AMOGH GARG - 2020UCO1688

Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Q.7	Q.8	Q.9	Q.10	Q.11	Q.12	Q.13	Q.14	Q.15
✓	✓	✓	✓	✓	✓		✓	✓	✓		✓		✓	

Q.2) P : Jack Passed Math ; Q : Jill Passed Math

a) $P \wedge Q$ (Jack and Jill both passed math)

b) $P \rightarrow (\sim Q)$ (If Jack passed math then Jill did not)

c) $P \vee Q$ is "Jack or Jill Passed in Maths".

Q.3) If student gets 95% on their final, then they will get an A in the course.

a) We can conclude that Sita got 95% in her finals

b) If Sita gets 92% in her final, then she will not get A, but may get some other grade.

c) If Sita does not get A, it means that Sita scored marks except from 95% in her finals.

Q.4) 1) $\{x : x+3 \in N\} \equiv \{-3, -2, -1, 0, ----\}$

It is set of all integers greater than equal to -3.

2) $\{x \in N : x+3 \in N\} \equiv \{0, 1, 2, 3, 4, ---\}$

It is set of all Natural numbers including 0.

3) $\{x : x \in N \vee -x \in N\} \equiv \{---, -2, -1, 0, 1, 2, 3, --\}$

It is set of all integers.

4) $\{x : x \in N \cap -x \in N\} \equiv \{0\}$

It is set of just one element which contains 0.

5) $\{x \in Z : x^2 \in N\} \equiv \{---, -2, -1, 0, 1, 2, 3, ---\}$

It is the set of all integers.

(2)

Q.6) $a_n = 2a_{n-1} - a_{n-2}$, Let $a_n = r^n$ be the solution of this equation

$\Rightarrow r^n = 2r^{n-1} - r^{n-2}$, Dividing both sides by r^{n-2}

$\Rightarrow r^2 - 2r + 1 = 0 \Rightarrow (r-1)^2 = 0$

$\Rightarrow r_1 = 1$ and $r_2 = 1$

$\therefore a_n = \alpha_1 (r_1)^n + \alpha_2 \times n \times (r_2)^n$

$a_0 = \alpha_1 + 0 = 3 \Rightarrow \boxed{\alpha_1 = 3}$

$a_1 = 3 + \alpha_2 \times 1 \times 1 = 4 \Rightarrow \boxed{\alpha_2 = 1}$

$\therefore$ Solution $\underline{a_n = 3 + n}$

Q.9) "a" – "h" → 8 letters are there

(a) Total words of length 5 $= (8)^5 = 32{,}768$

(b) No repeated letters $= \underbrace{{}^8C_5}_{\text{Choosing}} \times \underbrace{5!}_{\text{Arranging}} = 6720$

(c) Start with "aha" $= 8 \times 8 = 64$ (Filling the left out 2 blank spaces)

(d) End with "ahb" $= 8 \times 8 = 64$

Now, Start with "aha" or end with "ahb" or both

$= 64 + 64 - 1 = 127$ ("−1" because their intersection contains only 1 element i.e ahahb)

(e) Contain "bad" $= {}^3C_1 \times 8 \times 8 = 192$

$\therefore$ No repeats and do not contain "bad" $= 6720 - 192$

$= \underline{6528}$

Q.5) $A = \{2, 4, 6, 8\}$ and $|B| = 5$

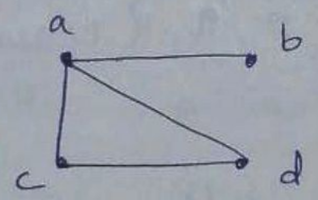
(a) $|A \cup B|$: → Smallest value is equal to 5 if B contains all elements of A, plus one extra element.
→ Largest value is 9 if B contains any 5 elements which are not in A.

(b) $|A \cap B|$: → Smallest value is 0 if A and B have no elements in common.
→ Largest value is 4, if B contains all elements of A, plus one extra element.

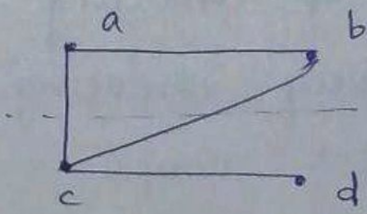
(c) $|A \times B|$: Smallest and largest value are both equal to 20, since $|A \times B| = |A| \times |B|$ in any case.

Q.8) (i) Range of $\{1, 2, 3\}$ is $\{a\}$
(ii) Domain of $\{a, b\}$ is $\{2, 4\}$
(iii) Domain of $\{d\}$ does not exist, since their is no pre-image of d in $\{1, 2, 3, 4, 5, 6\}$.

Q.10) G_1:



G_2:



Yes G_1 and G_2 are isomorphic graph since they have same connections.
If we flip the graph G_2 about the dotted line we will get G_1.

OR

We can interchange vertices "d and b", and, "a and c".
⇒ $E_2 = \{\{c, d\}, \{c, a\}, \{\cancel{d}b, a\}, \{a, b\}\}$
This becomes same as E1.
∴ G_1 and G_2 are isomorphic.

Q.14) Let us proof by contradiction.

Let $H_1 \cup H_2$ is subset of G since $H_1 \not\subset H_2$

$\Rightarrow \exists$ element $a \in H_1$, such that $a \notin H_1$.

similarly, $H_2 \not\subset H_1 \Rightarrow \exists$ element $b \notin H_2$ such that $b \in H_1$.

As $H_1 \cup H_2$ is group $\Rightarrow ab \in H_1 \cup H_2$

$\Rightarrow ab \in H_2$ or $ab \in H_1$

If $ab \in H_1$, $b = a^{-1}(ab) \in H$

as both a^{-1} and ab are element in sub-group H_1.

This contradicts our choice as element b.

similarly, if $ab \in H_2 \Rightarrow a = (ab)b^{-1} \in H_2$

which contradicts choice of a.

in both cases we reach a contradiction.

$\therefore$ $H1 \cup H2$ is not a sub group of G.

Q.12) Way of choosing first digit = 8 (Leaving out 7 from 1-9)
(i.e Hundred)

Way of choosing ten's digit = 9 (Leaving 7 from 0-9)

similarly choosing one's digit = 9

$\therefore$ Total ways = $8 \times 9 \times 9 = 648$

Q.1) $(3^3)^{41} \bmod 7 = (27)^{41} \bmod 7$

$= (28-1)^{41} \bmod 7$

Expanding : $\left[{}^{41}C_0 (28)^{41}(-1)^0 + \cdots + {}^{41}C_{41} (28)^0 (-1)^{41}\right] \bmod 7$

All 28 containing terms will be divisible by 7.

So, $x = -1 \bmod 7$

$= \underline{6}$ (Remainder can not be -ve)

Ans.) $\underline{6}$