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MM546/834 PDEs: theory, modeling and computing

Self–study and exercise problems, no 2

1. Recall and proof the Cauchy–Schwarz inequality in inner product spaces.
2. Formulate and proof the Poincare inequality for functions in $H_0^1(\mathbb{R})$.
3. Consider the second order linear PDE

$$\pm(Au_{xx} + 2Bu_{xy} + Cu_{yy}) = f \ .$$

   (a) Note that the type of the PDE does not depend on the sign of the PDE operator.

   (b) By partial integration (Greens first identity) identify the associated weak form

$$a(u,v) = \mp \int\int Au_x v_x + Bu_x v_y + Bu_y v_x + Cu_y v_y \, \mathrm{d}x\mathrm{d}y \ .$$

   (c) Note that

$$Av_x^2 + 2Bv_x v_y + Cv_y^2 = (v_x, v_y) \begin{pmatrix} A & B \\ B & C \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} \ .$$

   (d) Note that if $B^2 - AC < 0$, then the weak form $a(u,v)$ is H^1–ellictic.

4. Find any elliptic PDE where an analytic expression for its solution is known (f.ex. Sauer 8.3 Exercises). Compute approximations to the solution and estimate the order of convergence (MM533). Comment your observations.