AMOGH GULATI
2017019

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IQB Assignment - 1

Ans-1 For equilibrium

- $N(t+1) = N(t)$
- let equilibrium be attained at N^*
- at equilibrium

$$N^* = aN^*[1 - N^*]$$

$$N^* = aN^* - aN^{*2}$$

$$aN^* - N^* - aN^{*2} = 0$$

$$N^* = 0 \quad \text{or} \quad N^* = \frac{a-1}{2}$$

consider equilibrium point $N^* = \frac{a-1}{a}$
let it be perturbed by n_t

$$n_{t+1} + \frac{a-1}{a} = a\left(n_t + \frac{a-1}{a}\right)\left(1 - n_t + \frac{1-a}{a}\right)$$

$$n_{t+1} + \frac{a-1}{a} = (1 - an_t)\left(n_t + \frac{a-1}{a}\right) = n_t + \frac{a-1}{a} + \cancel{(1-a)n_t} - (a-1)$$

$$n_{t+1} = (1 - a + 1)n_t$$

$$n_{t+1} = \underline{(2-a)}\, n_t$$

- $2-a$ is the stretching factor

For stable asymptotes :-

$0 < \lambda < 1$ $\qquad \lambda = 2 - a$

So $\underline{1 < a < 2}$

For Stable oscillations :-

$-1 < \lambda < 0$

So $2 < a < 3$

For unstable oscillations $\lambda < -1$

$\underline{a > 3}$

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consider equilibrium point $N^* = 0$
introduce small pertubation

$(n_{t+1} + u) = a\, n_t (1 - n_t)$
$n_{t+1} = a n_t - \cancel{a n_t^2}$
$n_{t+1} = \underline{a}\, n_t$
stretching factor

- Stable asymptotes
$0 < \lambda < 1$
$0 < a < 1$
- stable oscillations
$-1 < \lambda < 0$
$-1 < a < 0$
- unstable oscillations
$\lambda < -1$
$a < -1$

Ans-3) $N(t+1) = N(t) \cdot e^{\gamma\left(1 - \frac{N(t)}{k}\right)}$

For equilibrium
- $N(t+1) = N(t)$
- let equilibrium be attained at N^*
$N^* = N^* \cdot (e)^{\gamma\left(1 - \frac{N^*}{k}\right)}$

$\boxed{N^* = 0}$ and $1 = 1\, e^{\gamma\left(1 - \frac{N^*}{k}\right)} = 1\, e^0$
So $\gamma\left(1 - \frac{N^*}{k}\right) = 0$

$\boxed{N^* = k}$

Consider $F(N) = N \cdot e^{\gamma\left(1 - \frac{N}{k}\right)}$
we can calculate stretching factor by finding $F'(N)$ at equilibrium points.

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$$F'(N) = 1\cdot e^{\gamma[1-\frac{N}{k}]} + N\cdot e^{\gamma[1-\frac{N}{k}]}\left(-\frac{\gamma}{k}\right)$$

at $N^* = 0$

$$F'(0) = e^{+\gamma} = \lambda$$

this is the stretching factor.

For stable asymptotes.

$0 < \lambda < 1$

$0 < e^{\gamma} < 1$

$-\infty < \gamma < 0$

For ~~stable~~ oscillations

$\cancel{1 < \lambda < 2}$ $\quad$ $\lambda < 0$

$\cancel{1 < e^{\gamma} < 2}$ $\quad$ $e^{\gamma} < 0$

$\cancel{1 < \gamma < \log 2}$ $\quad$ this would never happen.

~~For unstable oscillations~~ $\quad$ lets check the other point

$\cancel{\gamma > \log 2}$

~~at $N^* = k$~~ $\quad$ $N^* = k$

$\cancel{F'(k) = 1-\gamma = \lambda}$ $\quad$ $F'(k) = 1-\gamma = \lambda$

For stable asymptotes.

$0 < \lambda < 1$

$1 < 1-\gamma <$

$0 < \gamma < 1$

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For ~~stable~~ oscillations

~~$1 < \lambda < 2$~~

~~$1 < 1 - \gamma < 2$~~

$\lambda < 0$

$1 - \gamma < 0$

$1 < \gamma$

~~For unstable oscillations~~

Ans-4) $N(t) = \dfrac{K}{1 + \underbrace{\left(\dfrac{K - N(0)}{N(0)}\right)}_{\text{This is a constant, for simplicity sake take it to be } C_0} e^{-\gamma t}}$

$$N(t) = \frac{K}{1 + C_0 e^{-\gamma t}} \Rightarrow 1 + C_0 e^{-\gamma t} = \frac{K}{N(t)}$$

$$\Rightarrow C_0 e^{-\gamma t} = \frac{K}{N(t)} - 1 = \frac{K - N(t)}{C_0\, N(t)}$$

$$N(t+1) = \frac{K}{1 + C_0 e^{-\gamma(t+1)}}$$

$$= \frac{K}{1 + C_0 e^{-\gamma t} e^{-\gamma}} = \frac{K}{1 + \underbrace{C_0 e^{-\gamma t}}_{\text{replace}} e^{-\gamma}} = \frac{K}{1 + C_0 \left(\frac{K - N(t)}{N(t)}\right) e^{-\gamma}}$$

C_0

$$= \frac{K}{1 + \cancel{C_0}\left(\frac{K - N(t)}{N(t)}\right) e^{-\gamma}}$$