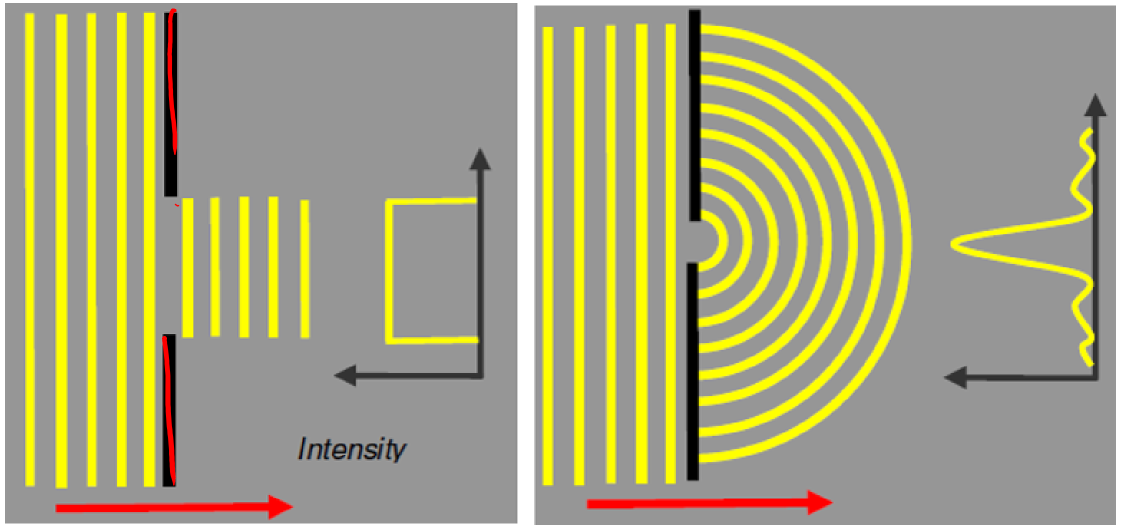
Diffraction of Light

The bend ing of light around the corners of an obstacle, the size of which is comparable to the wavelength of light.

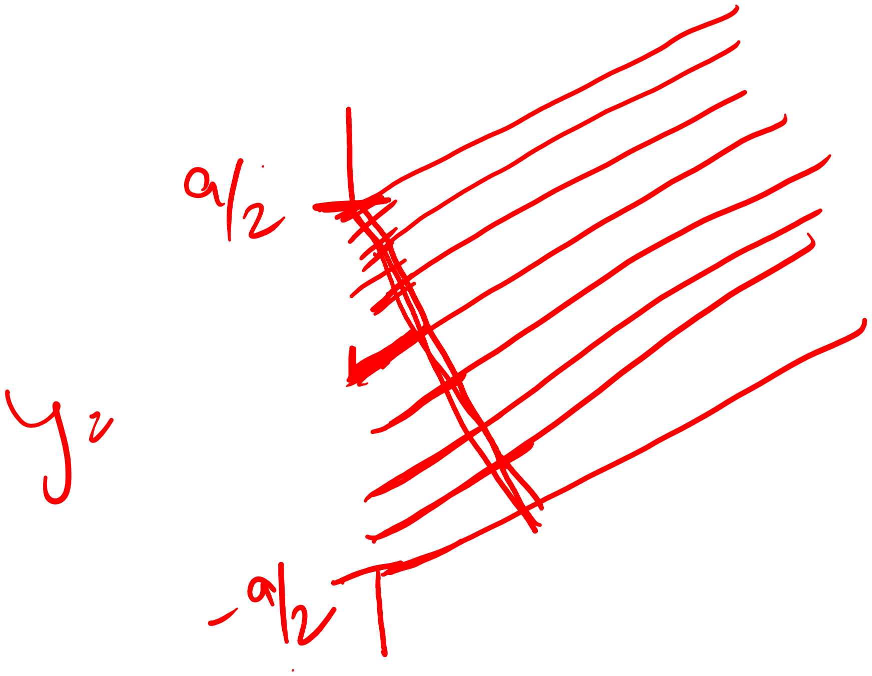
Or

The encroachment of light within the geometrical shadow is k/a diffraction of light

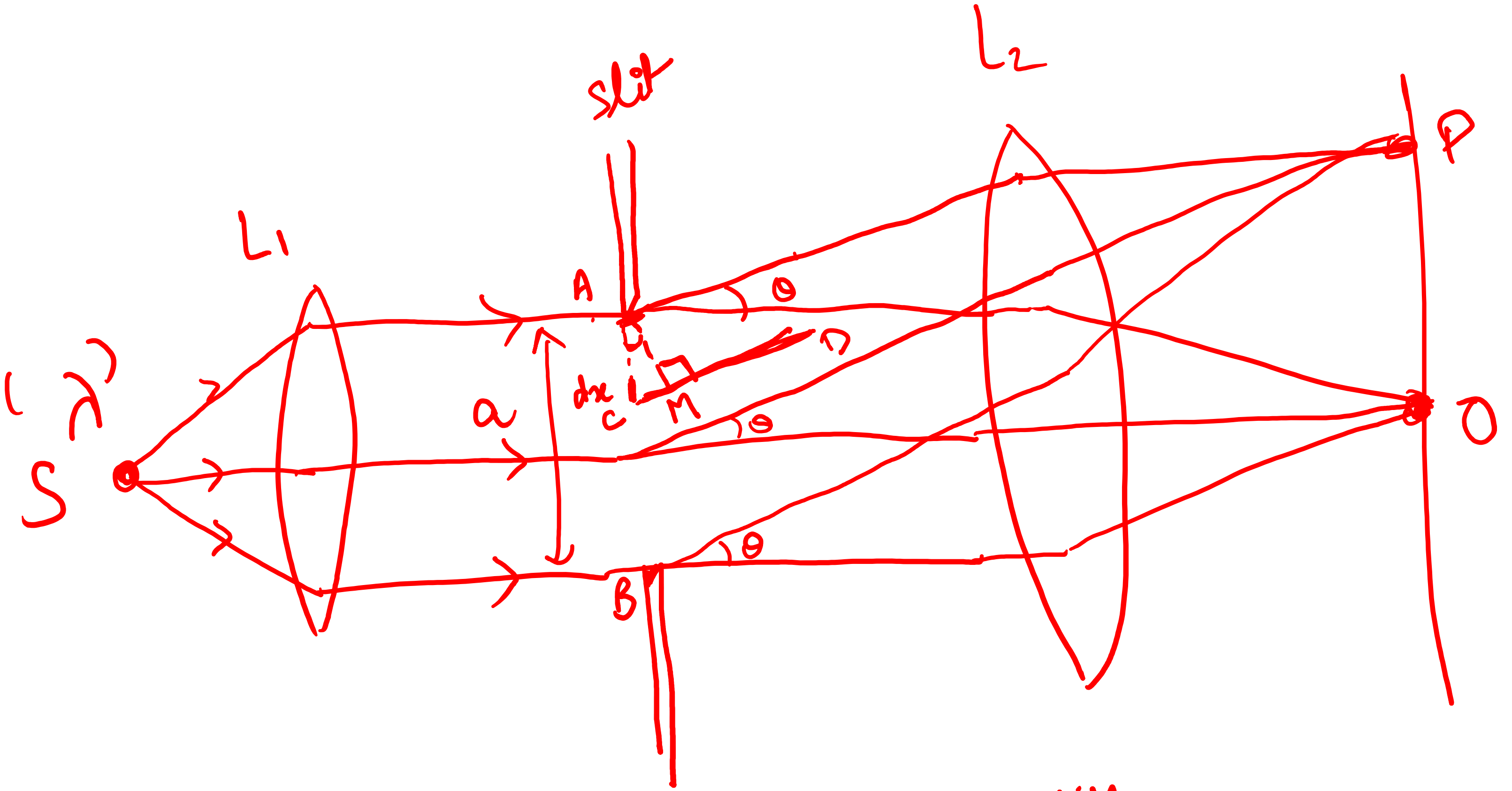


wide aperture-wave unperturbed sharp image of aperture seen

narrow aperture-wave diffracted diffraction pattern seen



Fraunhofer's Diffraction at a single



For unit length wavelet can be described as

$$y_0 = A\cos\omega t$$

Wavelet at a dist x. for a small element dx

$AB = a$

$AC = x$

dx

CM = path diff.

$CM = x\sin\theta$

$$dy = A\,dx\cos\left[\omega t + \frac{2\pi}{\lambda}x\sin\theta\right]$$

For total disturbance integrate the expression from $-\frac{a}{2}$ to $+a/2$

$$y = \int_{-a/2}^{a/2} dy = \int_{-a/2}^{a/2} A\cos\left(\omega t + \frac{2\pi x}{\lambda}\sin\theta\right)dx$$

$$= \int \left[A\cos\omega t\cos\left(\frac{2\pi x}{\lambda}\sin\theta\right)dx - A\sin\omega t\sin\left(\frac{2\pi x}{\lambda}\sin\theta\right)dx\right]$$

$$= A\cos\omega t\int_{-a/2}^{a/2}\cos\left(\frac{2\pi x}{\lambda}\sin\theta\right)dx - \underbrace{A\sin\omega t\int_{-a/2}^{a/2}\sin\left(\frac{2\pi}{\lambda}x\sin\theta\right)dx}_{0}$$

$$= A\cos\omega t\left[\sin\left(\frac{2\pi x\sin\theta}{\lambda}\right)\cdot\frac{\lambda}{2\pi\sin\theta}\right]_{-a/2}^{a/2}$$

$$= A\cos\omega t \left[\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)\right] \cdot \frac{\lambda}{\pi \sin\theta}$$

$$= A\cos\omega t \, \frac{\sin \frac{\pi a \sin\theta}{\lambda}}{\frac{\pi \sin\theta}{\lambda}} \frac{\times a}{\times a} = Aa\cos\omega t \, \frac{\sin\alpha}{\alpha}$$

where $\alpha = \frac{a\pi\sin\theta}{\lambda}$

A = amp of unit length

Aa = total amp. from whole slit $= A_0$

$$y = A_0 \frac{\sin\alpha}{\alpha} \cos\omega t = R\cos\omega t$$

R = resultant amp.

$= A_0 \frac{\sin\alpha}{\alpha}$

$\therefore$ Intensity at $P = I \propto R^2 = R^2$

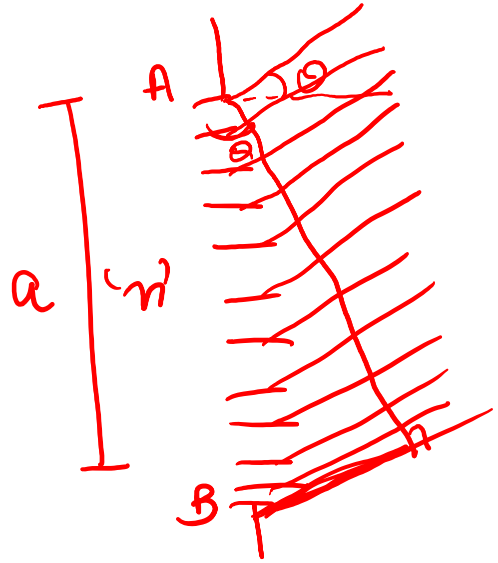
$$I = A_0^2 \frac{\sin^2\alpha}{\alpha^2} = \boxed{I_0 \frac{\sin^2\alpha}{\alpha^2}}$$

$$I_0 = A_0^2 = (Aa)^2$$

If $\underline{\underline{\theta = 0}}$ $\quad \alpha = \frac{\pi a \sin\theta}{\lambda} = 0$, $\boxed{I = I_0}$

$$\boxed{\because \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1}$$

Method II :- Geometric / Vector method.



If A is amp from one part

the nA = total amp. from slit = R_0

So phase diff varies from 0 to $\frac{2\pi}{\lambda} a\sin\theta$

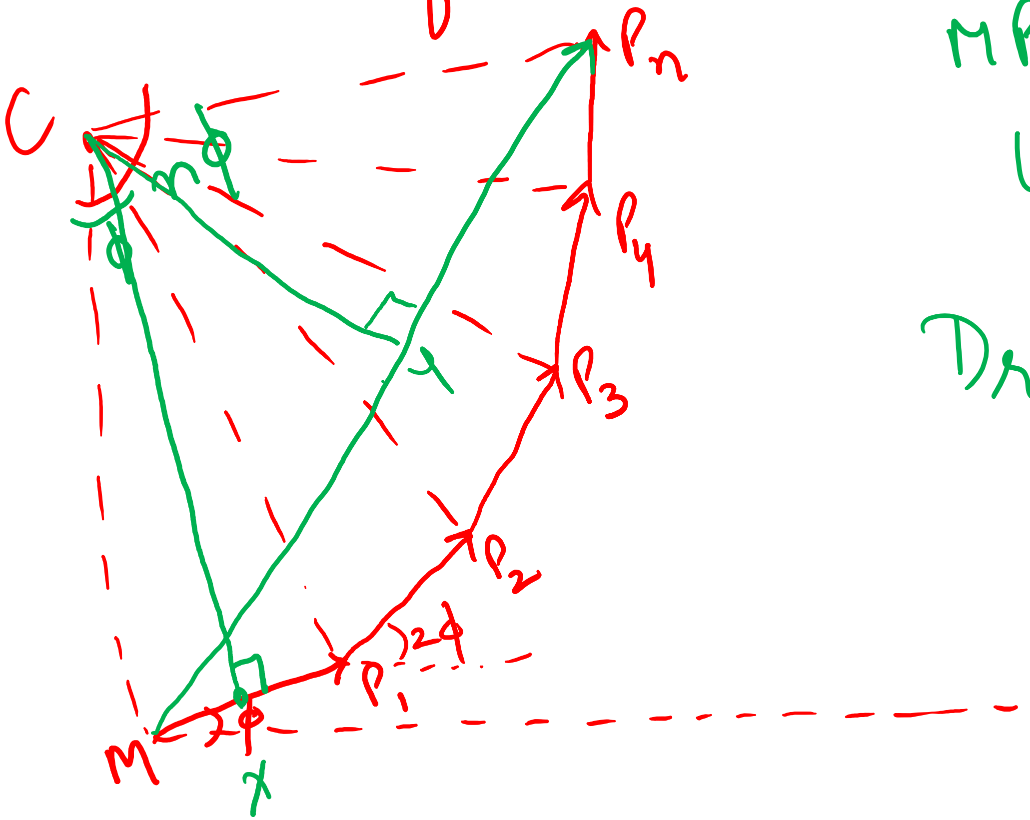
when going from A to B.

$\therefore$ phase diff b/w two consecutive wavelets will be

$$\boxed{\phi = \frac{2\pi}{n\lambda} a\sin\theta}$$

Length of vectors = Amplitude

dirn of " = ϕ



MP_n = Resultant

let magnitude = R_θ

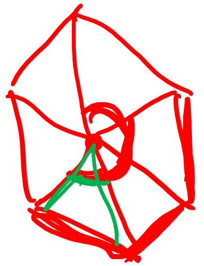
Draw $CX \perp MP_1$

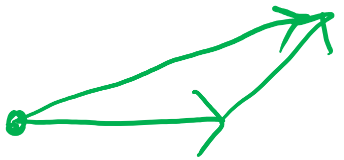
$CY \perp MP_n$

From ΔCMX, $\frac{MX}{MC} = \sin \phi/2$

$MX = MC \sin \phi/2 = \frac{A}{2}$ —— ①

$MP_1 = P_1P_2 = P_2P_3 = \ldots\ldots$





From ΔCYM $\quad \frac{MY}{MC} = \sin\frac{n\phi}{2}$ $\quad MY = MC\sin\frac{n\phi}{2} = \frac{1}{2}R_\theta$ — (2)

(2) ÷ (1) $\Rightarrow$ $R_\theta = A\frac{\sin(n\phi/2)}{\sin(\phi/2)}$

$$R_\theta = \frac{A\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi a\sin\theta}{n\lambda}\right)} = \frac{A\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)} = \frac{A\sin\alpha}{\alpha/n} = An\frac{\sin\alpha}{\alpha}$$

[∵ n is large, α is small.]

$$R_\theta = R_0\frac{\sin\alpha}{\alpha} \Rightarrow I = R^2 = R_0^2\frac{\sin^2\alpha}{\alpha^2} = \boxed{I_0\frac{\sin^2\alpha}{\alpha^2}}$$

1) Position of central maximum

$$\theta = 0 \qquad \alpha = 0 \qquad \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$$

$$I = I_0 = \text{maximum}$$

2) Position of minima

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \Rightarrow$$

$$\frac{\sin\alpha}{\alpha} = 0 \quad \sin\alpha = 0 \Rightarrow \alpha = \pm m\pi$$

$$m = 1, 2, 3, \ldots$$

$$\frac{\pi a \sin\theta}{\lambda} = \pm m\pi \Rightarrow \boxed{a\sin\theta = \pm m\lambda} \text{ cond}^n.$$

3) Position of your secondary max.:-

Direction →

$$\alpha = \pm(2m+1)\frac{\pi}{2}$$

$$\boxed{a\sin\theta = (2m \pm 1)\frac{\lambda}{2}}$$

$m = 1, 2, 3 \ldots\ldots$ ✓

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots\ldots$$

Intensities of max :-

$$I_1 = I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \left(\frac{\sin 3\pi/2}{3\pi/2}\right) = \frac{I_0}{22} \checkmark$$

$$I_2 = I_0 \left(\frac{\sin 5\pi/2}{5\pi/2}\right) = \frac{I_0}{61} \checkmark$$

$\therefore$ Intensity falls off very rapidly.

$$\boxed{1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \ldots\ldots}$$

To find the position of sec. max.

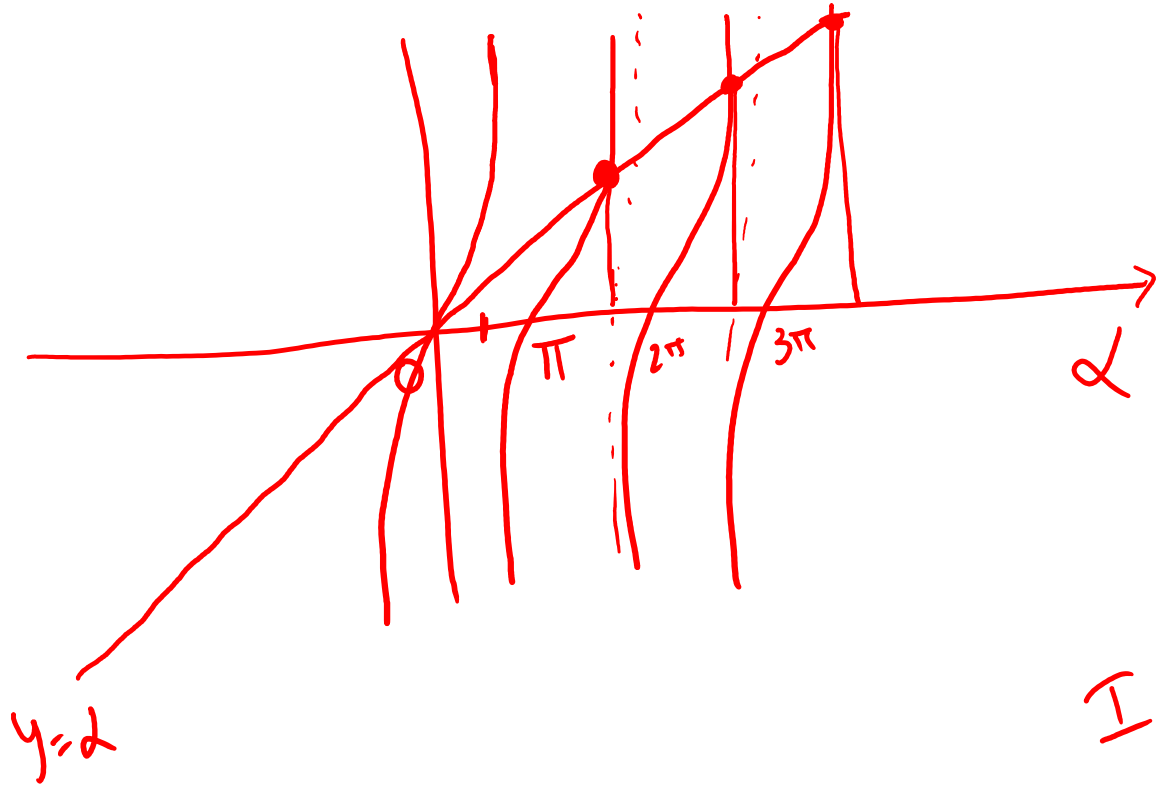
Differentiate I wrt α and set $= 0$

$$\frac{dI}{d\alpha} = 0 \Rightarrow \frac{d}{d\alpha}\left[I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0 \Rightarrow I_0\, 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

$$\Rightarrow \alpha\cos\alpha - \sin\alpha = 0 \Rightarrow \alpha\cos\alpha = \sin\alpha$$

$$\boxed{\alpha = \tan\alpha}$$

$$\boxed{y = \alpha} \qquad \boxed{y = \tan\alpha}$$

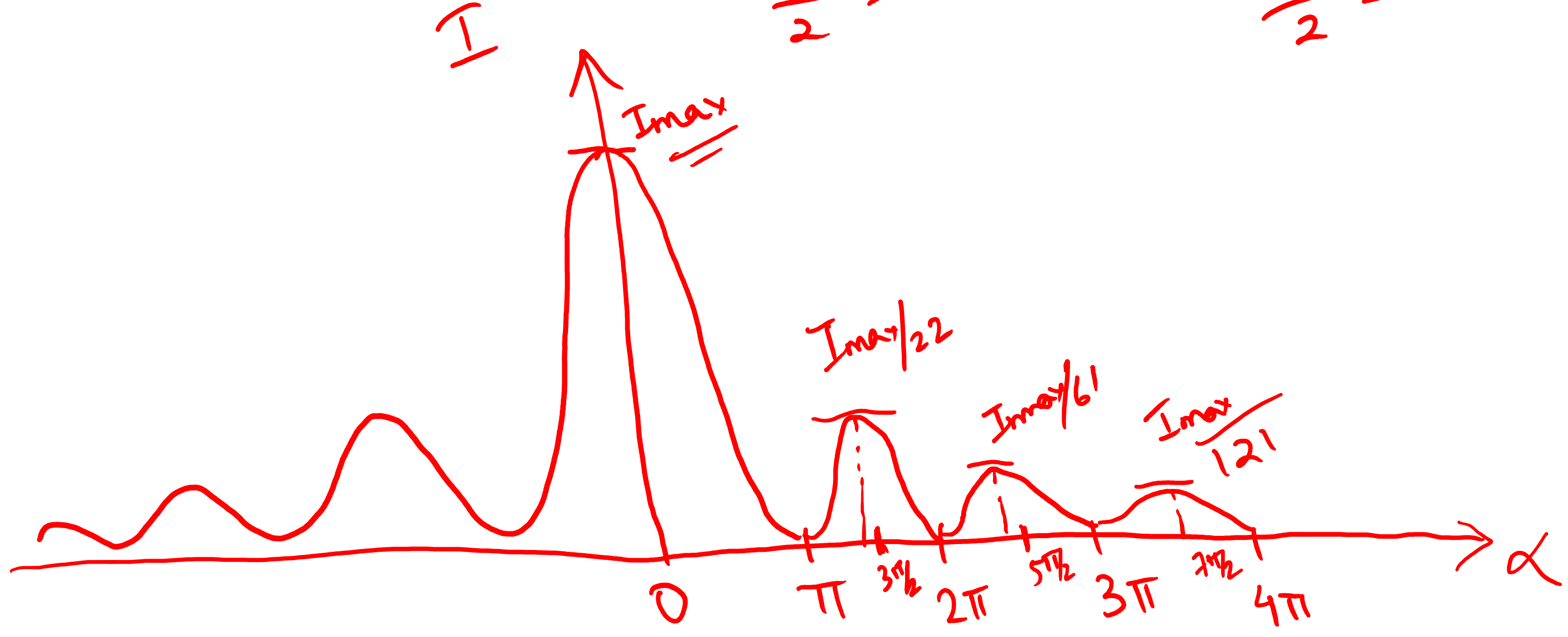


The intersection pts come at

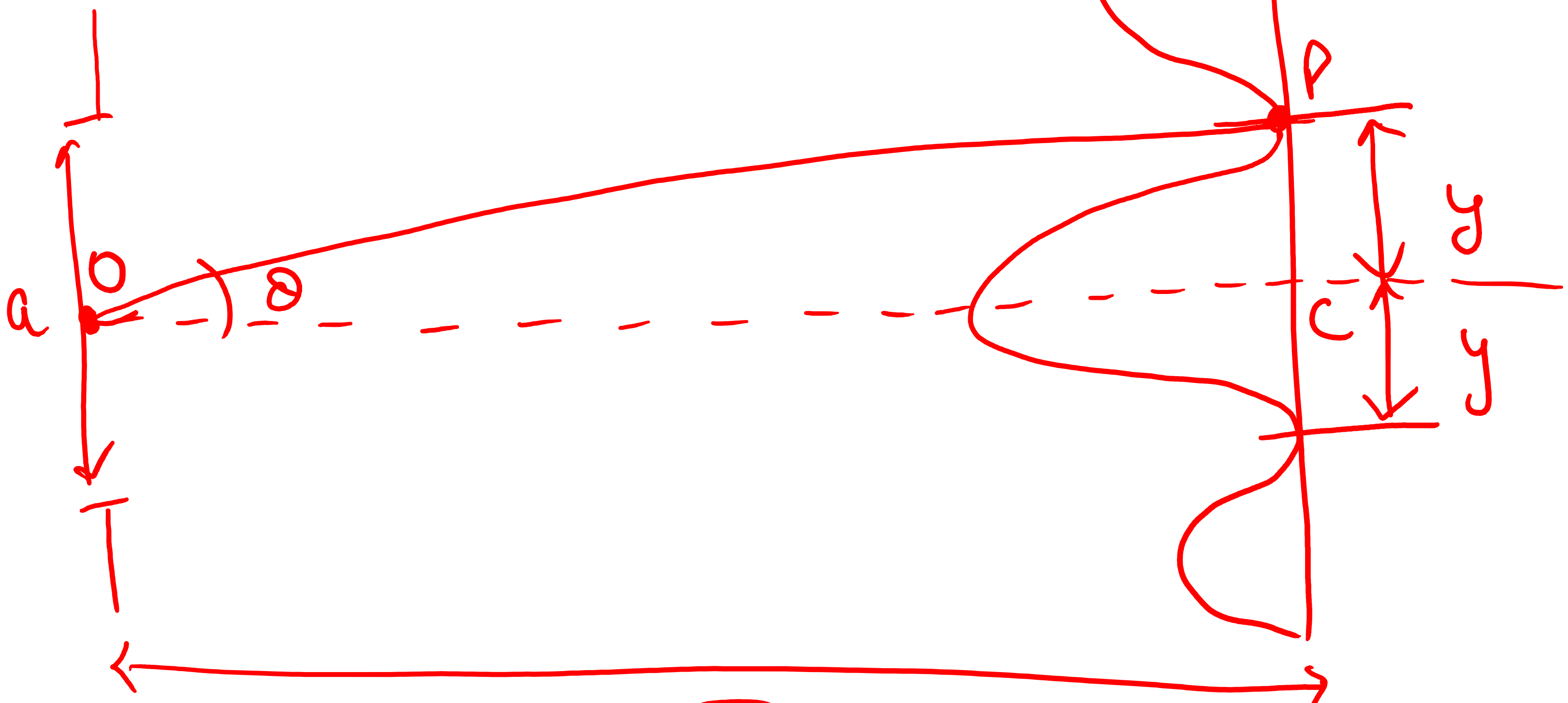
$\alpha_1 = 1.43\pi$ $\alpha_2 = 2.46\pi$

$\alpha_3 = 3.47\pi$

$\alpha = \frac{3\pi}{2} = 1.5\pi$ $\frac{5\pi}{2} = 2.5\pi$



Width of central mx. $= 2y$



D = focal length = f

ΔPOC

$PC = y$

$OC = D$

$$\boxed{\tan\theta \simeq \sin\theta = y/D}$$

$$a\sin\theta = \pm m\lambda$$

for 1st minimum

$m = 1$

$$\boxed{\sin\theta = \pm \lambda/a}$$

$$y = \pm\frac{\lambda D}{a} \Rightarrow \boxed{2y = \frac{2\lambda D}{a}}$$

$$\boxed{2y = \frac{2\lambda f}{a}}$$

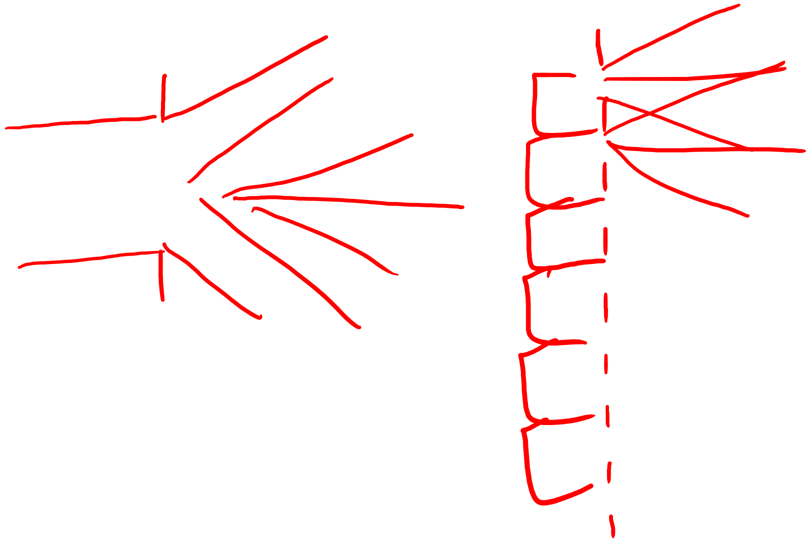
width $\propto$ 'λ' $\frac{1}{'a'}$

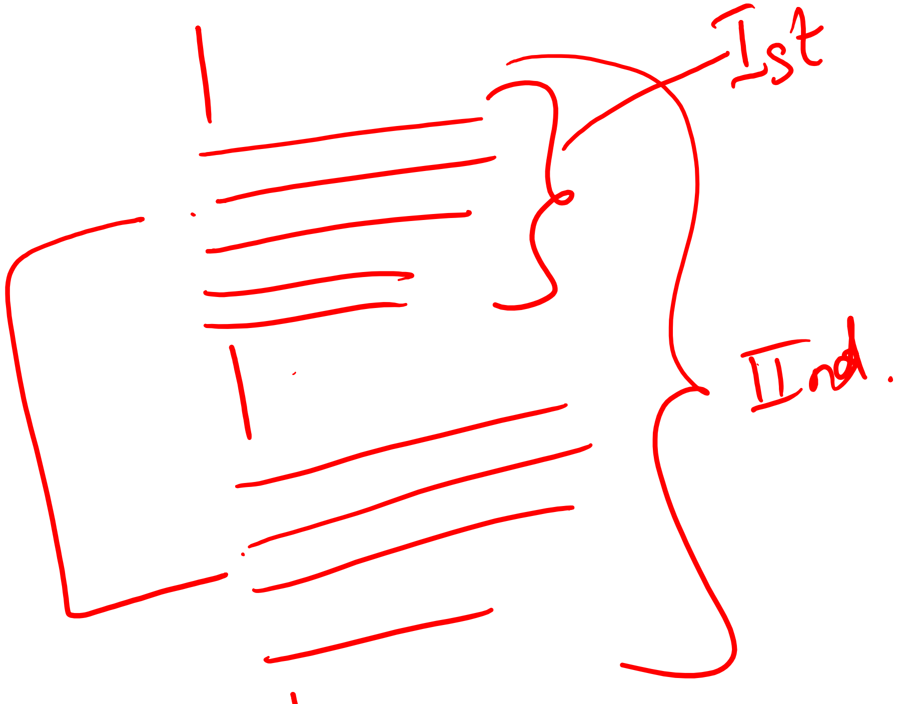
Effect of slit width

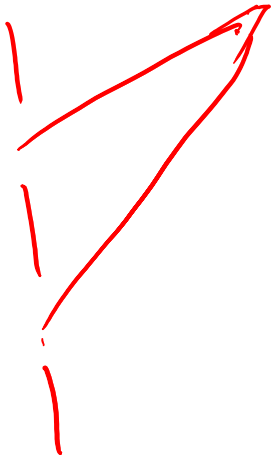
If a is large, θ will be small

max & min will lie very close to each other.

But if a is small $\Rightarrow \theta$ = large $\Rightarrow$ diff pattern will be distinct & clear.



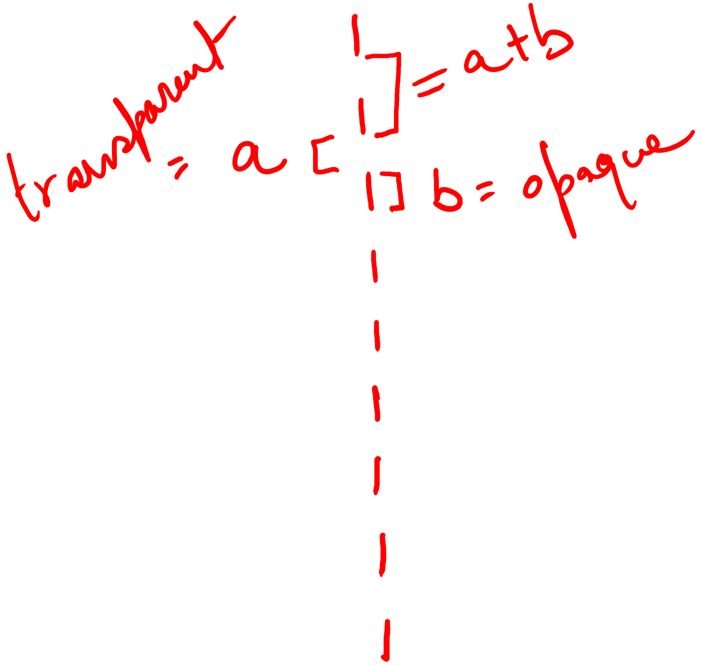




Plane transmission grating

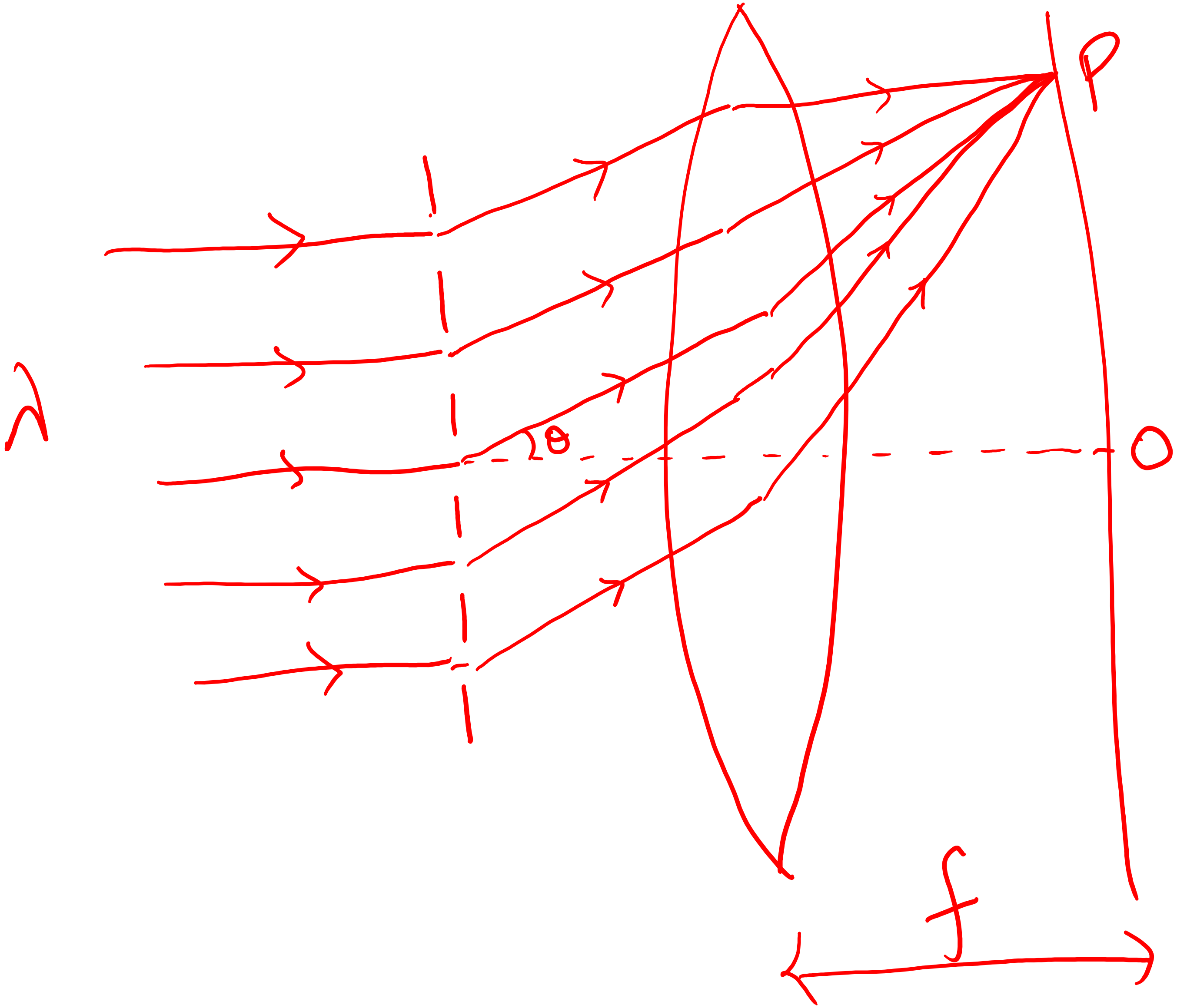
Diffraction due to N-slits

Diffraction grating - Arrangement of a large # of || and **equidistant** narrow rectangular slits of **equal** widths.

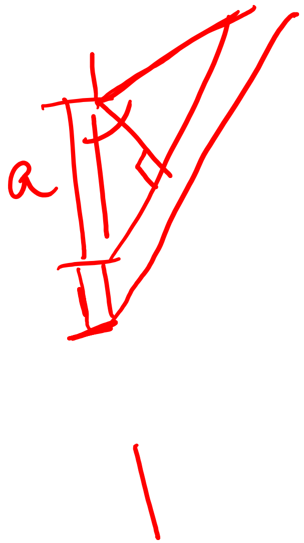


$a + b = d$ = grating element

Theory :-



Width of slit = a



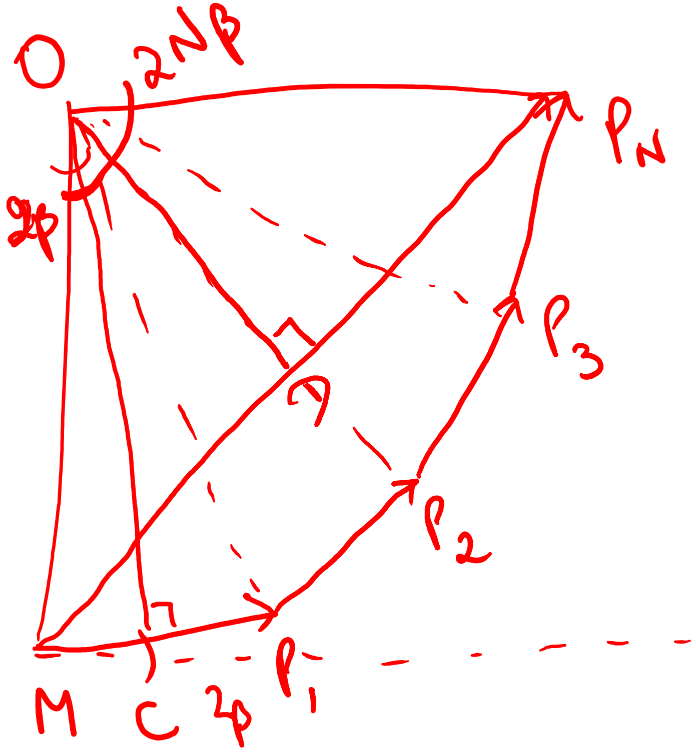
Expression of Intensity using vector method.

For one slit :- $A_1 = \dfrac{A_0 \sin\alpha}{\alpha}$; $\alpha = \dfrac{\pi}{\lambda} a\sin\theta$ $\Rightarrow$

Path diff for waves coming from two nearby slits

$$\Delta = (a+b)\sin\theta \quad , \quad \phi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$$

Consider N waves represented by N vectors of equal amp. $= A_1$, phase angle $= 2\beta$.



MP_N = Resultant vector / amp = A

Draw $OC \perp MP_1$

$OD \perp MP_N$

$$\frac{MP_N}{2} = OM \sin N\beta$$

$$\frac{MP_1}{2} = OM \sin \beta$$

$$\frac{MP_N}{MP_1} = \frac{\sin N\beta}{\sin \beta} \Rightarrow \boxed{A = A_1 \frac{\sin N\beta}{\sin \beta}}$$

$$A = A_0 \frac{\sin\alpha}{\alpha} \frac{\sin N\beta}{\sin\beta}$$

$$\boxed{I = \underbrace{A_0^2}_{} \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}}$$

I_0 = Max intensity for $\theta = 0$

Ist term

$I_0 \frac{\sin^2\alpha}{\alpha^2}$ = diffraction

2nd term

$\frac{\sin^2 N\beta}{\sin^2\beta}$ = interference term b/w diff slits.

Method II

Let amp. of each wave be $A_1 = A_0 \frac{\sin\alpha}{\alpha}$

$\boxed{e^{i\theta} = \underline{\cos\theta} + i\sin\theta}$

successive $\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$

Real part $e^{i\theta}$

Resultant y :-

$\boxed{y = A\cos(\omega t + \phi)}$

$$Y = A_1 \cos\omega t + A_1 \cos(\omega t + \phi) + A_1 \cos(\omega t + 2\phi) + \ldots N \text{ term}$$

$$= A_1 \{\cos\omega t + \cos(\omega t + \phi) + \ldots\ldots\} \Longrightarrow$$

$$= \underline{\underline{Re}} \left\{ A_1 \left\{ e^{i(\omega t)} + e^{i(\omega t + \phi)} + e^{i(\omega t + 2\phi)} + \ldots\ldots \right\} \right\}$$

Taking real part of $e^{i\theta}$

$$= A_1 e^{i\omega t}\left\{1 + e^{i\phi} + e^{2i\phi} + e^{3i\phi} \cdots - N\,\text{ter}\right\}$$

$$= A_1 e^{i\omega t}\left[\frac{1 - e^{iN\phi}}{1 - e^{i\phi}}\right]$$

Intensity $\Rightarrow YY^* \Rightarrow I = A_1^2\left[\frac{(1-e^{iN\phi})(1-e^{-iN\phi})}{(1-e^{i\phi})(1-e^{-i\phi})}\right]$

$$= A_1^2\left[\frac{1-\cos N\phi}{1-\cos\phi}\right] = A_0^2\frac{\sin^2\alpha}{\alpha^2}\left[\frac{\cancel{2}\sin^2\frac{N\phi}{2}}{\cancel{2}\sin^2\phi/2}\right] = \boxed{A_0^2\frac{\sin^2\alpha}{\alpha^2}\,\frac{\sin^2 N\beta}{\sin^2\beta}}$$

Case I : Principal Maxima

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta} \Rightarrow$$

For I_{max}, $\sin^2\beta = 0$ $\sin\beta = 0$ $\beta = \pm n\pi$, $n = 0, 1, 2, 3 \ldots$

then $\sin N\beta = 0$ giving $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0} =$ undefined.

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \pm N$$

$$\boxed{I_p = I_0 \frac{\sin^2\alpha}{\alpha^2} \cdot N^2}$$

$$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi \Rightarrow \boxed{(a+b)\sin\theta = \pm n\lambda}$$

For $n = 0$ we get central max.

Case II : Secondary minima

$I = 0 =$ min. when

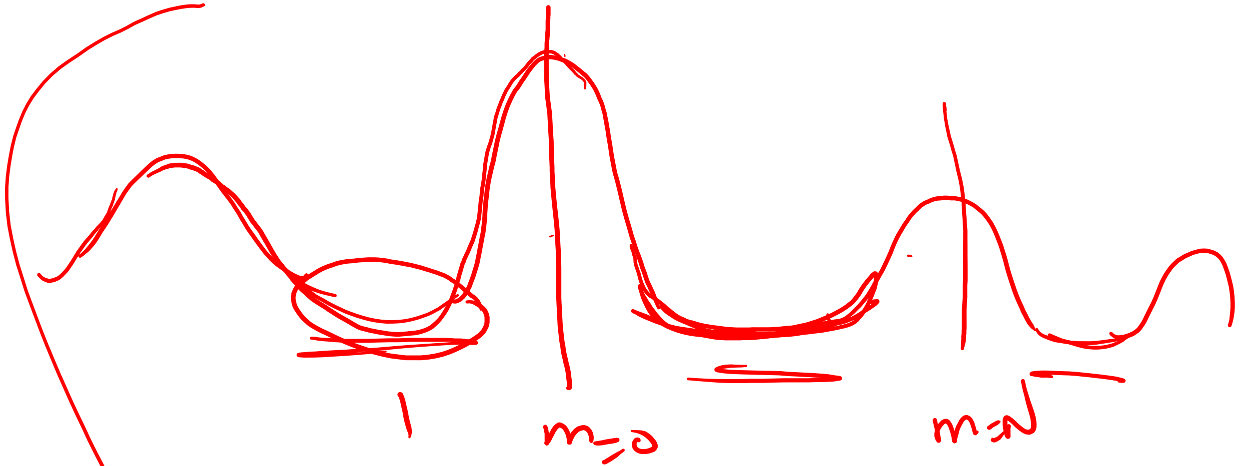
$\sin N\beta = 0$ but $\sin\beta \neq 0$

$N\beta = \pm m\pi$

$$\boxed{N(a+b)\sin\theta = \pm m\lambda}$$

$m = 1, 2, 3, 4, \ldots (N-1)$

If $m = 0$, $m = 'N'$, it corresponds to princ. max



Case III - Secondary Maxima

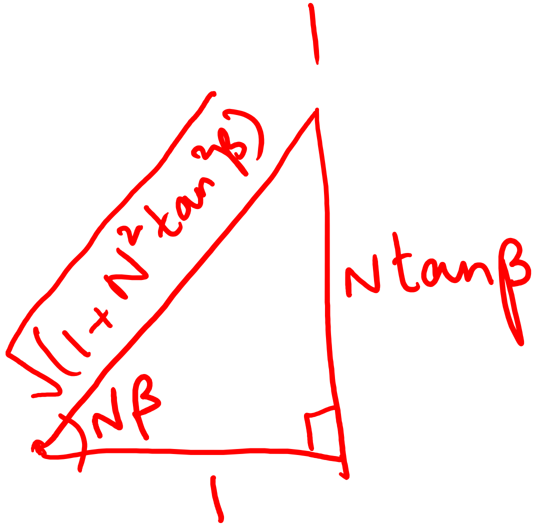
$$\frac{dI}{d\beta} = 0 = \underbrace{A_0^2 \frac{\sin^2\alpha}{\alpha^2}}\, 2\, \underbrace{\frac{\sin N\beta}{\sin\beta}} \left[\frac{N\sin\beta\cos N\beta - \sin N\beta\cos\beta}{\sin^2\beta} \right]$$

$$N\sin\beta\cos N\beta = \sin N\beta\cos\beta$$

$$\tan N\beta = N\tan\beta$$

$$\tan\theta = \frac{o}{a}$$

$$\theta = N\beta$$



Refering to the fig -

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta / (1+N^2\tan^2\beta)}{\sin^2\beta}$$

$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta}$$

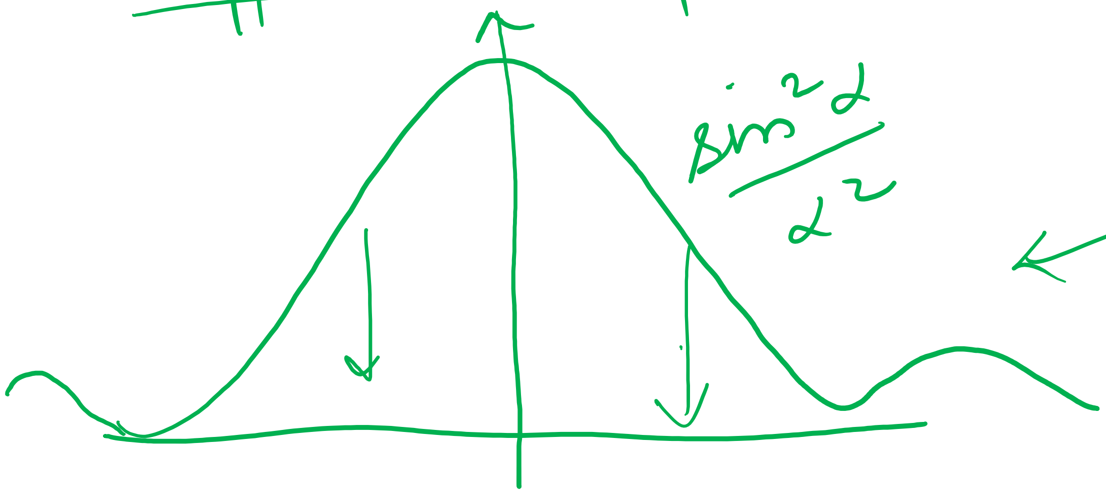
$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\cos^2\beta + \sin^2\beta - \sin^2\beta + N^2\sin^2\beta} = \frac{N^2}{1 + (N^2-1)\sin^2\beta}$$

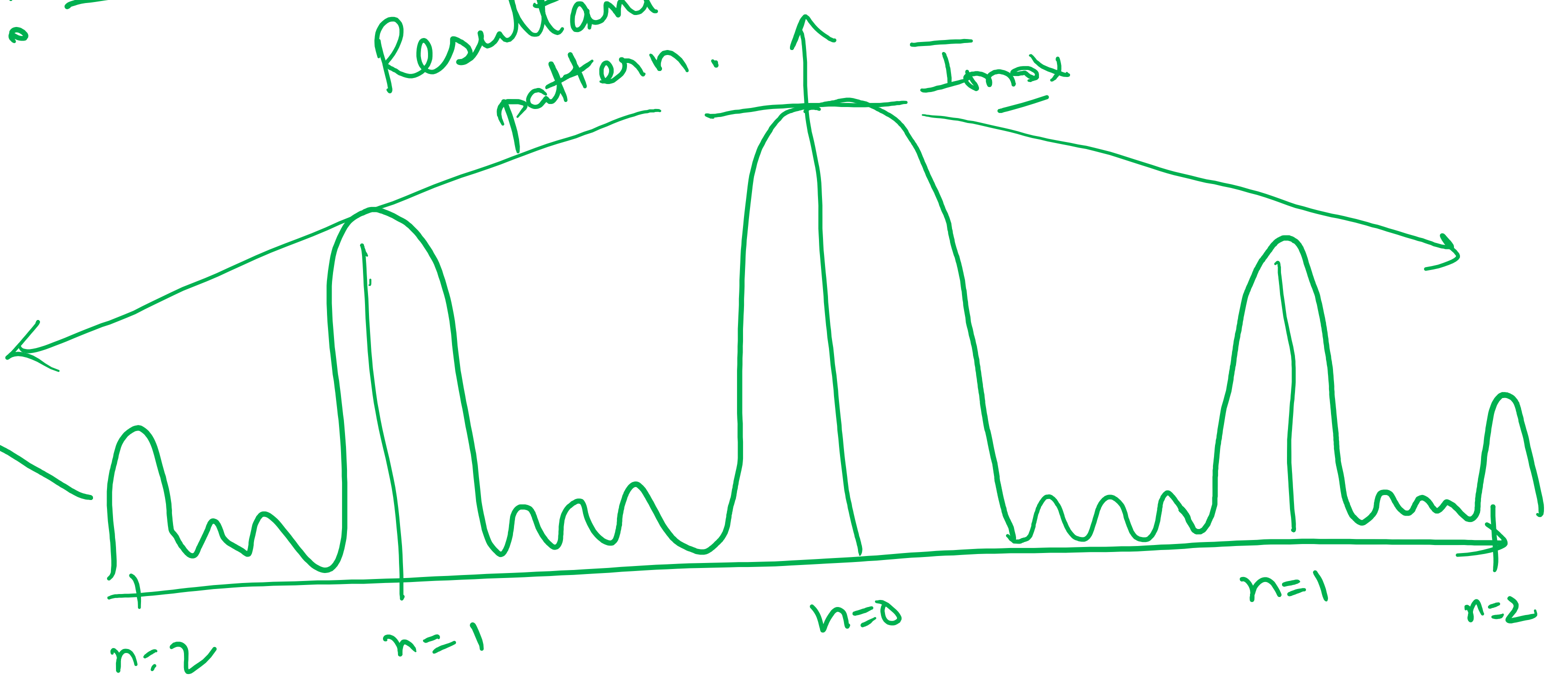
$$I_s = A_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

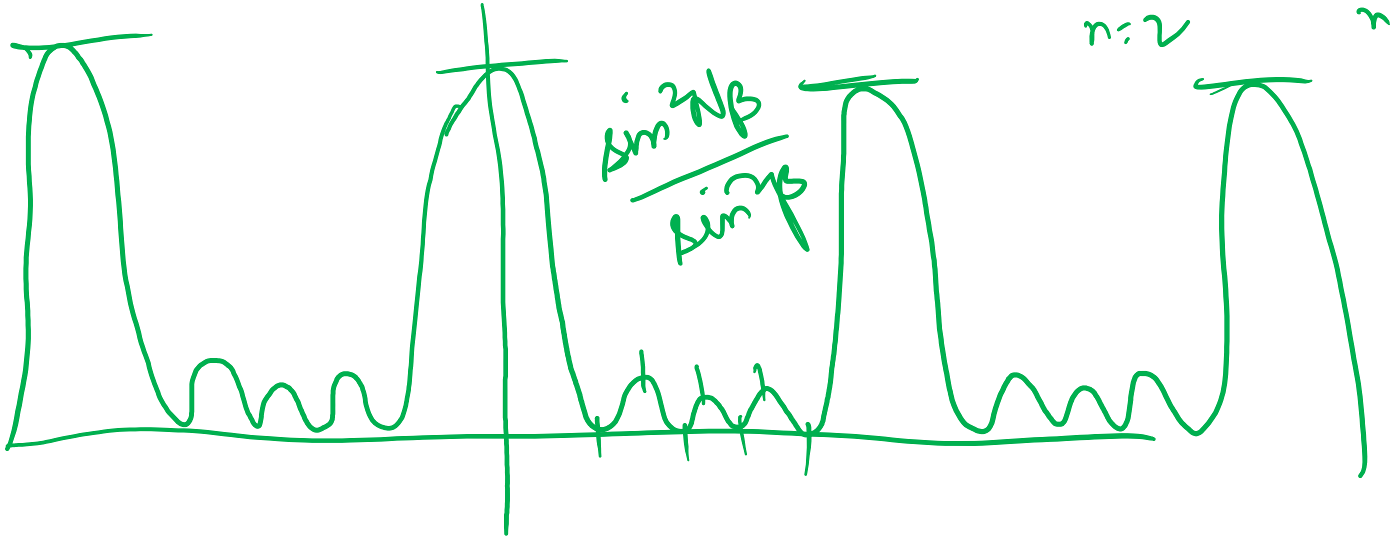
Comparing with I_p.

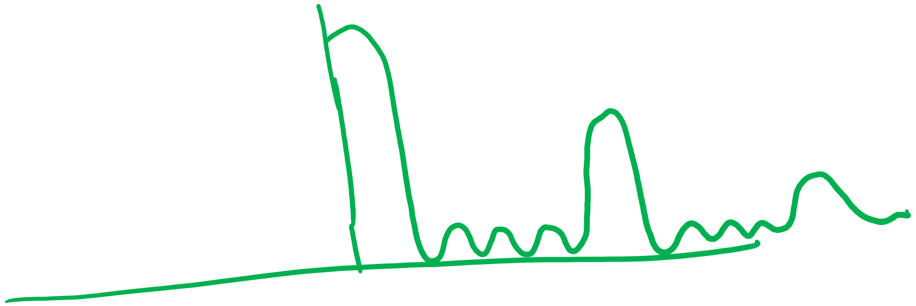
$$\boxed{\frac{I_s}{I_p} = \frac{1}{1+(N^2-1)\sin^2\beta}}$$

Diffraction pattern :-

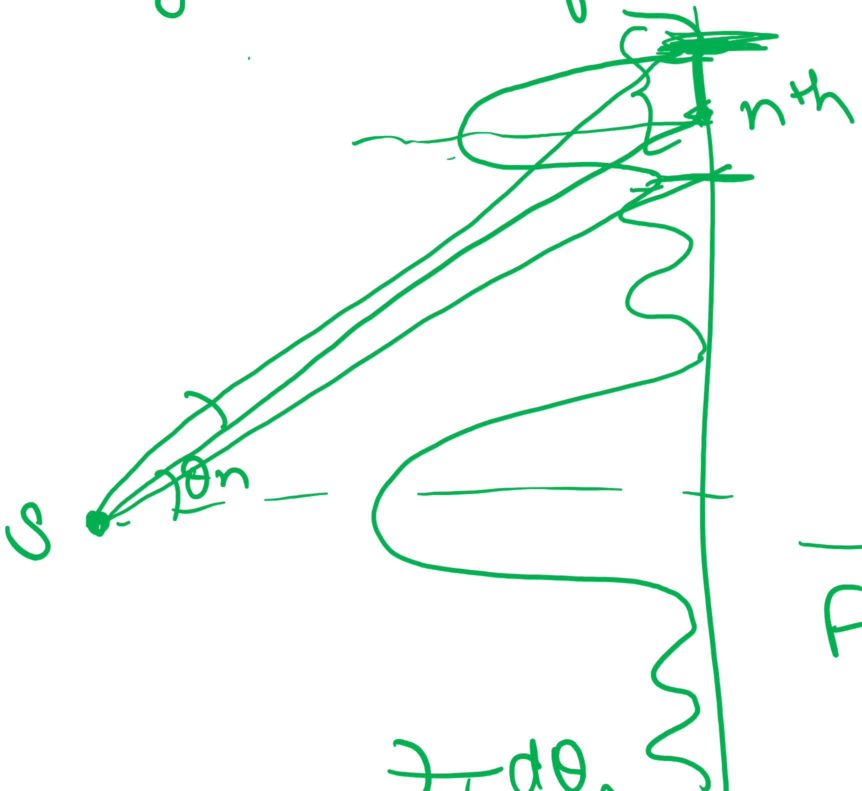








Angular Half width of Principal Max.



θ_n = central pt of n^{th} max

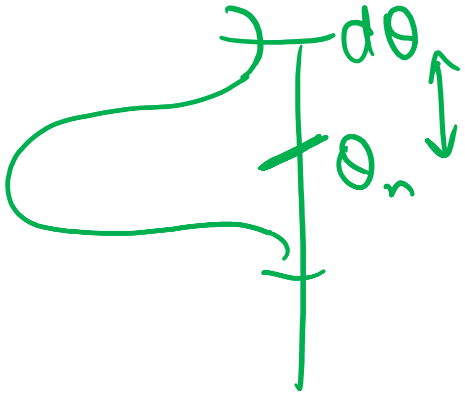
$\theta_n + d\theta$ = minimum pt on one side

$\theta_n - d\theta$ = " " " other side

$$\theta_n \pm d\theta$$

For n^{th} max :-

$$(a+b)\sin\theta_n = \pm n\lambda$$



For next min -

$$N(a+b)\sin(\theta_n \pm d\theta) = \pm m\lambda$$

$$\boxed{(a+b)\sin(\theta_n \pm d\theta) = n\lambda \pm \frac{\lambda}{N}}$$

Dividing →

$$\frac{\sin(\theta_n \pm d\theta)}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

$$\frac{\sin\theta_n \cos d\theta \pm \cos\theta_n \sin d\theta}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

$\cos d\theta \simeq 1 \qquad \sin d\theta \simeq d\theta$

$$\frac{\sin\theta_n \pm \cos\theta_n d\theta}{\sin\theta_n} = 1 \pm \frac{1}{nN} \Rightarrow \boxed{d\theta = \frac{\tan\theta_n}{nN}}$$

$\cancel{\sin\theta_n} \pm \cos\theta_n d\theta = \cancel{\sin\theta_n} \pm \frac{\sin\theta_n}{nN}$

$d\theta =$

Not in syllabus

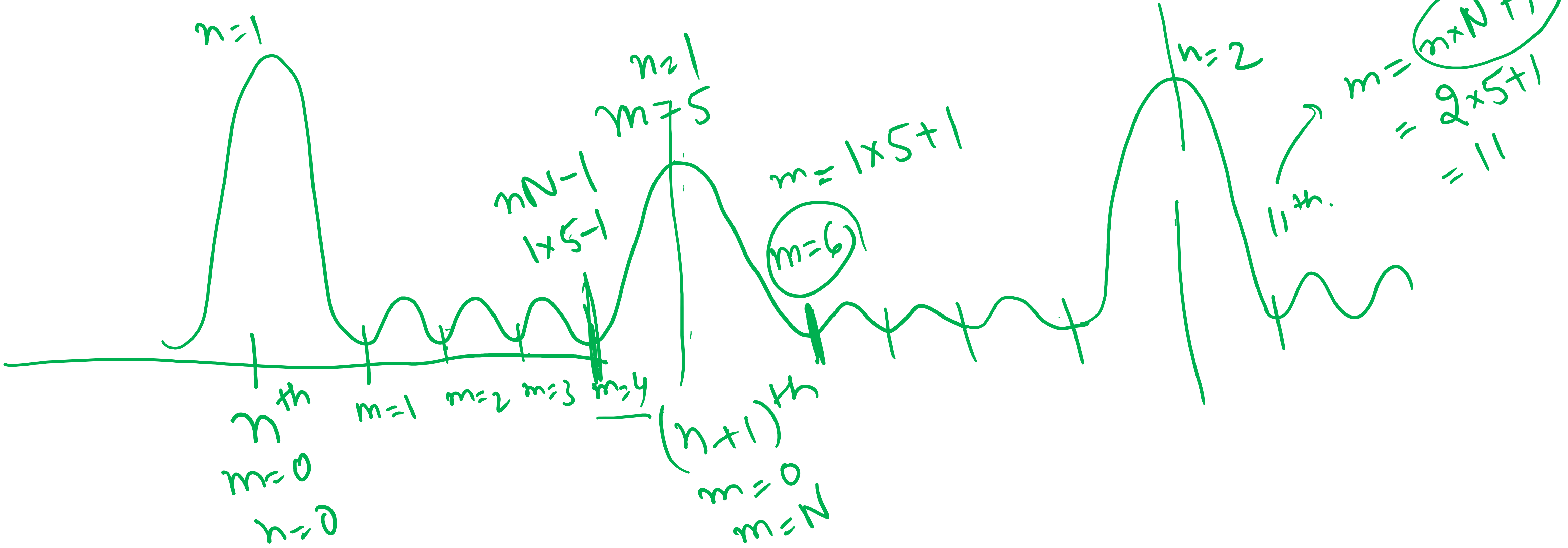
$$N(a+b)\sin\theta_n = m\lambda$$

n^{th} peak

$$N(a+b)\sin(\theta_n + d\theta) = (nN \pm 1)\lambda$$

$$(a+b)\sin(\theta_n + d\theta) = \left(n \pm \frac{1}{N}\right)\lambda$$

$n=0$ $m=1, 2, 3, 4$



Absent Spectra | Missing order spectra

1) $(a+b) \sin\theta = n\lambda$ (max. condn).

2) $a \sin\theta = m\lambda$ (min condn in single).

If both the condns are simultaneously met, then those particular spectra order of 'n' will be missing.

Divide $\Rightarrow$ $\boxed{\frac{(a+b)}{a} = \frac{n}{m}}$ req. condn.

If we take $b = a$ then,

$$n = 2m = 2, 4, 6, \ldots\ldots$$

$\therefore$ 2nd, 4th, 6th .... order spectra will be missing

If $b = 2a$, $n = 3m = 3, 6, 9 \ldots\ldots$

3rd, 6th, 9th spectra will be missing

Maximum no. of orders in a grating spectra

$$(a+b)\sin\theta = n\lambda \Rightarrow n = \frac{(a+b)\sin\theta}{\lambda}$$

Max. value of $\sin\theta = 1$ $\Rightarrow$ $\boxed{n_{max} = \frac{(a+b)}{\lambda}}$ $\Rightarrow$

If $(a+b)$ is b/w λ and 2λ then

$$(a+b) < 2\lambda \Rightarrow n_{max} < \frac{2\lambda}{\lambda} < 2$$

$$\boxed{\begin{aligned} n &= 0 \\ n &= 1 \end{aligned}}$$

Dispersive power of a grating - rate of change of the angle of diffraction with 'λ' of light.

$\left(\frac{d\theta}{d\lambda}\right)$ ✓

$$\boxed{(a+b)\sin\theta = n\lambda}$$

$$(a+b)\cos\theta\, d\theta = n\, d\lambda$$

$\theta_{red} > \theta_{violet}$

$$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

1) $\propto n \Rightarrow$ higher the n, higher the dispersion

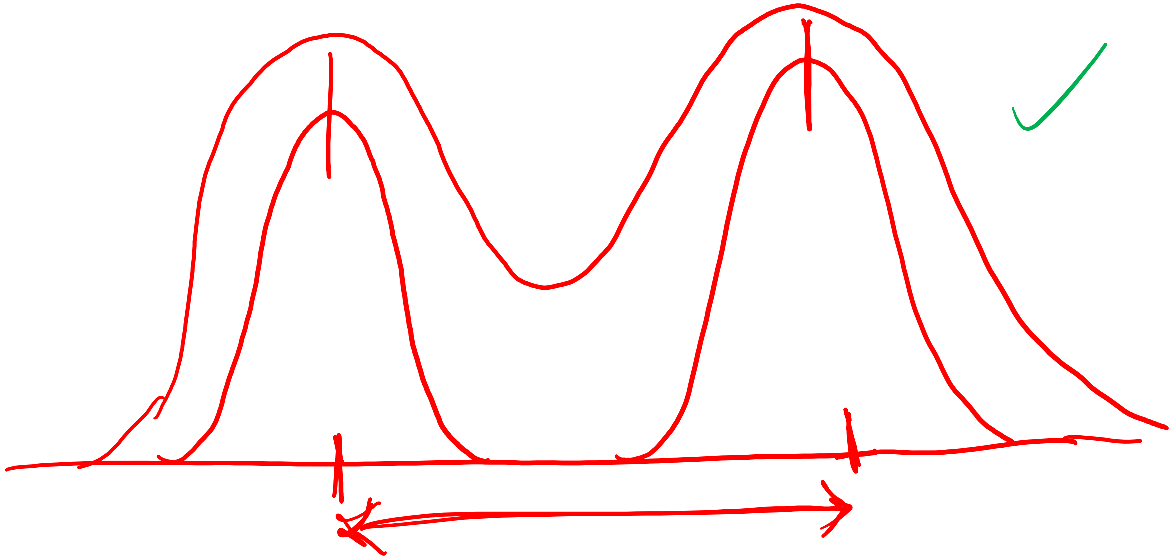
2) $\propto \frac{1}{a+b} \Rightarrow$ smaller $(a+b)$, " " " (wider).

Resolving power of a diffraction grating

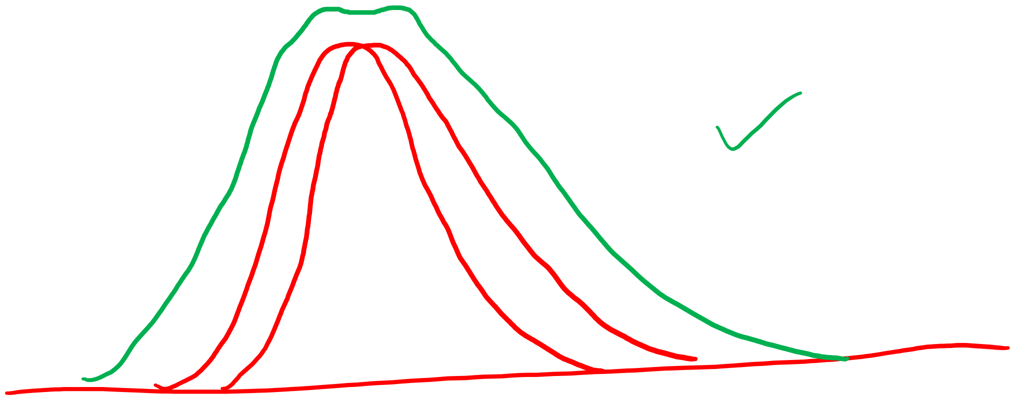
Resolution – is the ability to observe two nearby objects as separate or distinct.

$$\boxed{\frac{\lambda}{d\lambda}} = \text{Resolving power}$$

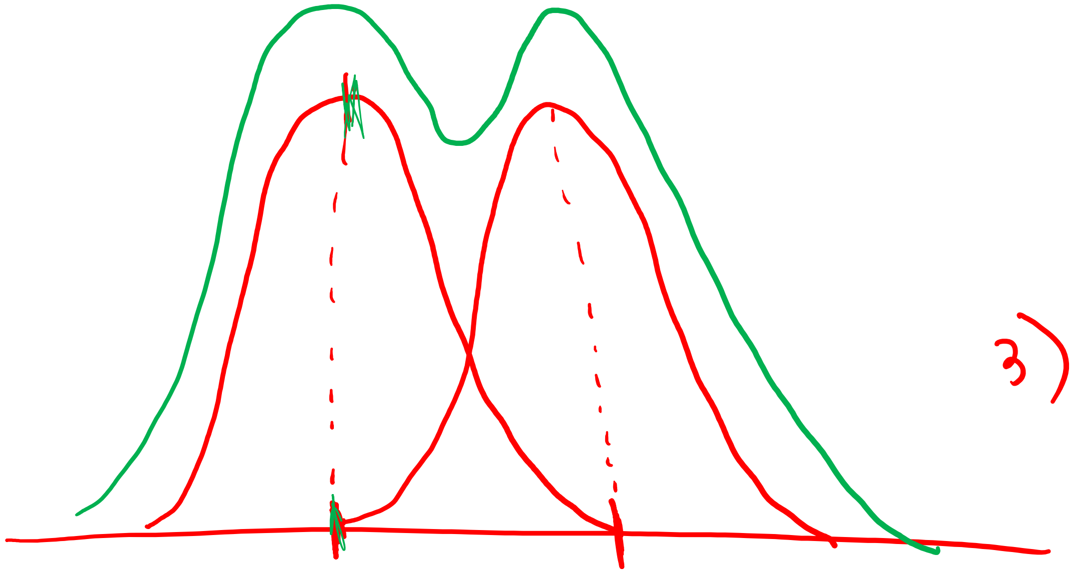
Rayleigh's Criterion :- Two nearby peaks (wavelengths) are said to be just resolved, when position of principal max. of one coincides with the first minimum of the other & viceversa.



1) over-resolved



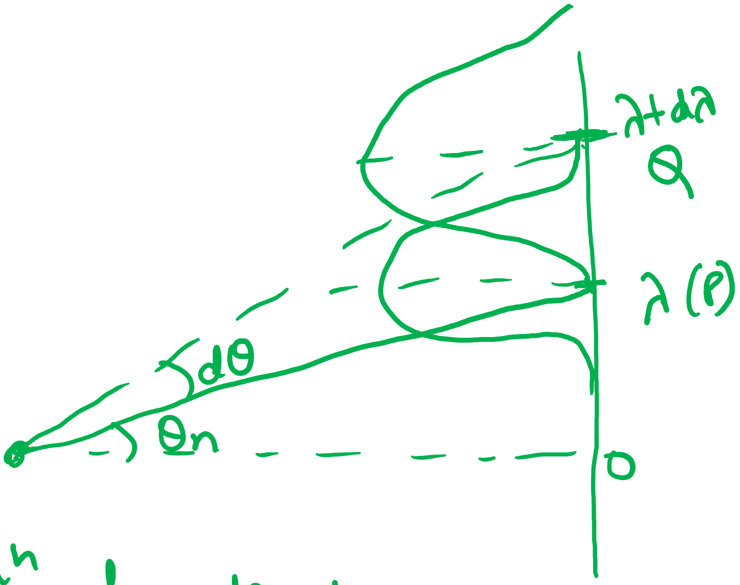
2) under-resolved.



3) just-resolved

$(a+b)\sin\theta_n = n\lambda$ max cond^n at point 'P'

$(a+b)\sin(\theta_n + d\theta) = n(\lambda + d\lambda)$ " " " 'Q' — 1



$N(a+b)\sin(\theta_n + d\theta) = m(\lambda)$ → min cond^n. for first peak at point 'Q'

$(a+b)\sin(\theta_n + d\theta) = n\lambda + \frac{\lambda}{N}$ — 2

2 = 1 $\Rightarrow$ $n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$

$$\boxed{\frac{\lambda}{d\lambda} = nN} \Rightarrow$$