

MM546/834 PDEs: theory, modeling and computing

Self-study and exercise problems, no 1

1. Get started with FEniCS! Install FEniCS on your computer and run `poisson_simple.py`.
2. Recall linear maps on finite dimensional vector spaces and their matrix representation (MM505).
3. Recall the concept of Hilbert spaces and completeness (MM533).
4. Recall that $L^2([0, 1])$ and $H^1([0, 1])$ are Hilbert- and Banachspaces (MM548).
5. Show that $(C^1[0, 1], \|\cdot\|_{H^1})$ is not complete. For example, consider the sequence

$$u_k(x) = \begin{cases} 2x & x \in \left[0, \frac{1}{2} - \frac{1}{2k}\right] \\ 1 - \frac{1}{2k} - 2k\left(x - \frac{1}{2}\right)^2 & x \in \left(\frac{1}{2} - \frac{1}{2k}, \frac{1}{2} + \frac{1}{2k}\right) \\ 2(1-x) & x \in \left[\frac{1}{2} + \frac{1}{2k}, 1\right] \end{cases}$$

and its limit $u \in H^1(0, 1)$

$$u = \lim_{k \rightarrow \infty} u_k = \begin{cases} 2x & x \in \left[0, \frac{1}{2}\right] \\ 2(1-x) & x \in \left(\frac{1}{2}, 1\right] \end{cases}$$