Module-1
Introduction to Linear Programming

Origin of Operation Research:

Operation Research is a scientific way to decision making which seek to determine how best to design and operate a system under scared resource. This subject came into existence into second world war. OR is defined as an experimental science which is devoted to observing understanding and predicting the behavior of purposeful man-machine systems.

Nature and Impact of OR:

OR involves ‘research on operations’. Thus operation research is applied to problems that concern how to conduct and co-ordinate the operations within an organizations. The nature of organization is immaterial and in fact OR has been applied extensively in such diverse areas as manufacturing, transportation, construction, tele communication, financial planning and health care. Therefore the breadth of application is usually wide. OR resembles the way research is conducted in established scientific field. It frequently attempts to find the best possible solution to the problem.

Operation Research has had an impressive impact on improving the efficiency of numerous organizations around the world. In the process, OR has made a significant contribution to increasing the productivity of the economics of various countries.

Main Phases of OR:

Phase 1: Formulation

This phase requires the problem to be formulated in the form of an appropriate model. This includes finding objective functions, constraints or restrictions, inter-relationships, possible alternate course of action, time limits for making decisions, ranges of controllable and uncontrollable variables which might affect the possible solutions. Hence one must be very careful while executing this phase.

Phase 2: Construction of a mathematical model

This phase is concerned with reformation of problem in an appropriate form which is useful in analysis. The most suitable model is a mathematical model representing the problem under study. A mathematical model should include decision variables, objective functions and constraints. The advantage of a mathematical model is that it describes the problem more concisely which makes the overall structure of the problem more comprehensible and it also helps to reveal important cause and effect relation.

Phase 3: Derivation of solutions from mathematical model

This phase is devoted to computation of those values of decision variables which maximize or minimize the objective function. It is always important to arrive at the optimal solution of the problem.

Phase 4: Testing the mathematical model and its solution

The completed model is tested for errors if any. The principle of judging the validity of the model is whether or not it predicts the relative effects of the alternative courses of action with sufficient accuracy to permit a sound decision. A good model should be applicable for a longer time and thus updates the model time to time taking into account the past, present and future specifications of the problem.

Phase 5: Establishing control over the solution

After the testing phase the next step is to install a well documented system for applying the model. It includes the solution procedure and operating procedure for implementation. This phase establishes a control over the solution with some degree of satisfaction. This phase also establishes a systematic procedure for detecting changes and controlling the situation.

Phase 6: Implementation

The implementation of controlled solution involves, the translation of models which results into operating instructions. It is important in OR to ensure that the solution is accurately translated into an operating procedure to rectify faults in the solution.

Advantages of OR:

Better Systems: Often, an O.R. approach is initiated to analyze a particular problem of decision making such as best location for factories, whether to open a new warehouse, etc. It also helps in selecting economical means of transportation, jobs sequencing, production scheduling, replacement of old machinery, etc.

Better Control: The management of large organizations recognize that it is a difficult and costly affair to provide continuous executive supervision to every routine work. An O.R. approach may provide the executive with an analytical and quantitative basis to identify the problem area. The most frequently adopted applications in this category deal with production scheduling and inventory replenishment.

Better Decisions: O.R. models help in improved decision making and reduce the risk of making erroneous decisions. O.R. approach gives the executive an improved insight into how he makes his decisions.

Better Co-ordination: An operations-research-oriented planning model helps in co-ordinating different divisions of a company.

Disadvantages of OR:

Dependence on an Electronic Computer: O.R. techniques try to find out an optimal solution taking into account all the factors. In the modern society, these factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations that can only be handled by computers.

Non-Quantifiable Factors: OR techniques provide a solution only when all the elements related to a problem can be quantified. All relevant variables do not lend themselves to quantification. Factors that cannot be quantified find no place in O.R. models.

Distance between Manager and Operations Researcher: O.R. being specialist's job requires a mathematician or a statistician, who might not be aware of the business problems. Similarly, a manager fails to understand the complex working of O.R. Thus, there is a gap between the two.

Money and Time Costs: When the basic data are subjected to frequent changes, incorporating them into the O.R. models is a costly affair. Moreover, a fairly good solution at present may be more desirable than a perfect O.R. solution available after sometime.

Implementation: Implementation of decisions is a delicate task. It must take into account the complexities of human relations and behavior.

Linear Programming:

It is a decision making technique under a given constraint that the relationship among the variable involved is linear.

Mathematical formulation of a linear programming:

A mathematical problem is an optimization problem in which the objective and constraints are given as mathematical functions and functional relationships. The procedure for mathematical formulation of a LPP consists of the following steps

Step1: write down the decision variables (Products) of the problem

Step2: formulate the objective function to be optimized (maximized or minimized) as linear function of the decision variables

Step3: formulate the other conditions of the problem such as resource limitation, market, constraints, and interrelations between variables etc., linear in equations or equations in terms of the decision variables.

Step4: add non-negativity constraints

The objective function set of constraint and the non-negative constraint together form a Linear Programming Problem.

Problems:

1. Consider a small manufacturer making two products A & B, two resources R1 and R2 are required to make these products. Each unit of product A requires 1 unit of R1 and 3 units of R2. Each units of B requires 1 unit of R1 and 2 units of R2. The manufacturer has 5 units of R1 and 12 units of R2 available. The manufacturer also makes a profit of Rs 6 per unit of product A sold and Rs 5 per unit of product B sold. Formulate the problem.

Solution:

Step1: Let the total number of units of A produced be 'x'.

Let the total number of units of B produced be 'y'.

Given: profit/one unit of A is Rs.6

Profit/x unit of A is Rs.6x

Profit/one unit of B is Rs.5

Profit/x unit of B is Rs.5x

Step2:Total profit z=6x+5y

Objective function is max z=6x+5y

Step3: Given that the products A and B requires 1 and 1 unit of R1 respectively with total availability of 5 units

i.e x+y<=5

Given that the products A and B requires 3 and 2 units of R2 respectively with total availability of 12 units

i.e 3x+2y<=12

Step4: The non negative conditions are:

x, y>=0

LP model:

Max z=6x+5y

STC x+y<=5

3x+2y<=12

x, y>=0

2. A Manufacture produces two types of models M1 and M2 each model of the type M1 requires 4 hrs of grinding and 2 hours of polishing, where as each model of the type M2 requires 2 hours of grinding and 5 hours of polishing. The manufactures have 2 grinders and 3 polishers. Each grinder works 40 hours a week and each polishers works for 60 hours a week. Profit on M1 model is Rs. 3.00 and on Model M2 is Rs 4.00.Whatever produced in week is sold in the market. How should the manufacturer allocate is production capacity to the two types models, so that he may make max in profit in week?

Solutions:

Step1: Let x1 be the number of units of model M1.

Let x2 be the number of units of model M2.

Step2: Objective function: Since, the profit on M1 and M2 is Rs.3.0 and Rs.4.0 Max Z =3x1 + 4x2

Step3: Constraint: there are two constraints one for grinding and other is polishing. No of grinders are 2 and the hours available in grinding machine is 40 hrs per week, therefore, total no of hours available of grinders is 2 X 40 = 80 hours No of polishers are 3 and the hours available in polishing machine is 60 hrs per week, therefore, total no of hours available of polishers is 3 X 60 = 180 hours

The grinding constraint is given by: $4x1+2x2 \le 80$ The Polishing Constraint is given by: $2x1 + 5x2 \le 180$

Non negativity restrictions are $x1, x2 \ge 0$ if the company is not manufacturing any products

The LPP of the given problem is

Max $Z = 3x1+4x2$

STC $4x1+2x2 \le 80$

$2x1+5x2 \le 180$

$x1, x2 \ge 0$

3. A farmer has 100 acre. He can sell all tomatoes. Lettuce or radishes he raise the price. The price he can obtain is Re 1 per kg of tomatoes, Rs 0.75 a head for lettuce and Rs 2 per kg of radishes. The average yield per acre is 2000kg tomatoes, 3000 heads of lettuce and 1000kgs of radishes. Fertilizer is available at Rs 0.5 per kg and the amount required per acre 100kgs each for tomatoes and lettuce, and 50kgs for radishes. Labor required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes, 6 man-days for lettuce. A total of 400 man days of labor available at Rs 20 per man day formulate the problem as linear programming problem model to maximize the farmer's total profit.

Solution:

Farmer's problem is to decide how much area should be allotted to each type of crop. He wants to grow to maximize his total profit. Let the farmer decide to allot X1, X2 and X3 acre of his land to grow tomatoes, lettuce and radishes respectively. So the farmer will produce 2000 X1kgs of tomatoes, 3000 X2head of lettuce and 1000 X3kgs of radishes. Profit=sales–cost=sales–(Labor cost +fertilizer cost) Sales = 1 x 2000 X1 + 0.75 x 3000 X2 + 2 x 1000 X3

Labor cost = 5x 20 X1 + 6 x 20 X2 + 5 x 20 X3

Fertilizer cost = 100x0.5 X1 + 0.5x 100 X2 + 0.5x50 X3

The LPP model is:

Max Z= 1850 X1 + 2080 X2 + 1875 X3 STC X1 + X2 + X3 $\le$ 100 5X1 + 6X2 + 5X3 $\le$ 400 X1 , X2 , X3 $\ge$ 0

4. A TV company has to decide on the minimum of 27 inches and 20 inches TV sets to be produced at one of its factories. The market research indicates that atmost 40, 27 inch TV sets and atmost 10, 20inch TV set can be sold per month. The maximum number of work hours available is 500hrs per month. A 27inch TV requires 20 work hours and a 20inch TV requires 10 work hours. Each 27inch TV is sold at a profit of Rs.120 and 20inch TV sold at a profit of Rs. 80, a wholesaler agreed to purchase all the TV sets produced, if the number do not exceed the max indicated by market research. Formulate the problem as an LP model.

Solution:

Let the total number of 27inches TV be 'x'
Let the total number of 20inches TV be 'y'
Given 1unit of 27inch TV produces a profit of Rs.120
'x' unit of 27inch TV produces a profit of Rs.120x
Given 1 unit of 20inch TV produces a profit of Rs.80
'y' unit of 20inch TV produces a profit of Rs.80y
Total profit= 120x+80y
Objective function z=120x+80y
Given that max sales of 27inch TV is 40 i.e x<=40
Given that max sales of 20inch TV is 10 i.e y<=10
One 27inch TV requires 20 work hours
x 27inch TV requires 20x work hours
One 20inch TV requires 10 work hours
y 20inch TV requires 10y work hours
Total work hour available is 500
i.e 20x+10y<=500
max sales/month 40+10=50
Total number of TV sets=x+y
Given wholesaler will purchase all the TV sets if the total does not exceed the maximum
i.e x+y<=50
LP model

Max z=120x+80y
STC x<=40
y<=10
120x+10y<=500
x+y<=50
where x>=0 y>=0

5. Egg contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and cost 12 paise per gram. Milk contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and costs 20 paise per gram. The daily requirements of vitamin A and vitamin B are 100 units and 120 units respectively. Find the optimal product mix.

	Egg	Milk	Min Requirements
Vitamin A	6	8	100
Vitamin B	7	12	120
Cost	12	20	

Solution:

Let x1 and x2 be the total cost of milk and egg produced respectively
The Objective function z=12x1+20x2
Vitamin A contents in egg and milk is 6 and 8 units respectively and minimum requirements is 100
i.e 6x1+8x2 >=100

Vitamin B contents in egg and milk is 7 and 12 units respectively and minimum requirements is 120

i.e $7x1+12x2 >=120$

The non negative constraints are: $x1,x2>=0$

The LP model is:

$$
\begin{aligned}
&\text{Max}=z=12x1+20x2\\
&\text{STC } 6x1+8x2>=100\\
&\qquad 7x1+12x2>=120\\
&\qquad x1,\ x2>=0
\end{aligned}
$$

Graphical Method:

The graphical procedure includes two steps

1. Determination of the solution space that defines all feasible solutions of the model.
2. Determination of the optimum solution from among all the feasible points in the solution space.

There are two methods in the solutions for graphical method

1. Extreme point method
2. Objective function line method

Steps involved in graphical method are as follows:

1. Consider each inequality constraint as equation.

2. Plot each equation on the graph as each will geometrically represent a straight line.

3. Mark the region. If the constraint is <= type then region below line lying in the first quadrant (due to non negativity variables) is shaded. If the constraint is >= type then region above line lying in the first quadrant is shaded.

4. Assign an arbitrary value say zero for the objective function.

5 Draw the straight line to represent the objective function with the arbitrary value.

6. Stretch the objective function line till the extreme points of the feasible region. In the maximization case this line will stop farthest from the origin and passing through at least one corner of the feasible region.

7. In the minimization case, this line will stop nearest to the origin and passing through at least one corner of the feasible region.

8. Find the co-ordination of the extreme points selected in step 6 and find the maximum or minimum value of Z.

Problems:

1. Solve the following LP problem using graphical method

Max: $z=6x+8y$

$5x+10y<=60$

$4x+4y<=40$

$x, y>=0$

Solution:

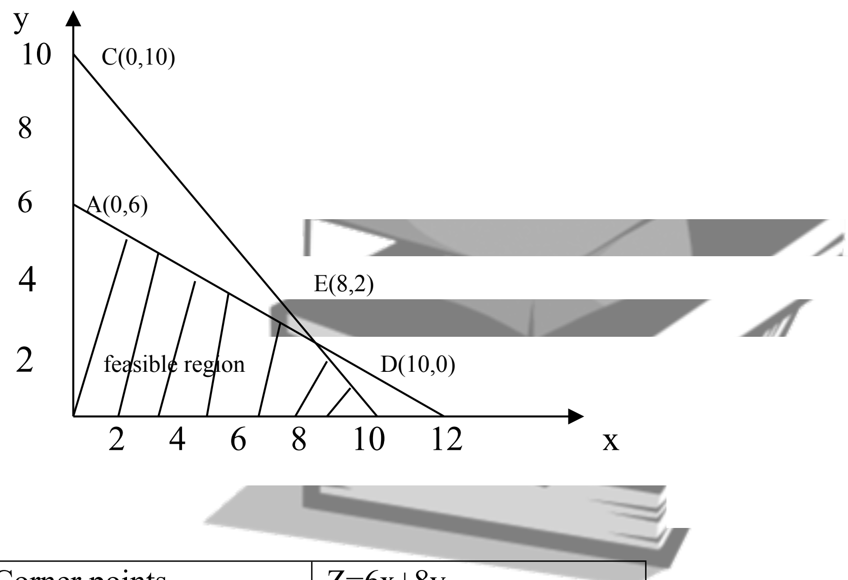
Replace all inequality by equality

$5x+10y=60$ =>when $x=0$ $y=6$

when $y=0$ $x=12$ The points are: A(0,6) and B(12,0)

$4x+4y=40$ => when $x=0$ $y=10$

when $y=0$ $x=10$ The points are C(0,10) and D(10,0)



Corner points	Z=6x+8y
A(0,6)	48
D(10,0)	60
E(8,2)	64

Here the maximum value of z is attained at the corner point E(8,2), which is the point of intersection of lines $5x+10y=60$ and $4x+4y<=40$. Hence the required solution is $x=8,y=2$ and the max value $z=64$

2. Solve the following LPP by graphical method:

Minimize $z=20x+10y$

$x+2y <= 40$

$3x+y >= 30$

4x+3y >=60

Solution:

Replace all inequalities by equality

x+2y = 40 when x=0, y=20

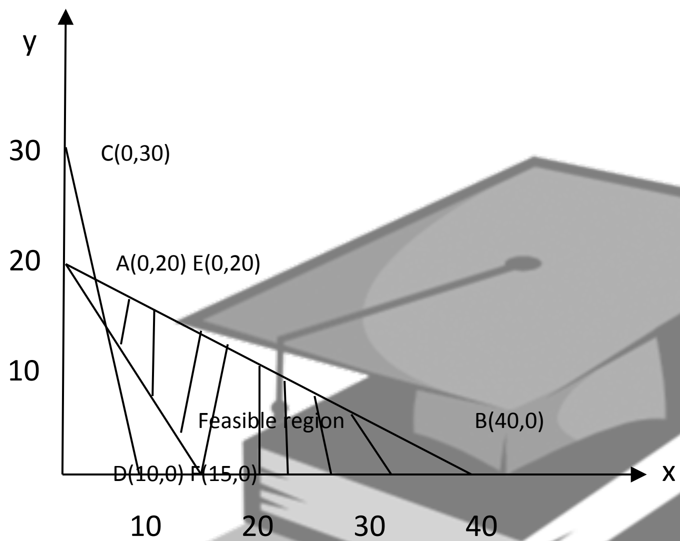
when y=0, x=40 The points are A(0,20) and B(40,0)

3x+y =30 when x=0, y=30

when y=0, x=10 The points are C(0,30) and D(10,0)

4x+3y =60 when x=0, y=20

when y=0, x=15 The points are E(0,20) and F(15,0)



Corner points	Z=20x+10y
G(4,18)	260
F(15,0)	300
B(40,0)	800
H(6,12)	240

Here the minimum value of z is attained at the corner point H(6,12), which is the point of intersection of lines 3x+y=30 and 4x+3y=60. Hence the required solution is x=6, y=12 and the min value z=240

3. Solve the following LPP

Maximize z=3x+2y

x-y >= 1

$x-y >= 3$

$x,y =0$

⇨ The solution space is unbounded. In fact the maximum value of Z occurs at infinity. Hence the problem does not have a feasible solution.