Disorder in Two Dimensional XY Models on a Square Lattice

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Introduction

The 2D XY model acts as a toy model to study an important class of phase transitions called the Berzinski-Kosterlitz-Thouless(BKT) Transitions. These model many 2D systems like thin films of superfluids, arrays of Josephson junctions, some high-temperature superconductors, and the behavior of exciton-polaritons in semiconductor microcavities.

The increasing need to try and use novel effects in 2D systems for technological applications, like making efficient lasers out of exciton-polariton systems[1] or using Josephson junction arrays as quantum computers, makes the study of the BKT transition very important

The 2D XY model

The 2D XY on a square lattice is governed by the microscopic Hamiltonian

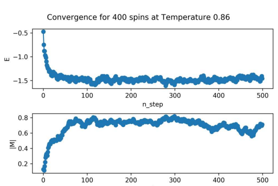
$$H = -\sum_{\langle ij \rangle} J_{ij}\vec{s}_i \cdot \vec{s}_j$$

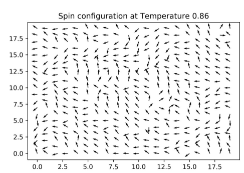
Where the sum is over the nearest neighbors and $\vec{s}$ 2D unit vectors representing the direction of the spins, and J is a positive number representing the strength of coupling between the two spins

The coupling strength J_{ij} is drawn from a probability distribution which models the disorder in the system

Methods

- The standard Metropolis algorithm is used to calculate various equilibrium properties and dynamics of the 2D XY model on a 20x20 square lattice with a constant coupling constant
- a spin is always flipped if the energy of the resulting configuration is lower
- A spin is flipped with a probability of $\exp\left(\frac{-\Delta E}{k_b T}\right)$ if the new configuration is energetically unfavourable
- This process is repeated till the thermodynamic variables like energy stop changing appreciably
- In a single Monte-Carlo sweep there is an attempt to kick each spin in the lattice into a new direction
- The system is allowed to equilibrate using 2000 Monte-Carlo sweeps at a given temperature

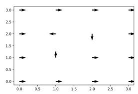


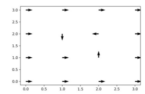


BKT Transition: Monte-Carlo Simulation

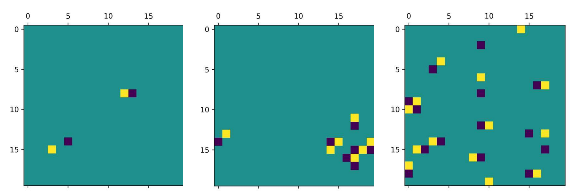
Vortices are topological defects that involve spin configurations with a non zero curl, that is the line integral of the vector field representing spin direction is non zero along a closed loop. This definition is appropriately extended to a discrete case and every possible 2 x 2 square (the smallest closed loop in a square lattice) is checked for the presence of a vortex.

These vortices behave like charged particles living on a 2D surface with charge being proportional to their vorticity.


An antivortex

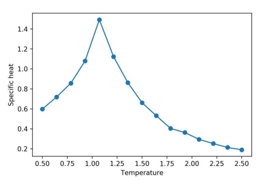

A vortex

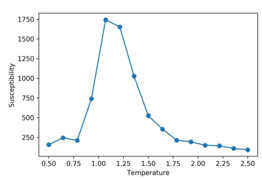
The BKT transition describes the topological phase transition from a low temperature where the vortices are tightly bound to a high temperature phase where they are free


(a) (b) (c)

Position of vortices for T=0.89 (a), T=1.095 (b) and T=1.026 (c)

At low temperatures, vortices (represented in yellow) and anti-vortices (represented in purple) are low in number and are tightly bound. As the temperature increases across the critical temperature, a large number of vortices(however, vorticity remains zero) are generated which interact with each other, making the notion of pairing weak.

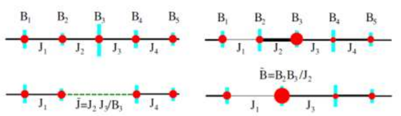




Plots of specific heat and the susceptibility of the system also shows peaks at the temperature around which vortex pair unbinding starts, confirming the presence of a phase transition.

Strong Disorder Renormalization

Phase transitions in disordered systems are studied using a renormalization technique called strong disorder renormalization. For lattice systems, this involves removing high-energy states that are irrelevant to phase transitions. An Equivalent lattice is now constructed to reflect the fact that these high-energy states have been ignored.



An example of SDRG on a transverse field Ising model [2]

When the coupling constant is strong, an equivalent lattice has the two spins merged into a super-spin with a different magnetic field perpendicular to the spins. Similarly, when a strong magnetic field perpendicular to the spins is present, an equivalent lattice changes the spin to a bond between the two adjacent sites.

The change in lattice also alters the probability distributions for coupling parameters. When this step is repeated so that the maximum energy scale becomes equal to the thermal energy, all the coupling of the lattice can be neglected, and standard methods of statistical physics can be applied to the resulting independent system

Conclusions and Future work

We have been able to simulate the BKT phase transition for a clean 2D XY model on a square lattice using classical Monte-Carlo methods. Vortex unbinding has also been observed as temperature is raised.

We plan to use the strong disorder renormalization technique to study the same transition in a model with line impurities, which will provide accurate information about real world systems.

We also plan to extend the study to vortex dynamics and rate of vortex production when the system is quenched from high temperature to low temperature

References

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