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MM546/834 PDEs: theory, modeling and computing

Self–study and exercise problems, no 5

Let us consider the Sturm–Liouville eigenvalue problem

$$-y'' = \lambda y \ , \quad y(0) = y(1) = 0$$

and its discrete version $A^h u^h = \lambda^h u^h$, where

$$A^h = \frac{1}{h^2} \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{n-1 \times n-1} \ , \quad nh = 1 \ .$$

1. Verify that
$$\lambda_k^h = \frac{2}{h^2}(1 - \cos(k\pi h)) \ , \quad k = 1, 2, \ldots, n-1$$
are the eigenvalues with eigenvectors
$$u_k^h = (\sin(k\pi h), \sin(k\pi 2h), \ldots, \sin(k\pi(n-1)h)^T$$
to the discrete version and compare them to those of the Sturm–Liouville eigenvalue problem.
2. Comment on the stiffness of the semi–discrete diffusion equation $u_t = -A^h u$ by comparing the largest to the smallest eigenvalue.
3. Recall the stability of explicit and implicit Euler schemes for stiff initial value problems.