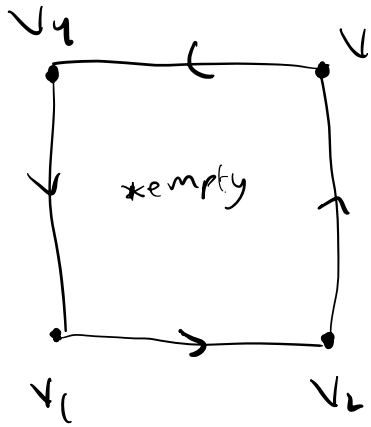


$e_1 = [v_1, v_2]$

$e_2 = [v_3, v_4]$

$e_3 = [v_3, v_4]$

$e_4 = [v_4, v_1]$

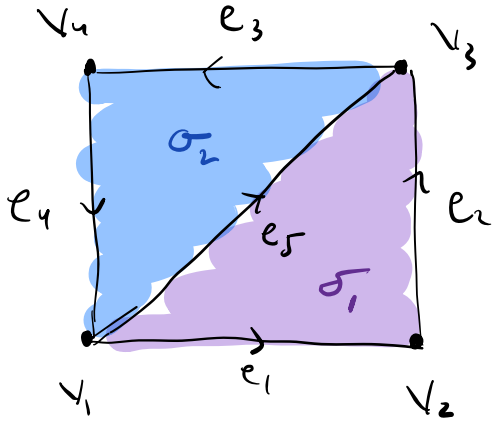
$|K|$ = union of edges of square

$$C_0(K) = \{ n_1 v_1 + n_2 v_2 + n_3 v_3 + n_4 v_4 \mid n_i \in \mathbb{Z} \}$$

$$C_1(K) = \{ n_1 e_1 + n_2 e_2 + n_3 e_3 + n_4 e_4 \mid n_i \in \mathbb{Z} \}$$

$C_p(K) = 0 \quad \forall p > 1$

(our polytope only has 0 and 1 simplexes



$\sigma_1 = [v_1, v_2, v_3]$

$\sigma_2 = [v_1, v_3, v_4]$

$$L = \{ v_1, v_2, v_3, v_4, v_1v_2, v_2v_3, v_3v_4, v_4v_1, v_1v_3, v_1v_2v_3, v_1v_3v_4 \}$$

$Z_1(L)$ generated by $\{ e_1 + e_2 - e_5, e_3 + e_4 + e_5 \}$

$\cong \mathbb{Z} \oplus \mathbb{Z}$

$B_1(L) = \langle e_1 + e_2 - e_5, e_3 + e_4 + e_5 \rangle = Z_1(L)$

$\therefore H_1(L) = 0$