UNIT-I

Physics

- Classical mechanics (Newtonian mechanics)
- Quantum mechanics (Modern Physics)

Classical Mechanics:-

(i) classical mechanics deals with macroscopic object

(ii) classical mechanics is based on Newton's law

(iii) In classical mechanics future behaviour of the particle can be completely known if position of particle is known.

Quantum Mechanics:-

(i) Quantum mechanics deals with microsopic particle

(ii) In Quantum mechanics there is always some uncertainity in the determination of position and momentum of particle.

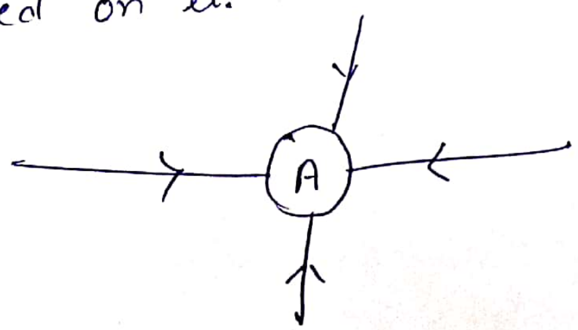
Newton's law of Motion:-

(i) First law of Motion:-

A body must continue in its state of rest or of uniform motion moving along a straight line unless acted upon by an external force.

for E.g

Suppose a body A, and three forces F_1, F_2 and F_3 are acted on it.



$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

or we can say that body is in equilibrium.

$$\left.\begin{array}{l} \text{Rest} \rightarrow \text{Rest} \\ \text{Motion} \rightarrow \text{Motion} \end{array}\right\} \text{Until no External force is applied on it.}$$

Second Law of Motion:-

The rate of change of momentum is proportional to the impressed force and takes place in the direction of force.

$$F_{Net} = \frac{\Delta P}{\Delta t}$$

Third Law of Motion:-

To Every action there is equal and opposite reaction.

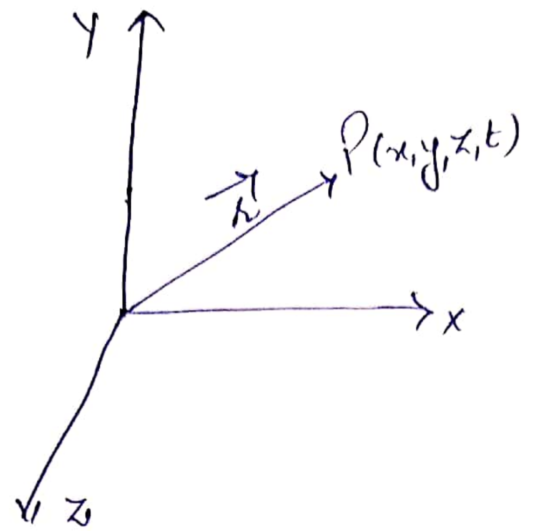
$$\vec{F_{12}} = \vec{F_{21}}$$

Frame of Reference

It is a coordinated System

or.

A coordinate system attached to a rigid body and describe the position of any particle in space relative to it, then such a coordinate system is known as frame of reference.



Space-time frame of reference.

Frame of Ref

inertial or unaccelerated frame of reference

Non-inertial or accelerated frame of reference

Inertial frame of reference:-

(i) acceleration ($a = 0$), velocity (v = constant)

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 0, \quad (\vec{F} = m\vec{a} = 0)$$

(ii) Newton's law of inertia is valid

Non- Inertial frame of reference:-

(i) Newton's law of inertia is not valid

(ii) accelerated ($a \neq 0$, velocity = change or variable)

(iii) Body Rotating.

What is theory of Relativity:-

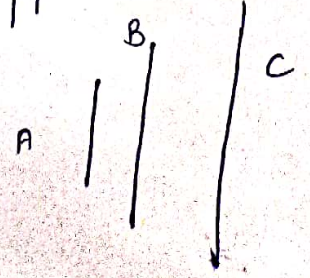
The theory which deals with the relativity of motion and rest.

Theory of Relativity

- Special theory of relativity
- general theory of relativity

All the measurements in this universe are relative

length:- Suppose we draw three lines A, B, C



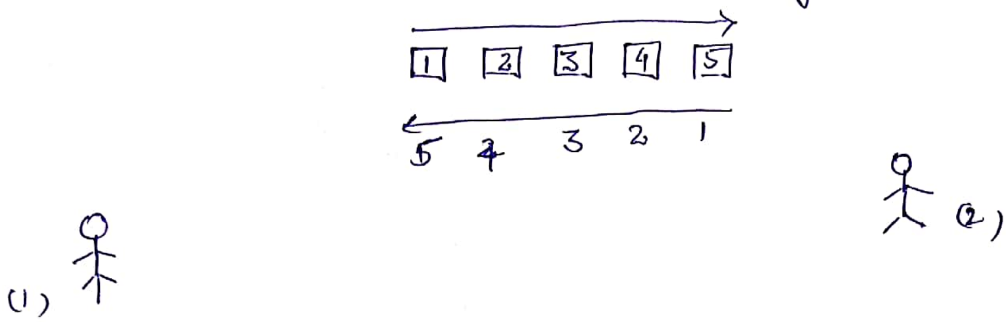
A is smaller than B and C

→ B is greater than A but smaller than C

→ C is greater than B and C.

Position:-

Suppose five students seating on a bench



Two peoples are standing on the opposite direction of bench

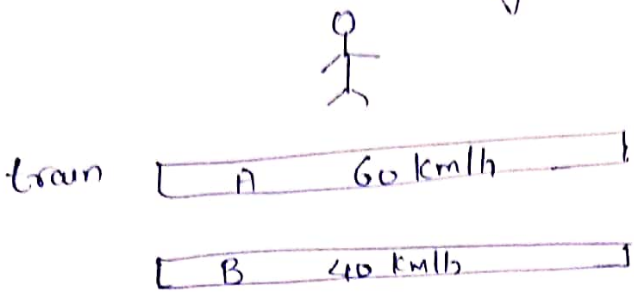
E.g Suppose two friends standing on the opposite banks of a river facing each other. There is one house on the bank of the river. One friend says that it is toward left while other friend says it is towards right.

Size:-

Suppose we two balls one is normal ball or hockey ball and other is football, so we can say that football is bigger than hockey

Velocity:-

Suppose that a man standing on platform



for a man speed of train is A = 60 km/h
B = 40 km/h.

a man seating on a train A

according to his platform is going backwards from 60 km/h

Speed of train B is les 20 km/h less than A

a man seating on train B

according to this speed of train A is 20 km/h more than B

Time:-

Suppose 5 people live in different five country

India USA UK China

there is time lag in all the five country

Event:-

The interactions of the objects are termed as events or happening

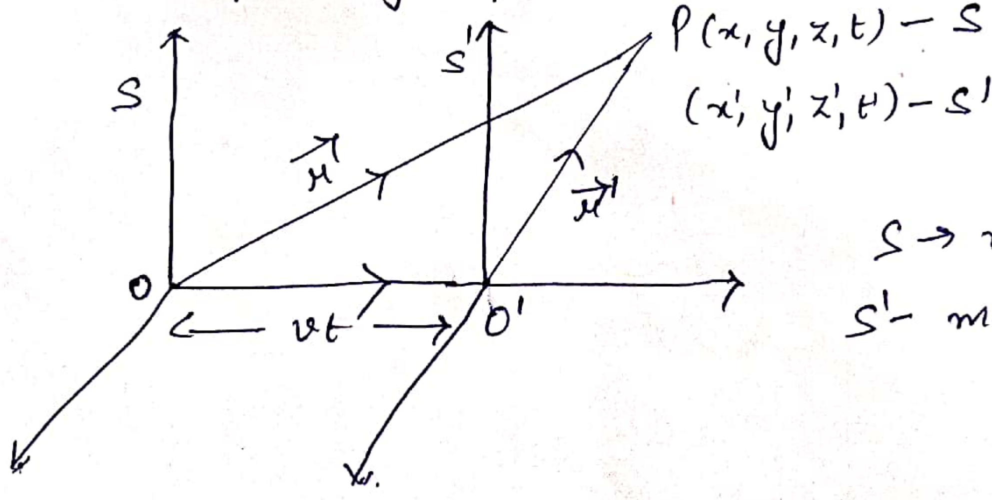
Event does not belongs to a particular inertial reference frame. An event is just something that happens.

Galilean transformation Equation

→ A point or an event is observed from different frames. then it has different coordinates

→ The Equations which ~~deals~~ relate the coordinate of two inertial frames of reference are known as Galilean transformation equation

→ Suppose two frame of reference S and S'



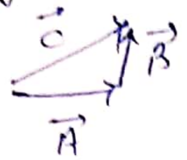
$S \rightarrow$ rest

$S' -$ motion

$S \rightarrow (x, y, z, t)$

$S' \rightarrow (x', y', z', t')$

According to triangle law of vector



$$\vec{r} = \vec{r}' + \vec{v}t$$

$$\vec{r}' = \vec{r} - \vec{v}t$$

$\vec{r}$ and $\vec{r}'$ are the position vector

$$\vec{r} = (x\hat{i} + y\hat{j} + z\hat{k})$$

$$\vec{r}' = (x'\hat{i} + y'\hat{j} + z'\hat{k})$$

$$x'\hat{i} + y'\hat{j} + z'\hat{k} = x\hat{i} + y\hat{j} + z\hat{k} - vt\hat{i}$$

$$= (x - vt)\hat{i} + y\hat{j} + z\hat{k}$$ relating L.H.S to R.H.S

Expressing in the coordinate from

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$ Galilean transformation equations

Invariant :-

A function, quantity or property which remains unchanged when a specified transformation is applied.

Consequences of Galilean transformation Equation:

(1) Transformation of length:-

S and S' are two inertial frame of reference.

S' frame is moving with velocity v relative to S along the x-axis.

S → (x_1, y_1, z_1) and (x_2, y_2, z_2). are coordinate of rods in S frame.

(x_1', y_1', z_1') and (x_2', y_2', z_2') are coordinate of rods in S' frame.

Length in S frame-

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

using distance formula.

Length in S' frame

$$l' = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1') + (z_2' - z_1')^2}$$

$$= \sqrt{(x_2 - vt - x_1 + vt)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(x_2' - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$l' = l$$

Transformation of velocity :-

u and u' are the velocity in S and S' frame of reference

$$x' = x - vt$$

diff this Eq w.r.t time

$$\frac{d}{dt}(x') = \frac{d}{dt}(x - vt)$$

$$\frac{dx'}{dt} = \frac{dx}{dt} - \frac{vdt}{dt} \quad (v \text{ is constant})$$

$$\boxed{u_x' = u_x - v}$$

before transformation $\Downarrow$ $\Downarrow$ after transformation

So velocity is variant or not envariant

Transformation of acceleration :-

We know from velocity transformation under G.T.

$$x' = x - vt$$

$$\frac{d(x')}{dt} = \frac{dx}{dt} - \frac{v\,d(t)}{dt} \quad (v \text{ is constant})$$

$$u_x' = u_x - v \quad (\text{velocity transformation Eq})$$

$$\frac{du_x'}{dt} = \frac{du_x}{dt} - \underbrace{\frac{dv}{dt}}_{=0} \quad \left\{ \text{acceleration } a = \frac{d^2x}{dt^2} \right\}$$

$$a_x' = a - 0 \quad \{\text{acceleration is invariant}\}$$

$$\boxed{a_x' = a}$$

Prove that Newton's law of motion invariant.

Newton's law of motion:-

Let F be the force acting on a mass m in frame S.

So $F = ma$
m ↳ Mass, a ↳ acceleration

F' be the force acting on a mass m and acceleration a' in frame S'

$F' = ma'$
a' ↳ acceleration in S'

as we prove earlier

acceleration is invariant so.

$a' = a$.

$F = F'$

the law is invariant to Galilean transformation.

(ii) The law of Conservation of momentum:-

If a number of particles collide against each other, their total momentum after collision is the same as their total momentum before collision, irrespective of the collision being elastic or inelastic.

Two particles of masses m_1 and m_2 in frame S moving with velocity u_1 and u_2 respectively before collision and with velocity v_1 and v_2 after collision

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad -(1)$$

u_1' and u_2' are the velocity before collision and v_1' and v_2' are the velocity after collision in S' frame

$$m_1 u_1' + m_2 u_2' = m_1 v_1' + m_2 v_2' \quad -(2).$$

$$u_1 = u_1' + v, \quad u_2 = u_2' + v, \quad v_1 = v_1' + v, \quad v_2 = v_2' + v.$$

Put these value in eq (2)

$$m_1 (u_1' + v) + m_2 (u_2' + v) = m_1 (v_1' + v) + m_2 (v_2' + v)$$

$$m_1 u_1' + \cancel{m_1 v} + m_2 u_2' + \cancel{m_2 v} = m_1 v_1' + \cancel{m_1 v} + m_2 v_2' + \cancel{m_2 v}$$

$$\boxed{m_1 u_1' + m_2 u_2' = m_1 v_1' + m_2 v_2'}$$

This proves that law of conservation of momentum is also invariant Galilean transformation.

Law of Conservation of ~~mom~~ energy:-

Two particles of masses m_1 and m_2 in frame S moving with velocity u_1 and u_2 respectively before collision and with velocity v_1 and v_2 after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + E \quad -(1)$$

ly u_1' and u_2' are the velocity before collision and v_1' and v_2' are the velocity after collision in S' frame

$$\frac{1}{2}m_1u_1'^2 + \frac{1}{2}m_2u_2'^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 + E \quad -(2)$$

Where E is part of the Energy of the particles (before the collision appearing. in some other form)

$$\frac{1}{2}m_1(u_1'^2+v)^2 + \frac{1}{2}m_2(u_2'+v)^2 = \frac{1}{2}m_1(v_1'+v)^2 + \frac{1}{2}m_2(v_2'+v)^2$$
$$+E, \quad -(3)$$

L.H.S

$$\frac{1}{2}\left[m_1\{u_1'^2+v^2+2u_1'v) + m_2(u_2'^2+v^2+2u_2'v)\right]$$

$$\frac{1}{2}\{(m_1u_1'^2 + m_1v^2 + 2u_1'vm_1) + (m_2u_2'^2 + m_2v^2 + 2u_2'vm_2)\}$$

R.H.S

$$\frac{1}{2}\left[(m_1v_1'^2 + m_1v'^2 + 2m_1v_1'v) + m_2v_2'^2 + m_2v^2 + 2m_2v_2'v\right] + E$$

L.H.S

$$\frac{1}{2}\left\{\left(m_1 u_1'^2 + 2u_1' v m_1\right) + \left(m_2 u_2' + 2u_2' v m_2\right)\right\}$$

$$\frac{1}{2}\left[m_1 u_1'^2 + m_2 u_2'^2 + 2v\left(u_1' m_1 + u_2' m_2\right)\right] \quad -(4)$$

R.H.S

$$\frac{1}{2}\left\{m_1 v_1'^2 + m_2 v_2'^2 + 2v\left(m_1 v_1' + m_2 v_2'\right)\right\} \quad -(5)$$

Earlier we prove that law of Conservation of momentum is invariance

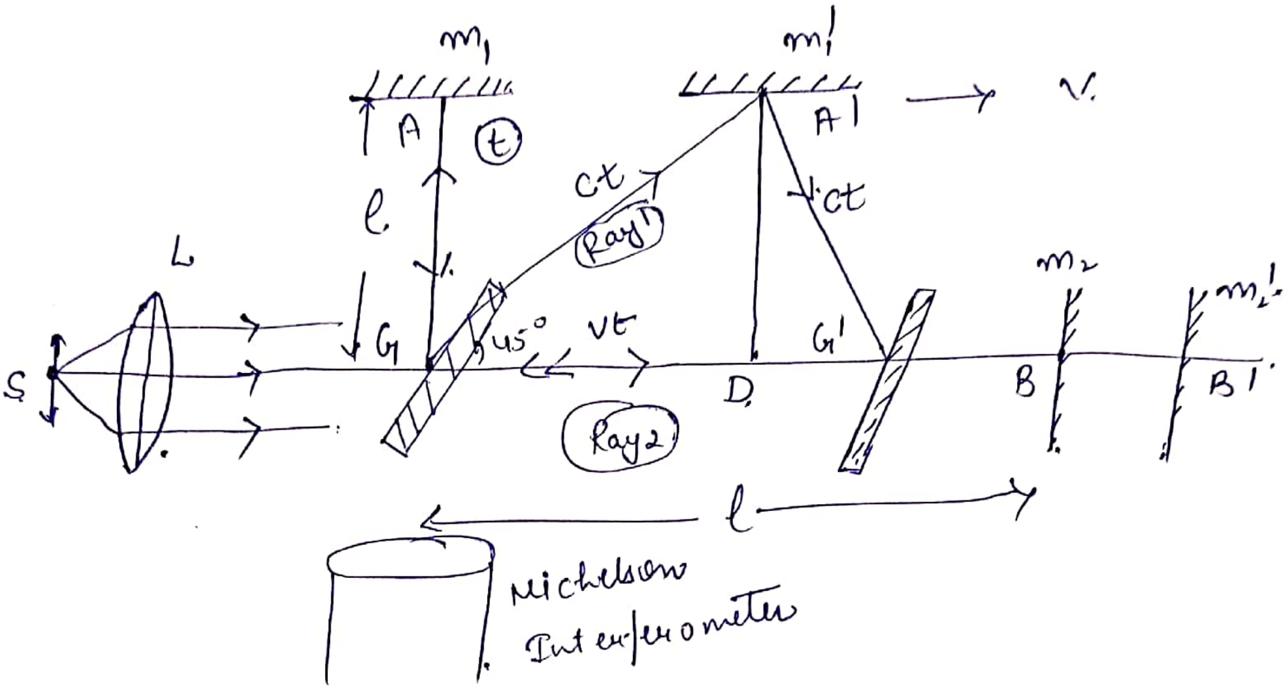
so $\quad m_1 v_1' + m_2 v_2' = m_1 u_1' + m_2 u_2'$

$$\boxed{\frac{1}{2}\left\{m_1 u_1'^2 + m_2 u_2'^2\right\} = \frac{1}{2}\left\{m_1 v_1'^2 + m_2 v_2'^2\right\}}$$

So low of Conservation of energy is invariant to Galilean transformation.

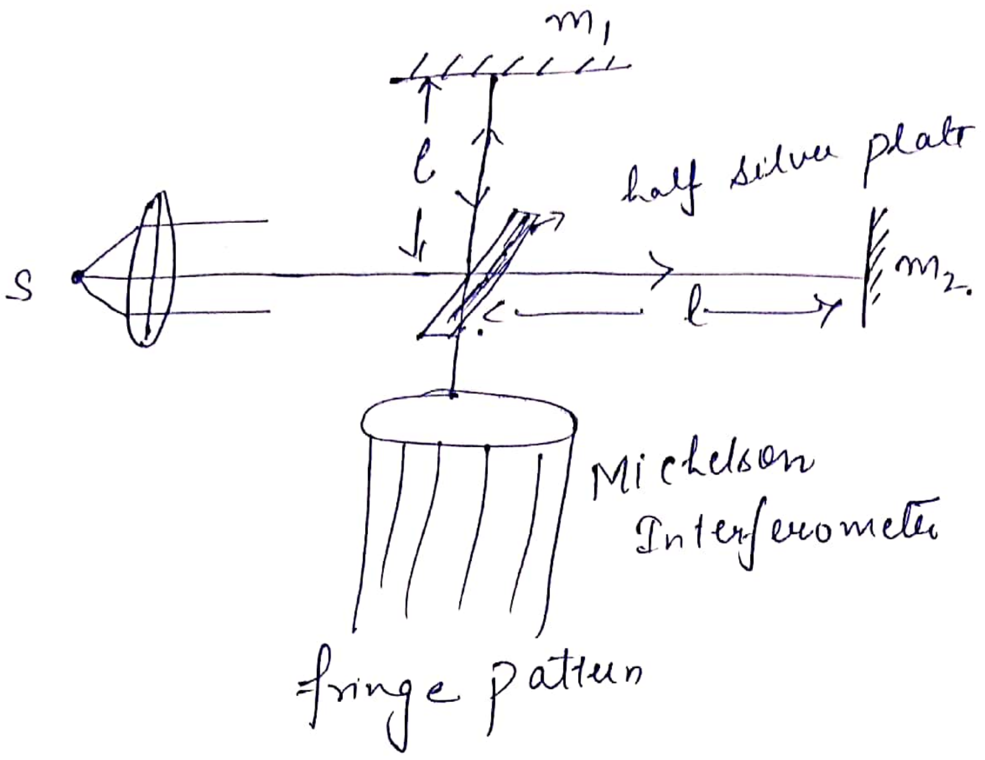
Michelson– Morley Experiment:-

To measure relative motion b/w Earth and Ether to verify the presence of Ether Medium.



Ether:- Ether is a hypothetical medium. It is used to. Explain how light waves reach us from the Sun. through free Space. Ether is considered as a transparent medium, highly elastic which provides a fixed. frame of reference called absolute frame of reference

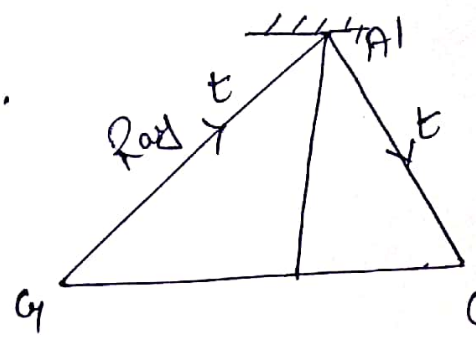
Actual Setup:-



S is a source of monochromatic light, a parallel beam from which falls upon a thin, parallel-sided glass plate G., thinly silvered on the back surface.

G and G'

Analysis.



Let us consider time taken by ray ① to travel.

path $GA'G'$ is t_1

Hence $\boxed{t_1 = 2t}$

from Δ GDA'

$$(GA')^2 = (GD)^2 + (A'D)^2$$

$$(ct)^2 = (vt)^2 + (l)^2$$

$$c^2t^2 = v^2t^2 + l^2$$

$$c^2t^2 - v^2t^2 = l^2$$

$$t^2 = \frac{l^2}{c^2 - v^2}$$

$$t^2 = \frac{l^2}{c^2\left(1 - \frac{v^2}{c^2}\right)}$$

$$\boxed{t = \frac{l}{c\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$t = \frac{l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

using Binomial theorem and neglecting higher term

$$t = \frac{l}{c}\left(1 + \frac{1v^2}{2c^2} + \cdots\right)$$

$$t = \frac{l}{c}\left(1 + \frac{v^2}{2c^2}\right) \quad - (2)$$

Put the value of t in eq (1)

$$t_1 = 2t$$

$$t_1 = \frac{2l}{c}\left(1 + \frac{v^2}{2c^2}\right) \quad - (3)$$

$$t_1 = \frac{2l}{c}\left(1+\frac{V^2}{2c^2}\right)$$

for Ray 2 GG'

Total time t_2 = Time (G to B and B' to G_1')

$$t_2 = T_{G \text{ to } B} + T_{B' \text{ to } G_1'}$$

apparatus $\xrightarrow{V}$

light $\longrightarrow$

$$= \frac{\text{Distance}}{\text{Velocity}} + \frac{\text{Distance}}{\text{Velocity}}.$$

$$= \frac{l}{c-V} + \frac{l}{c+V}$$

$$t_2 = \frac{l(c+V) + l(c-V)}{(c^2-V^2)}.$$

$$t_2 = \frac{lc + \cancel{lV} + lc - \cancel{lV}}{(c^2-V^2)}$$

$$t_2 = \frac{2lc}{(c^2-V^2)}$$

$$t_2 = \frac{2lc}{c^2\left(1-\frac{V^2}{c^2}\right)}$$

$$\boxed{t_2 = \frac{2l}{c\left(1-\frac{V^2}{c^2}\right)}}$$

$$t_2 = \frac{2l}{c}\left(1-\frac{V^2}{c^2}\right)^{-1}$$

using Binominal. Expansion.

Scanned with CamScanner

$$\boxed{t_2 = \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right)} \quad - \quad (3)$$

Time difference

$$\Delta t = t_2 - t_1$$

$$= \frac{2l}{c}\left(1+\frac{v^2}{c^2}\right) - \frac{2l}{c}\left(1+\frac{v^2}{2c^2}\right)$$

$$= \cancel{\frac{2l}{c}} + \frac{2l}{c}\left(\frac{v^2}{c^2}\right) - \cancel{\frac{2l}{c}} - \frac{lv^2}{c^3}$$

$$\boxed{\Delta t = \frac{lv^2}{c^3}}$$

time difference

$$\boxed{\Delta t = \frac{lv^2}{c^3}} \quad - \quad (4)$$

Path difference:-

$$\cancel{\Delta t = \frac{lv^2}{c^3}}$$

of path difference b/w two rays

$$n = \frac{\text{Path diff}}{\text{wavelength}}$$

$$n = \frac{c(\Delta t)}{\lambda}$$

$$= \frac{clv^2}{c^2\lambda} = \frac{lv^2}{c^2\lambda}$$

Now apparatus turned 90°.

Path diff equal to one wavelength λ, the pattern shifts through 1 fringe

$$n\lambda = \frac{2Du^2}{c^2}$$

$$\boxed{n = \frac{2Du^2}{c^2\lambda}}$$

orbital velocity u of the earth and hence that of the apparatus, equal to 18.5 miles or 30 km/sec

or 3×10^6 cm

$\lambda = 6\times10^{-5}$ cm

$$\frac{2\times1100\times9\times10^{12}}{9\times10^{20}\times6\times10^{-5}}$$

$$= 0.37$$

$$\approx 0.4$$

Theoretical calculation

In partical No fringe shift was found.

Negative Results of Michelson Morley Exp

fringe shift = 0.4 fringe

(1) Ether Drag theory → Earth rotates around the Sun and its own axis. Hence it drags ether so there is no relative motion b/w earth and Ether

(2) Lorentz - fitzgerald length Contraction Hypothesis
length of moving object will be contracted along the direction of motion by a factor $l = l_0\sqrt{1-\frac{v^2}{c^2}}$

(3) Constancy of Speed of light:
Speed of light always constant so the path direction b/w two rays. Hence no fringe shift occurs

Einstein Postulates of Relativity:-

Two Postulates

1st Postulates:

All Newton's law of motion and classical physics laws are valid and remain constant in all inertial frame of reference

2nd Postulates:-

(Constancy of speed of light)
Speed of light remains constant in all inertial frame of Reference

Einstein theory of Relativity	Galilean theory Relativity
(1) Speed of light constant	(1) Speed changes after transformation.
(2) low / High Speed valid	(2) low Speed (in compto light)
(3) Time is not Constant $t \neq t'$	(3) Time is Constant $t = t'$

Lorentz Transformation Equation:-

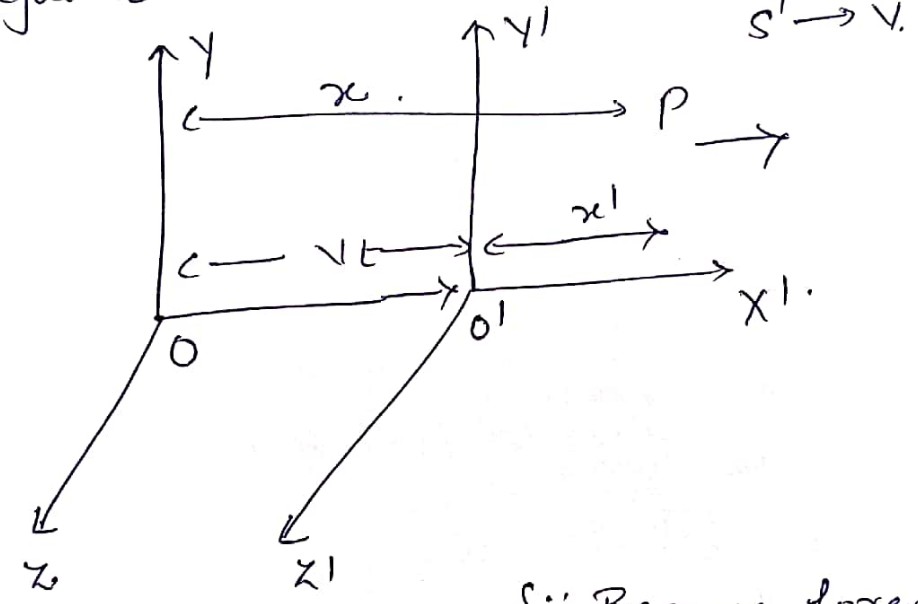
According to Lorentz Transformation. Eq the measurements in X direction made in frame S must be linearly Proportional to that made in S' frame. Hence a constant K should be there so.

Consideration

S → rest frame

S' → moving frame with V along X - axis

O and O' → Two observers S and S' frame of refrence



{∵ Because Lorentz consider that Einstein equation is true}

$x' = K(x - vt)$ —①

lly
inverse

$x = K(x' + vt')$ —②

Using Einsteins I^{st} Postulate:-

All physical laws are valid and same for all inertial frame of reference.

So Put value of x' (eq ①) in eq ②

$$x = k[k(x-vt)+vt']$$

$$x = k[k(x-vt)+vt']$$

$$\frac{x}{k} = [kx - kvt + vt']$$

$$vt' = \frac{x}{k} - kx + kvt$$

$$\cancel{t'} = \cancel{\frac{x}{kv}} - \cancel{\frac{kx}{k}} + \cancel{\frac{kvt}{k}}$$

$$\cancel{t'} = \frac{x}{kv} \cancel{-x+vt}$$

$$t' =$$

$$t' = \frac{x}{kv} - \frac{kx}{v} + \frac{kvt}{v}$$

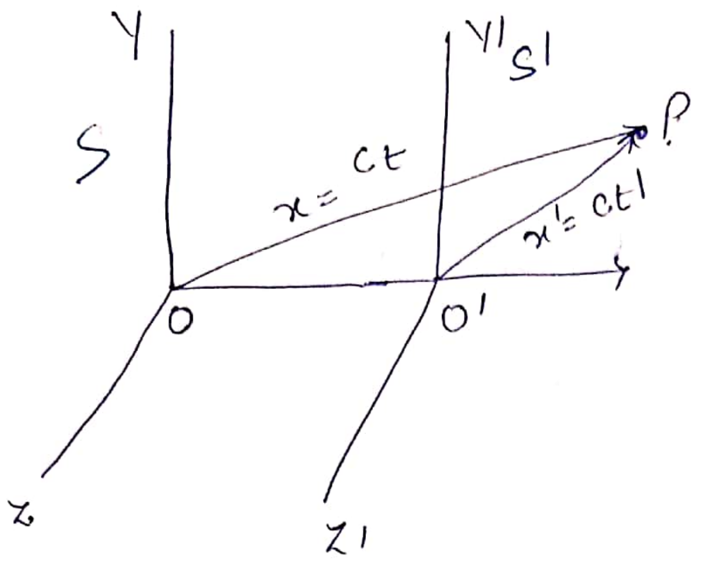
$$t' = kt + \frac{x}{kv} - \frac{kx}{v}$$

$$t' = kt - \frac{kx}{v} + \frac{x}{kv}$$

$$\boxed{t' = kt - \frac{kx}{v}\left(1 - \frac{1}{k^2}\right)} \quad ③$$

Using Einstein 2nd postulate

Constancy of Speed of light



Laser light fall at a point P from origin O and O'

Put these values in Eq ① and Eq (2) from eq ①

$$ct' = k(ct - vt) \quad — ④$$

from Eq ② $\{ x = k(x' + vt') \}$

$$ct = k(ct' + vt') \quad — ⑤$$

To find the value of Constant k.

Multiply ④ and ⑤

$$c^2tt' = k^2(ct - vt)(ct' + vt')$$

$$c^2\cancel{tt'} = k^2\cancel{tt'}(c - v)(c + v)$$

$$c^2 = k^2(c^2 - v^2)$$

$$k^2 = \frac{c^2}{c^2 - v^2}$$

$$k^2 = \frac{\cancel{c^2}}{\cancel{c^2}\left(1 - \frac{v^2}{c^2}\right)}$$

$$k = \frac{(c^2)^{1/2}}{\sqrt{c^2 - v^2}}$$

$$k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{--- (6)}, \quad \boxed{\frac{1}{k^2} = 1-\frac{v^2}{c^2}} \quad \text{--- (7)},$$

Put values from (6) and (7) into (1) and (3)

from Eq (1)

$$\boxed{x' = \frac{x-vt}{\sqrt{1-v^2/c^2}}} \quad \text{--- (8)}$$

from Eq (3)

$$t' = k\left\{t - \frac{x}{v}\left(1-\frac{1}{k^2}\right)\right\}$$

$$t' = \frac{\left\{t - \frac{x}{v}\left\{1-\left(1-\frac{v^2}{c^2}\right)\right\}\right\}}{\sqrt{1-v^2/c^2}}$$

$$t' = \frac{\left\{t - \frac{x}{v}\left(\cancel{1}-\cancel{1}+\frac{v^2}{c^2}\right)\right)}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{t' = \frac{t-\frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} \quad \text{--- (9)}$$

and As there is no motion y and z axis

Hence $y' = y$ and --- (10)

$z' = z.$ --- (11)

(8), (9), (10), (11) are Lorentz transformation Eq.

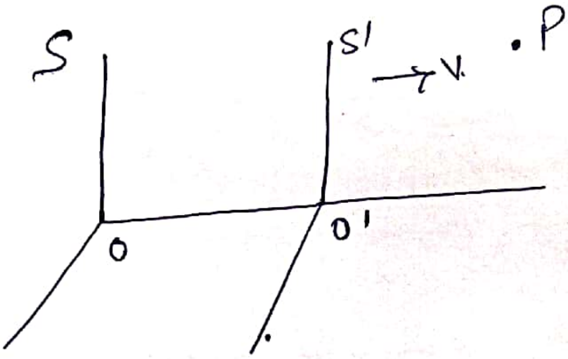
$$\boxed{\begin{aligned} x' &= \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \\ t' &= \frac{t - \frac{xv}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \end{aligned}}$$

Lorentz transformation Equation — (A)

Inverse Lorentz transformation:-

from Lorentz transformation Eq. (A)

Now.



observer is setting on O'

$$x = \frac{x' + vt'}{\sqrt{1-v^2/c^2}}$$

$$t = \frac{t' + \frac{x'v}{c^2}}{\sqrt{1-v^2/c^2}}$$

$$y = y'$$

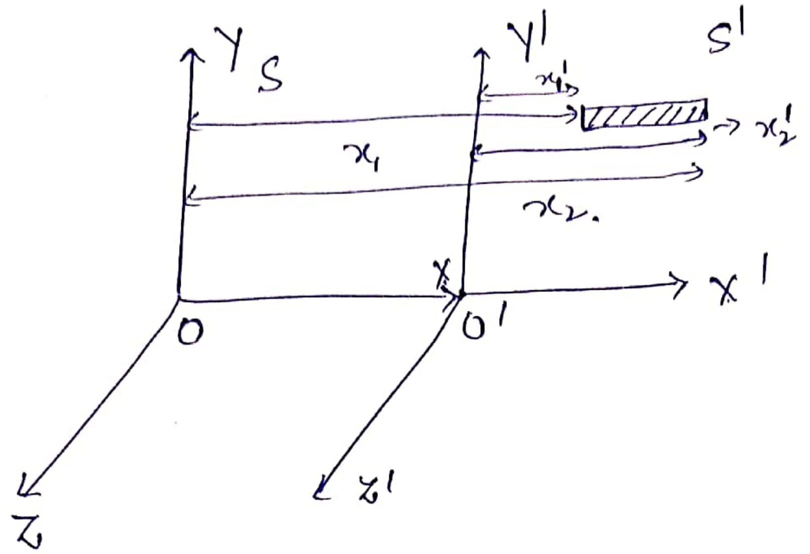
$$z = z'$$

replace x' by x

$x' \to x$
$x \to x'$
$t \to t'$
$t' \to t$
$v \to -v$

~~Effect of Special~~

Lorentz Contraction:-



S – rest frame

S' → moving ref frame (velocity v along x axis)

No motion along Y and z direction

l_0 → proper length (actual length)

l → length Contraction

from fig:

$$l_0 = x_2' - x_1' \quad \text{— (1)} \quad (\text{from } S' \text{ frame})$$

$$l = x_2 - x_1 \quad \text{— (2)} \quad (\text{from } S \text{ frame})$$

We know from Lorentz transformation Eq

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}}$$

So

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}$$

Put both the value in eq (1)

$$l_0 = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}} - \left(\frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \right)$$

$$= \frac{1}{\sqrt{1 - v^2/c^2}} \left\{ x_2 - vt - (x_1 - vt) \right\}$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[x_2 - \cancel{vt} - x_1 + \cancel{vt} \right]$$

$$= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[x_2 - x_1 \right]$$

$$\{ x_2 - x_1 = l \}$$

$$l_0 = \frac{l}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{l = l_0 \sqrt{1 - \frac{v^2}{c^2}}}$$ length Contraction.

↳ Contracted length

Condition - I

$V <<< C$

$$l = l_0 \sqrt{1 - \frac{V^2}{C^2}}$$

$$\sqrt{1-\frac{V^2}{C^2}} = 1 \qquad \frac{V^2}{C^2} \approx 0.00$$

$l = l_0$

No Contraction

Case - II

$V = C$

$$l = l_0 \sqrt{1 - \frac{C^2}{C^2}}$$

$l = 0$

$\boxed{\text{Not - Valid}}$

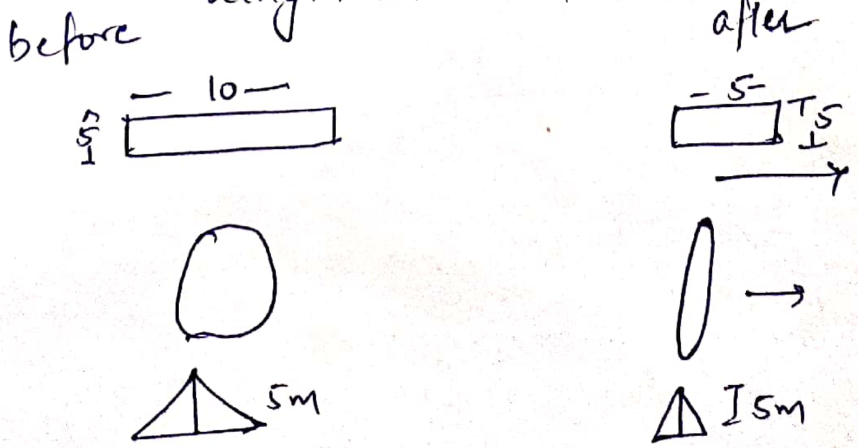
Case - III when V is near C,

$V \approx C$

$$l = l_0 \sqrt{1 - 0.000-}$$

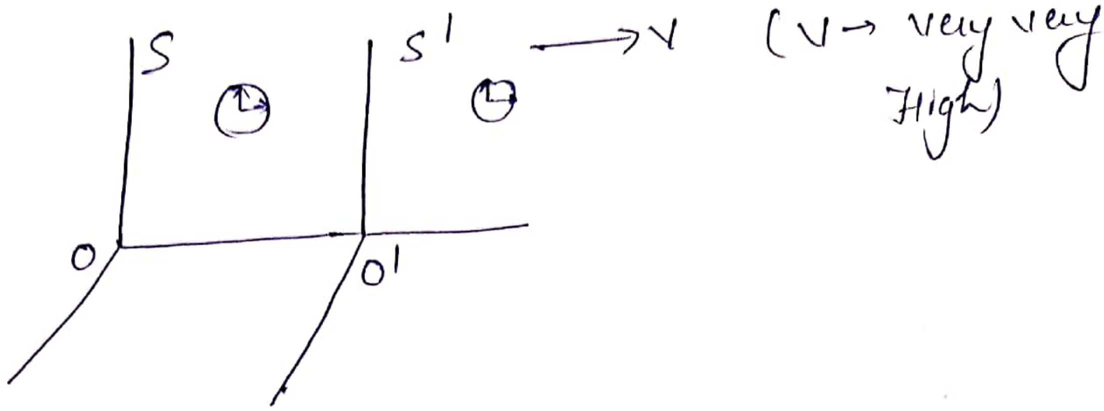
$\boxed{l < l_0}$

length will be contracted.



Jis direction mai human body move karti hai Usi direction mai body contract hogi.

Time Dilation and Twin Paradox:-



When both frame at rest $\Delta t = 0$

frame S' moving with velocity v or ($v \approx c$)

Time dilation:- A clock in a moving frame of reference (S') measure a longer time interval (i.e needles move slowly) between two events while for the same event the clock in stationary frame measure short time interval. This is known as time dilation.

According to Lorentz time dilation

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t \rightarrow$ Dilated time

$t_0 \rightarrow$ Proper time

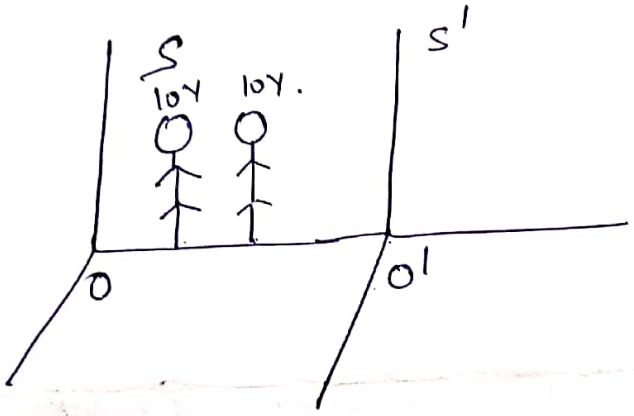
$$\boxed{t > t_0}$$

When clock is on frame S' and moving with velocity V.

S frame observer (rest)

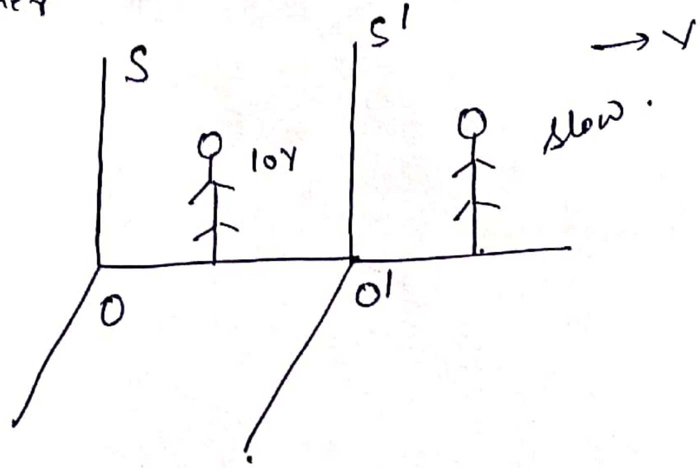
According frame S → clock is moving slow (longer time)

<u>Twin paradox:-</u>



Twin Brother

after 10 year

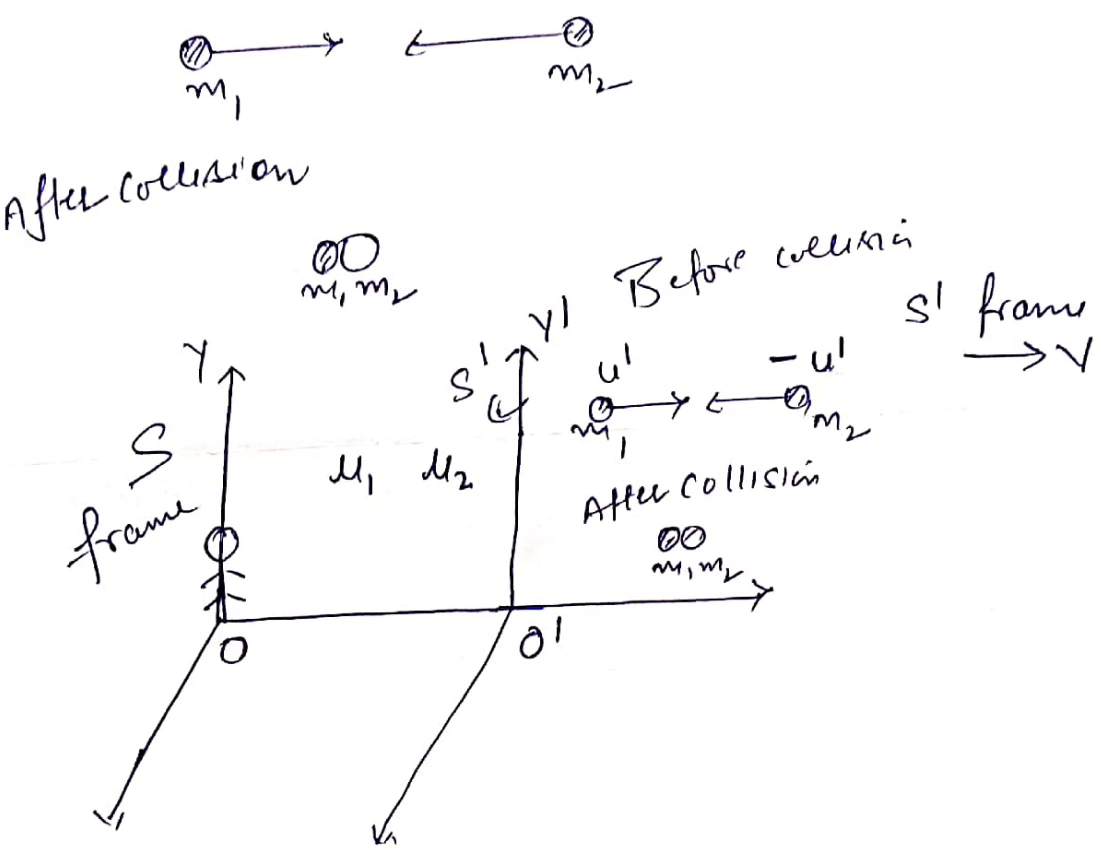


After 10 year
S - frame - 20 year

S' (motion)
S' - frame - 12 year.

Biological clock slow (when body moves with from a very-very high speed)

Variation in mass with velocity :-



Consider two bodies of masses m_1 and m_2 moving in frame S' towards each other with velocities u' and $-u'$ respectively and consider u_1 and u_2 are the velocities of same bodies relatives to an observer in S frame

From velocity addition theorem

$$\boxed{U = \frac{u'+v}{1+\frac{u'v}{c^2}}}$$

So $$\boxed{u_1 = \frac{u'+v}{1+\frac{u'v}{c^2}}} \quad -(1)$$

$$\boxed{u_2 = \frac{-u'+v}{1-\frac{u'v}{c^2}}} \quad -(2)$$

from O's observer conservation of mass and momentum.

Momentum before collision = momentum after collision

$$m_1u_1 + m_2u_2 = m_1V + m_2V$$

$$m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right) + m_2\left(\frac{-u'+v}{1-\frac{u'v}{c^2}}\right) = m_1V + m_2V$$

$$m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}} - V\right) = Vm_2 - m_2\left(\frac{-u_1'+v}{1-\frac{u'v}{c^2}}\right)$$

$$m_1\left(\frac{u'+\cancel{v} - \cancel{v} - \frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right) = m\left[\frac{v\left(1-\frac{u'v}{c^2}\right) + u_1' - v}{1-\frac{u'v}{c^2}}\right]$$

$$m_1\left(\frac{u' - \frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right) = m_2\left\{\frac{\cancel{v} - \frac{u'v^2}{c^2} + u_1' - \cancel{v}}{1-\frac{u'v}{c^2}}\right\}$$

$$\frac{m_1 \cancel{\left\{ u'\left(1-\frac{v^2}{c^2}\right)\right\}}}{1+\frac{u'v}{c^2}} = \frac{m_2 \cancel{\left\{ u'_1\left\{1-\frac{v^2}{c^2}\right\}\right\}}}{1-\frac{u'v}{c^2}}$$

$$\boxed{\frac{m_1}{m_2} = \frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}}} \quad - (3)$$

To find $\left(1+\frac{u'v}{c^2}\right)$ we solve

$$1-\frac{u_1^2}{c^2} = 1-\frac{1}{c^2}\left[\frac{u'+v}{1+\frac{u'v}{c^2}}\right]^2$$

$$1-\frac{1}{c^2}\frac{(u'+v)^2}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= 1-\frac{\frac{1}{c^2}(u'+v)^2}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= 1-\frac{\frac{1}{c^2}\left(u'^2+v^2+2u'v\right)}{\left(1+\frac{u'v}{c^2}\right)^2}.$$

$$= \frac{\left(1+\frac{u'v}{c^2}\right)^2-\frac{1}{c^2}\left(u'+v^2+2u'v\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{1 + \frac{u'^2 v^2}{c^4} + \frac{2u'v}{c^2} - \frac{1}{c^2}\left(u'^2 + v^2 + 2u'v\right)}{\left(1 + \frac{u'v}{c^2}\right)^2}$$

$$= \frac{1 + \frac{u'^2 v^2}{c^4} + \cancel{\frac{2u'v}{c^2}} - \frac{u'^2}{c^2} + \frac{v^2}{c^2} - \cancel{\frac{2u'v}{c^2}}}{\left(1 + \frac{u'v}{c^2}\right)^2}$$

$$= \frac{1 + \frac{u'^2 v^2}{c^4} - \frac{u'^2}{c^2} - \frac{v^2}{c^2}}{\left(1 + \frac{u'v}{c^2}\right)^2}$$

$$1 - \frac{u_1^2}{c^2} = \frac{\left(1 - \frac{v^2}{c^2}\right) - \frac{u'^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)}{\left(1 + \frac{u'v}{c^2}\right)^2}$$

$$\boxed{1 - \frac{u_1^2}{c^2} = \frac{\left(1 - \frac{v^2}{c^2}\right)\left(1 - \frac{u'^2}{c^2}\right)}{\left(1 + \frac{u'v}{c^2}\right)^2}} \quad — (4)$$

$$\left(1+\frac{u'v}{c^2}\right)^2 = \frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}$$

$$1+\frac{u'v}{c^2} = \left[\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}\right]^{1/2} \quad -(5)$$

Sly

$$1-\frac{u_2^2}{c^2} = \left[\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1-\frac{u'v}{c^2}\right)^2}\right]$$

$$\left(1-\frac{u'v}{c^2}\right) = \left[\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{\left(1-\frac{u_2^2}{c^2}\right)}\right]^{1/2} \quad -(6)$$

Put values from Eq (5) and Eq (6) in Eq (3)

$$\frac{m_1}{m_2} = \frac{\sqrt{\cancel{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}\Big/\left(1-u_1^2/c^2\right)}}{\sqrt{\cancel{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}\Big/\left(1-u_2^2/c^2\right)}}$$

$$\boxed{\frac{m_1}{m_2} = \sqrt{\frac{\left(1-\frac{u_2^2}{c^2}\right)}{\left(1-\frac{u_1^2}{c^2}\right)}}} \quad -(7)$$

Let us consider m_2 is moving with velocity zero velocity

so m_2 is on rest

$u_2 = 0$

$$\frac{m_1}{m_2} = \sqrt{\frac{1-0}{1-\frac{u_1^2}{c^2}}}$$

$$m_1 = \frac{m_2}{\sqrt{1-\frac{u_1^2}{c^2}}}$$

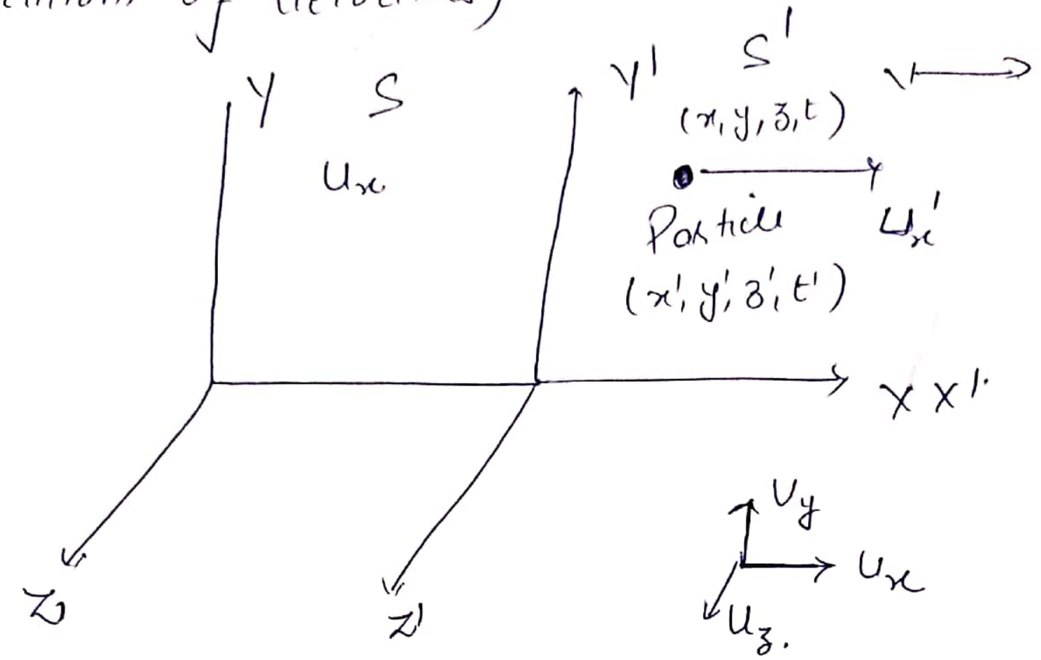
for Simple / Easy notations

$\boxed{m_1 = m}$ $\boxed{m_2 = m_0}$ $\boxed{u_1 = v}$

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Variation in mass with velocity

Velocity addition Theorem:- (Relativistic additions of Velocities)



for S frame of ref

$$U_x = \frac{dx}{dt} \rightarrow \text{Small change in distance w.r.t '} t \text{'} \quad (1)$$

$$U_y = \frac{dy}{dt}, \quad U_z = \frac{dz}{dt}$$

for S' frame of ref

$$\boxed{U_x' = \frac{dx'}{dt'}} \quad -(2)$$

from Lorentz transformation Eq

$$x' = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad -(3), \qquad t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1-v^2/c^2}} \quad -(4)$$

$$y' = y, \quad z' = z$$

To find eq (2) values dx' and dt'

diff (3) and (4)

$$\boxed{dx' = \frac{dx - v\,dt}{\sqrt{1 - v^2/c^2}}} \quad -(5)$$

$$\boxed{dt' = \frac{dt - \frac{v}{c^2}dx}{\sqrt{1 - v^2/c^2}}} \quad -(6)$$

from eq (2)

$$U_x' = \frac{\frac{dx - v\,dt}{\cancel{\sqrt{1 - v^2/c^2}}}}{\frac{dt - \frac{v}{c^2}dx}{\cancel{\sqrt{1 - v^2/c^2}}}} = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx}$$

$$U_x' = \frac{dx - v\,dt}{dt - \frac{v}{c^2}dx}$$

divide this eq by dt Numerator and denominator.

$$U_x' = \frac{\frac{dx}{dt} - \frac{v\,\cancel{dt}}{\cancel{dt}}}{\frac{\cancel{dt}}{\cancel{dt}} - \frac{v}{c^2}\frac{dx}{dt}}$$

$$U_x' = \frac{U_x - v}{1 - \frac{v}{c^2}U_x}, \qquad \boxed{U_x' = \frac{U_x - v}{1 - \frac{vU_x}{c^2}}}$$

General

$\cancel{u'} = \cancel{u-v}$ $\quad u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}}$.

Hence for transformation theory

$$\boxed{U = \dfrac{u'+v}{1+\dfrac{u'v}{c^2}}} \quad -(7)$$

$u' \to u$
$u \to u'$
$v \to -v$

this is the final
Expession known as rel add of velocity

Condition 1

when $v = c$

$$U = \frac{u'+c}{1+\frac{u'c}{c^2}} = \frac{u'+c}{1+\frac{u'}{c}} = \left(\frac{\cancel{u'+c}}{\cancel{c+u'}}\right)c = c$$

c remain constant.

Condition 2
$u' = c$

$$U = \frac{c+v}{1+\frac{cv}{c^2}} = \left(\frac{\cancel{c+v}}{\cancel{c+v}}\right)c = c$$

Condition

$V = c, \quad u' = c$

$$U = \frac{c + c}{1 + \frac{c^2}{c^2}} = \frac{2c}{1+1} = \frac{\cancel{2}c}{\cancel{2}} = c$$

$$\boxed{U = c}$$

Show that Space time interval is invariant under Lorentz transformation.

OR

Show that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under L.T

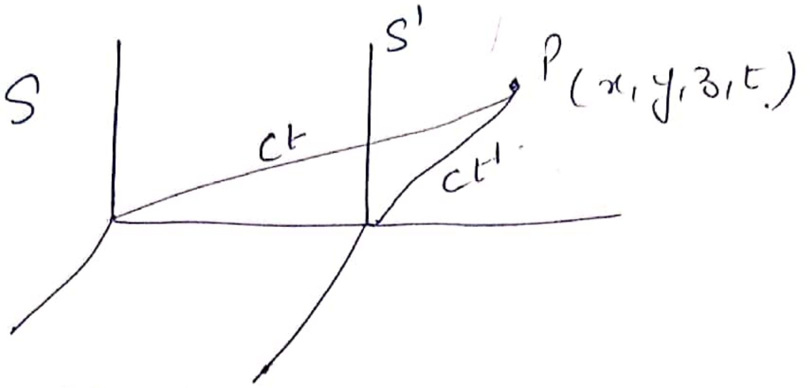
We known

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z'$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

If Space time interval invariant then

$$x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t'^2$$



$$\text{Time} = \frac{\text{Distance}}{\text{velocity}}$$

$$t = \frac{(x^2+y^2+z^2)^{1/2}}{c}$$

$$x^2+y^2+z^2 = c^2t^2$$

$$x^2+y^2+z^2 - c^2t^2 = 0.$$

R.H.S

$$\Rightarrow \quad x'^2 + y'^2 + z'^2 - c^2t'^2 = \quad \text{—} (1)$$

Using L.T.E put values in Eq (1)

$$\left[\frac{x-vt}{\sqrt{1-v^2/c^2}}\right]^2 + y^2+z^2 - c^2\frac{\left(t-\frac{xv}{c^2}\right)^2}{\left(\sqrt{1-v^2/c^2}\right)^2}$$

$$\frac{(x-vt)^2}{\left(1-\frac{v^2}{c^2}\right)} - \frac{c^2\left(t-\frac{xv}{c^2}\right)^2}{\left(1-\frac{v^2}{c^2}\right)} + y^2+z^2$$

$$\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left[x^2+v^2t^2-2xvt - \left\{c^2t^2 + c^2x\frac{x^2v^2}{c^4} - 2\cancel{c^2}t\frac{xv}{\cancel{c^2}}\right\}\right]$$

$$+ y^2+z^2$$

$$\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left[x^2+v^2t^2-\cancel{2xvt}-c^2t^2-\frac{x^2v^2}{c^2}+\cancel{2xvt}\right]+y^2+z^2$$

$$\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left[x^2+v^2t^2-c^2t^2-\frac{x^2v^2}{c^2}\right]+y^2+z^2.$$

$$\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left\{x^2-c^2t^2+v^2t^2-\frac{x^2v^2}{c^2}\right\}+y^2+z^2$$

$$\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left\{(x^2-c^2t^2)+\frac{v^2}{c^2}\left[c^2t^2-x^2\right]\right\}+y^2+z^2$$

$$\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left\{(x^2-c^2t^2)-\frac{v^2}{c^2}\left[x^2-c^2t^2\right]\right\}+y^2+z^2.$$

$$\frac{(x^2-c^2t^2)\cancel{\left\{1-\frac{v^2}{c^2}\right\}}}{\cancel{\left(1-\frac{v^2}{c^2}\right)}}+y^2+z^2$$

$$\boxed{x^2+y^2+z^2-c^2t^2=L.H.S}$$

Space time interval is invariant.

Relativistic Relation b/w Energy and momentum.

from Einstein's Mass Energy Relation

$$E = mc^2$$

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \quad -(1)$$

and from $p = mv. = \dfrac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$

Squaring Both side

$$p^2(c^2 - v^2) = m_0^2 v^2 c^2$$

$$p^2c^2 - p^2v^2 = m_0^2 v^2 c^2$$

$$p^2c^2 = m_0^2 v^2 c^2 + p^2 v^2$$

$$p^2c^2 = v^2\left[m_0^2 c^2 + p^2\right]$$

$$\boxed{v^2 = \frac{p^2c^2}{m_0^2c^2 + p^2}} \quad -(2)$$

Put the value of v^2 in eq (1)

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{1}{\cancel{c^2}}\left\{\frac{p^2 \cancel{c^2}}{p^2 + m_0^2 c^2}\right\}}}$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \left\{\frac{p^2}{p^2 + m_0^2 c^2}\right\}}}.$$

$$E = \frac{m_0 c^2}{\left(\frac{\cancel{p^2} + m_0^2 c^2 - \cancel{p^2}}{p^2 + m_0^2 c^2}\right)^{1/2}}$$

$$E = \frac{m_0 c^2}{\left(\frac{m_0^2 c^2}{p^2 + m_0^2 c^2}\right)^{1/2}}$$

Squaring both sides

$$E^2 = \frac{m_0^2 c^4 (p^2 + m_0^2 c^2)}{(m_0^2 c^2)}$$

$$E^2 \cancel{m_0^2} c^2 = \cancel{m_0^2} c^4 (p^2 + m_0^2 c^2)$$

$$\boxed{E^2 = p^2 c^2 + m_0^2 c^4} \quad - (3)$$

Massless particle

Those particle which have zero rest mass (i.e $m_0 = 0$)

e.g photon

$$\lambda = \frac{h}{p}, \quad p = \frac{h}{\lambda} \quad - (1)$$

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \quad — (2)$$

$$P = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

Comp (1) and (2)

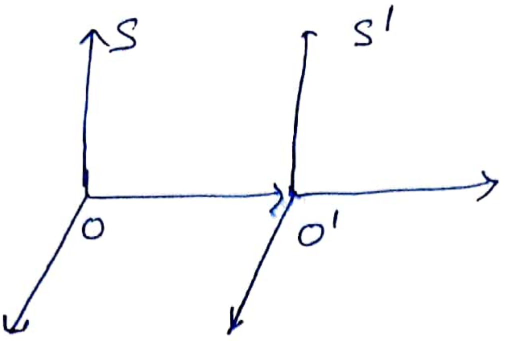
$$\frac{h}{\lambda} = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

for photon $v = c$.

$$\boxed{m_0 = 0}$$

So rest mass of Photon is zero and photon is a massless particle

Relativity of Simultaneity :-



Two frame of reference

$$t' = \frac{t - \frac{vx}{c}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_1' = \frac{t_1 - \frac{vx_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad , \quad t_2' = \frac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1-v^2/c^2}} - \frac{(v/c^2)(x_2 - x_1)}{\sqrt{1-v^2/c^2}}$$

Since, the events are simultaneous in frame S_1 so $t_2 - t_1 = 0$

$$\Delta t' = \frac{(v/c^2)(x_2 - x_1)}{\sqrt{1-v^2/c^2}}.$$

$$\Delta t' \neq 0$$

It means that events which are simultaneous the frame S observed by O

Mass energy equivalence

Let us consider a particle of rest mass m_0. mass m when acted upon by a force F to produce velocity v in the the Same direction

$dw = F \cdot dx$ kinetic energy or dk

$dw = dk = F \cdot dx$

according to 2nd law, $F = \frac{dP}{dt}$, P is the momentum $F = \frac{d}{dt}(mv)$

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$dk = m\frac{dv}{dt}dx + v\frac{dm}{dt}dx$$

$$= m\frac{dx}{dt}dv + v\frac{dx}{dt}dm$$

$$dk = mv\,dv + v^2 dm \qquad \text{— (a)}$$

relativistic formula

$$M = \frac{m_0}{\sqrt{1-v^2/c^2}} \quad = \quad m^2(c^2 - v^2) = m_0^2 c^2 \text{ — (1)}$$

diff Eq (2) we get

$$2mc^2 dm - 2mv^2 dm - 2m^2 v\,dv = 0$$

$$c^2 dm = mv\,dv + v^2 dm. \qquad \text{— (2)}$$

Compare (a) and (2)

$dk = c^2 dm.$

kinetic energy is changing from 0 to K.

due to change in its mass from m_0 to m

$$\int_0^K dK = \int_{m_0}^{m} c^2 dm$$

$$K = c^2(m - m_0)$$

Total Energy E = rest mass energy + relativistic K.E

$$= m_0c^2 + (m - m_0)c^2$$

$$\boxed{E = mc^2}$$

Einstein mass energy relation

Expression of Relativistic Momentum

$P = mv$, Relativistic mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$P = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$F = \frac{dP}{dt} = \frac{d}{dt}\frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

$$m_0 \frac{d}{dt}\left[v\left(1 - \frac{v^2}{c^2}\right)^{-1/2}\right]$$

$$= \boxed{m_0 \frac{dv}{dt}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}}$$ – Newton's Second Law