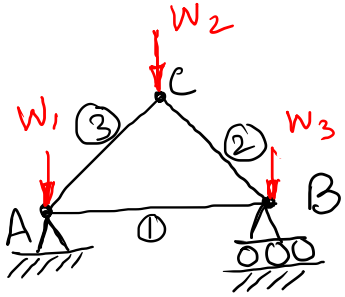
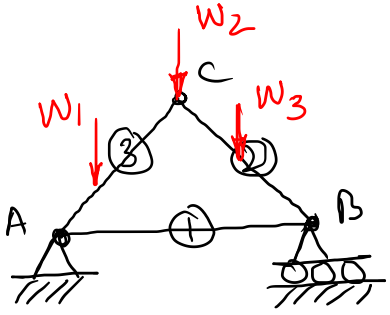
Analysis of plane Trusses

Engineering structures:

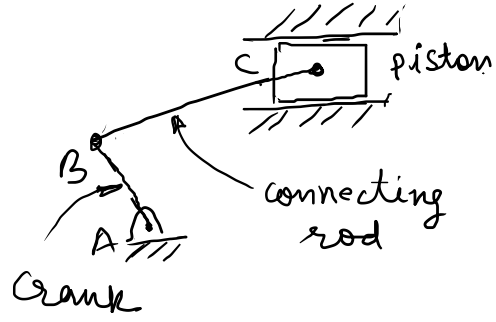
1. Truss



2. Frame



3. Machine



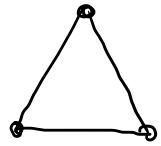
(A) Perfect Truss

$$m = 2j - 3$$

m = No. of members

j = No. of Joints

(i)



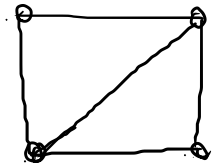
$m = 3$

$j = 3$

$3 = 2 \times 3 - 3$

$3 = 3$

(ii)



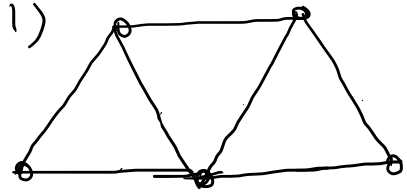
$m = 5$

$j = 4$

$5 = 2 \times 4 - 3$

$5 = 5$

(iii)



$m = 7$

$j = 5$

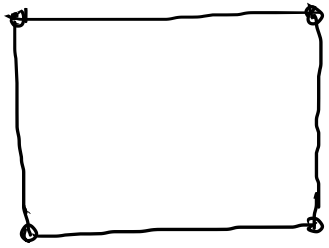
$7 = 2 \times 5 - 3$

$7 = 7$

(B) Imperfect / Deficient Truss

$$m \neq 2j-3$$

(i)



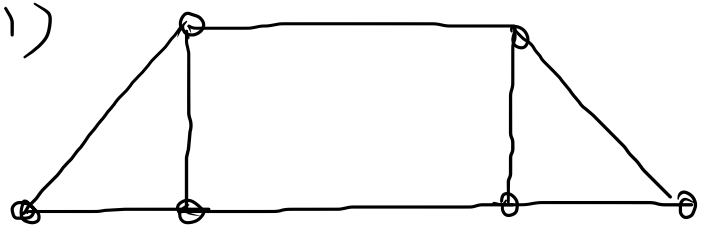
$m = 4$

$j = 4$

$4 \neq 2 \times 4 - 3$

$4 \neq 5$

(ii)



$m = 8$

$j = 6$

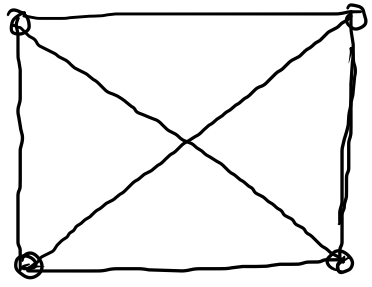
$8 \neq 2 \times 6 - 3$

$8 \neq 9$

(c) Redundant Truss

$$m \neq 2j - 3$$

(i)



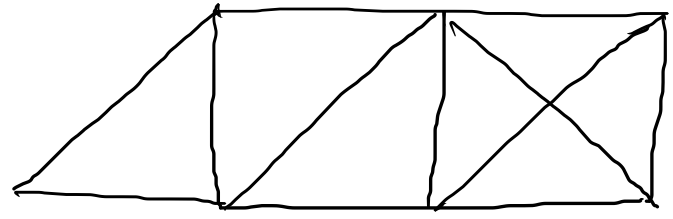
$m = 6$

$j = 4$

$6 \neq 2 \times 4 - 3$

$6 \neq 5$

(ii)



$m = 12$

$j = 7$

$12 \neq 2 \times 7 - 3$

$12 \neq 11$

Methods of Analysis

(1.) Analytical Methods
- (i) Method of joints
- (ii) Method of sections

(2.) Graphical Method

(i) Method of Joints

- Every joint is treated as a free body in equilibrium and we can apply equations of equilibrium.

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

Q:1 Determine the forces in all the members of a truss.

Sol:

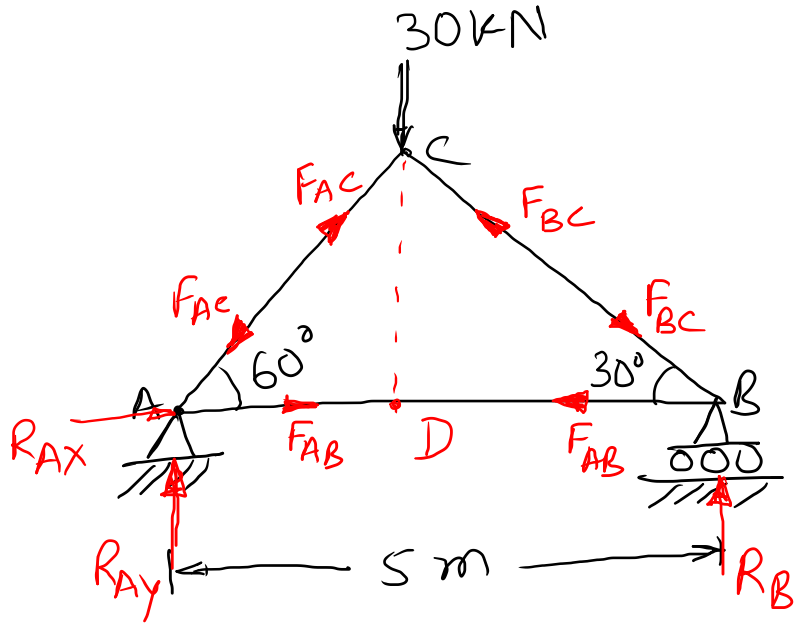
$\Sigma F_x = 0, \quad R_{AX} = 0$

$\Sigma F_y = 0, \quad R_{AY} + R_B = 30$

$\Sigma M_A = 0, \quad R_B \times 5 - 30 \times 1.25 = 0$

$\Rightarrow R_B = 7.5\,KN$

$R_{AY} = 22.5\,KN$



In ΔABC

$\cos 60^\circ = \frac{AC}{AB}$

$\Rightarrow AC = AB\cos 60^\circ$

$= 2.5\,m$

ΔADC

$\cos 60^\circ = \frac{AD}{AC}$

$AD = AC\cos 60^\circ$

$= 1.25\,m$

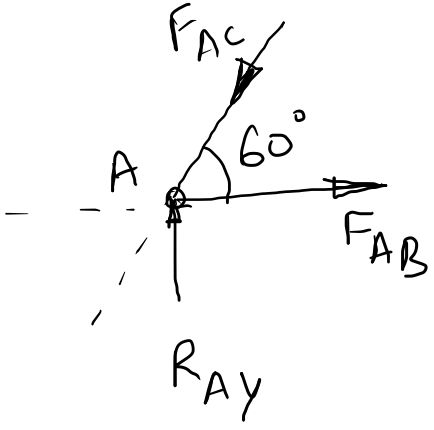
consider Joint 'A'

$\Sigma F_x = 0, \quad F_{AB} - F_{AC}\cos 60^\circ = 0$

$\Sigma F_y = 0, \quad R_{AY} - F_{AC}\sin 60^\circ = 0$

$\Rightarrow F_{AB} = 12.99\,KN$ (Tensile)

$F_{AC} = 25.97\,KN$ (compressive)

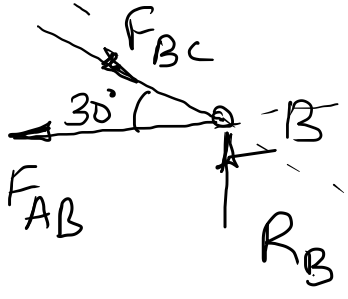


consider Joint 'B'

$\Sigma F_x = 0, \quad F_{BC}\cos 30^\circ - F_{AB} = 0$

$\Sigma F_y = 0, \quad R_B - F_{BC}\sin 30^\circ = 0$

$\Rightarrow F_{BC} = 15\,KN$ (compressive)



Q: 2 Determine forces in members of the truss.

Sol^{n}:

$$\Sigma F_x = 0, \quad R_{BX} - 15 = 0$$

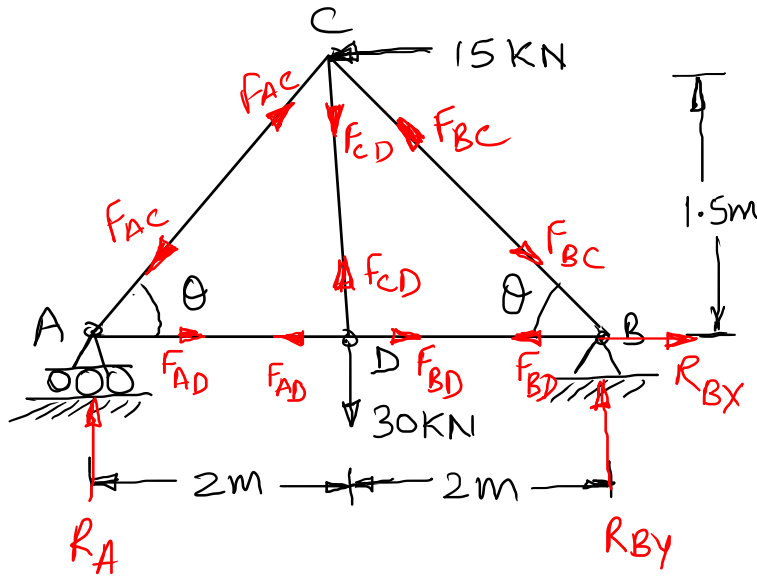
$$R_{BX} = 15\,KN$$

$$\Sigma F_y = 0, \quad R_A + R_{BY} - 30 = 0$$

$$\Sigma M_B = 0, \quad -R_A \times 4 + 15 \times 1.5 + 30 \times 2 = 0$$

$$R_{BY} = 9.375\,KN$$

$$R_A = 20.625\,KN$$



In Δ ADC

$$AC = \sqrt{AD^2 + DC^2} = 2.5\,m$$

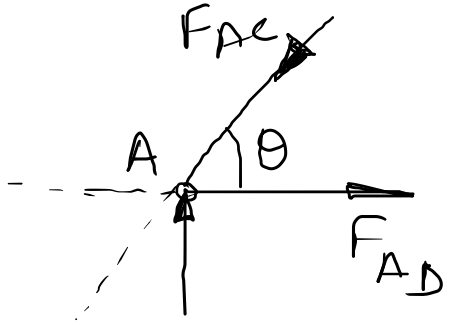
$$\sin\theta = 0.6$$

$$\cos\theta = 0.8$$

consider Joint 'A'

$\Sigma F_x = 0, \quad F_{AD} - F_{AC}\cos\theta = 0$

$\Sigma F_y = 0, \quad R_A - F_{AC}\sin\theta = 0$

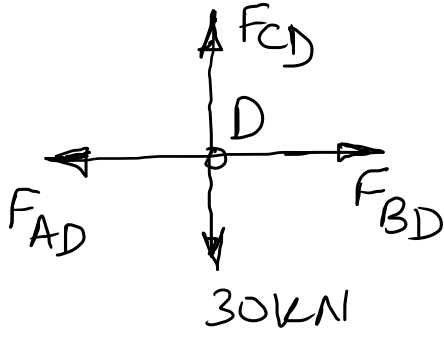


$F_{AD} = 27.50\,KN(T), \quad F_{AC} = 34.38\,KN(C)$

Consider Joint 'D'

$\Sigma F_x = 0, \quad F_{BD} = F_{AD} = 27.50\,KN(T)$

$\Sigma F_y = 0, \quad F_{CD} = 30\,KN(T)$

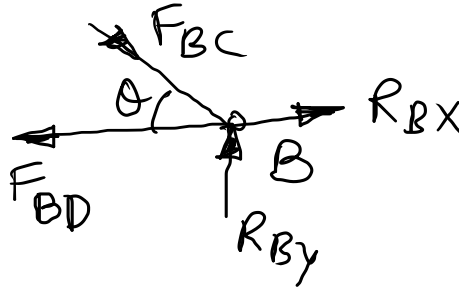


consider Joint 'B'

$\Sigma F_x = 0, \quad R_{BX} - F_{BD} + F_{BC}\cos\theta = 0$

$\Sigma F_y = 0, \quad R_{BY} - F_{BC}\sin\theta = 0$

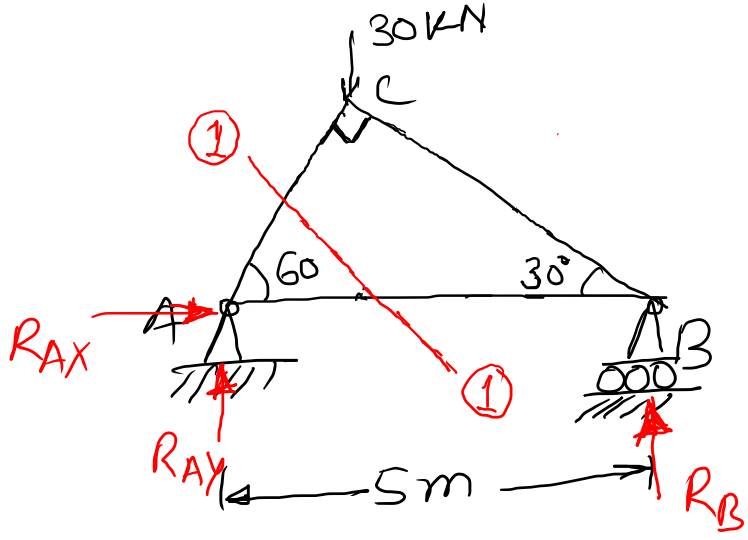
$F_{BC} = 15.63\,KN(C)$



(ii) Method of sections

- The truss is split into two parts by passing an imaginary cutting / section plane.
- The section plane should not cut more than three members in which the forces are to be determined.
- Do not pass section plane from the joints.
- This method is particularly convenient in the analysis of large truss in which forces in only few members are required.

Q:1 Determine the forces in members of the truss



Sol^n.

$\Sigma F_x = 0$, $R_{AX} = 0$

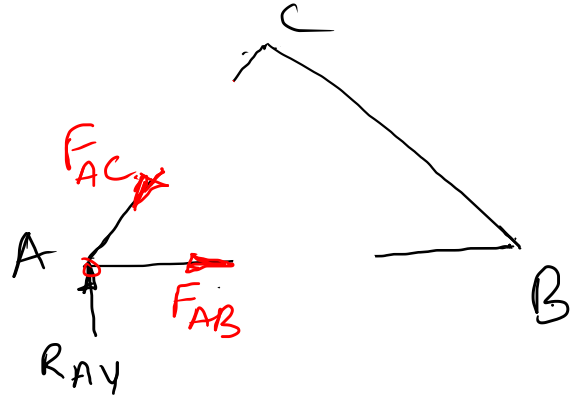
$\Sigma F_y = 0$, $R_{AY} + R_B = 30$

$\Sigma M_A = 0$, $R_B \times 5 - 30 \times 1.25 = 0$

$\Rightarrow R_B = 7.5$ KN

$R_{AY} = 22.5$ KN

Let the cutting plane (1)-(1) be passed which devides the truss into two parts.

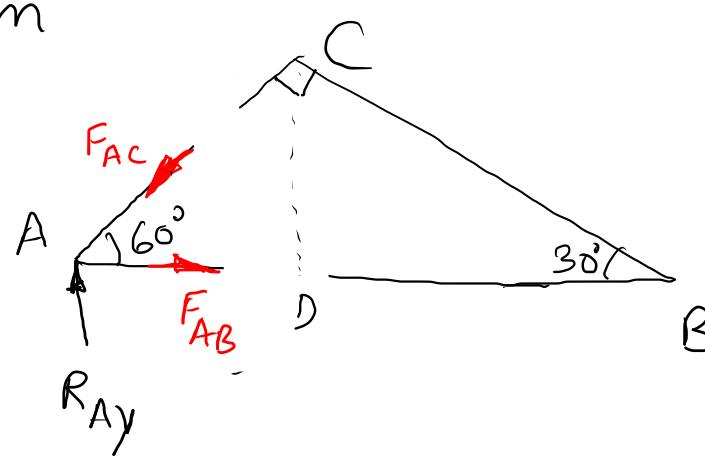


Left part of the truss is in equilibrium under the action of:

(i) Reaction, R_{AY}

(ii) unknown force F_{AB}

(iii) unknown force F_{AC}



Taking moment of these forces about 'B'

$$\Sigma M_B = 0, \quad -R_{AY} \times AB + F_{AC} \times BC = 0$$

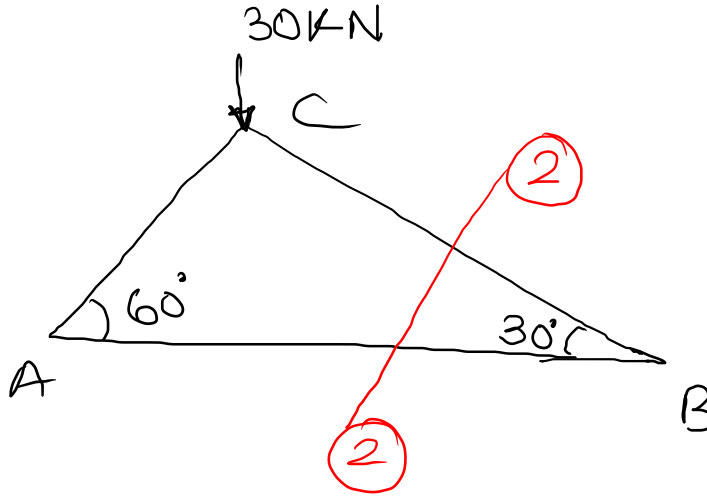
$$\Rightarrow F_{AC} = 25.98\,KN\,(C)$$

Taking moment of these forces about 'C'

$$\Sigma M_C = 0, \quad -R_{AY} \times AD + F_{AB} \times CD = 0$$

$$\Rightarrow F_{AB} = 12.99\,KN\,(T)$$

To determine force in member BC, another section ②-② is passed which cuts the members AB & BC.

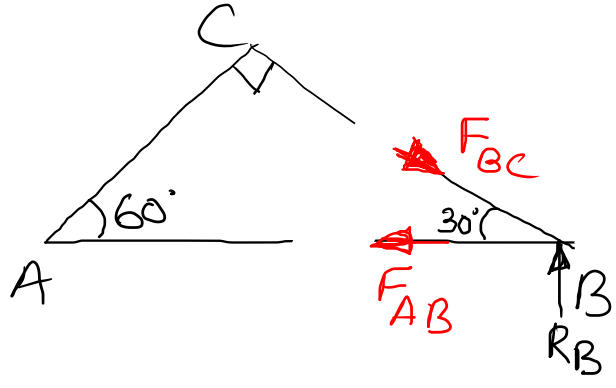


consider equilibrium of right part of the truss under the action of:

(i) Reaction, R_B

(ii) force F_{AB}

(iii) unknown force F_{BC}



Taking moment of these forces about 'A':

$$\Sigma M_A = 0, \quad R_B \times AB - F_{BC} \times AC = 0$$

$$\Rightarrow F_{BC} = 15\,KN\,(C)$$