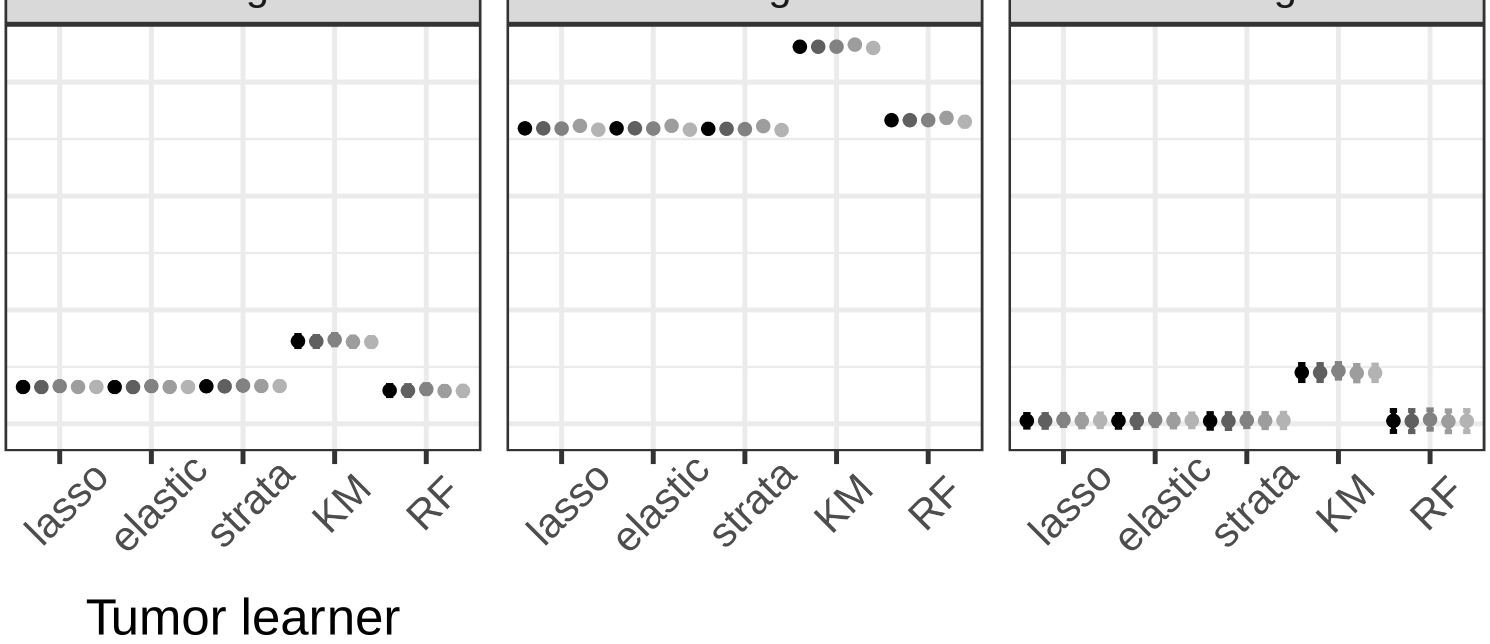

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