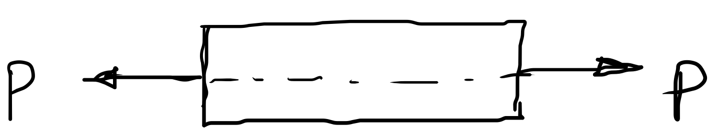
Simple stress and strain

- **Stress (σ)** $= \dfrac{\text{Force}}{\text{Area}} = \dfrac{N}{mm^2}$

(1.) Simple stress / direct stress

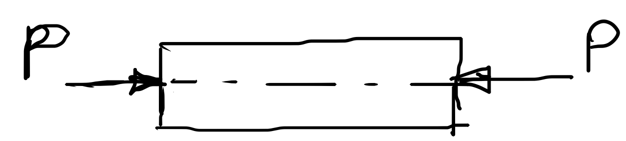
(i) Tensile Stress



$$\sigma_T = \frac{P}{A}$$

$$= \frac{\text{Tensile force}}{\text{Area of cross section}}$$

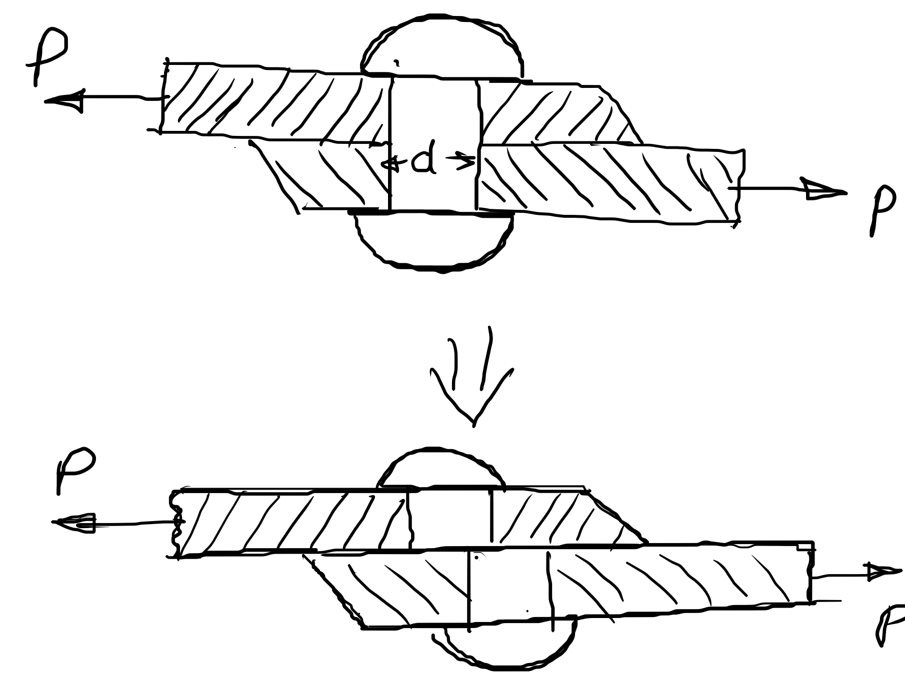
(ii) compressive stress



$$\sigma_C = \frac{P}{A}$$

$$\frac{\text{compressive Force}}{\text{Area of cross section}}$$

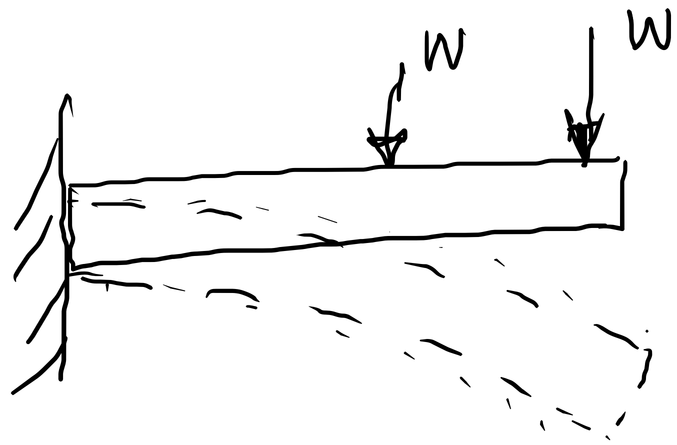
(iii) Shear stress



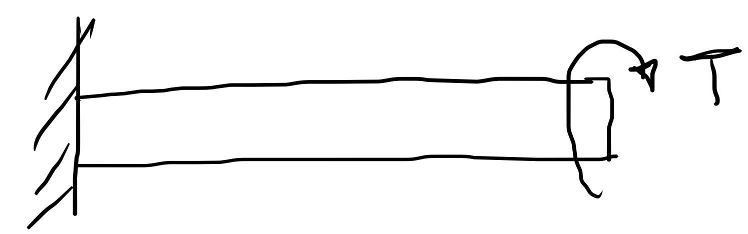
$$\tau = \frac{\text{Shear force}}{\text{Shear Area}} = \frac{P}{\frac{\pi}{4}d^2}$$

(2.) Indirect stresses

(i) Bending stress



(ii) Torsional stress



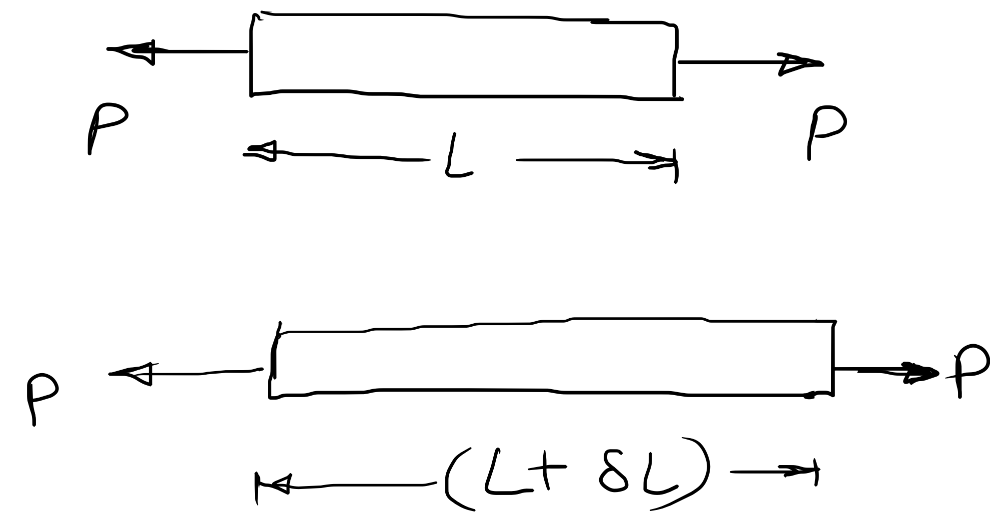
(3.) Combined stresses

Any possible combination of above two types.

$$\text{Strain}(e) = \frac{\text{change in dimension}}{\text{original dimension}}$$

(i) Tensile strain $= \dfrac{(L + \delta L) - L}{L}$

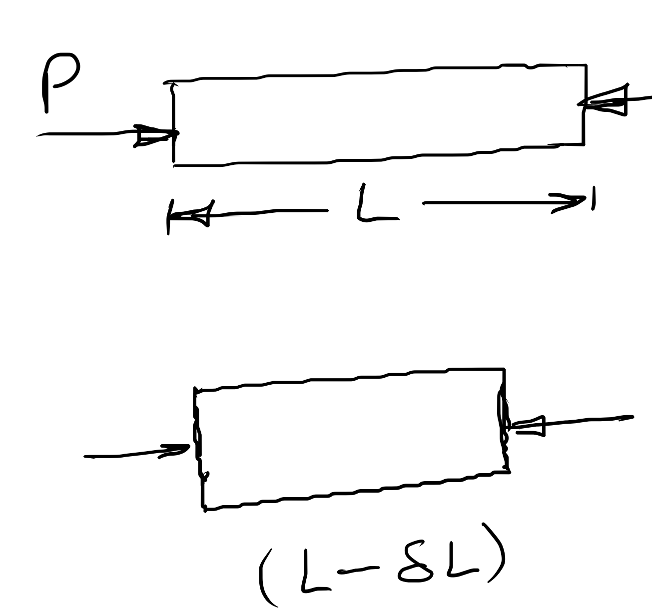
$$e_T = \frac{\delta L}{L}$$



(ii) compressive strain $= \dfrac{(L - \delta L) - L}{L}$

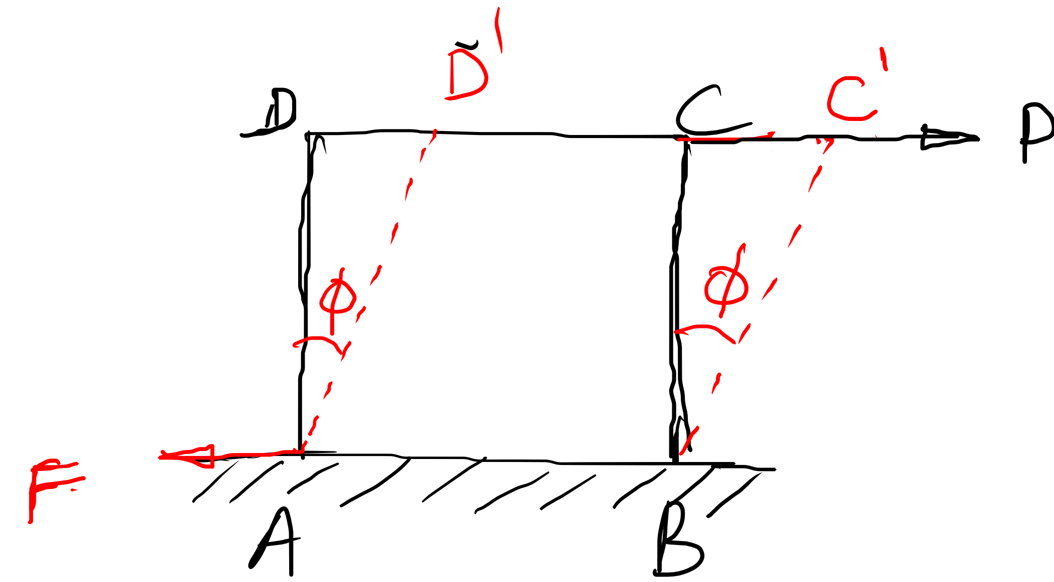
$$e_c = \left| -\left(\frac{\delta L}{L}\right) \right|$$

$$= \left(\frac{\delta L}{L}\right)$$



(iii) Shear strain (ϕ)

Angular deformation produced in the body due to shear force is called shear strain.



In $\Delta\, A D' D$

$$\tan\phi = \frac{DD'}{AD} = \frac{CC'}{BC}$$

$\tan\phi \simeq \phi$ (since ϕ is very small)

$$e_s = \phi$$

(iv) volumetric strain (e_v) = $\dfrac{\text{change in volume}}{\text{original volume}}$

$$e_v = \frac{\delta V}{V}$$

Hook's Law

within elastic limit,

Stress $\propto$ strain

$\sigma \propto e$

$\sigma = E.e$

E = constant of proportionality
= young's modulus
= modulus of elasticity

(1.) Young's modulus, $E = \dfrac{\sigma}{e} = \dfrac{\sigma_T}{e_t}$ or $\dfrac{\sigma_c}{e_c}$

(2.) Modulus of rigidity, C, N or G = $\dfrac{\text{Shear stress}}{\text{Shear strain}} = \dfrac{\tau}{e_s}$

(3.) Bulk modulus, K = $\dfrac{\text{normal stress}}{\text{Volumetric Strain}} = \dfrac{\sigma_n}{e_v}$

Q.1 A steel wire 2m long and 3mm in diameter is extended by 0.75mm when a load is suspended from the wire. If the same load is suspended from a brass wire, 2.5 m long and 2 mm in diameter is extended by 4.64 mm. Determine the modulus of elasticity of brass if that of steel be $2 \times 10^5\ N/mm^2$.

Soln:

For steel wire

$l_s = 2m$

$d_s = 3mm$

$\delta l_s = 0.75mm$

$E_s = 2\times10^5\ N/mm^2$

For Brass wire

$l_b = 2.5m$

$d_b = 2mm$

$\delta l_b = 4.64\ mm$

$E_b = ?$

For Hook's Law

$$\sigma = E.e$$

$$\frac{P}{A} = E.\frac{\delta l}{l}$$

$$\Rightarrow P = \frac{AE.\delta l}{l}$$

For steel wire

$$P = \frac{\pi}{4} d_s^2 . E_s . \frac{\delta l_s}{l_s} = \frac{\pi}{4}(3^2)\ 2\times10^5 \times \frac{0.75}{2000} \quad \text{—(1)}$$

For Brass wire

$$P = \frac{\pi}{4} d_b^2 . E_b . \frac{\delta l_b}{l_b} = \frac{\pi}{4}(2)^2 \times E_b \times \frac{4.64}{2500} \quad \text{—(2)}$$

Since a common force P is applied to both steel and Brass wires

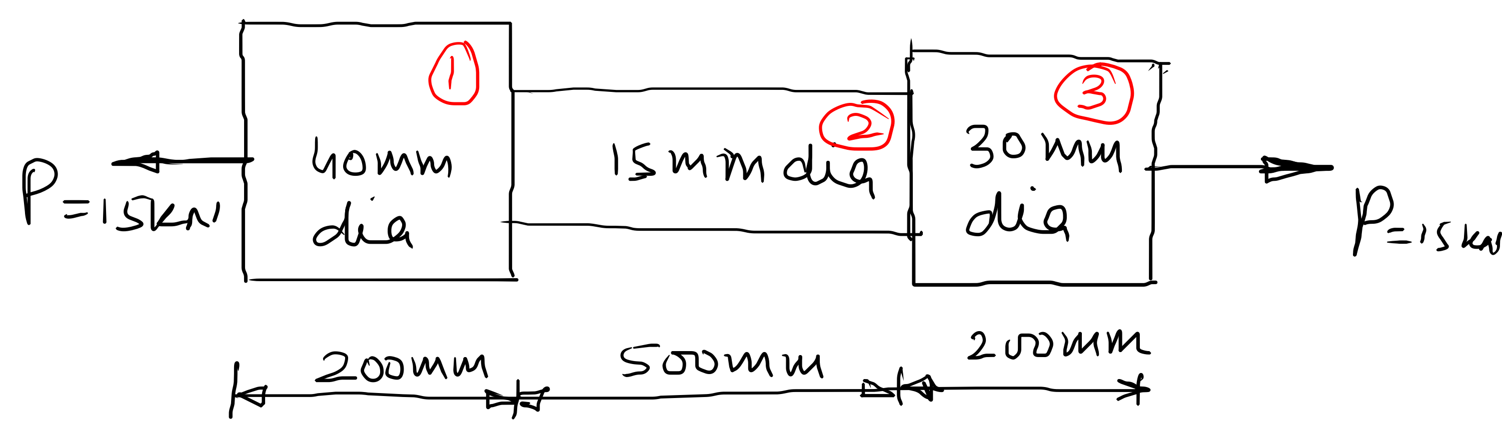
$$\therefore \frac{\pi}{4}(3)^2 \times 2 \times 10^5 \times \frac{0.75}{2000} = \frac{\pi}{4}(2)^2 \times E_b \times \frac{4.64}{2500}$$

$$\therefore E_b = 0.909 \times 10^5 \text{ N/mm}^2$$

Q:2 Find total elongation of the bar.

Take, $E = 200\ GN/m^2$

$1G = 10^9$



Sol^n:

$\sigma = E.e$

$\frac{P}{A} = E.\frac{(\delta l)}{l}$

$\Rightarrow \delta l = \frac{Pl}{AE}$

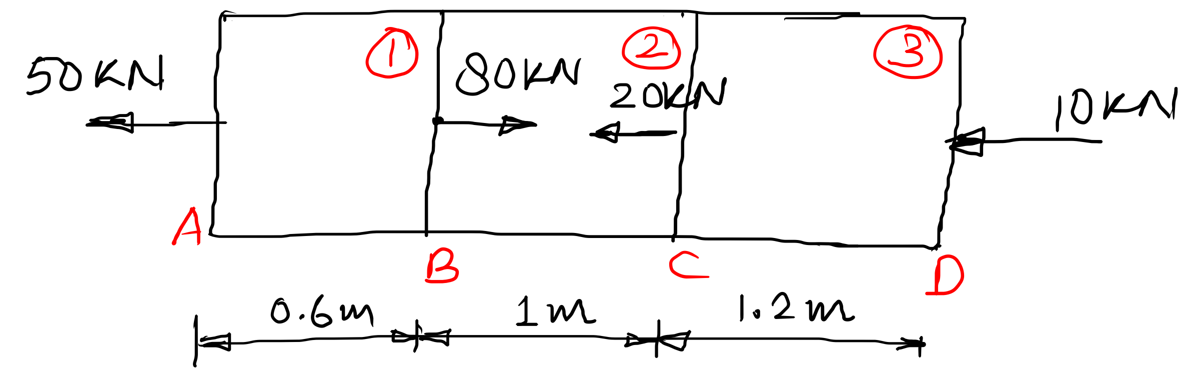
Total elongation of the bar

$\delta l_T = \delta l_1 + \delta l_2 + \delta l_3$

$= \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E} = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$

$\delta l_T = 0.2454\ mm$

Q:3 Find the total elongation of the bar, Take, $E = 100\ GN/m^2$



Cross section = $1000\,mm^2$

Sol^n:

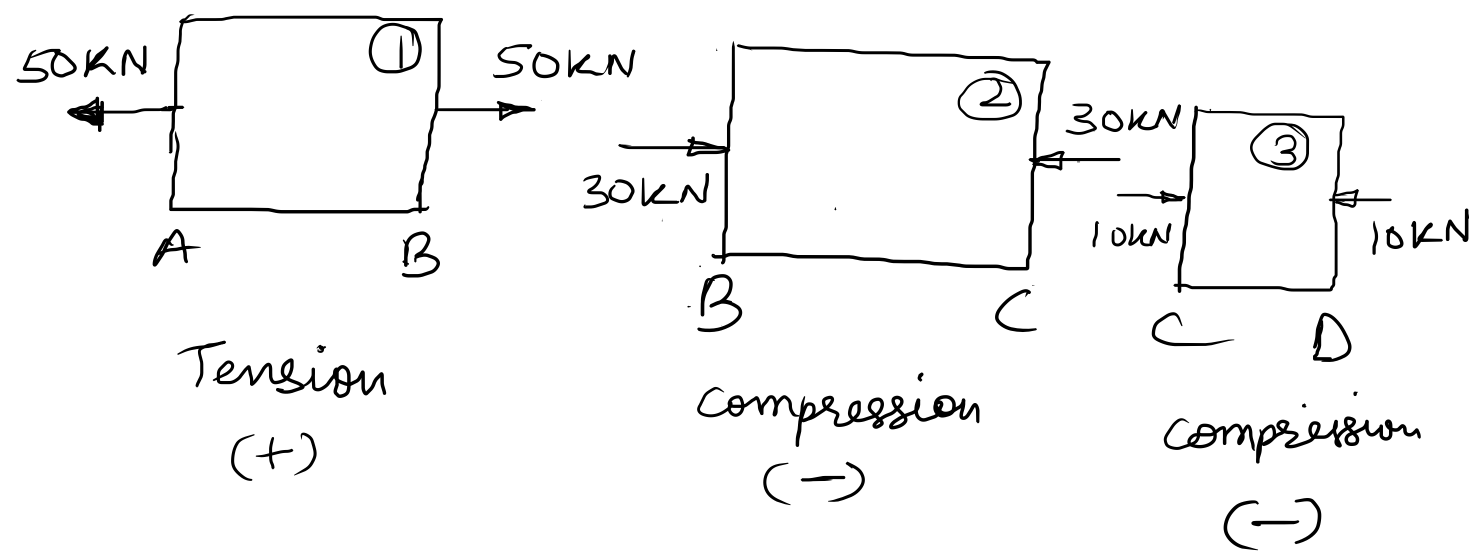
$$\delta l = \frac{Pl}{AE}$$

$$\delta l_T = \delta l_1 - \delta l_2 - \delta l_3$$

$$= \frac{1}{AE}\left[P_1 l_1 - P_2 l_2 - P_3 l_3\right]$$

$$= -0.12\,mm$$

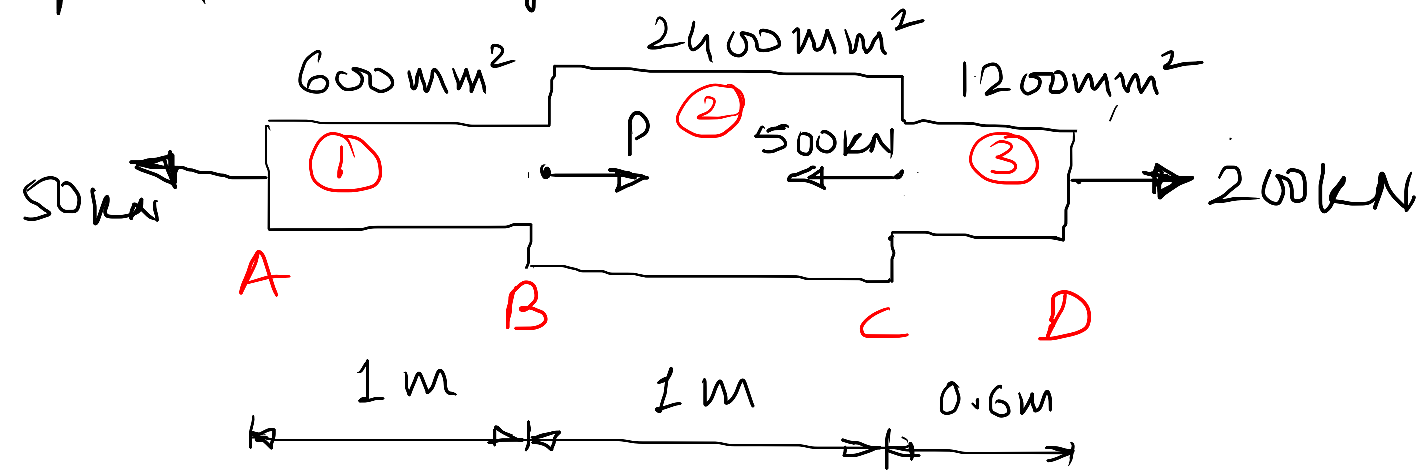
$$\delta l_T = 0.12\,mm \text{ (shorten)}$$



Q: 4 Find: (i) force 'P' necessary for equilibrium

(ii) Total elongation

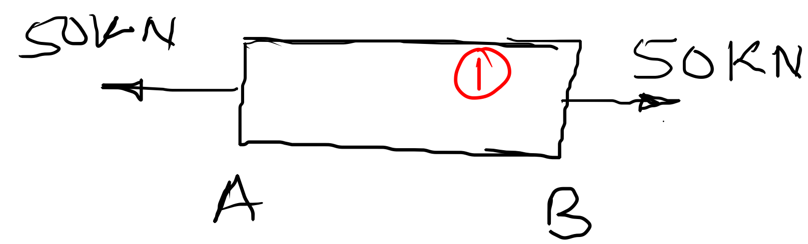
Take $E = 210 \, GN/m^2$



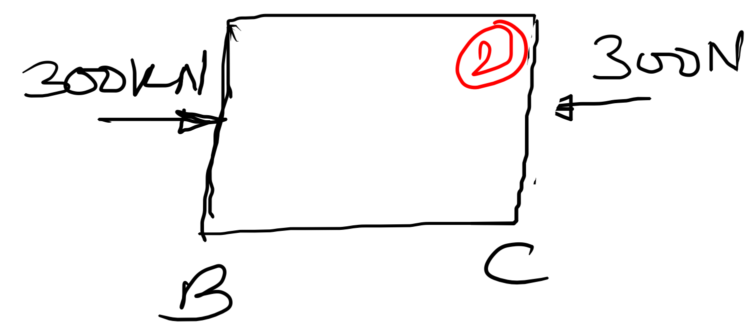
Sol:

$$50 + 500 = P + 200$$

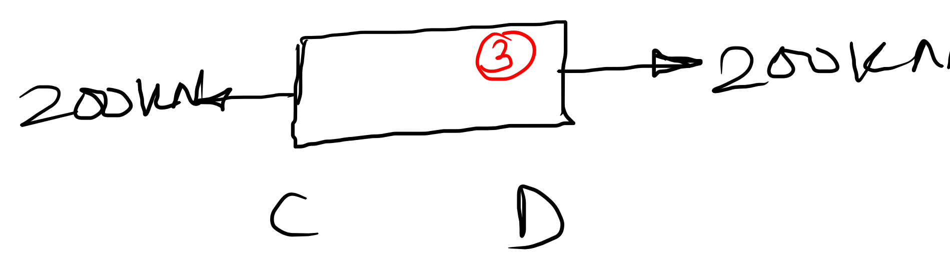
$$\Rightarrow P = 350 \, kN$$



Tension (+)



compression (−)



Tension (+)

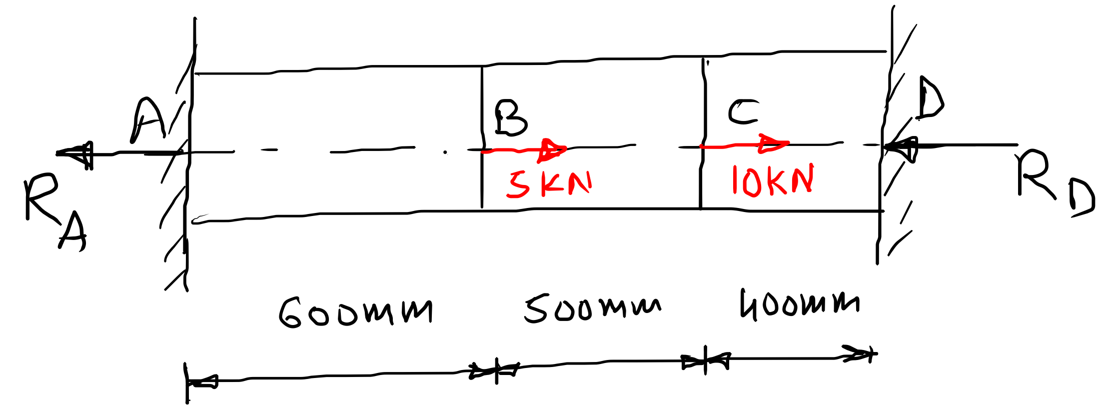
$$\delta l = \frac{Pl}{AE}$$

$$= \frac{1}{E}\left[\frac{P_1 l_1}{A_1} - \frac{P_2 l_2}{A_2} + \frac{P_3 l_3}{A_3}\right]$$

$$\delta l_T = 0.278 \, mm$$

Q:5 A straight uniform bar AD is clamped at both the ends. Initially the bar is stress free. Find the stress in each part of the bar.

Area = $1000\ mm^2$



Sol^n:

For equilibrium of the bar

$$R_A + R_D = 5 + 10 = 15\ KN \quad \text{--- (1)}$$

Total elongation of bar = 0

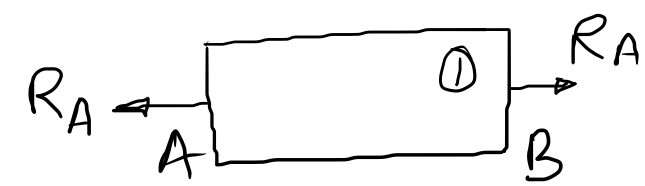
$$\delta l = \delta l_1 + \delta l_2 - \delta l_3 = 0$$

$$\frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} - \frac{P_3 l_3}{AE} = 0$$

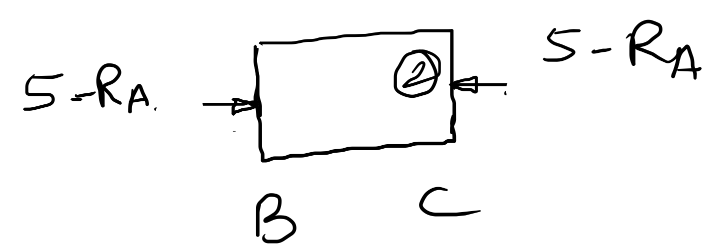
$$R_A \times 600 - (5 - R_A)\,500 - (15 - R_A)\,400 = 0$$

$$\Rightarrow R_A = 5.67\ KN$$

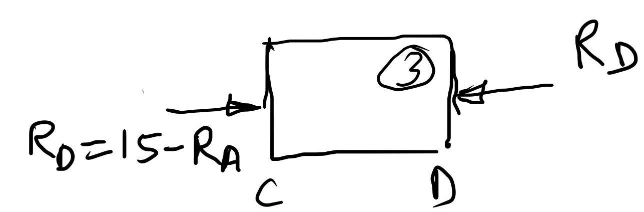
$$R_D = 9.33\ KN$$



(+)



(−)



(−)

stress in part AB, $\sigma_{AB} = \dfrac{5.67\,kN}{1000\,mm^2} = 5.67\,N/mm^2$ (T)

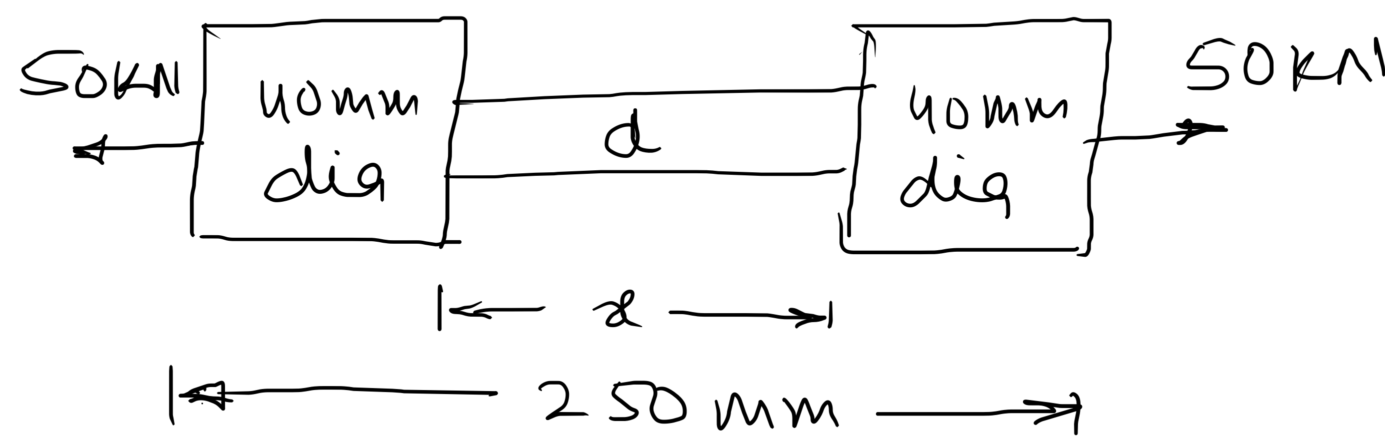
stress in part BC, $\sigma_{BC} = \dfrac{0.67\,kN}{1000\,mm^2} = 0.67\,N/mm^2$ (T)

stress in part CD, $\sigma_{CD} = \dfrac{9.33\,kN}{1000\,mm^2} = 9.33\,N/mm^2$ (C)

Q:6 Find diameter of the middle portion if the stress is limited to $130 MN/m^2$.

Find length of the middle portion if the total elongation of the bar is 0.15 mm.

Take, $E = 200 GN/m^2$.

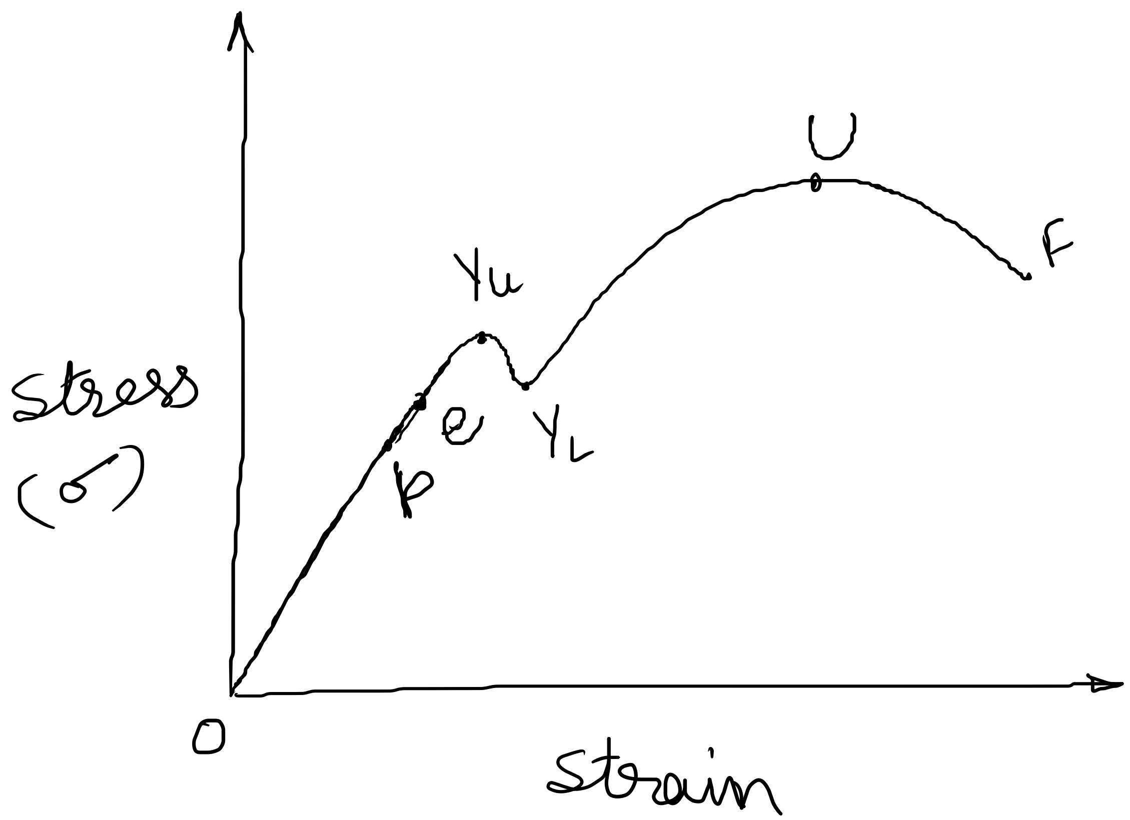


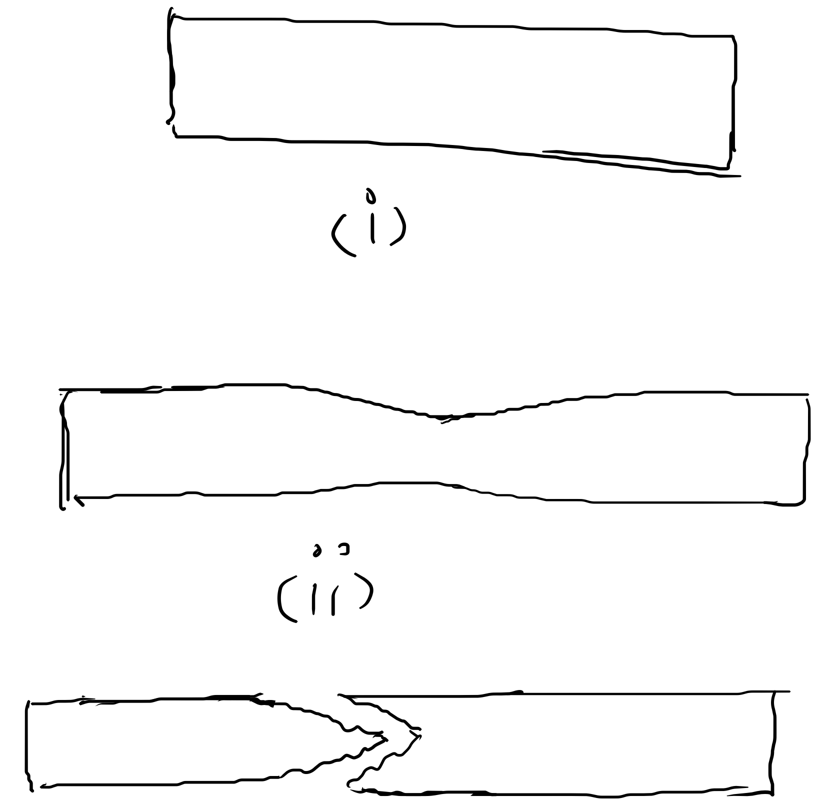
Solⁿ:

$d = 22.1 \text{ mm}$

$x = 222 \text{ mm}$

Stress - strain curve in Tension for a Ductile Material





Cup/cone type of failure

Stress- strain curve for Ductile Material (Mild steel)

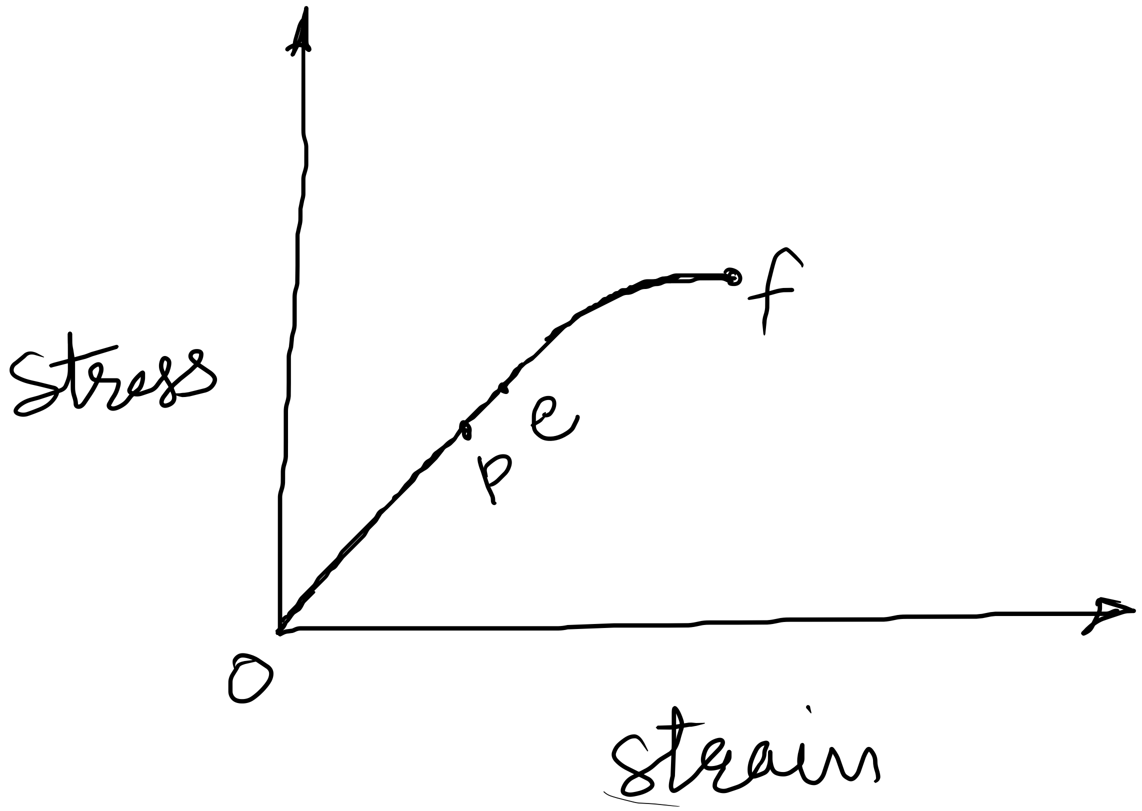
p – limit of proportionality, U – ultimate stress

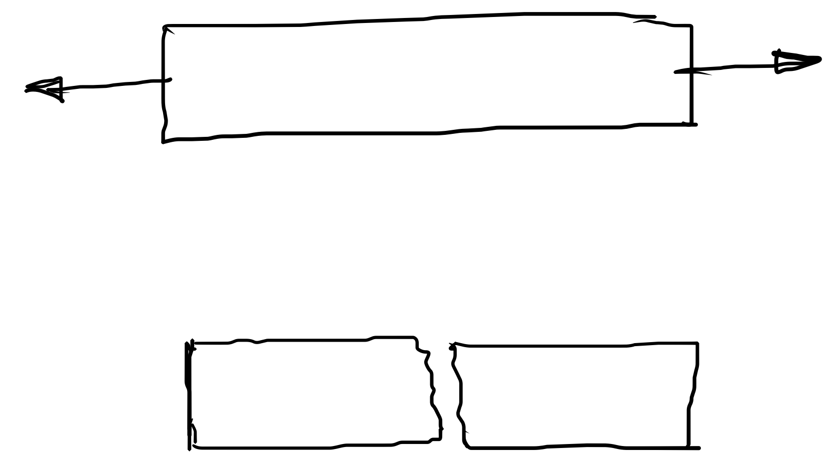
e – elastic limit, F – Failure stress

Y_U – upper yield point

Y_L – Lower yield point

Stress-strain curve in Tension for a Brittle material





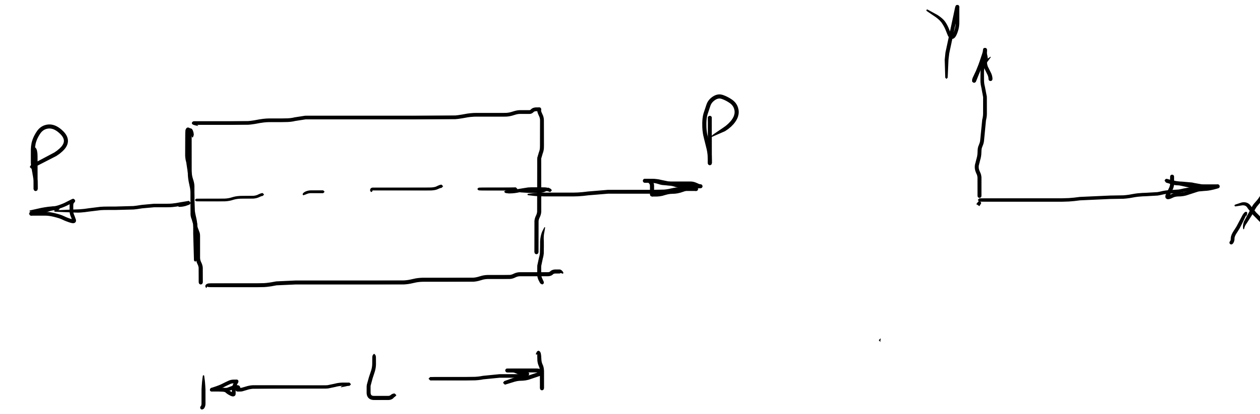
Brittle fracture

stress-strain curve for Brittle Material
(Cast Iron)

p - limit of proportionality
e - elastic limit
f - failure stress

Poisson's Ratio

- Linear / primary strain along direction of the force



- Lateral / secondary strain in the direction perpendicular to the force.

$$\therefore \text{Poisson's Ratio}, \mu = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{1}{m}$$

Relations between the Elastic modulii

(1.) $C = \dfrac{mE}{2(m+1)}$

(2.) $K = \dfrac{mE}{3(m-2)}$

(3.) $E = \dfrac{9KC}{3K+C}$

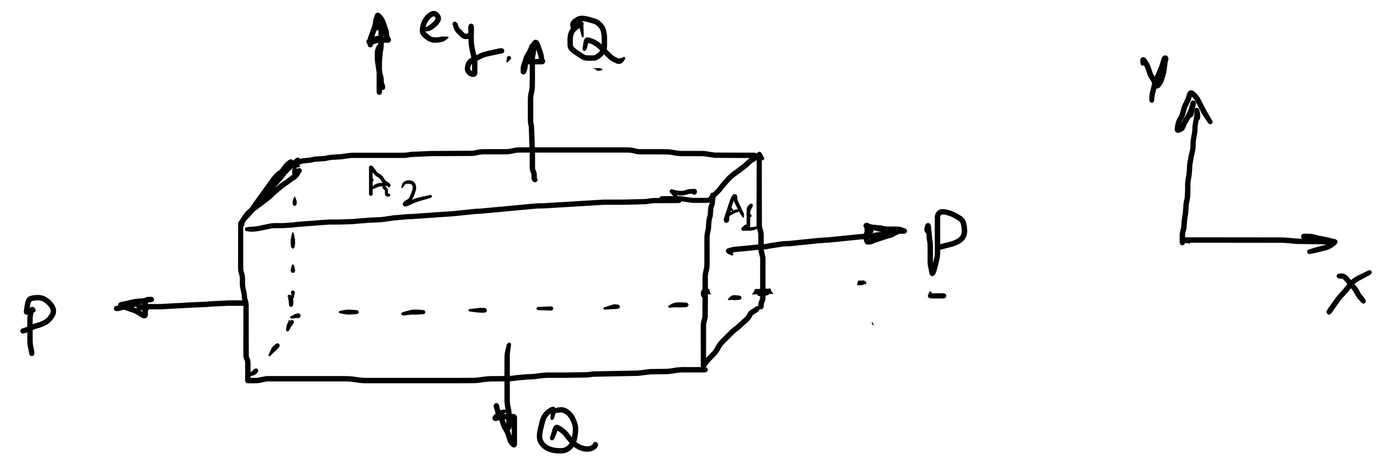
C – modulus of rigidity

E – modulus of elasticity

K – Bulk modulus

$\dfrac{1}{m} = \mu$ (poisson's ratio)

Q:1



$$\sigma_x = \frac{P}{A_1}, \quad \sigma_y = \frac{Q}{A_2}$$

Linear strain along x-direction, $e_x = \frac{\sigma_x}{E}$

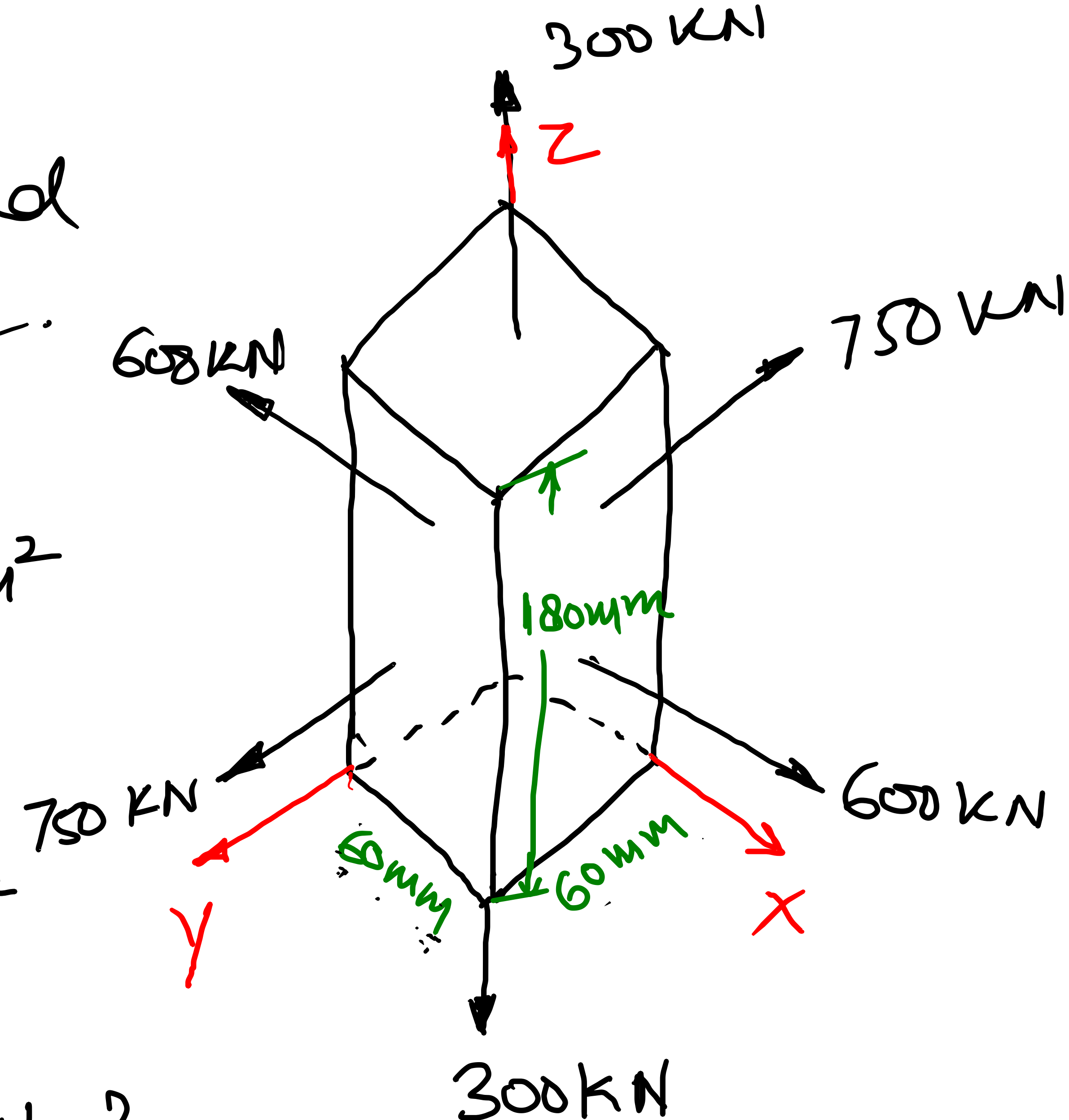
Linear strain along y-direction, $e_y = \frac{\sigma_y}{E}$

Net strain along x-axis $= \underset{(T)}{\frac{\sigma_x}{E}} - \underset{(C)}{\frac{\sigma_y}{E}.\mu}$

Net strain along y-axis $= \underset{(T)}{\frac{\sigma_y}{E}} - \underset{(C)}{\frac{\sigma_x}{E}.\mu}$

Q:2 $E = 200\,GN/m^2$, $\mu = 0.3$

Find change in dimensions and change in volume of the bar.



Sol^n

$$\sigma_x = \frac{600 \times 10^3}{180 \times 60 \times 10^{-6}} = 55.55\,MN/m^2 \;(T)$$

$$\sigma_y = \frac{750 \times 10^3}{180 \times 60 \times 10^{-6}} = 69.44\,MN/m^2 \;(T)$$

$$\sigma_z = \frac{300 \times 10^3}{60 \times 60 \times 10^{-6}} = 83.33\,MN/m^2 \;(T)$$

Net strain along x-axis

$$(e_x)_{net} = \frac{\sigma_x}{E} - \frac{\sigma_y}{E}\mu - \frac{\sigma_z}{E}.\mu = 4.85 \times 10^{-5}$$

Increase in dimension parallel to x-axis

$$dx = 4.85 \times 10^{-5} \times 60 = 0.00291 \text{ mm}$$

Net strain along y-axis

$$(e_y)_{net} = \frac{\sigma_y}{E} - \frac{\sigma_x}{E} \cdot \mu - \frac{\sigma_z}{E} \cdot \mu = 1.389 \times 10^{-4}$$

$\therefore$ Increase in dimension parallel to y-axis

$$dy = 1.389 \times 10^{-4} \times 60 = 0.00833 \text{ mm}$$

Net strain along z-axis

$$(e_z)_{net} = \frac{\sigma_z}{E} - \frac{\sigma_x}{E} \cdot \mu - \frac{\sigma_y}{E} \cdot \mu = 2.292 \times 10^{-4}$$

$\therefore$ Increase in dimension parallel to z-axis

$$dz = 2.292 \times 10^{-4} \times 180 = 0.0412 \text{ mm}$$

Change in volume

volumetric strain, $e_v = \frac{\delta V}{V}$

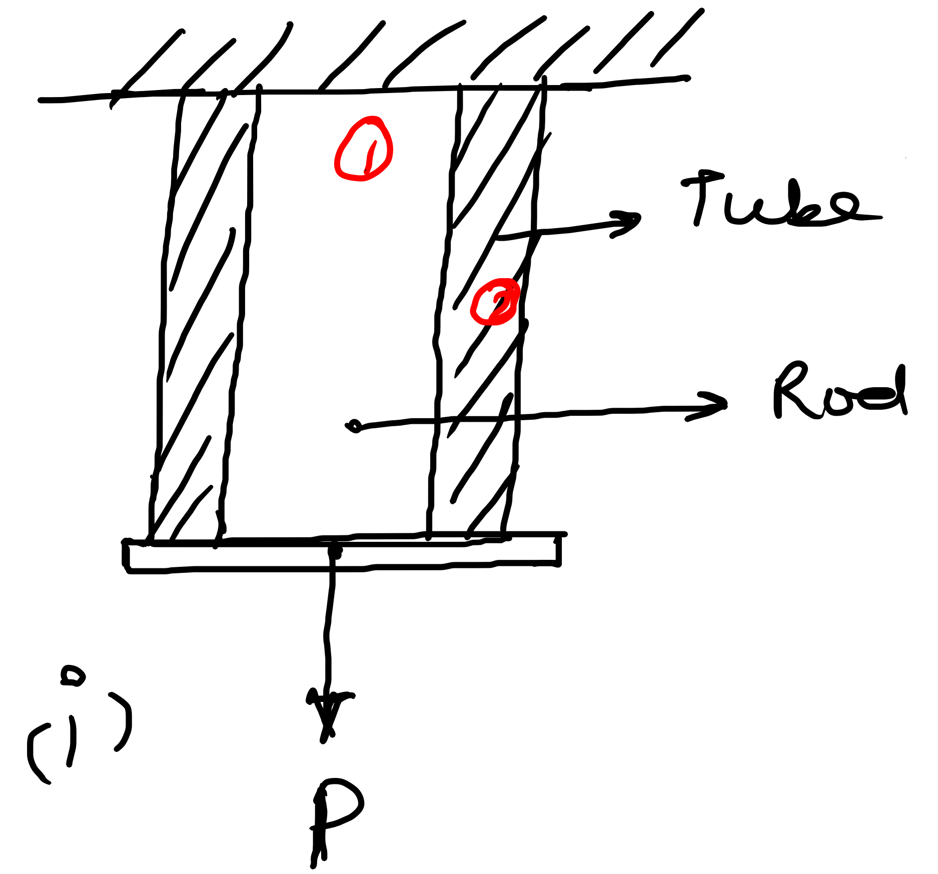
$$e_v = (e_x)_{net} + (e_y)_{net} + (e_z)_{net} = 4.166 \times 10^{-4}$$

$$\therefore \delta V = e_v \cdot V = 4.166 \times 10^{-4} \times (60 \times 60 \times 180)\ mm^3$$

$$= 269.95\ mm^3$$

Stress and strain in the compound bars

$$P = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2$$



strains produced in tube and rod are equal.

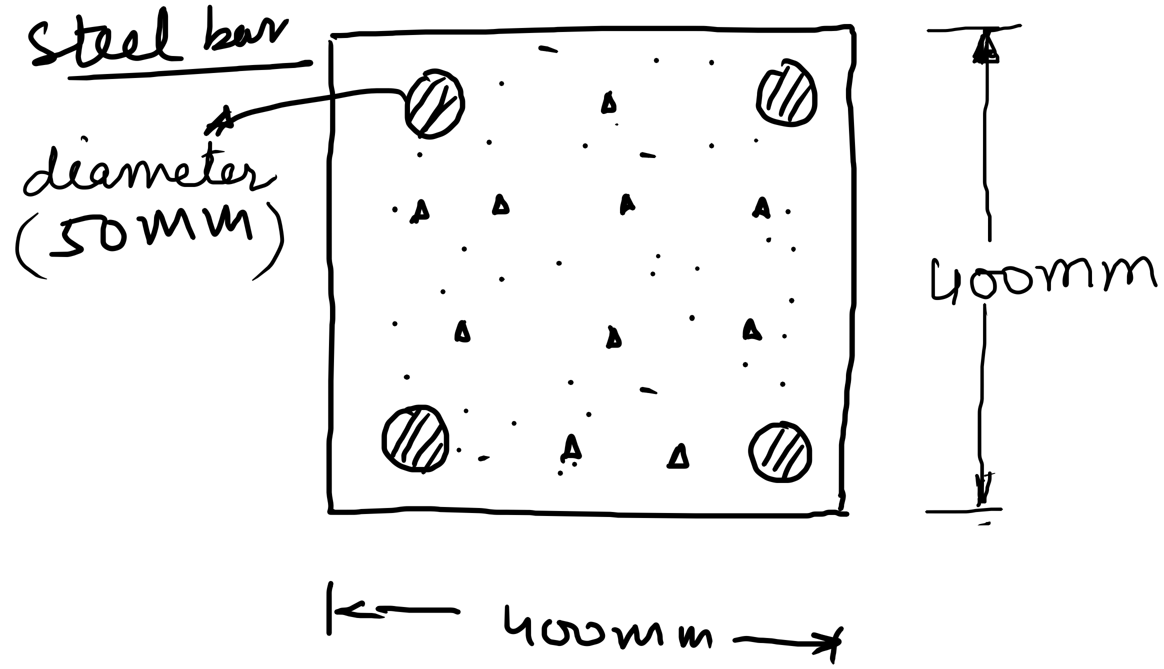
$$e_1 = e_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

Q: 1 If the column carries a compressive load of 300 kN determine :

(i) Load carried by concrete and steel bars.

(ii) The compressive stress produced in concrete and steel bars.



(i) cross-section of the column

Take, $E_s = 15 E_c$

Sol. Area of the column $= 400 \times 400 \times 10^{-6}$

$= 0.16\, m^2$

Area of steel bars (A_s) $= 4 \times \frac{\pi}{4} (0.05)^2$

$= 0.00785\, m^2$

$\therefore$ Area of concrete, $A_c = 0.16 - 0.00785 = 0.1521 m^2$

$$e_s = e_c$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\boxed{E_s = 15 E_c}$$

$$\boxed{\sigma_s = 15 \sigma_c} \quad \text{--- (1)}$$

Load shared by four steel bars + Load shared by concrete = 300 kN

$$P_s + P_c = 300 \text{ kN}$$

$$\sigma_s A_s + \sigma_c A_c = 300 \text{ kN} \quad \text{--- (2)}$$

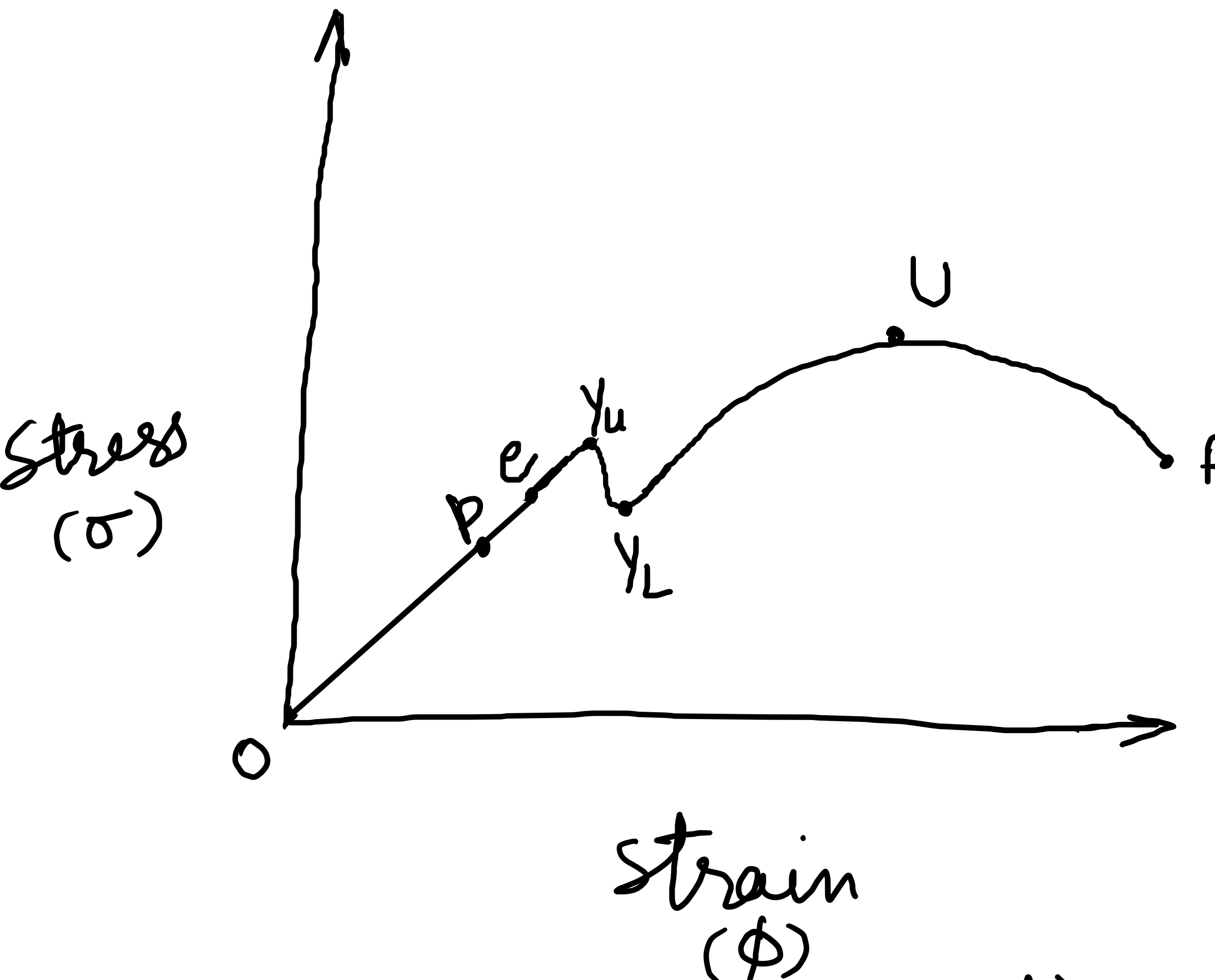
$$15 \sigma_c \times 0.00785 = \sigma_c \times 0.1521$$

$\sigma_c = 1.11 MN/m^2 \qquad \sigma_s = 15 . \sigma_c = 16.65 MN/m^2$

$$P_s = \sigma_s \cdot A_s = 131\ kN$$

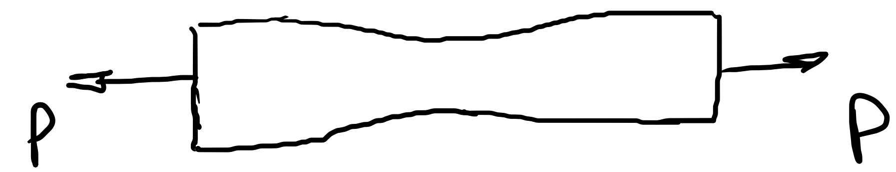
$$P_c = \sigma_c \cdot A_c = 169\ kN$$

Stress-strain curve for a Ductile Material

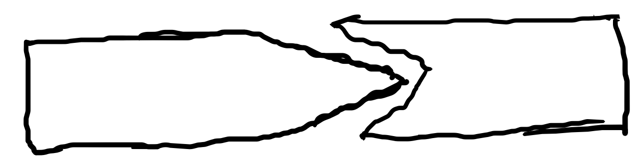


p – limit of proportionality
e – elastic limit
Y_u – upper yield point
Y_L – Lower yield point
U – ultimate stress
F – Fracture point

(i) Before loading

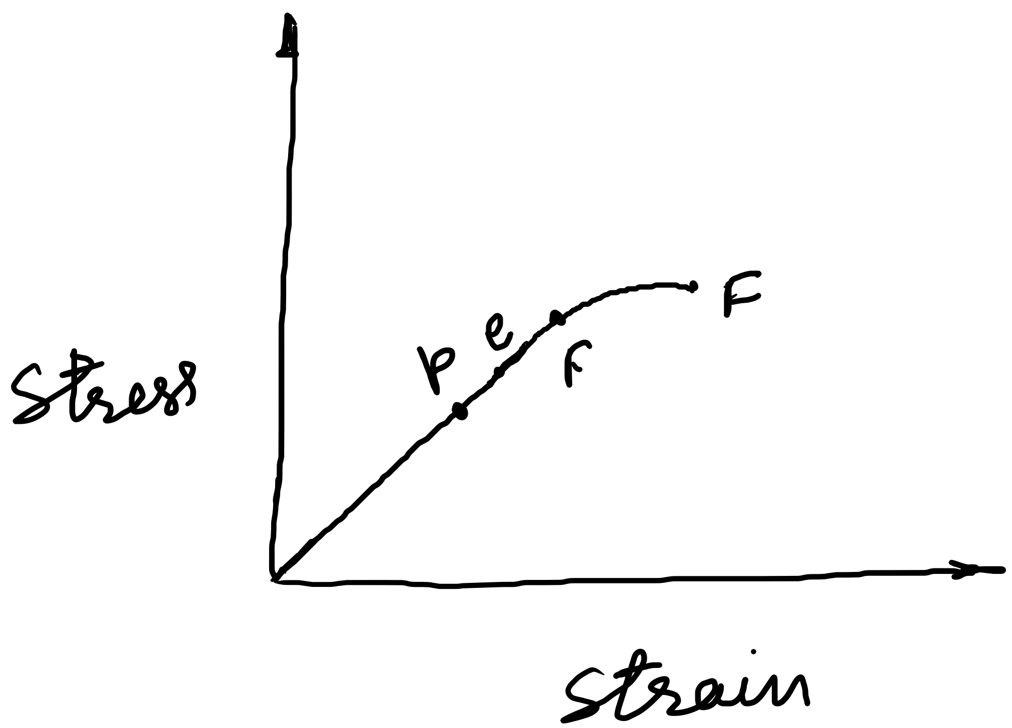


(ii) Neck formation



(iii) cup/cone fracture
or ductile fracture

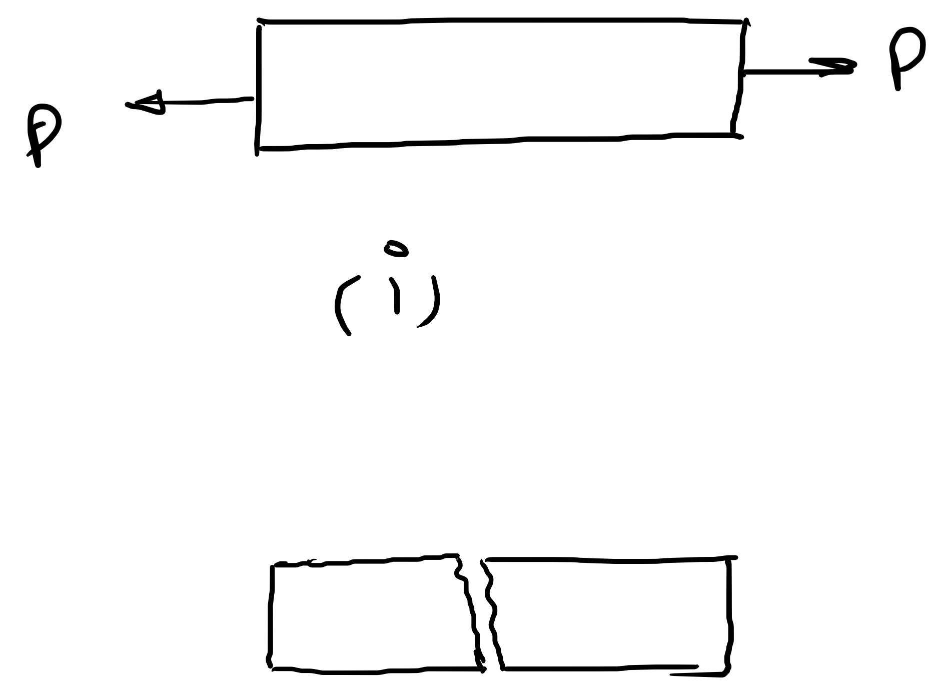
Stress - strain curve for a Brittle Material



p – limit of proportionality

e – elastic limit

f – fracture point



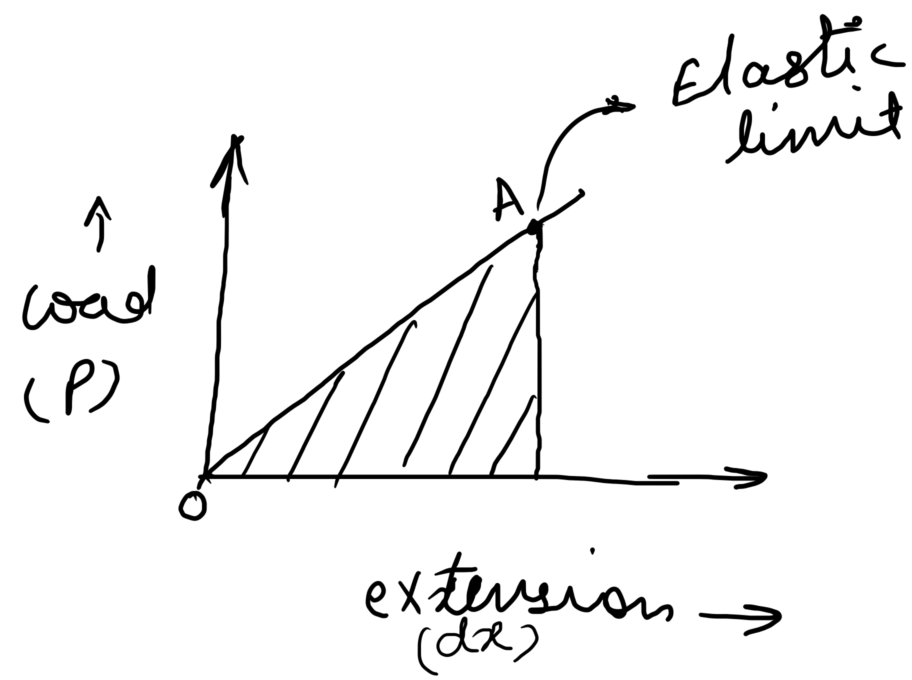
(i)

(ii) Brittle fracture

(1.) Strain energy:

- Resilience = Total energy stored in the material within elastic limit
- Proof Resilience = Max. energy stored in the material within elastic limit.
- modulus of Resilience = $\dfrac{\text{Proof resilience}}{\text{Volume of Body}}$

Strain energy stored in a body when load is applied gradually



work done by load P = Area of load-extension curve

$$W = \frac{1}{2} \times dx \times P$$

$$= \frac{1}{2} \cdot \frac{\sigma}{E} \cdot L \times \sigma \times A$$

$$= \frac{\sigma^2}{2E} (A \times L)$$

Strain energy, $U = \dfrac{\sigma^2}{2E} \times$ Volume of the body

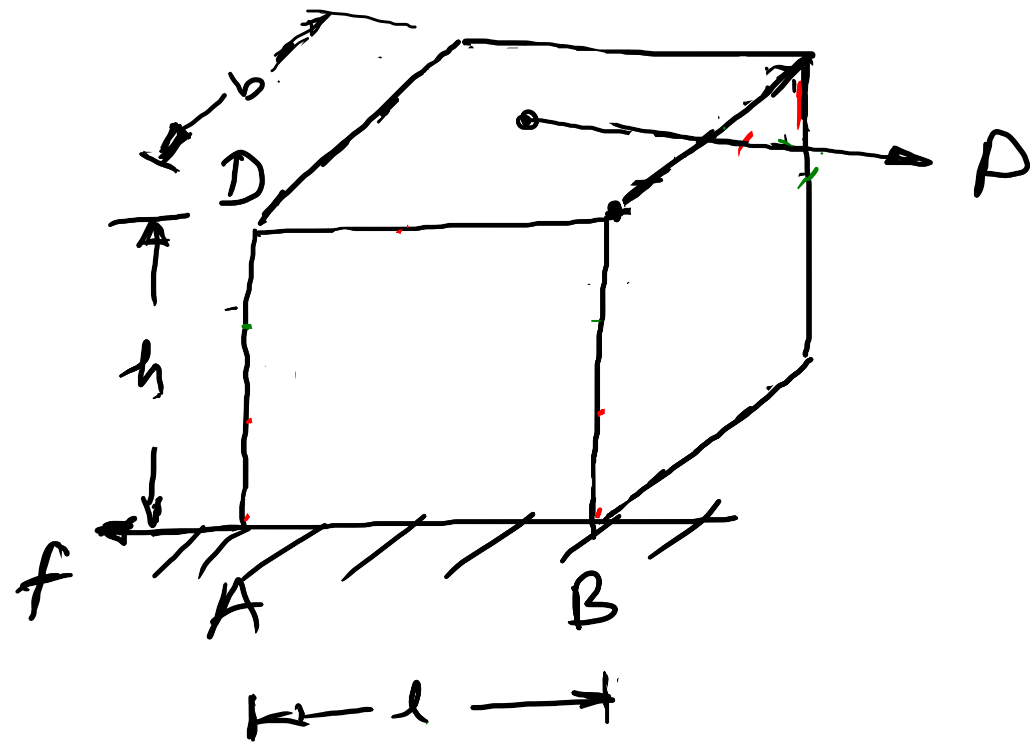
$$\sigma = \frac{P}{A} \quad \text{--- (1)}$$

$$e = \frac{dx}{L}$$

$$e = \frac{\sigma}{E}$$

$$\therefore dx = \frac{\sigma L}{E} \quad \text{--- (2)}$$

(2) Shear strain energy



$$\text{Shear stress } \tau = \frac{\text{shear force}}{\text{shear Area}} = \frac{\vec{P}}{A}$$

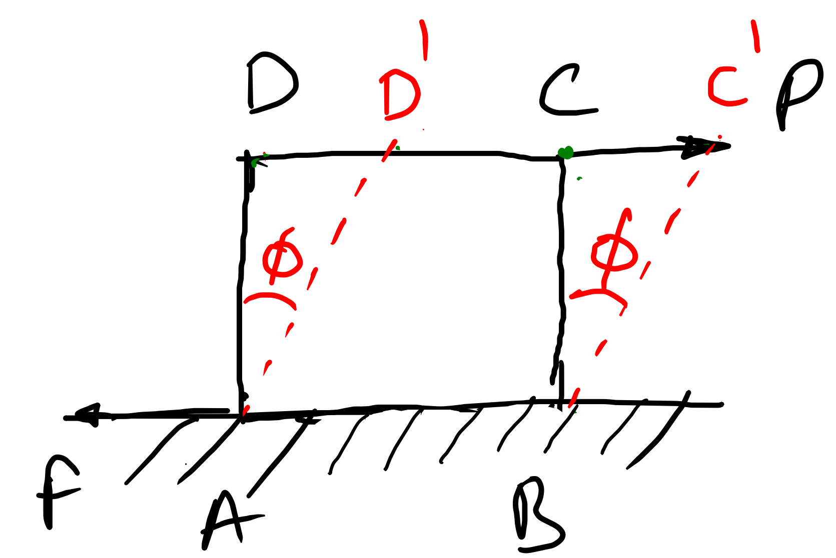
$$\text{shear Area } A, = l \times b$$

$$P = \tau \times l \times b \quad \text{—} (1)$$

$$\text{Shear strain}, \phi = \frac{CC'}{CB}$$

$$\boxed{\tan\phi \simeq \phi}$$

$$CC' = \phi \cdot CB = \phi \cdot h \quad \text{—} (2)$$



If we apply shear force P gradually.

$$\text{Average load} = \frac{0+P}{2} = \frac{P}{2}$$

Work done by load = Average load × distance moved

$$= \frac{P}{2} \times CC'$$

$$= \frac{\tau \times l \times b}{2} \times \phi \times h$$

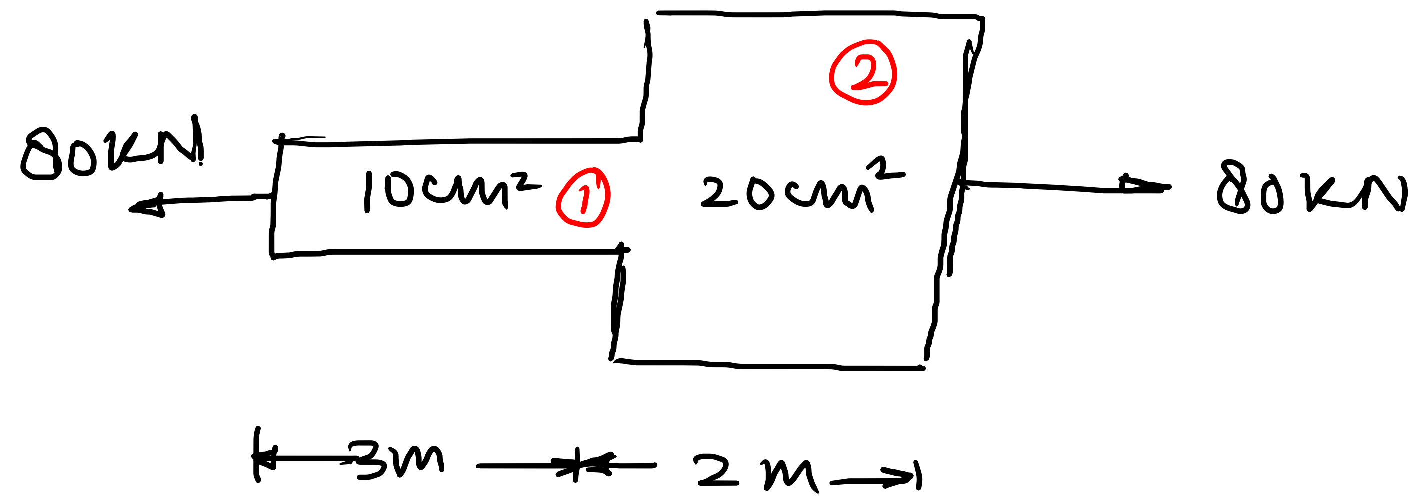
$$\boxed{\phi = \frac{\tau}{C}}$$

$$= \frac{\tau}{2} \times \frac{\tau}{C} (l \times b \times h)$$

$\therefore$ Shear strain energy $= \dfrac{\tau^2}{2C} \times$ volume of the Body

Q. 1 Find the total strain energy stored in the bar. compare this value with that obtained in a uniform bar of same length, same volume under the same load.

Take, $E = 2\times10^5\ N/mm^2$

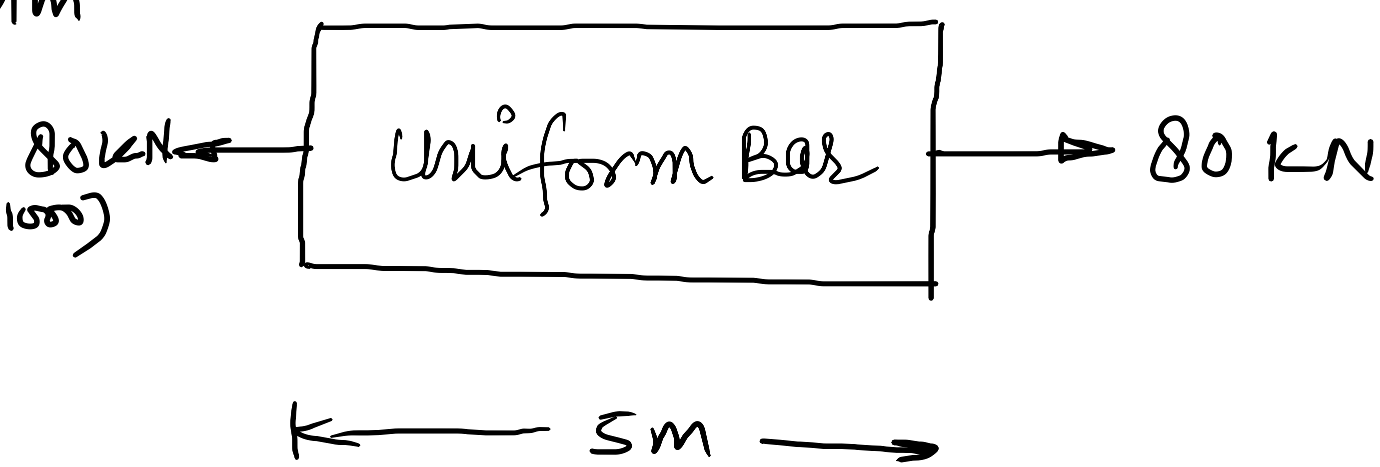


(i)

Sol^n:

$$\sigma_1 = \frac{P}{A_1} = \frac{80\times10^3}{10\times10^2} = 80\ N/mm^2$$

$$\sigma_2 = \frac{P}{A_2} = \frac{80\times10^3}{20\times10^2} = 40\ N/mm^2$$



(ii)

$$U_1 = \frac{\sigma_1^2}{2E}\times V_1 = \frac{80^2}{2\times2\times10^5}\times(10\times10^2\times3\times1000)$$

$$= 48000\ N\text{-}mm$$

$$= 48\ N\text{-}m$$

$$U_2 = \frac{\sigma_2^2}{2E} \times V_2 = \frac{(40)^2}{2 \times 2 \times 10^5} \times (20 \times 10^2 \times 2000) = 16 \text{ N-m}$$

$\therefore$ Total strain energy $U = U_1 + U_2 = 48 + 16 = 64$ N-m

(ii) <u>uniform Bar</u>

$V_1 = A_1 L_1 = 10 \times 300 = 3000 \text{ cm}^3$

$$\boxed{V = V_1 + V_2}$$

$V_2 = A_2 L_2 = 20 \times 200 = 4000 \text{ cm}^3$

Volume of uniform bar $V = V_1 + V_2 = 7000 \text{ cm}^3$.

Area of uniform bar, $A = \dfrac{V}{L} = \dfrac{7000}{500} = 14 \text{ cm}^2$

stress in uniform bar, $\sigma = \dfrac{P}{A} = \dfrac{80 \times 10^3}{1400} = 57.143 \text{ N/mm}^2$

Strain energy in uniform bar, $U = \dfrac{\sigma^2}{2E} \times$ volume

$$= \frac{(57.143)^2}{2 \times 2 \times 10^5} \times 7 \times 10^6$$

$$= 57.143 \text{ N-m}$$