IMADA Spring 2021
SDU Jamie Gabe

Advanced Linear Algebra (MM562/MM853)
Information sheet 1
Programme for weeks 5 and 6

Welcome to the the course Advanced Linear Algebra. You are currently reading the first weakly information sheet which contains information about course in general, and the lectures and exercise classes for the first two weeks.

Practical information

The book. We are using Henrik Schlichtkrulls lecture notes *Advanced Vector Spaces* which are available on ItsLearning under “Plans” and then “Resources and activities” for Week 5.

As you will notice, the notes do not contain exercises, so those I will either include directly on the weekly information sheets, or take from Halmos’ classical book *Finite Dimensional Vector Spaces* which can be downloaded via the SDU library.

The lectures and exercises. The course will stretch over the entire semester, with one lecture per week and exercises every second week. The lectures will be delivered by Jamie Gabe and the exercises by Max Holst Mikkelsen.

Due to the pandemic, the teaching is (for the time being) online on Zoom, live, at the specified time. The Zoom information for the luctures is:

- Meeting ID: 696 7403 9882
- Password: LinAlg
- Direct link,

Programme for week 5 and 6

Lectures.

- Week 5: Advanced Vector Spaces, Lecture 1.
- Week 6: Advanced Vector Spaces, Lecture 2 (we may end up postponing page 17-19 for next week; let’s see).

Exercises.

Exercises related to the lecture in week 5:

- Show that, in a vector space $(V, +, \cdot)$ over a field $\mathbb{F}$, for all $x \in V$ and $\alpha \in \mathbb{F}$ one has $\alpha \cdot x = 0$ iff $\alpha = 0_{\mathbb{F}}$ or $x = 0_V$. Moreover, show that $(-1_{\mathbb{F}}) \cdot x = -x$. Here $0_{\mathbb{F}}$, $1_{\mathbb{F}}$ denote the additive and multiplicative neutral elements in $\mathbb{F}$ and 0_V the zero vector in V.

- Let $p \in \mathbb{N}$ be a prime. Show that if V is a vector space over $\mathbb{Z}_p$, then $\underbrace{x + x + \cdots + x}_{p \text{ times}} = 0$ for every $x \in V$.

- Halmos: §1, exercise 5.

- Halmos: §4, exercise 4.

- Halmos: §7, exercise 1, 2, 5.

- (From the Exam 2020): Argue that the set $B' = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$ constitutes a basis for $\mathbb{R}^2$.

Exercises related to the lecture in week 6:

- Consider the vector space $\mathbb{C}^5$ and denote by $P\colon \mathbb{C}^5 \to \mathbb{C}^5$ the map $P(x_1, x_2, x_3, x_4, x_5) = (x_1, x_2, 0, 0, 0)$. Show that P is linear and determine its kernel, range, nullity and rank. Moreover, show that $\mathbb{C}^5 / \ker(P) \simeq \mathbb{C}^2$.

- Halmos: Section 22, exercise 1.

- Let U, V be vector spaces over a field $\mathbb{F}$ and let $A \in \mathrm{Hom}(U, V)$ be an isomorphism. Show that the set theoretical inverse map $A^{-1}\colon V \to U$ is linear and hence an isomorphism as well.

- Recall that $C(\mathbb{R}, \mathbb{R})$ denotes the vector space (over $\mathbb{R}$) of continuous functions from $\mathbb{R}$ to $\mathbb{R}$.
  (a) Show that $C^1(\mathbb{R}, \mathbb{R}) := \{f \in C(\mathbb{R}, \mathbb{R}) \mid f \text{ continuously differentiable}\}$ is a subspace of $C(\mathbb{R}, \mathbb{R})$.
  (b) Show that the map $f \mapsto f'$ defines a linear map $D\colon C^1(\mathbb{R}, \mathbb{R}) \to C(\mathbb{R}, \mathbb{R})$ and determine its kernel and range.
  (c) Contemplate why the result in (b) does not contradict Theorem 2.10.