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MM546/834 PDEs: theory, modeling and computing

Self–study and exercise problems, no 4

1. Let us consider the BVP
$$-u''(x) + u(x) = 0, \quad x \in (0,1) \ ,$$
with $u(0) = 1 = u(1)$. From the variational formulation determine the FE-solution
$$u_\Delta(x) = u_0\varphi_0(x) + u_1\varphi_1(x) + u_2\varphi_2(x)$$
on the 3-point mesh $x_0 = 0$, $x_1 = 1/2$ and $x_2 = 1$, with the piecewise linear hat-functions $\varphi_i(x_j) = \delta_{ij}$.

2. Consider the BVP
$$u''(x) = 6x \ , \quad x \in (0,1) \ ,$$
with $u(0) = 0$ and $u(1) = 1$. Determine the finite element solution
$$u_\Delta(x) = u_0\varphi_0(x) + u_1\varphi_1(x) + u_2\varphi_2(x)$$
on the 3-point mesh $x_0 = 0$, $x_1 = 1/2$ and $x_2 = 1$, with the piecewise linear hat-functions $\varphi_i(x_j) = \delta_{ij}$.

3. Consider the PDE
$$-\nabla \cdot [p(x)\nabla u(x)] + r(x)u(x) = f(x) \ , \quad x \in \Omega \subset \mathbb{R}^n$$
with positive coefficients $p \geq c_0 > 0$, $r \geq 0$ and boundary condition $u(x) = 0$ on $\partial\Omega$.
   (a) Derive Galerkin's weak formulation for this problem.
   (b) Show that the problem is H^1–elliptic.