Coriolis Force
Negligible on equator

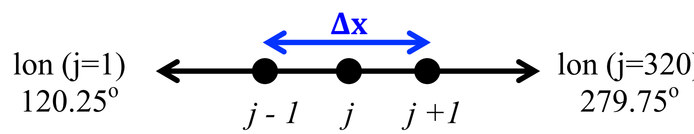
$$\frac{\partial u}{\partial t} + \boxed{u\frac{\partial u}{\partial x}} + \boxed{v\frac{\partial u}{\partial z}} + \boxed{w\frac{\partial u}{\partial z}} = \boxed{-\frac{1}{\rho}\frac{\partial p}{\partial x}} + \cancel{2\Omega 9 sin\varphi} + \boxed{F_x}$$

Pressure Gradient   Friction

x = longitude	(degrees)	u = Eastward sea water velocity	(meters/ second)
y = latitude	(degrees)	v = Northward sea water velocity	(meters/ second)
z = depth	(meters)	w = Upward sea water velocity	(meters/ second)

Calculating $u * \frac{du}{dx}$ term:

Longitude indexing variable = j



$$\Delta u_j = u_{j+1} - u_{j-1}$$

$\Delta x_j = lon_{j+1} - lon_{j-1}$ = 1 degree longitude
Δx is a constant regardless of where we are spatially in the gridded data so for calculations, using:
Δx = 111320 meters

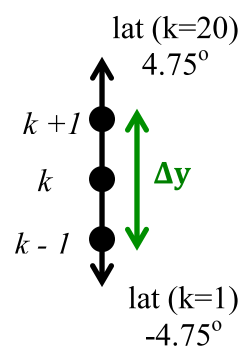
$$(u * \frac{du}{dx})_j = u_j * \Delta u_j \;/\; \Delta x$$

Units: $\frac{meters}{second} * \frac{meters}{second} * \frac{1}{meters} = \frac{meters}{second^2}$

Note: when using central differencing method, you need data flanking your point of interest. This is not possible at the edge of the domain. Therefore, we start the 'for loop' at j = 2 and end it at j = 319.

Calculating $v * \frac{du}{dy}$ term:

Latitude indexing variable = k



$$\Delta u_k = u_{k+1} - u_{k-1}$$

$\Delta y_k = lat_{k+1} - lat_{k-1}$ = 1 degree latitude
Δy is a **NOT** constant everywhere but treating it as such:
Δy = 110574 meters

$$(v * \frac{du}{dy})_k = v_k * \Delta u_k \;/\; \Delta y_k$$

Units: $\frac{meters}{second} * \frac{meters}{second} * \frac{1}{meters} = \frac{meters}{second^2}$

Note: (see Note above) we start the 'for loop' at k = 2 and end it at k = 19.

Calculating $w * \frac{du}{dz}$ term:

Depth indexing variable = d

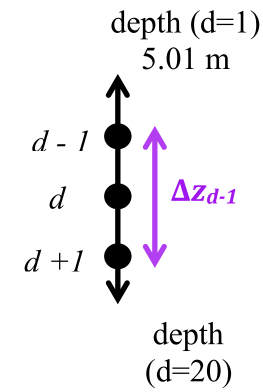
$$\Delta u_d = u_{d-1} - u_{d+1}$$

$$\Delta z_d = depth_{d-1} - depth_{d+1}$$

$$\left(w * \frac{du}{dz}\right)_d = w_d * \Delta u_d \;/\; \Delta z_{d-1}$$

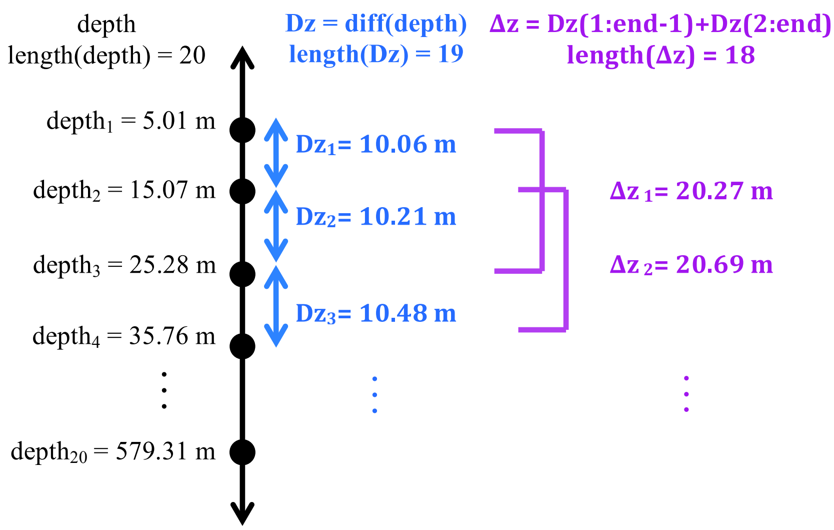
Δz is **NOT** constant everywhere
the calculation is below

Units: $\frac{meters}{second} * \frac{meters}{second} * \frac{1}{meters} = \frac{meters}{second^2}$



Note: (see Notes above) we start the ‘for loop’ at d = 2 and end it at d = 19.

Calculating Δz:



Note: when indexing Δz, use d-1. Example: When calculating the $w * \frac{du}{dz}$ term for depth(2) use $\Delta z(1)$. This is because there is no Dz or Δz for the surface and last depth (due to central differencing approach) and thus the number of elements is reduced going from ‘depth’ to Dz to Δz.

Calculating F_x term:

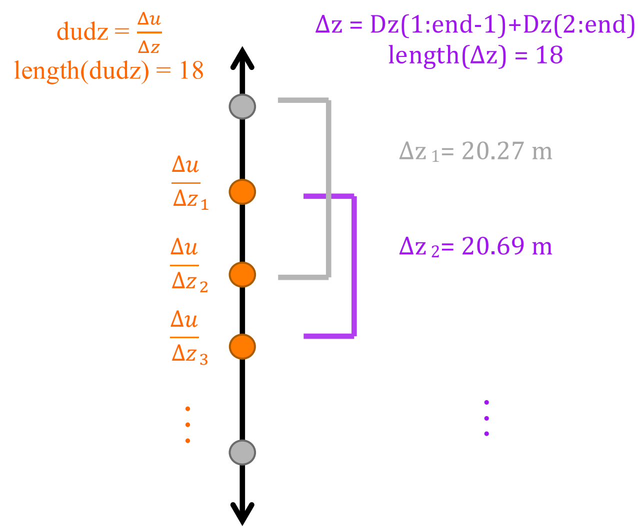
Second depth indexing variable = dd

$$F_x = \nu * \frac{d^2u}{dz^2} \qquad \nu = \text{kinematic viscosity}\ (\frac{\text{meters}^2}{\text{second}}) = \frac{\mu}{\rho} \qquad \mu = \text{dynamic viscosity} = 0.001\ \text{Pa}\cdot\text{s}$$

$$\rho = \text{density} = 1028\ \frac{\text{kilogram}}{\text{meters}^3}$$

$$\frac{d^2u}{dz^2}_{dd-1} = \frac{\Delta \frac{\Delta u}{\Delta z}_{dd-1}}{\Delta z_{dd}}$$

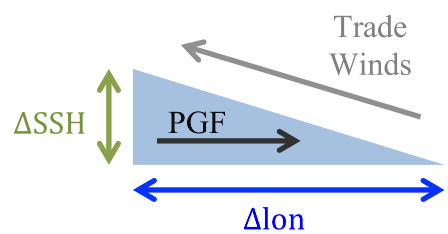
$$\Delta \frac{\Delta u}{\Delta z}_{dd-1} = \frac{\Delta u}{\Delta z}_{dd-1} - \frac{\Delta u}{\Delta z}_{dd+1}$$



Units: $\frac{\boldsymbol{meters^2}}{\boldsymbol{second}} * \frac{\mathbf{1}}{\boldsymbol{meters*seconds}} = \frac{\boldsymbol{meters}}{\boldsymbol{seconds^2}}$

Note: there are 18 elements in the dudz vector (2 fewer than the depth vector). See notes above regarding central differencing method yields 2-fewer points. Ergo the 'for loop' starts at dd = 2 and ends at dd = 17.

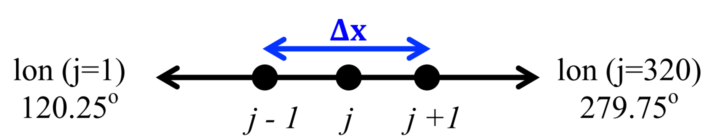
Calculating $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ term:



$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = -g * \frac{\Delta z}{\Delta x}$$

g = gravitational acceleration = 9.81 $\frac{meters}{seconds^2}$

$\Delta z = \text{SSH}_{j+1} - \text{SSH}_{j-1}$



Units: $\frac{\boldsymbol{meters}}{\boldsymbol{seconds^2}} * \frac{\boldsymbol{meters}}{\boldsymbol{meters}} = \frac{\boldsymbol{meters}}{\boldsymbol{seconds^2}}$

Note: Pressure gradient is constant through depth…

On the Equator (no Coriolis term):

$$\frac{\partial u}{\partial t} = F_x - (\frac{1}{\rho}\frac{\partial p}{\partial x} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial z} + w\frac{\partial u}{\partial z})$$

Unit conversion to $\frac{\text{meters}}{\text{second} \cdot \text{century}}$

$$\frac{\text{meters}}{\text{s}^2} \cdot \frac{60\text{ s}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{24\text{ hr}}{1\text{ day}} \cdot \frac{365\text{ days}}{1\text{ yr}} \cdot \frac{100\text{ years}}{1\text{ century}} = \frac{\text{meters}}{\text{second} \cdot \text{century}}$$

Unit Conversion for trend in velocity, which is in $\frac{\text{meters}}{\text{second} \cdot \text{month}}$

$$\frac{\text{meters}}{\text{month} \cdot \text{second}} \cdot \frac{12\text{ months}}{1\text{ yr}} \cdot \frac{100\text{ years}}{1\text{ century}} = \frac{\text{meters}}{\text{second} \cdot \text{century}}$$