week1 - mm852

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1. Ex
   1000 paths of 1-dimensional wiener process $W(t) \in [0,1]\,, \quad$ time-step $= \frac{1}{500}$.
   Eval the function $u\left(t, W(t)\right) = e^{t+\frac{1}{2}W(t)}$. Plot for 5 individual paths.
   we use the definition of a wiener process: By 2.1(1), we know the process starts at

   $W(t_0 = 0) = 0$

   $W(t_1) = W(t_1 = \frac{1}{500}) - W(0) \sim N\left(0, \frac{1}{500}\right)$

   $W(t_2) = W(t_2 = \frac{2}{500}) - W(t_1 = \frac{1}{500}) \sim N\left(0, \frac{1}{500}\right)$ we want to draw this from this norm

Ex2.

1. $X_{n+1} = X_n + f(t_n, X_n)\Delta_n + g(t_n, X_n)\Delta_n W$, n=0, ..., N-1,

d X $= \gamma X dt + \mu X dW, \quad X(0) = X_0,$
$\gamma = 1, \mu = 0.6 \text{and} X_0 = 0.5 \in [0,2]\,.$
Simulate 100k trajectories each with 512 equidistant steps. $N = 2^9$

$$dX = \gamma X dt + \mu X dW, \quad X(0) = X_0, \tag{1}$$
$$= X dt + 0.6 X dW \quad X(0) = 0.5 \qquad \text{and with EM we get} \tag{2}$$
$$X_{n+1} = X_n + \left(X_n + 0.6 X_n\right)\Delta_n W_n \tag{3}$$
$$\tag{4}$$