Calculus I, Math 1510 — Assignment #1

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Section 1.1, Problem 7

Determine the domain and range of $g(x) = 3x^2 - 10$.

Solution

Domain: g is defined for all values of x. Its domain is the set of all real numbers, written:

$$D = (-\infty, \infty) \quad \text{or} \quad \mathbb{R}$$

Range: Because $x^2 \geq 0$ for all x, it follows that $3x^2 - 10 \geq 10$, which implies that the range of f is:

$$R = [-10, \infty]$$

Section 1.1, Problem 9

Water tower A cylindrical water tower with a radius of 10 m and a height of 50 m is filled to a height of h m. The volume V of water (in cubic meters) is given by the function $g(h) = 100\pi h$. Identify the independent and dependent variables for this function, and then determine an appropriate domain.

Solution

Independent variable: The independent variable is the variable associated with the domain. Therefore in $100\pi h$ the independent variable is h.

Dependent variable: The dependent variable is the variable associated with the range. In $V = g(h)$ the dependent variable is V.

Appropriate domain: As the height of the water tower is 50 m, the domain is 0 (when the tower is empty) to 50 (when the tower is full):

$$D = 0 \leq h \leq 50$$

Section 1.1, Problem 23

State the domain and range of the function

$$f(x) = \frac{x^2 - 5x + 6}{x - 2}$$

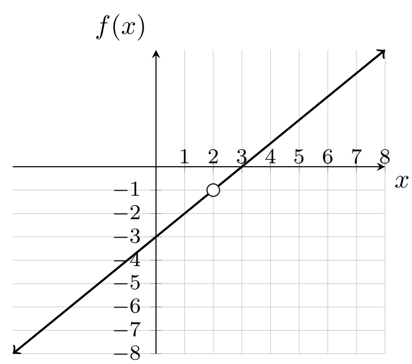
Solution

Domain: The domain of the function is all real numbers that don't cause the denominator of the expression to evaluate to 0. Therefore the domain of the function is:

$$D = \{x : x \neq 2\}$$

Range: The range of the function is all real numbers except the number -1 because y is undefined at $x = 2$ and that's where it *would* be -1.

$$R = \{y : y \neq -1\}$$



Section 1.1, Problem 27

State the domain of the function

$$h(u) = \sqrt[3]{u - 1}$$

Solution

As a cubed root is an odd numbered root, the result when evaluated will always equal a real number. Therefore the domain of the function is all real numbers:

$$D = (-\infty, \infty) \quad \text{or} \quad \mathbb{R}$$

Section 1.1, Problem 31

Launching a rocket A small rocket is launched vertically upward from the edge of a cliff 80 ft above the ground at a speed of 96 ft/s. Its height (in feet) above the ground is given by $h(t) = -16t^2 + 96t + 80$, where t represents time measured in seconds.

a. Assuming the rocket is launched at $t = 0$, what is an appropriate domain for h?

b. Graph h and determine the time at which the rocket reaches its highest point. What is the height at that time

Solution

a. We can find the domain of the function by using the quadratic equation

$$x = \frac{-96 \mp \sqrt{96^2 - 4(-16)(80)}}{2(-16)}$$
$$x = \frac{-96 \mp 119.733}{-32}$$
$$x = -.7416, \quad x = 6.742$$

Since the rocket is launched at $t = 0$, the appropriate domain is

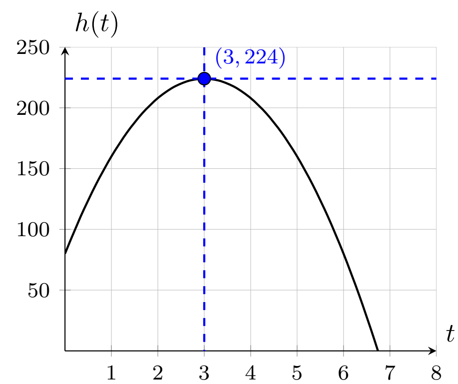
$$\{x : 0 \leq x \leq 6.742\}$$

b. We can calculate the maximum of a parabola using $x = \frac{-b}{2a}$.

$$\frac{-96}{2(-16)} = 3$$

When we evaluate $h(3)$ we get

$$y = -16(3)^2 + 96(3) + 80$$
$$y = -144 + 288 + 80$$
$$y = 224$$



Section 1.1, Problem 33 & 34

Let $f(x) = x^2 - 4$, $g(x) = x^3$, and $F(x) = 1/(x-3)$. Simplify or evaluate the following expressions.

33. $g(1/z)$

Solution

$$y = (1/z)^3$$
$$y = 1/z^3$$

34. $F(y^4)$

Solution

$$1/y^4 - 3$$

Section 1.1, Problem 47 & 48

Let $f(x) = |x|$, $g(x) = x^2 - 4$, $F(x) = \sqrt{x}$, and $G(x) = 1/(x-2)$. Determine the following composite functions and give their domains.

47. $f \circ g$

Solution

$$y = f(g(x))$$
$$y = f(x^{-}4)$$
$$y = |x^2 - 4|, D = \text{all real numbers or } \mathbb{R}$$

48. $g \circ f$

Solution

$$y = g(f(x))$$
$$y = g(|x|)$$
$$y = |x^2 - 4|, D = \text{all real numbers or } \mathbb{R}$$

Section 1.1, Problem 57 & 58

Let $g(x) = x^2 + 3$. Find a function f that produces the given composition.

57. $(f \circ g)(x) = x^4 + 6x^2 + 9$

Solution

$x^4 + 6x^2 + 9$ factors to $(x^2 + 3)(x^2 + 3)$ or $(x^2 + 3)^2$. In order to produce the desired result with an argument of $g(x) = x^2 + 3$, $f(x)$ should be:

$$f(x) = x^2$$

58. $(f \circ g)(x) = x^4 + 6x^2 + 20$

Solution

This problem is very similar to the above #57 if we rewrite the result of the composition as $x^4 + 6x^2 + 9 \mathbf{+ 11}$. We can factor in the same way as above and then just add 11 to the result: $(x^2 + 3)(x^2 + 3) + 11$. So the resulting $f(x)$ is

$$f(x) = x^2 + 11$$

Section 1.1, Problem 65 & 66

Simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for the following functions

65. $f(x) = x^2$

Solution

First we note that $f(x+h) = (x+h)^2$ or $x^2 + 2hx + h2$. We can substitute this expression into the difference quotient and simplify:

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{(x^2+2hx+h^2)-x^2}{h} \\
&= \frac{2hx+h^2}{h} \\
&= \frac{2hx}{h} + \frac{h^2}{h} \\
&= 2x + h
\end{aligned}$$

66. $f(x) = 2x^2 - 3x + 1$

Solution

First we note that $f(x+h) = 2(2+h2)^2 - 3(x+h) + 1$. We can substitute this expression into the difference quotient and simplify:

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{f(x+h) = 2(2+h2)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h} \\
&= \frac{2(x+h)(x+h) - 3x - 3h + 1 - (2x^2 - 3x + 1)}{h} \\
&= \frac{2(x^2 + 2hx + h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} \\
&= \frac{\cancel{2x^2} + 4hx + 2h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{2x^2} + \cancel{3x} - \cancel{1}}{h} \\
&= \frac{4\cancel{h}x + 2h^{\cancel{2}\,h} - 3\cancel{h}}{\cancel{h}} \\
&= 4x + 2h - 3
\end{aligned}$$

Section 1.2, Problem 1

Give four ways in which functions may be defined and represented.

Solution

1. Formulas
2. Graphs
3. Tables
4. Words

Section 1.2, Problem 3

Determine the function f represented by the graph of the line $y = f(x)$ in the figure. (Given points are $(0, -1)$ & $(3, -1)$)

Solution

From the point $(0, -1)$ we can determine that the y intercept is -1. We can also determine the slope using $m = \dfrac{y_2 - y2}{x_2 - x_1}$:

$$\frac{-3 - (-1)}{3 - 0} = -\frac{2}{3}$$

Therefore the function f is

$$f = -\frac{3}{2}x - 1$$

Section 1.2, Problem 23

Bald eagle population After DDT was banned and the Endangered Species Act was passed in 1973, the number of bald eagles in the United States increased dramatically. In the lower 48 states, the number of breeding pairs of bald eagles increased at a nearly linear rate from 1875 pairs in 1986 to 6471 pairs in 2000.

a. Use the data points for 1986 and 2000 to find a linear function p that models the number of breeding pairs from 1986 to 2000 ($0 \leq t \leq 14$).

b. Using the function in part (a), approximately how many breeding pairs were in the lower 48 states in 1995?

Solution

a. As we are dealing with the domain $(0 \leq t \leq 1)4$, our starting year (1986) can be considered to be $t = 0$. So given the point for 1986 - $(0, 1875)$ we see that our y intercept is 1875. We can consider our second point from 2000 to be $(14, 6471)$. Using the formula for slope we can calculate our slope

$$m = \frac{6471 - 1875}{14 - 0} = 328.285$$

The linear function can then be expressed as

$$y = 328.285x + 1875$$

b. Using our domain $(0 \leq t \leq 14)$, we can see that the year 1995 can be expressed as 9. We can determine approximately how many breeding pairs were in 1995 by evaluating $f(9)$ using the formula above

$$\begin{aligned} f(9) &= 328.285(9) + 1875 \\ &= 4829.565 \end{aligned}$$

Section 1.2, Problem 25 - 26

Defining piecewise functions Write a definition of the function whose graph is given.

25. Solution

We can see that $x = 3$ from $x \leq 3$ just by looking at the graph. For $x > 3$, we can determine the definition of the function by first calculating the slope using points from the graph, and then finding the y intercept using substitution.

Slope: Using the points $(4, 5)$ and $(5, 7)$ we can find the slope with

$$\frac{7 - 7}{5 - 4} = 2$$

y **Intercept:** We can solve for the y intercept using (4, 5):

$$\begin{aligned} 2(4) + b &= 5 \\ b &= 5 - 8 \\ &= -3 \end{aligned}$$

Now we can express the function definition for $x > 3$ as $2x - 3$. So the whole piecewise function is

$$f(x) = \begin{cases} -3 & \text{if } x \leq 3 \\ 2x - 3 & \text{if } x > 3 \end{cases}$$

26. Solution

There are two cases for this piecewise function: $(x < 3)$ and $(x \geq 3)$.

$x < 3$: We can see that the y intercept is 1 and we can calculate the slope using points $((0, 1)$ and $(3, 4))$ on the graph: $\dfrac{4-1}{3-0} = 1$. The first case of the function can then be expressed as

$$f(x) = x + 1$$

$x \geq 3$: Using substitution we can find the slope with the following points $(3, 2), (6, 1)$

$$\frac{1-2}{6-3} = -\frac{1}{3}$$

We can then determine the y intercept by substituting the point $(3, 2)$

$$\begin{aligned} f(3) = -\frac{1}{3}(3) + b &= 2 \\ b &= -1 - 2 \\ &= -3 \end{aligned}$$

The second case of the function can then be expressed as

$$f(x) = -\frac{1}{3} - 3$$

We can then express the entire piecewise function like this

$$f(x) = \begin{cases} x + 1 & \text{if } x < 3 \\ -\frac{1}{3} - 3 & \text{if } x \geq 3 \end{cases}$$

Section 1.2, Problem 33 - 34

Piecewise linear functions Graph the following functions.

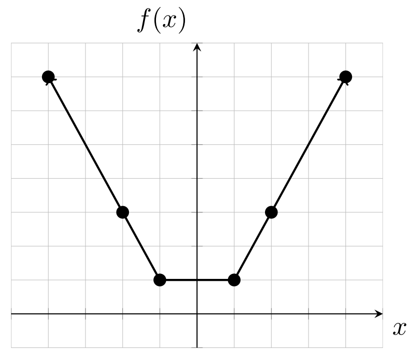
33.

$$f(x) = \begin{cases} -2x - 1 & \text{if } x < -1 \\ 1 & \text{if } -1 \leq x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$$

Solution

I plotted some sensible (x, y) coordinates using the functions provided.

$$
\begin{aligned}
&x = -4, \quad y = -2(-4) - 1 = 7 \\
&x = -2, \quad y = -2(-2) - 1 = 3 \\
&x = -1, \quad y = 1 \\
&x = 1, \quad y = 1 \\
&x = 2, \quad y = 4 - 1 = 3 \\
&x = 4, \quad y = 8 - 1 = 7
\end{aligned}
$$



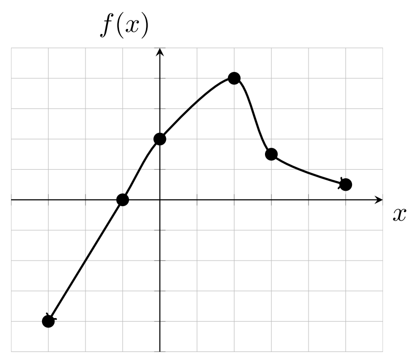
34.

$$
f(x) = \begin{cases} 2x + 2 & \text{if } x < 0 \\ x + 2 & \text{if } 0 \leq x \leq 2 \\ 3 - \frac{x}{2} & \text{if } x > 2 \end{cases}
$$

Solution

I plotted some sensible (x, y) coordinates using the functions provided.

$$
\begin{aligned}
&x = -3, \quad y = -2(-3) - 2 = -4 \\
&x = -1, \quad y = 0 \\
&x = 0, \quad y = 2 \\
&x = 2, \quad y = 4 \\
&x = 3, \quad y = \frac{6}{2} - \frac{3}{2} = 1\frac{1}{2} \\
&x = 5, \quad y = \frac{6}{2} - \frac{5}{2} = \frac{1}{2}
\end{aligned}
$$



Section 1.2, Problem 49 - 51

Area functions Let $A(x)$ be the area of the region bounded by the t-axis and the graph of $y = (t)$ from $t = 0$ to $t = x$. Consider the following functions and graphs.

a. Find $A(2)$ **b.** Find $A(6)$ **c.** Find a formula for $A(x)$

49. $f(t) = 6$

Solution

$A(x)$ is in the area bounded by the graph of f and the t-axis from $t = 0$ to $t = x$.

a. $A(2) = 2(6) = 12$

b. $A(6) = 6(6) = 36$

c. if $f(t) = 6$ then
$A(x) = f(t) * x$ and
$A(x) = 6$

50. $f(t) = \dfrac{t}{2}$

Solution

We can use the formula for the area of a triangle

a. $A(2) = \dfrac{1}{2}(2)\left(\dfrac{2}{2}\right) = 1$

b. $A(6) = \dfrac{1}{2}(6)\left(\dfrac{6}{2}\right) = 9$

c.

$$\begin{aligned} A(x) &= \frac{1}{2}x\left(\frac{x}{2}\right) \\ &= \frac{1}{2}\left(\frac{x^2}{2}\right) \\ &= \frac{x^2}{4} \end{aligned}$$

51. $f(t) = \begin{cases} 2t - 8 & \text{if } t \leq 3 \\ 2 & \text{if } t > 3 \end{cases}$

a.

b.

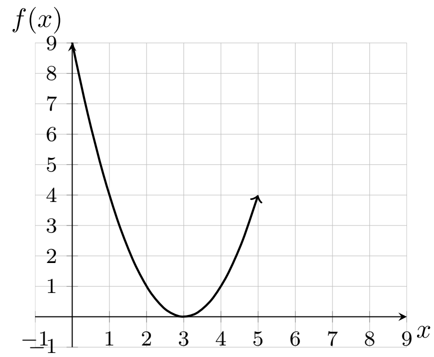
c.

Section 1.2, Problem 55

Transformations of $f(x) = x^2$ Use shifts and scalings to transform the graph of $(x) = x^2$ into the graph of g. Use a graphing utility to check your work.

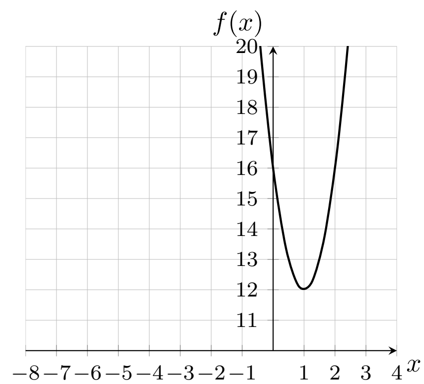
a.

$$\begin{aligned} g(x) &= f(x-3) \\ &= (x-3)(x-3) \\ &= x^2 - 6x + 9 \end{aligned}$$



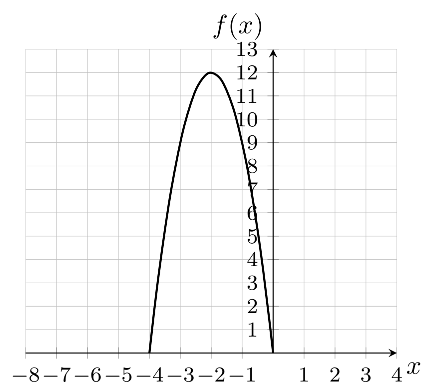
b.

$$\begin{aligned} g(x) &= f(2x-4) \\ &= (2x-4)(2x-4) \\ &= 4x^2 - 8x + 16 \end{aligned}$$



c.

$$\begin{aligned} g(x) &= -3f(x-2) + 4 \\ &= -3(x-2)(x-2) + 4 \\ &= -3(x^2 - 4x + 4) + 4 \\ &= -3x^2 + 12x - 4 + 4 \\ &= -3x^2 + 12x \end{aligned}$$



d.

$$
\begin{aligned}
g(x) &= 6f\left(\frac{x-2}{3}\right)+1 \\
&= 6\left(\frac{x-2}{3}\right)\left(\frac{x-2}{3}\right)+1 \\
&= 6\left(\frac{x^2-4x+4}{9}\right)+1 \\
&= \frac{6x^2-24x+24}{9}+1 \\
&= \frac{2x^2-8x+8}{3}+1
\end{aligned}
$$