

$$Z_1(k) : 0 =$$

$$\partial_p (n_1 e_1 + n_2 e_2 + n_3 e_3 + n_4 e_4)$$

$$= (n_4 - n_1) v_1 + (n_1 - n_2) v_2 \\ + (n_2 - n_3) v_3 + (n_3 - n_4) v_4$$

$\Leftrightarrow$

$$n_4 - n_1 = 0$$

$$n_1 - n_2 = 0$$

$$n_2 - n_3 = 0$$

$$n_3 - n_4 = 0$$

$\Rightarrow$

$$n_1 = n_4 = n_2 = n_3$$

$$Z_1(k) = \{n(e_1 + e_2 + e_3 + e_4)$$

$$| n \in \mathbb{Z} \} \cong \mathbb{Z}$$

$$B_1(k) : \partial_2 C_2(k) = 0$$

$$\Rightarrow H_1(k) = \frac{Z_1(k)}{B_1(k)} = \mathbb{Z}$$

$$H_2(L) = \frac{Z_2(L)}{B_2(L)}$$

$$Z_2(L) = ?$$

$$0 = \partial_2 (n_1 \sigma_1 + n_2 \sigma_2)$$

$$= n_1 \partial_2 \sigma_1 + n_2 \partial_2 \sigma_2 =$$

$$n_1 ([v_2, v_3] - [v_1, v_3])$$

$$+ [v_1, v_2]) + n_2 ([v_3, v_4]$$

$$- [v_1, v_4] + [v_1, v_3]) =$$

$$n_1 (e_2 - e_5 + e_1) +$$

$$n_2 (e_3 - e_4 + e_5)$$

$=$

$$n_1 e_2 + (n_2 - n_1) e_5$$

$$+ n_1 e_1 + n_2 e_3 +$$

$$- n_2 e_4 = 0$$

$$\Rightarrow n_1 = n_2 = 0$$

$$H_2(L) = 0 / B_2(L) = 0$$