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Fuzzy Clustering Using A Compensated Fuzzy Hopfield Network

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Abstract. Hopfield neural networks are well known for cluster analysis with an unsupervised learning scheme. This class of networks is a set of heuristic procedures that suffers from several problems such as not guaranteed convergence and output depending on the sequence of input data. In this paper, a Compensated Fuzzy Hopfield Neural Network (CFHNN) is proposed which integrates a Compensated Fuzzy C-Means (CFCM) model into the learning scheme and updating strategies of the Hopfield neural network. The CFCM, modified from Penalized Fuzzy C-Means algorithm (PFCM), is embedded into a Hopfield net to avoid the NP-hard problem and to speed up the convergence rate for the clustering procedure. The proposed network also avoids determining values for the weighting factors in the energy function. In addition, its training scheme enables the network to learn more rapidly and more effectively than FCM and PFCM. In experimental results, the CFHNN method shows promising results in comparison with FCM and PFCM methods.

Key words: clustering analysis, FCM, PFCM, Hopfield Neural Network, fuzzy sets

1. Introduction

In many fields, such as segmentation, pattern recognition, and vector quantization, the clustering process is an indispensable step in these problems. Clustering algorithms attempt to organize the training patterns into clusters so that patterns within a cluster are more similar to each other than those belonging to different clusters. Since the range of clustering application is so large, there is no fundamental clustering problem formulation due to variant relationship between the input objects. There are several algorithms based upon the least mean squares criterion for clustering problem such as *K*-means (*C*-means) [1], ISODATA [2], FCM [3–6], and PFCM [7–8]. Generally speaking, *K*-means and ISODATA, in which a sample belongs to only one cluster, are called hard clustering methods, while FCM and PFCM are called fuzzy clustering methods in which every sample belonging all clusters with different degrees of membership.

The Hopfield Neural Network [9]–[10] is a well-known technique used for solving optimization problems based on the Lyapunov energy function. In this paper, a new Hopfield-model net called CFHNN is proposed for the clustering problem. In CFHNN, the CFCM modified from PFCM is embedded into a Hopfield

Neural Network so that the parallel implementation for the clustering problem is feasible. The cluster problem can be cast as an optimal problem that may also be regarded as a minimization of a criterion defined as a function of the least square Euclidean distance between the training pattern and the cluster center. The CFHNN is trained to classify the input patterns into feasible cluster when the defined energy function converges to near-global minimum. The training patterns are mapped to a two-dimensional Hopfield Neural Network. Also, the CFCM technique is used to update the clustering performance and to eliminate searching for the weighting factors. In CFHNN, the columns represent the number of clusters and the rows represent the training patterns. In the context of the clustering problem, each training pattern is represented by a neuron that is fully connected by the other neurons. After a number of iterations, neuron states are refined to reach near-optimal result when the defined energy function is converged. However, a training pattern does not necessarily belong to only one cluster. Instead, a certain membership grade belonging to a proper class is associated with every pattern. In addition to the fuzzy reasoning strategy, a compensated term is added as input bias to update the performance of training results. Consequently, the energy function can be quickly converged into a near-global minimum in order to produce satisfactory clusters. Compared with conventional techniques, the major strength of the presented CFHNN is that it is computationally more efficient due to the inherent parallel structures. In a simulated study, the CFHNN has shown the capability for clustering problems and gave promising results.

The remainder of this paper is organized as follows. Section 2 describes the FCM, PFCM, and CFCM algorithms. In Section 3, the clustering problem using a CFHNN is demonstrated. Section 4 discusses the convergence of the CFHNN on the mathematical derivations. Some experimental results and the performance comparison of FCM, PFCM, and the CFHNN methods are given in Section 5. Finally some conclusions are presented in Section 6.

2. Fuzzy Clustering Techniques

Fuzzy clustering strategies are mathematical tools for detecting similarities between members of a collection of samples. Since the introduction of the fuzzy set theory in 1965 by Zadeh [11], it has been applied in a variety of fields. The theory of fuzzy logic provides a mathematical framework to capture the uncertainties associated with human cognition processes.

The FCM clustering algorithm was first introduced by Dunn [5], and the related formulation and algorithm was extended by Bezdek [6]. The purpose of the FCM approach, like the conventional clustering techniques, is to minimize the criteria in the least squared error sense. For $c \geqslant 2$ and m any real number greater than

1, the algorithm choses $\mu_x : Z \rightarrow [0, 1]$ so that $\sum_x \mu_x = 1$ and $\varpi_i \in R^d$ for $i = 1, 2, \ldots, c$ to minimize the objective function

$$J_{\mathrm{FCM}} = \frac{1}{2} \sum_{x=1}^{n} \sum_{i=1}^{c} (\mu_{x,i})^m \parallel z_x - \varpi_i \parallel^2, \tag{1}$$

where $\mu_{x,i}$ is the value of the ith membership grade on the xth sample z_x. The cluster centers $\varpi_1, \ldots, \varpi_j, \ldots, \varpi_c$ can be regarded as prototypes for the clusters represented by the membership grades. For the purpose of minimizing the objective function, the cluster centers and membership grades are chosen so that a high degree of membership occurs for samples close to the corresponding cluster centers. The FCM algorithm, a well-known and powerful method in clustering analysis, is reviewed as follows.

2.1. FCM ALGORITHM

Step 1. Initialize the cluster centers $\varpi_i (2 \leqslant i \leqslant c)$, fuzzification parameter $m (1 \leqslant m < \infty)$, and the value $\varepsilon > 0$. Give a fuzzy c-partition $U^{(0)}$.

Step 2. Calculate the membership matrix $U = [\mu_{x,i}]$ using Equation (2) as below.

$$\mu_{x,i} = \frac{\left(\frac{1}{(d_{x,i})^2}\right)^{1/(m-1)}}{\sum_{i=1}^{c} \left(\frac{1}{(d_{x,i})^2}\right)^{1/(m-1)}}, \tag{2}$$

where $d_{x,i}$ is the Euclidean distance between the training pattern z_x and the class center ϖ_i.

Step 3. Update the class centers

$$\varpi_i = \frac{1}{\sum_{i=1}^{n} (\mu_{x,i})^m} \sum_{x=1}^{n} (\mu_{x,i})^m z_x. \tag{3}$$

Step 4. Compute $\Delta = \max(| U^{(t+1} - U^{(t)} |)$. If $\Delta > \varepsilon$, then go to Step 2; otherwise go to Step 5.

Step 5. Find the results for the final cluster centers.

The value m, prechosen as any value from 1 to ∞, is called the fuzzification parameter (or exponential weight), and it reduces the noise sensitivity in the computation of the class centers. In addition, the effect for $\mu_{x,i}$ is dependent on the value m. The larger the value $m (m > 1)$, the higher the fuzziness will be.

Another strategy of the fuzzy clustering method with the addition of a penalty term, called PFCM algorithm, was demonstrated by Yang [7–8]. It is an FCM of generalized type depending upon the penalized term in accordance with the value of ϖ. It was shown by Yang that the PFCM algorithm is more meaningful and effective than the FCM method. The PFCM objective function is reviewed as follows:

$$\begin{aligned} J_{\mathrm{PFCM}} &= \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\parallel z_x-\varpi_i\parallel^2-\frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_i \\ &= J_{\mathrm{FCM}}-\frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_i, \end{aligned} \tag{4}$$

where α_i is a proportional constant of class 1 and $v(\geqslant 0)$ is also a constant. When $v=0$, J_{PFCM} equals J_{FCM}. The penalty term, $-\frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_i$, is added to the objective function, and α_i, ϖ_i, and $\mu_{x,i}$ are defined as

$$\alpha_i=\frac{\sum_{x=1}^{n}\mu_{x,i}^{m}}{\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}};\ i=1,\ 2,\ldots,\ c, \tag{5}$$

$$\varpi_i=\frac{\sum_{x=1}^{n}\mu_{x,i}^{m}z_{x,i}}{\sum_{x=1}^{n}\mu_{x,i}^{m}}, \tag{6}$$

which is same as Equation (3), and

$$\begin{aligned} &\mu_{x,i}=\left(\sum_{\ell=1}^{c}\frac{(\parallel z_x-\varpi_i\parallel^2-v\ln\alpha_i)^{1/(m-1)}}{(\parallel z_x-\varpi_\ell\parallel^2-v\ln\alpha_\ell)^{1/(m-1)}}\right)^{-1}; \\ &x=1,\ 2,\ldots,\ n;\ i=1,\ 2,\ldots,\ c. \end{aligned} \tag{7}$$

Then the PFCM algorithm is presented as follows.

2.2. PFCM ALGORITHM

Step 1. Randomly set cluster center $\varpi_i(2\leqslant i\leqslant c)$, fuzzification parameter $m(1\leqslant m<\infty)$, and the value $\varepsilon>0$. Give a fuzzy c-partition $U^{(0)}$.

Step 2. Compute $\alpha_i^{(t)}$, $\varpi_i^{(t)}$ with $U^{(t-1)}$ using Equations (5) and (6). Calculate the membership matrix $U=[\mu_{x,i}]$ with $\alpha_i^{(t)}$, $\varpi_i^{(t)}$ using Equation (7).

Step 3. Compute $\Delta = \max(| U^{(t+1)} - U^{(t)} |)$. If $\Delta > \varepsilon$, then go to Step 2; otherwise go to Step 4.

Step 4. Find the results for the final cluster centers.

Yang has proved the convergence of J_{PFCM} in reference [7]. But the penalty degree is too heavy to converge rapidly. The penalty term $-\frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\ln\alpha_1$ in PFCM is replaced by a compensated term $+\frac{1}{2}v\sum_{x=1}^{n}\sum_{i-1}^{c}\mu_{x,i}^{m}\tanh(\alpha_i)$ and then the energy function and membership function in a so-called Compensated Fuzzy C-Means (CFCM) algorithm are defined as

$$\begin{aligned} J_{\text{CFCM}} &= \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m} \parallel z_x - \varpi_i \parallel^2 + \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\tanh(\alpha) \\ &= J_{\text{FCM}} + \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\tanh(\alpha_i) \end{aligned} \tag{8}$$

and

$$\mu_{x,i} = \left(\sum_{\ell=1}^{c}\frac{(\parallel z_x - \varpi_i \parallel^2 + v\tanh(\alpha_i))^{1/(m-1)}}{(\parallel z_x - \varpi_\ell \parallel^2 + v\tanh(\alpha_\ell))^{1/(m-1)}}\right)^{-1}; \quad x = 1, 2, \ldots, n;\ i = 1, 2, \ldots, c, \tag{9}$$

where α_1 and v are the same definition as Equation (4). Equation (8) can be rewritten as

$$\begin{aligned} J_{\text{CFCM}} &= J_{\text{FCM}} + \frac{1}{2}v\sum_{x=1}^{n}\sum_{i-1}^{c}\mu_{x,i}^{m}\tanh(\alpha_i) \\ &= J_{\text{FCM}} - \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}^{m}\tanh(-\alpha_i). \end{aligned} \tag{10}$$

Due to $0 < \alpha_i < 1$, from Figure 1 we can find $\tanh(-\alpha_i) \subset \ln(\alpha_i)$ which implies that J_{CFCM} can also be convergent.

3. Compensated Fuzzy Hopfield Network

The Hopfield neural network, with its simple architecture and parallel potential, has been applied in many fields [12–19]. Chung [13] et al. used the discrete Hopfield Neural Network with competitive learning (called CHNN) to polygonal approximation. In [13], the winner-take-all scheme has been adopted in the 2-dimensional discrete Hopfield Neural Network to eliminate the need for finding weighting factors in the energy function. Lin et al. [14–15] proposed a Fuzzy Hopfield Neural Network (called FHNN) to medical image segmentation. In the conventional Hopfield Network or CHNN, a neuron (*x*, *i*) in a firing state indicates

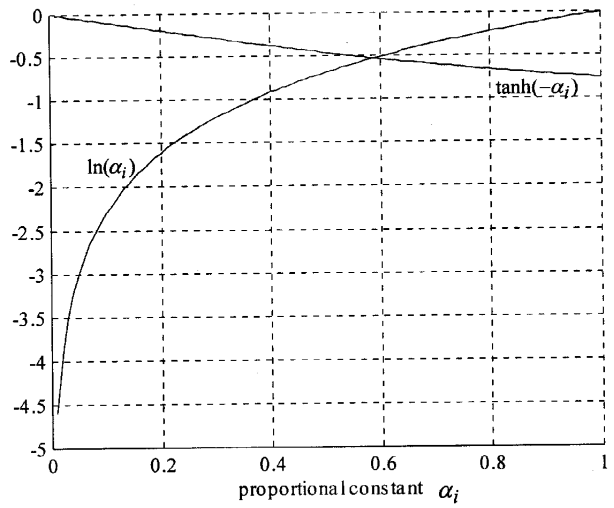

Figure 1. The curves of $\ln(\alpha_i)$ and $\tanh(-\alpha_i)$ within $0 \leqslant \alpha_i \leqslant 1$.

that sample z_x belongs to class i. But, in the FHNN, a neuron (x, i) in a fuzzy state indicates that sample z_x belongs to class i with a degree of uncertainty described by a membership function. Following the same idea of FHNN, the author also applied the discrete Hopfield Neural Network with Compensated Fuzzy C-Means strategy to clustering analysis. In this paper, the two-dimensional neuron array is used. Each row is assigned to a training pattern and each column is regarded as a cluster center in the CFHNN. If the number of clusters c is prechosen, the CFHNN consists of $n \times c$ neurons which can be conceived as a two-dimensional array which occupies n rows and c columns.

The CFHNN uses the Hopfield Neural Network architecture with CFCM strategy to classify the training patterns to generate feasible clusters. In order to increase the capability of the proposed approach, the energy function is formulated on the basis of a within-class scatter matrix, a concept widely used in pattern classification, where the within-class scatter matrix is defined by the average Euclidean distance between training pattern and cluster center within the same cluster. Let $\mu_{x,i}$ be the fuzzy state of the (x, i)th neuron and $W_{x,i;y,i}$ present the interconnected weight between neuron (x, i) and neuron (y, i). A neuron (x, i) in the network receives weighted inputs $W_{x,i;y,i}$ from each neuron (y, i) and a bias $I_{x,i}$ from output. The total input to neuron (x, i) is computed as

$$\mathrm{Net}_{x,i} = \left| z_x - \sum_{y=1}^{n} W_{x,i;y,i} (\mu_{y,i})^m \right|^2 + I_{x,i}. \tag{11}$$

The modified Lyapunov energy function of the two-dimensional Hopfield Neural Network using CFCM strategy, also defined in reference [14] is given by

$$E=\frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i})^{m}\left|z_x-\sum_{y=1}^{n}W_{x,i;y,i}(\mu_{y,i})^{m}\right|^{2}+\frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}I_{x,i}(\mu_{x,i})^{m}, \tag{12}$$

where $|\cdot|$ is the average Euclidean distance between training patterns to cluster center, $\sum_{y=1}^{n} W_{x,i;y,i}$ is the total weighted input received from neuron (y,i) in row i, $\mu_{x,i}$ is the output state at neuron (x,i), and m is the fuzzification parameter. Each column of this modified Hopfield network represents a class and each row represents a training pattern. The network reaches a stable state when the modified Lyapunov energy function is minimized. For example, a neuron (x,i) in a maximum membership state indicates that training pattern z belongs to class i.

The objective function, used to generate a suitable clustering that has a minimum average distance between training pattern and the cluster centroid within class, is given by

$$\begin{aligned} E \;=\; & \tfrac{A}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i})^{m}\left|z_x-\sum_{y=1}^{n}\frac{1}{\sum_{h=1}^{n}(\mu_{h,i})^{m}}z_y(\mu_{y,i})^{m}\right|^{2}+ \\ & +\tfrac{B}{2}\left[\left(\sum_{x=1}^{n}\sum_{i=1}^{c}\mu_{x,i}\right)-n\right]^{2}+\tfrac{1}{2}\upsilon\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i}^{m}\tanh(\alpha_i), \end{aligned} \tag{13}$$

where E is the total intra-class scatter energy that accounts for the scattered energies distributed by all training patterns in the same class with a membership grade, and both z_x and z_y are the training patterns at rows x and y, respectively. The proportional constant of class i, α_i is the same definition as J_{CFCM}.

The first term in Equation (13) is the within-class scatter energy that is the average distance between training patterns and the cluster centroid over c clusters. The second term guarantees that the n training patterns in $\mathbf{Z}$ can only be distributed among these c clusters. More specifically, the second term imposes constraints on the objective function and the first term minimizes the intra-class Euclidean distance from training patterns to the cluster centroid in any given cluster. The last term is the same compensated term as the definition in Equation (8) of CFCM algorithm.

As mentioned in [13], the quality of classification results is very sensitive to the weighting factors. Searching for optimal values for these weighting factors is expected to be tedious and time-consuming. To alleviate this problem, a 2-D Hopfield Neural Network with compensated Fuzzy C-Means clustering strategy, called CFHNN is proposed so that the constraint terms in Equation (13) can be handled more efficiently. All the neurons on the same row compete with one another to determine which neuron is the maximum membership value belonging to class i. In other words, the summation of the membership grade of states in the same row equals 1, and the total membership states in all n rows equal n. It is also ensured that all training patterns will be classified into these c classes. The modified Hopfield Neural Network CFHNN enables the scatter energy function to converge rapidly into a minimum value. Then, the scatter energy of the CFHNN can be further simplified as

$$E = \frac{1}{2}\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i})^m \left| z_x - \sum_{y=1}^{n}\frac{1}{\sum_{h=1}^{n}(\mu_{h,i})^m} z_y (\mu_{y,i})^m \right|^2 + \\ + \frac{1}{2} v \sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i})^m \tanh(\alpha_i). \tag{14}$$

By using Equation (14), which is a modification of Equation (13), the minimization of energy E is greatly simplified since it contains only two terms and hence the requirement of having to determine the weighting factors A and B vanishes. Comparing Equation (14) with the modified Lyapunov function Equation (12), the synaptic interconnection weights and the bias input can be obtained as

$$W_{x,i;y,i} = \frac{1}{\sum_{h=1}^{n}(\mu_{h,i})^m} z_y, \tag{15}$$

and input bias

$$I_{x,i} = v \tanh(\alpha_i). \tag{16}$$

By introducing Equations (15) and (16) into Equation (11), the input to neuron (x, i) can be expressed as

$$\mathrm{Net}_{x,i} = \left| z_x - \sum_{y=1}^{n}\frac{1}{\sum_{h=1}^{n}(\mu_{h,i})^m} z_y (\mu_{y,i})^m \right|^2 + v \tanh(\alpha_i). \tag{17}$$

Consequently, the state (i.e., membership function) for the neuron (x, i) row is given as

$$\mu_{x,i} = \left[\sum_{j=1}^{c} \left(\frac{\mathrm{Net}_{x,i}}{\mathrm{Net}_{x,j}} \right)^{1/m-1} \right]^{-1} \quad \text{for all } i. \tag{18}$$

Using Equations (5), (17) and (18), the CFHNN can classify c clusters in a parallel manner that is described as follows.

3.1. CFHNN ALGORITHM

Step 1. Input a set of training pattern $\mathbf{Z} = \{z_1, z_2, \ldots, z_n\}$, fuzzification parameter $m(1 \leqslant m < \infty)$, the number of clusters c, and initialize the states for all neurons $U = [\mu_{x,i}]$ (membership matrix).

Step 2. Compute α_i and weighted matrix using Equations (5), and (15), respectively.

Step 3. Calculate the input to each neuron (x, i):

$$\mathrm{Net}_{x,i} = \left| z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1}^{n}(\mu_{h,i})^m} z_y(\mu_{y,i}^m \right|^2 + v \tanh(\alpha_i).$$

Step 4. Apply Equation (18) to update the neurons' membership values in a synchronous manner:

$$\mu_{x,i} = \left[\sum_{j=1}^{c} \left(\frac{\mathrm{Net}_{x,i}}{\mathrm{Net}_{x,j}} \right)^{1/m-1} \right]^{-1}, \text{ for all } i.$$

Step 5. Compute $\Delta = \max(| U^{(t+1)} - U^{(t)} |)$. If $\Delta >$, then go to Step 2, otherwise go to Step 6.

Step 6. Find the clusters for the final membership matrix.

In Step 3, the inputs are calculated for all neurons. In Step 4, the Compensated Fuzzy C-Means clustering method is applied to determine the fuzzy states with the synchronous process. Here, a synchronous iteration is defined as an updated fuzzy state for all neurons.

4. Convergence of the CFHNN

It is always true that a stable state has to be converged in CFHNN evolutions. So, proof of the convergence of the CFHNN is described as follows. The scatter energy function is first considered.

$$\begin{aligned} E \;=\; & \tfrac{1}{2} \sum_{x=1}^{n} \sum_{i=1}^{c} (\mu_{x,i})^m \left| z_x - \sum_{y=1}^{n} \tfrac{1}{\sum_{h=1}^{n}(\mu_{h,i})^m} z_y(\mu_{y,i})^m \right|^2 \\ & + \tfrac{1}{2} v \sum_{x=1}^{n} \sum_{i=1}^{c} (\mu_{x,i})^m \tanh(\alpha_i) \end{aligned} \tag{19}$$

due to $0 \leqslant (\mu_{x,i})^m \leqslant 1$ and $0 \leqslant \tanh(\alpha_i) \leqslant 1$; which implies that

$$E \leqslant \frac{1}{2} \sum_{i=1}^{c} \sum_{x=1}^{n} \left[z_x - \sum_{y=1}^{n} \frac{1}{\sum_{h=1)}^{n}(\mu_{h,i})^m} z_y(\mu_{y,i})^m \right]^2 + \frac{1}{2} v \cdot n \cdot c. \tag{20}$$

Equation (20) shows that the objective energy is less than or equal to half the total distance between training patterns to the cluster centers plus a constant $\frac{1}{2}v \cdot n \cdot c$. This proves that E is bounded from below.

Equation (19), like Equation (8), is based on least-squared errors criteria, and it is rewritten as follows:

$$E = \frac{1}{2}\sum_{i=1}^{c}\sum_{x=1}^{n}(\mu_{x,i})^m \mid z_x - \varpi_i \mid^2 + \frac{1}{2}v\sum_{x=1}^{n}\sum_{i=1}^{c}(\mu_{x,i})^m \tanh(\alpha_i) \tag{21}$$

and

$$\varpi_i = \sum_{y=1}^{n}\frac{1}{\sum_{h=1}^{n}(\mu_{h,i})^m} z_y(\mu_{y,i})^m, \tag{22}$$

where ϖ_i (center of cluster i) is the total interconnection weight received from all neurons y in the same column i. As proved in references [8] and described previously, the energy of CFHNN equals J_{CFCM} as follows:

$$E = J_{\mathrm{CFCM}}. \tag{23}$$

Thus, the reassignment of a membership degree belonging to cluster i in training sample z_x will result in a decrease of the objective energy function whenever z_x is located closer to a feasible cluster center. In addition, the compensated term will speed up the decrease of the objective function. Consequently, the CFHNN will converge to a satisfactory result after several iterations of updating the reassignment matrix.

5. Experimental Results

To see the performance of the FCM, PFCM, and the proposed algorithm CFHNN, the Butterfly example given by Ruspini [20] and Jou [21] is considered. Table I lists 15 input patterns in R^2. Patterns 7, 8, and 9 construct a bridge between the wings of the butterfly. Parameters $c = 2$, $\varepsilon = 0.01$, $v = 0.55$ (the best value indicated in reference [7]), $1 < m \leqslant 2$ are fixed respectively and initial membership grades are randomly set for all methods in this paper. The membership grades of different algorithms are listed with $m = 1.25$ and 2.0, respectively, in Table I. In Table I, the membership grades are nearly symmetric with respect to pattern z_8 in both data coordinate directions for all algorithms. The larger m, the fuzzier the membership grades of the final partition become. In all algorithms in this paper, the energy functions converge to a local minimum within 15 iterations. But the proposed approach is the fastest one in convergence rate with average iterations. The convergence results are shown in Figures 2 and 3 with $m = 1.25$ and 2.0, respectively. In addition, it can reasonably be expected that the membership grade

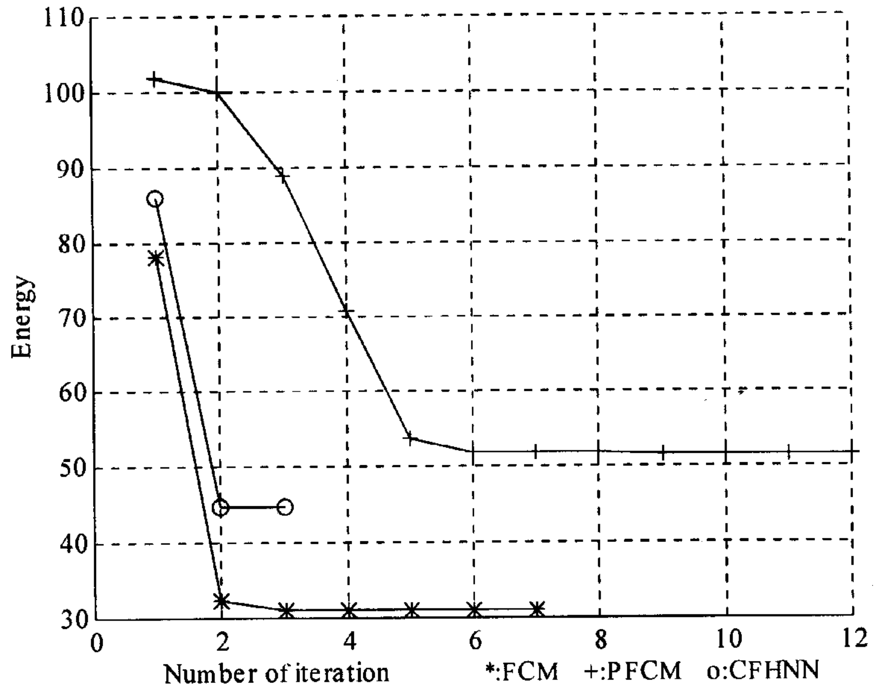

Figure 2. The convergent curve with $m = 1.25$ and $c = 2$ in different algorithms.

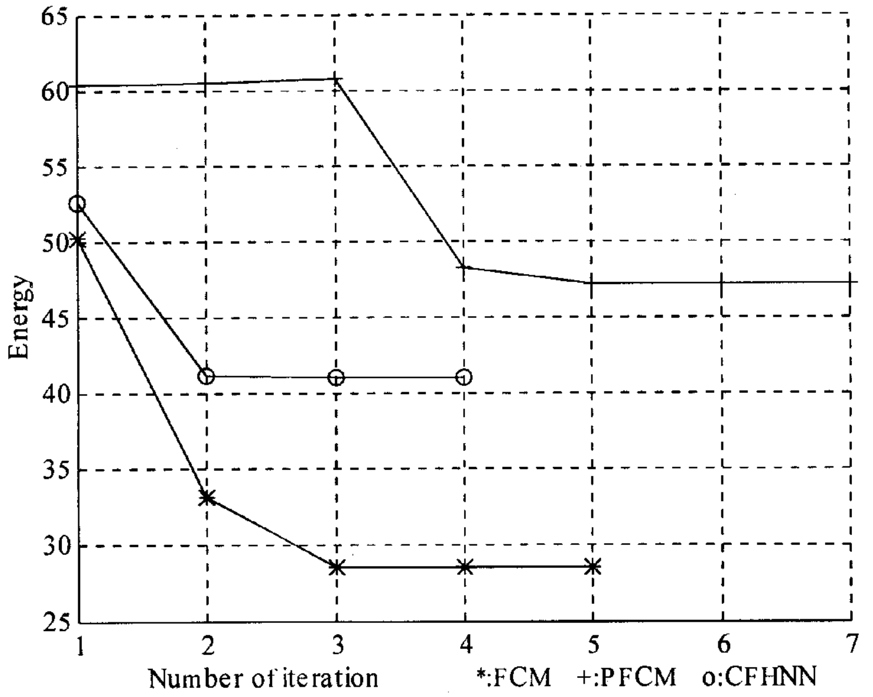

Figure 3. The convergent curve with $m = 2.0$ and $c = 2$ in different algorithms.

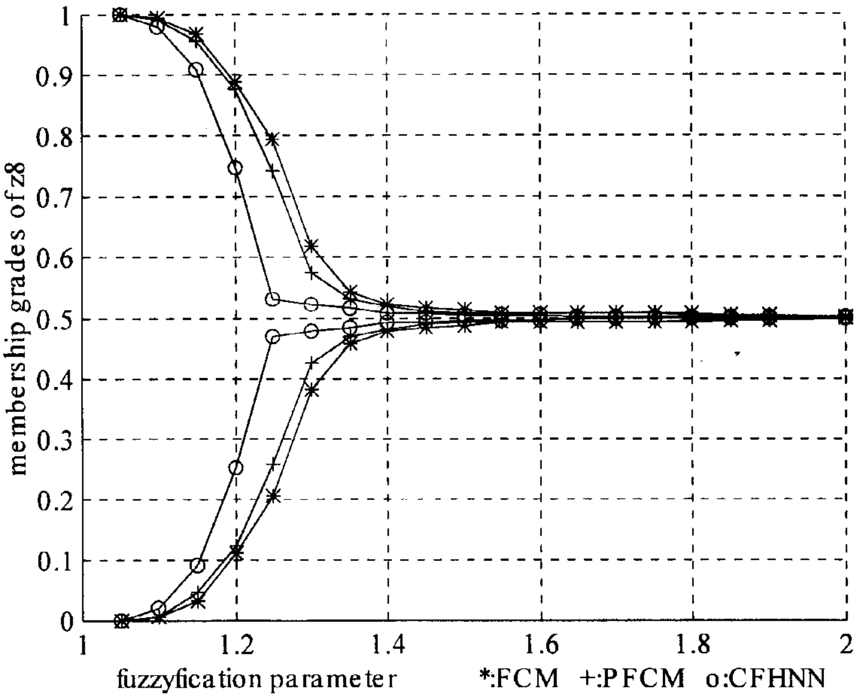

Figure 4. The membership grade curve of pattern z_8 with $c = 2$ in different algorithms.

of pattern z_8 with respect to both clusters should be close to 0.5 with a smaller ε for all algorithms in this paper. Anyway, the CFHNN can rapidly result in a symmetric manner for the membership grade of pattern z_8 with respect to both clusters. This fact can be shown in Figure 4. In Figure 4, the curves above membership grade = 0.5 indicate the membership grades of pattern z_8 belonging to cluster 1 while the curves under membership grade = 0.5 denote the membership grades of pattern z_8 belonging to cluster 2 with distinct m from 1.05 to 2.0 for all strategies in this paper. In accordance with Figure 4, the symmetric manner of membership grades can be rapidly reached for the critical patterns in partition clusters using CFHNN.

6. Discussion and Conclusions

In this paper, an approach using the Hopfield Neural Network imposed by a Compensated Fuzzy C-Means mechanism (so-called Compensated Fuzzy Hopfield Neural Network) is proposed for fuzzy clustering analysis. Thus, the efforts of determining the optimal values of the weighting factors on the penalty terms may be avoided. Accordingly, the computation of this proposed algorithm can be speeded up while the convergence of network is still guaranteed. In this way, there is no need to change the algorithm except for the dimensions of the neurons. The symmetric manner of membership grades can be more rapidly reached for the critical patterns

Table I. The training of Butterfly and membership grades with $c = 2$ after convergence in different algorithms

Patterns		$m = 1.25$						$m = 2.0$					
		FCM		PFCM		CFHNN		FCM		PFCM		CFHNN	
x	z_x	μ_1	μ_2	μ_1	μ_2	μ_1	μ_2	μ_1	μ_2	μ_1	μ_2	μ_1	μ_2
1	(0,0)	0.000	1.000	0.000	1.000	0.000	1.000	0.022	0.978	0.034	0.966	0.029	0.971
2	(0,2)	0.000	1.000	0.000	1.000	0.000	1.000	0.001	0.999	0.006	0.994	0.003	0.997
3	(0,4)	0.000	1.000	0.000	1.000	0.000	1.000	0.022	0.978	0.034	0.966	0.029	0.971
4	(1,1)	0.000	1.000	0.000	1.000	0.000	1.000	0.003	0.997	0.016	0.984	0.010	0.990
5	(1,2)	0.000	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.007	0.993	0.003	0.997
6	(1,3)	0.000	1.000	0.000	1.000	0.000	1.000	0.003	0.997	0.016	0.984	0.010	0.990
7	(2,2)	0.000	1.000	0.000	1.000	0.000	1.000	0.020	0.980	0.060	0.940	0.043	0.957
8	(3,2)	0.792	0.208	0.738	0.262	0.555	0.445	0.502	0.498	0.498	0.502	0.498	0.502
9	(4,2)	1.000	0.000	1.000	0.000	1.000	0.000	0.981	0.019	0.939	0.061	0.956	0.044
10	(5,1)	1.000	0.000	1.000	0.000	1.000	0.000	0.997	0.003	0.984	0.016	0.990	0.010
11	(5,2)	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000	0.993	0.007	0.997	0.003
12	(5,3)	1.000	0.000	1.000	0.000	1.000	0.000	0.997	0.003	0.984	0.016	0.990	0.010
13	(6,0)	1.000	0.000	1.000	0.000	1.000	0.000	0.978	0.022	0.966	0.034	0.971	0.029
14	(6,2)	1.000	0.000	1.000	0.000	1.000	0.000	0.999	0.001	0.994	0.006	0.997	0.003
15	(6,4)	1.000	0.000	1.000	0.000	1.000	0.000	0.978	0.022	0.966	0.034	0.971	0.029

in partition clusters by using CFHNN than with FCM and PFCM. In addition, this proposed algorithm has great potential in parallel implementation for real-time applications.

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