Towards Disorder in Two Dimensional XY Models on a Square Lattice

Amogh Waghmare (EP19B018) IDDD in Quantum Science and Technology
Guide: Dr.Anil Prabhakar co-Guide: Dr.Rajesh Naraynanan
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Introduction

The 2D XY model acts as a toy model to study an important class of phase transitions called the Berzinski-Kosterlitz-Thouless(BKT) Transitions. These model many 2D systems like

- thin films of superfluids,
- arrays of Josephson junctions
- behavior of exciton-polaritons in semiconductor microcavities.

The increasing use novel effects in 2D systems for technological applications, like making efficient lasers out of exciton-polariton systems[1] or using Josephson junction arrays as quantum computers, makes the study of the BKT transition very important

The 2D XY model

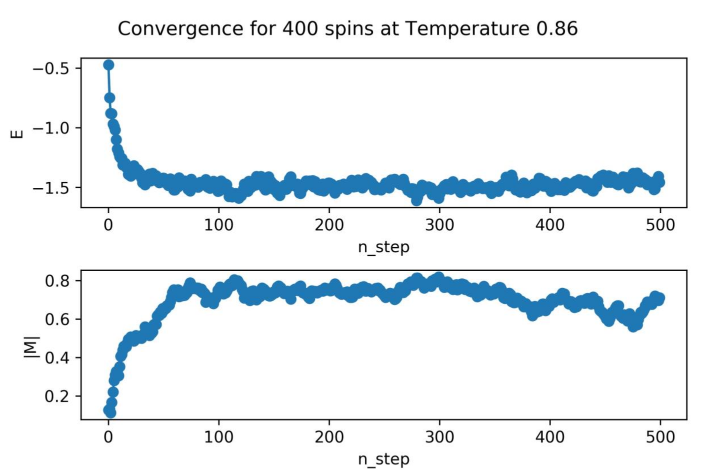
- the microscopic Hamiltonian for the model is given by

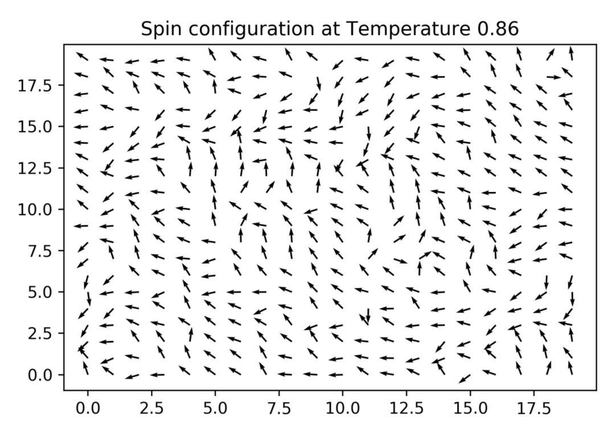
$$H = -\sum_{\langle ij \rangle} J_{ij} \vec{s}_i \cdot \vec{s}_j$$

- Where the sum is over the nearest neighbors and $\vec{s}$ are 2D unit vectors representing the direction of the spins, and J is a positive number representing the strength of coupling between the two spins
- The coupling strength J_{ij} is drawn from a probability distribution which models the disorder in the system

Methods

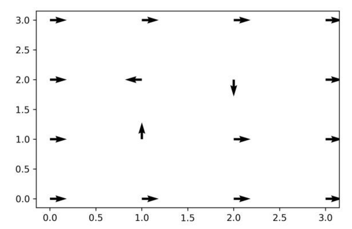
- The standard Metropolis algorithm is used to calculate various equilibrium properties and dynamics of the 2D XY model on a 20x20 square lattice with a constant J and periodic boundary conditions.
- A random change in spin direction is accepted with a probability of $\exp\left(\frac{-\Delta E}{k_b T}\right)$ if the new configuration is energetically unfavourable, and is always accepted if the change causes lowering of energy.
- In a single Monte-Carlo sweep there is an attempt to kick each spin in the lattice into a new direction once. The system is allowed to come to equilibrium by allowing about 2000 such sweeps.

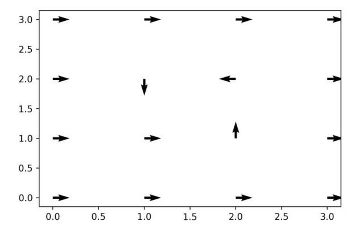




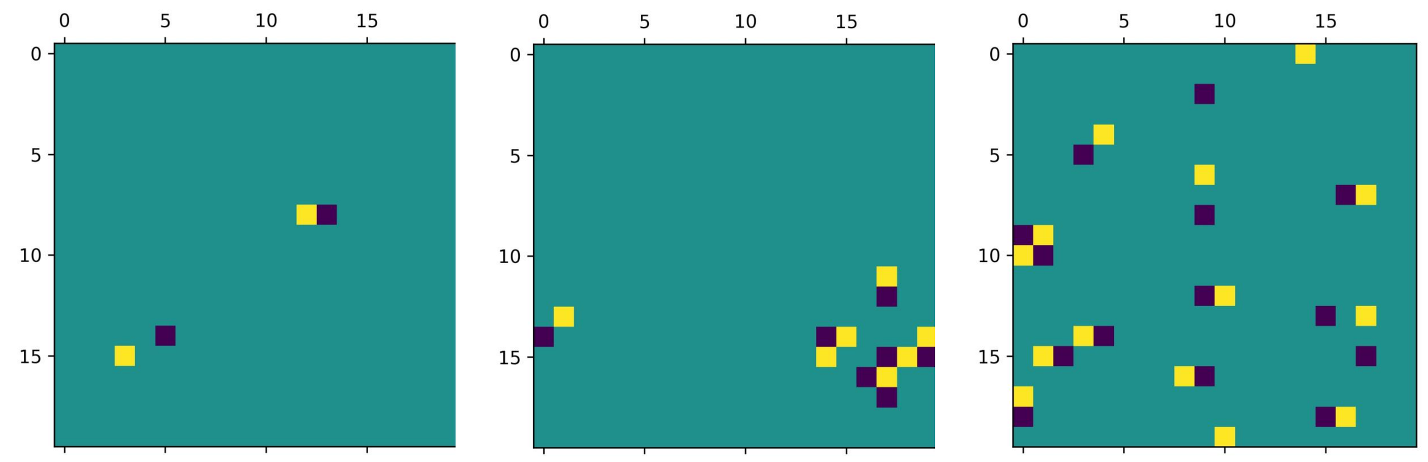
BKT Transition in XY model without disorder: Simulation

- Vortices are topological defects that involve spin configurations with a non zero curl, that is the line integral of the vector field representing spin direction is non zero along a closed loop. In the discreet case the sum of change in the angle of spins(constrained between $-\pi$ to π) around a loop becomes non-zero.
- These vortices behave like charged particles living on a 2D surface with charge being proportional to their vorticity.


An antivortex

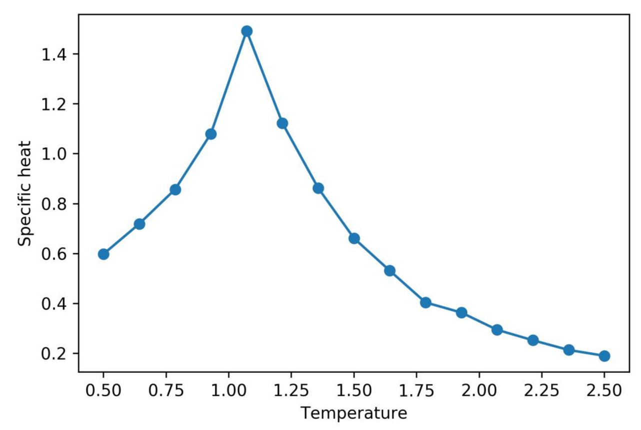

A vortex

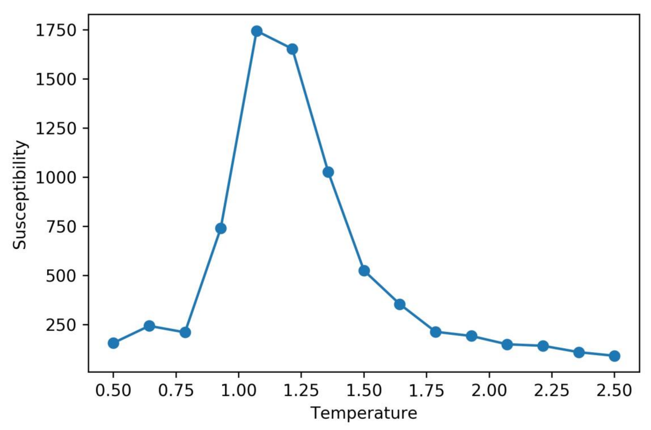
- The BKT transition describes the topological phase transition from a low temperature where the vortices are tightly bound to a high temperature phase where they are free


(a) (b) (c)

Position of vortices for T=0.89 (a), T=1.095 (b) and T=1.026 (c)

- At low temperatures, vortices (represented in yellow) and anti-vortices (represented in purple) are low in number and are tightly bound. As the temperature increases across the critical temperature, a large number of vortices(vorticity remains zero) are generated which interact with each other, making the notion of pairing weak.





- Plots of specific heat and the susceptibility of the system also shows peaks at the temperature around which vortex pair unbinding starts, showing the presence of a phase transition.

Effects of Disorder

- Many Systems follow a power law scaling near the critical point, that is the properties of the system like magnetization, specific heat etc. have a form:

$$\left(\frac{T - T_c}{T_c}\right)^{\gamma}$$

- The set of exponents γ are same for a wide variety of systems irrespective of their microscopic details. Such systems are said to be the part of the same universality class.
- Presence of disorder in a system can change its universality class, make the power law exponents γ disorder dependent or change the power law scaling to a logarithmic scaling
- For an one dimensional array of Josephson junctions, Altman and Refael [2] have shown that disorder can make the systems critical exponents disorder dependent.
- Since a classical 2D XY model with correlated disorder(coupling constants vary randomly in one direction but are perfectly correlated in the other) can be mapped to this system[3], we can use Monte-Carlo simulations to study the details of this crossover from normal exponents to disorder dependent exponents.

Conclusions and Future work

- We have been able to simulate the BKT phase transition for a 2D XY model on a square lattice using classical Monte-Carlo methods. Vortex unbinding has also been observed as temperature is raised.
- We plan to introduce correlated disorder in our Monte-Carlo simulations and try and reproduce the results of Fawaz and Voijta[3]
- We also want to extend these results by investigating the effects of disorder on the rate of vortex formation in a thermal quench in this 2D XY model and look for an analogous quench for the one dimensional Josephson junction model.

References

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