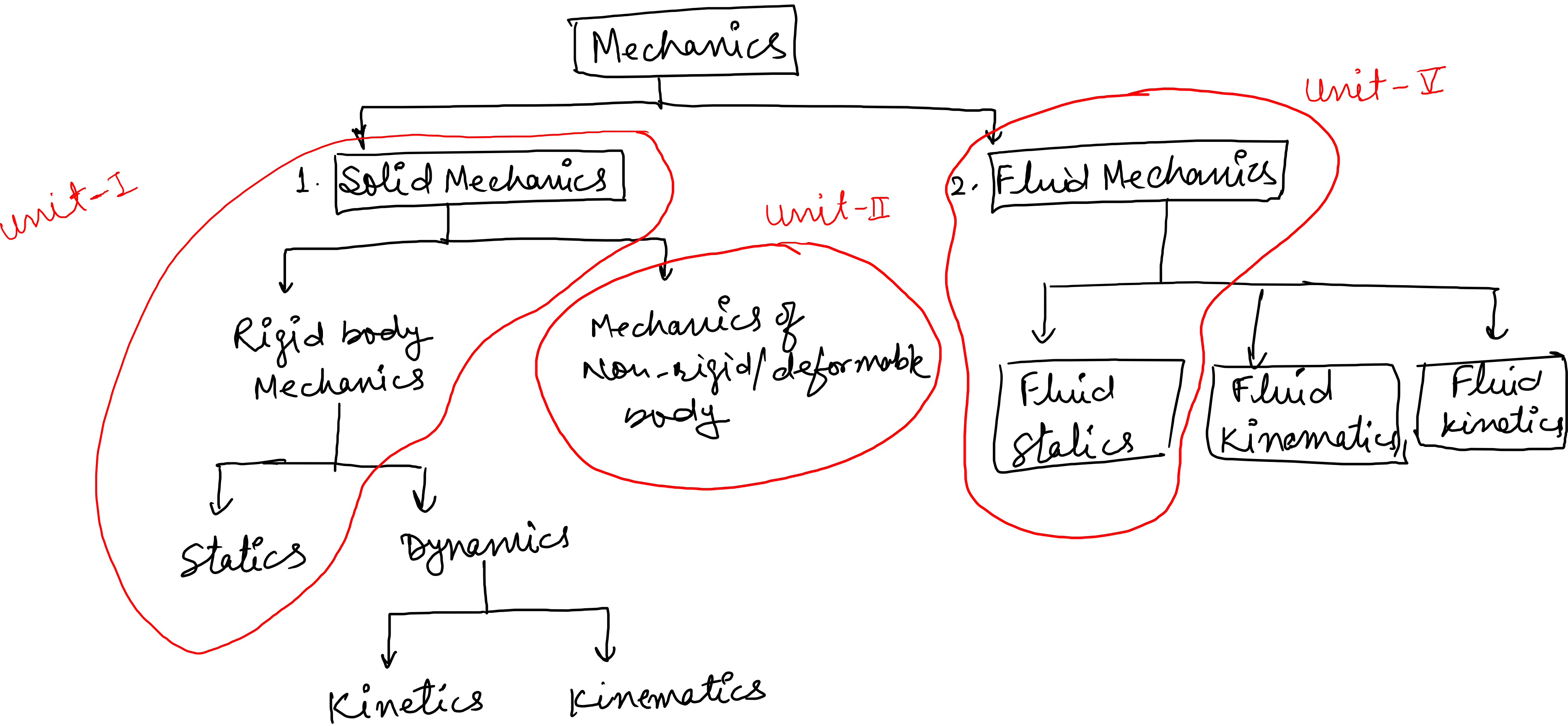
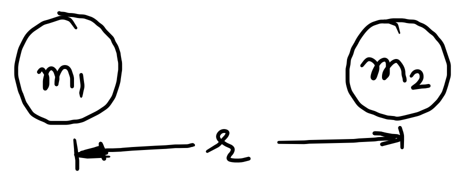

Mechanics
1. Solid Mechanics
unit-I
Rigid body Mechanics
Statics
Dynamics
Kinetics
Kinematics
unit-II
Mechanics of Non-rigid/deformable body
2. Fluid Mechanics
unit-V
Fluid Statics
Fluid Kinematics
Fluid kinetics

Laws of Mechanics:

1. Newton's First Law of motion $\rightarrow$ Law of inertia
2. " Second Law " $\rightarrow$ $F \propto \frac{dp}{dt} \Rightarrow F = ma$
3. " Third law " $\rightarrow$ Law of action-reaction

4. Gravitational law of attraction

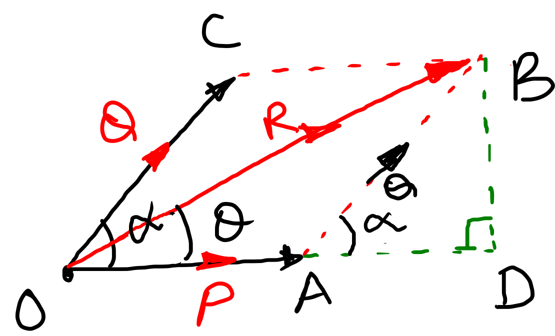
$$F \propto m_1 m_2 \quad \text{—(i)}$$



$$F \propto \frac{1}{r^2} \quad \text{—(ii)}$$

$$F \propto \frac{m_1 m_2}{r^2} \Rightarrow F = \frac{G.m_1 m_2}{r^2}, \quad G = 6.67 \times 10^{-11} \frac{Nm^2}{Kg^2}$$

5. Parallelogram Law of forces



In Δ ODB

$$OB^2 = OD^2 + BD^2$$

$$R^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

In Δ ADB

$$AD = Q\cos\alpha$$

$$BD = Q\sin\alpha$$

Magnitude, $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$ — (1)

Direction, $\tan\theta = \dfrac{Q\sin\alpha}{P + Q\cos\alpha}$ — (2)

Case-I: $\alpha = 0^\circ$

$R = P + Q$

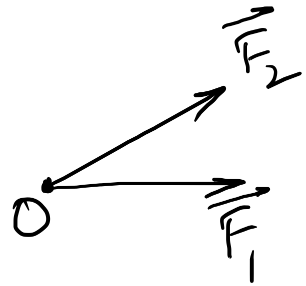
Case-II: $\alpha = 180^\circ$

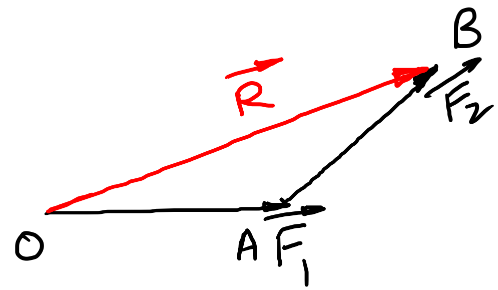
$R = P - Q$

Case-III: $\alpha = 90^\circ$

$R = \sqrt{P^2 + Q^2}$

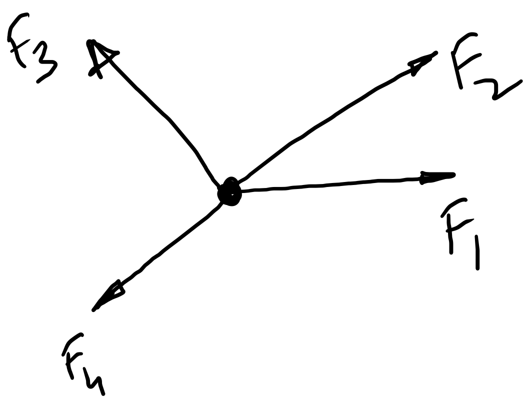
6. Triangle Law of forces

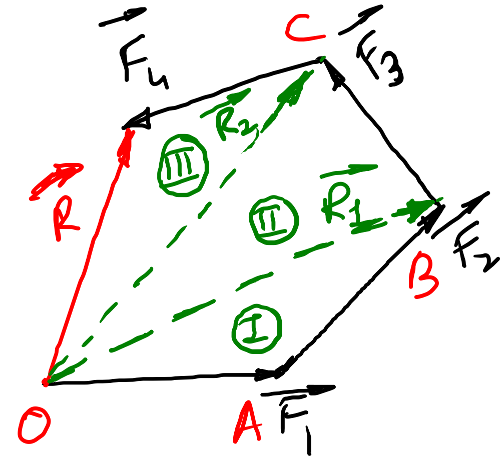




$$\vec{R} = \vec{F_1} + \vec{F_2}$$

7. Polygon law of forces

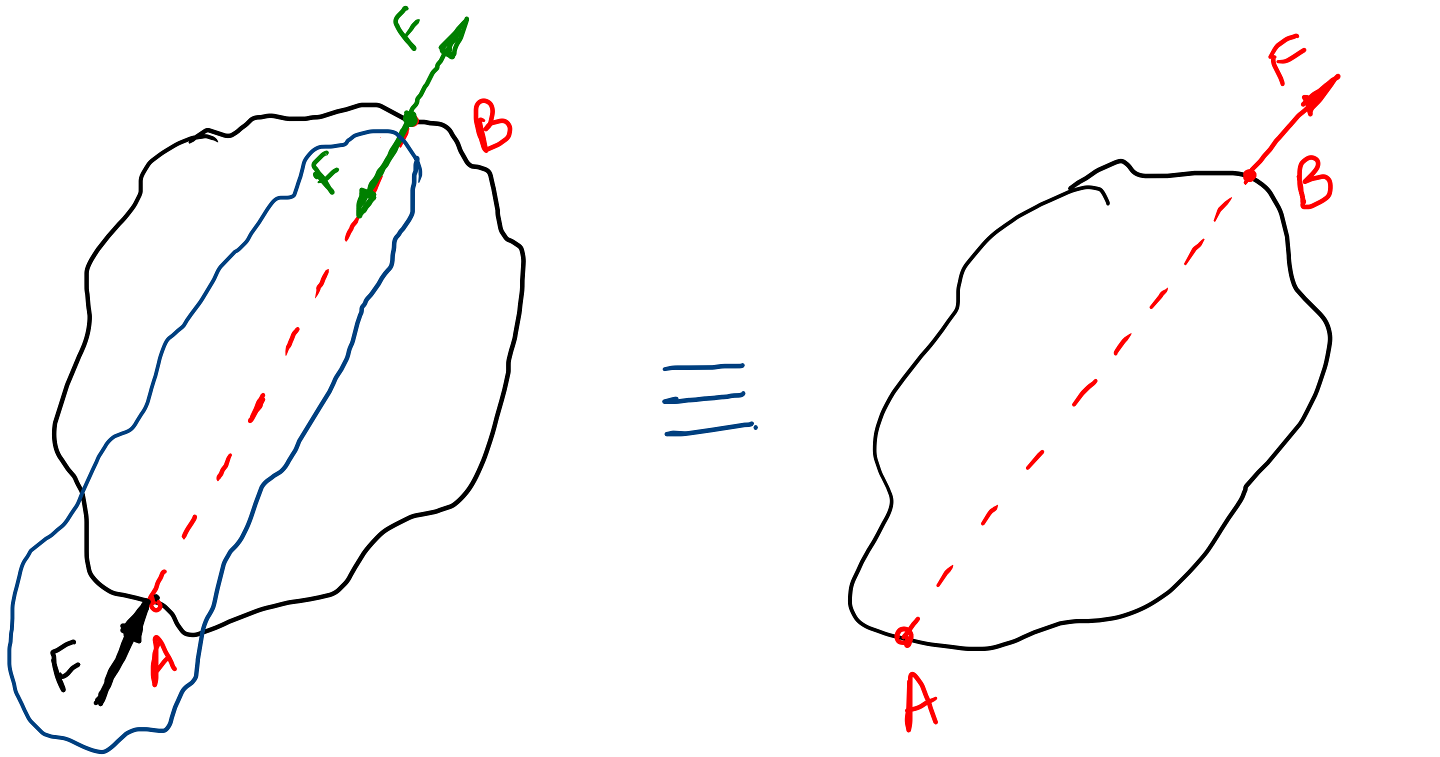




$$\vec{R_1} = \vec{F_1} + \vec{F_2}$$
$$\vec{R_2} = \vec{R_1} + \vec{F_3}$$
$$\vec{R} = \vec{R_2} + \vec{F_4}$$

$$\boxed{\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}}$$

8. Principle of transmissibility of forces



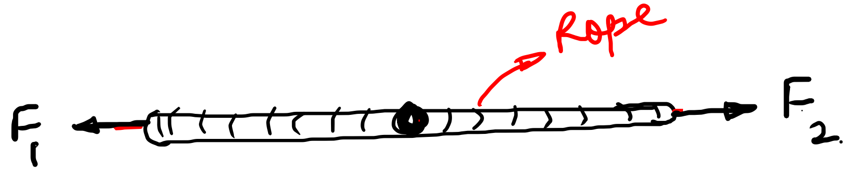
System of Forces

1. Co-planar or 2D - system
   - (i) Collinear
   - (ii) Concurrent
   - (iii) Parallel
     - Like
     - Unlike
   - (iv) Non-concurrent & Non-parallel

2. Non-coplanar or 3D - System
   - (i) Concurrent
   - (ii) Parallel
     - Like
     - Unlike
   - (iii) Non-concurrent & Non-parallel

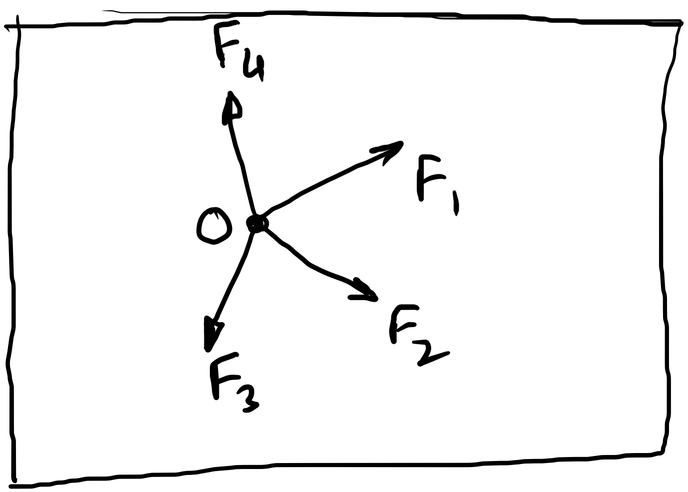
(1.) Coplanar forces (2D)

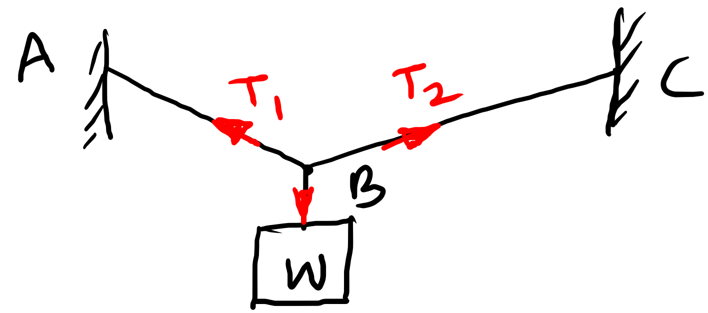
(i) collinear

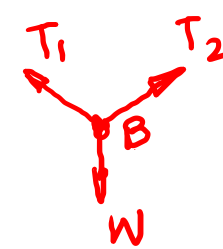


- A game of tug of war

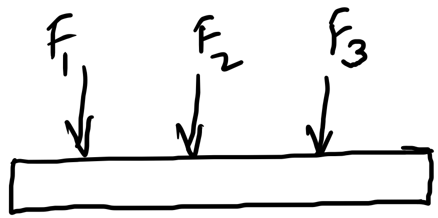
(ii) concurrent



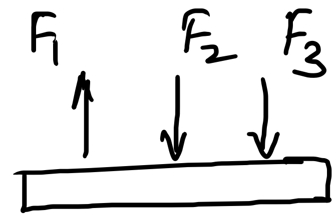




(iii) Parallel

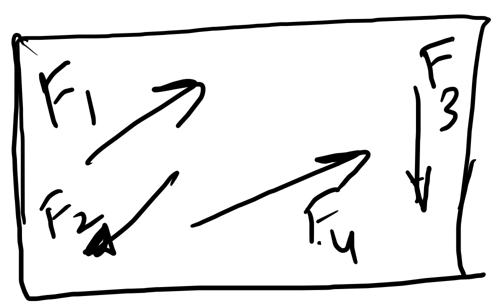


Like



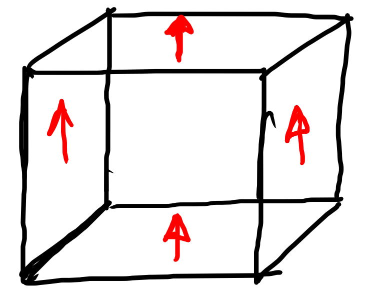
unlike

(iv) Non concurrent and Non parallel



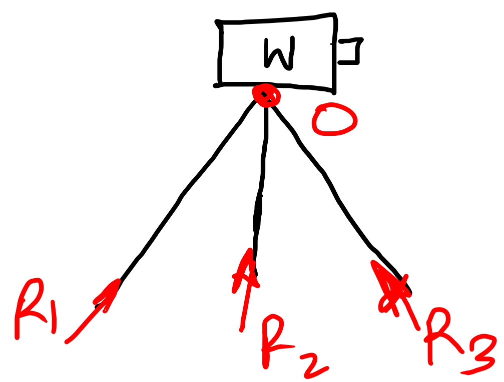
(2.) Non-coplanar forces

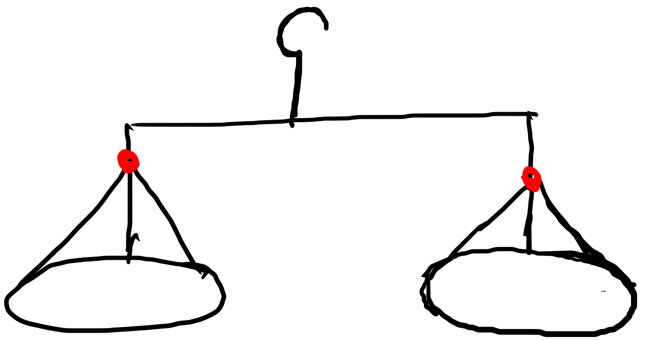
(i) Parallel



(ii) concurrent

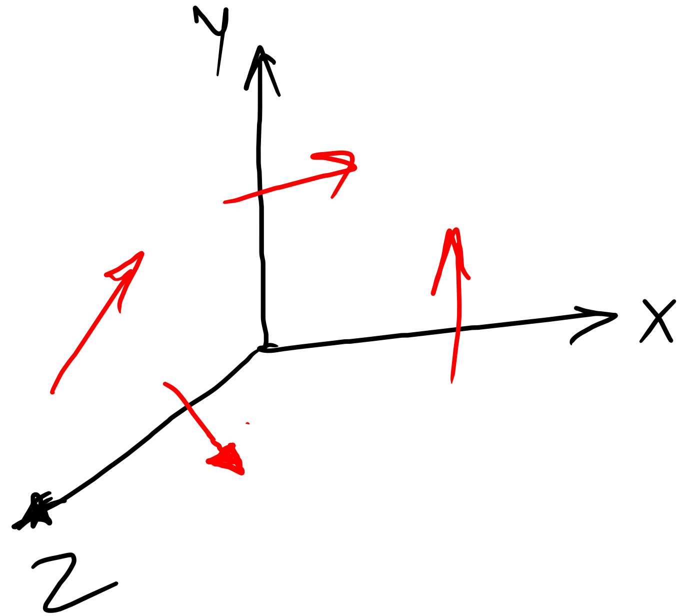
• A tripod stand of camera





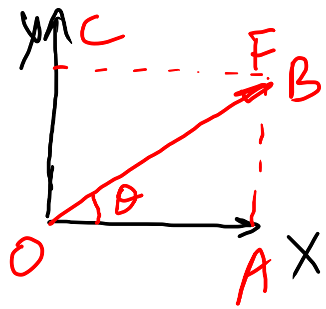
weighing balance

(iii) Non concurrent and Non parallel



Resolution of a forces

(1.) Rectangular resolution



$$\frac{OA}{OB} = \cos\theta$$

$$F_x = OA = F\cos\theta$$

$$F_y = OC = F\sin\theta$$

(2.) Oblique resolution

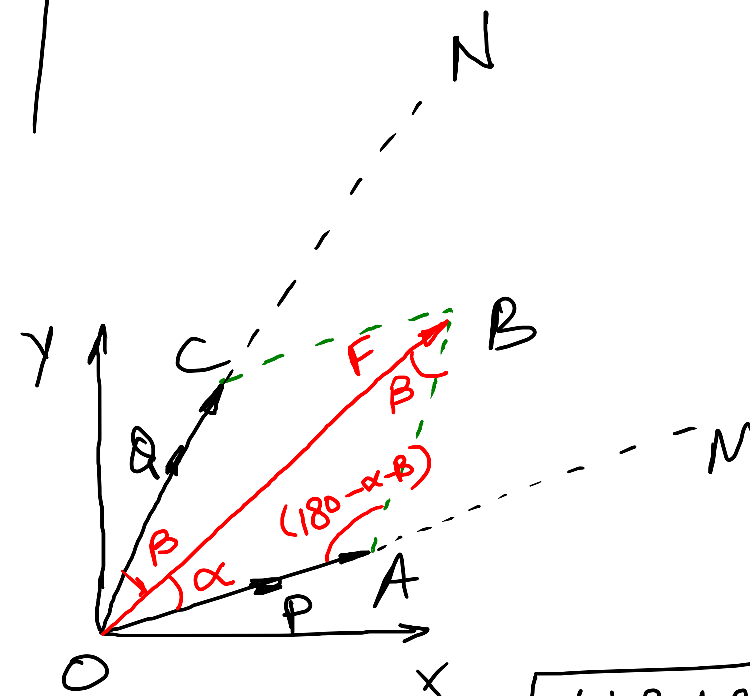
Apply sine rule of triangle

consider $\underline{\Delta\, OAB}$

$$\frac{OA}{\sin\beta} = \frac{AB}{\sin\alpha} = \frac{OB}{\sin(180^\circ - \alpha - \beta)}$$

$$P = \frac{F\sin\beta}{\sin(\alpha+\beta)}$$

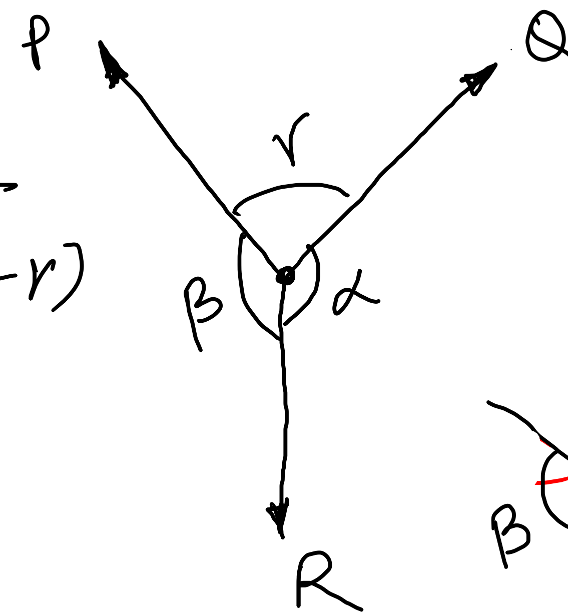
$$Q = \frac{F\sin\alpha}{\sin(\alpha+\beta)}$$



$$\boxed{\alpha + \beta \neq 90^\circ}$$

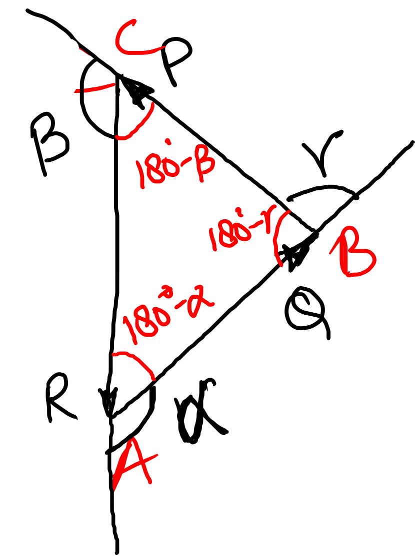
Lami's theorem

Apply sine rule of triangle

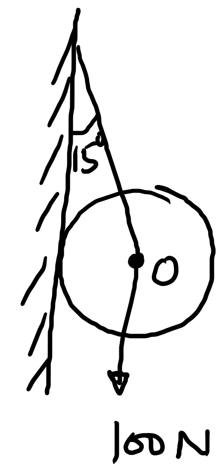


$$\frac{AB}{\sin(180^\circ-\beta)} = \frac{BC}{\sin(180^\circ-\alpha)} = \frac{CA}{\sin(180^\circ-\gamma)}$$

$$\Rightarrow \frac{Q}{\sin\beta} = \frac{P}{\sin\alpha} = \frac{R}{\sin\gamma}$$



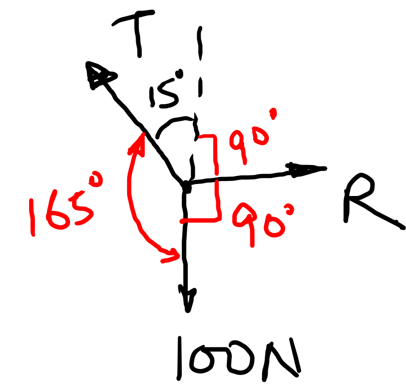
Q: 1 Find Tension in the string and reaction.



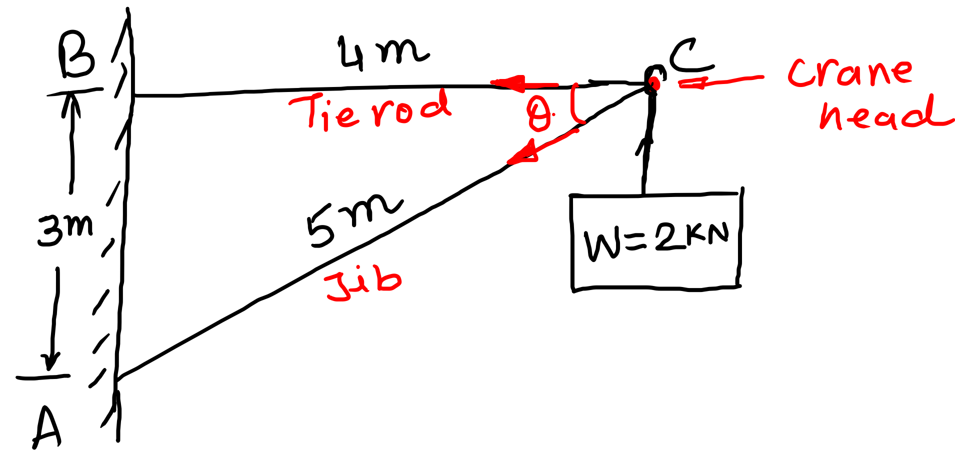
Sol^n:

$$\frac{T}{\sin 90^\circ} = \frac{R}{\sin 165^\circ} = \frac{100}{\sin 105^\circ}$$

$$\Rightarrow \left.\begin{aligned} T &= 103.52\,N \\ R &= 26.795\,N \end{aligned}\right]$$



Q; 2 Find tension in the tie rod and axial force in the jib.



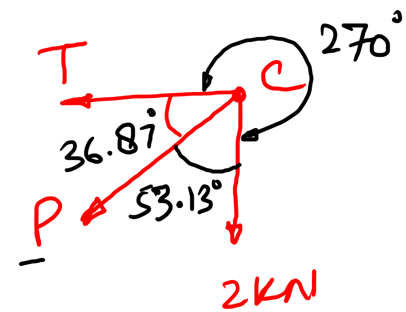
Jib crane apparatus

Sol^n

$$\sin\theta = \frac{3}{5}$$

$$\theta = 36.87^\circ$$

Apply Lami's Theorem



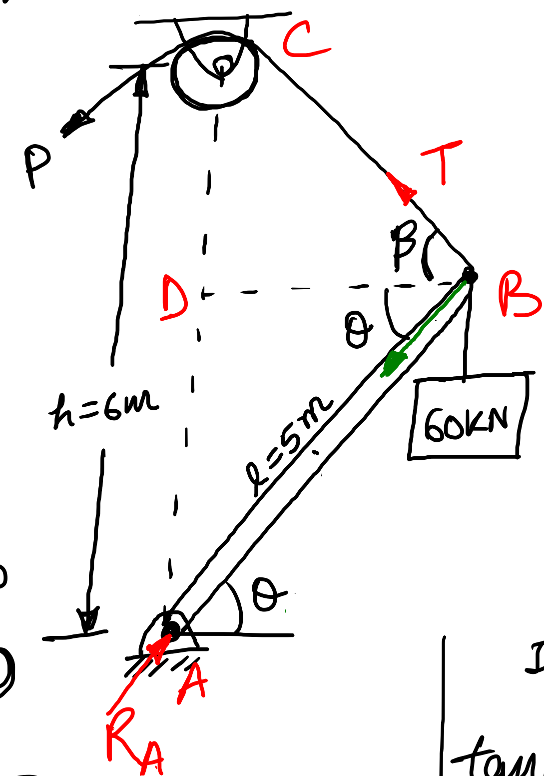
$$\frac{T}{\sin 53.13^\circ} = \frac{P}{\sin 270^\circ} = \frac{2KN}{\sin 36.87^\circ}$$

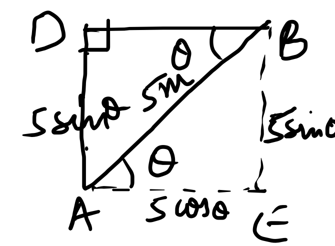
$\Rightarrow$ $T = 2.667$ KN

$P = -3.33$ KN

$\therefore$ $P = 3.33$ KN (from A to C)

Q:3 Find axial force in the bar AB and the limiting value of tension when the bar approaches vertical position.





Sol^n:

$\Sigma F_x = 0$, $R_A \cos\theta - T\cos\beta = 0$

$$T\cos\beta = R_A\cos\theta \quad \text{— (1)}$$

$\Sigma F_y = 0$, $R_A \sin\theta + T\sin\beta - 60\,kN = 0$

$$T\sin\beta = W - R_A\sin\theta \quad \text{— (2)}$$

$$\tan\beta = \frac{W - R_A\sin\theta}{R_A\cos\theta} \quad \text{— (3)}$$

$$\frac{W - R_A\sin\theta}{R_A\cancel{\cos\theta}} = \frac{6 - 5\sin\theta}{5\cancel{\cos\theta}}$$

In ΔBCD

$$\tan\beta = \frac{CD}{BD} = \frac{AC - AD}{BD} = \frac{6 - 5\sin\theta}{5\cos\theta} \quad \text{— (4)}$$

$$6R_A - 5R_A \cancel{\sin\theta} = 5W - 5R_A \cancel{\sin\theta}$$

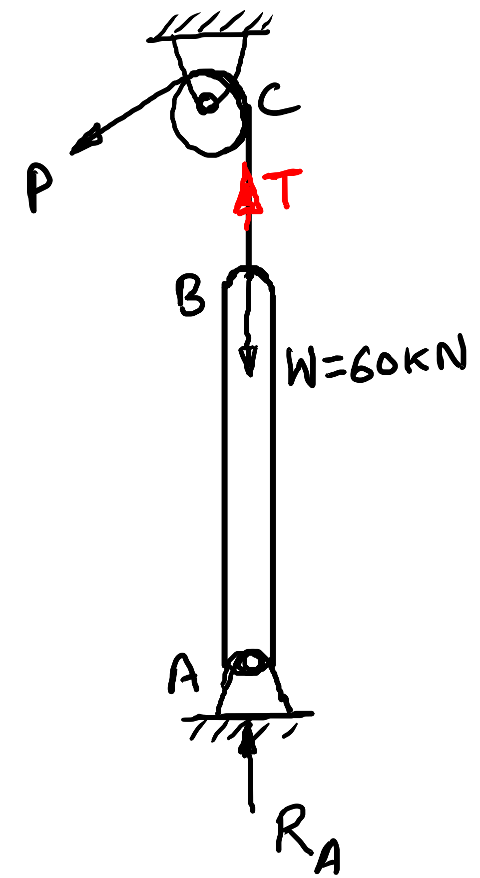
$$\Rightarrow 6R_A = 5W$$

$$\boxed{R_A = 50\,KN}$$

Limiting value of Tension

$$T + R_A = 60\,KN$$

$$\boxed{T = 10\,KN}$$



Q:4 A partical is acted upon by the following forces:

(i) 40N pull 30° from North towards East

(ii) 50N push 45° " South " West

(iii) 20N push 60° " North " west

(iv) 60N push 60° " south " East

Determine magnitude and direction of the resultant force on the partical.

Sol:

$\Sigma F_x =$

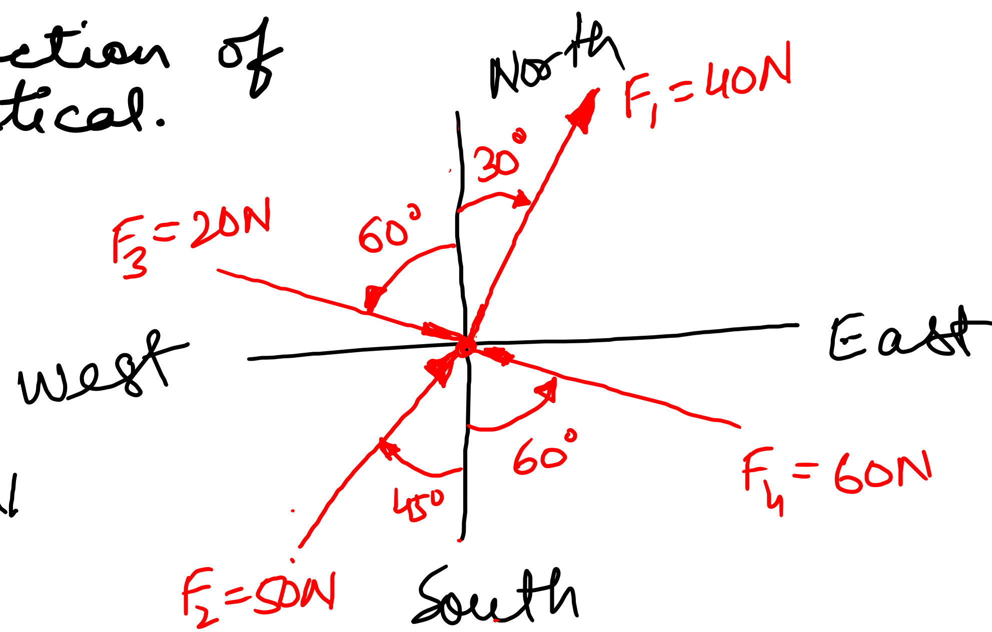
$\Sigma F_y =$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

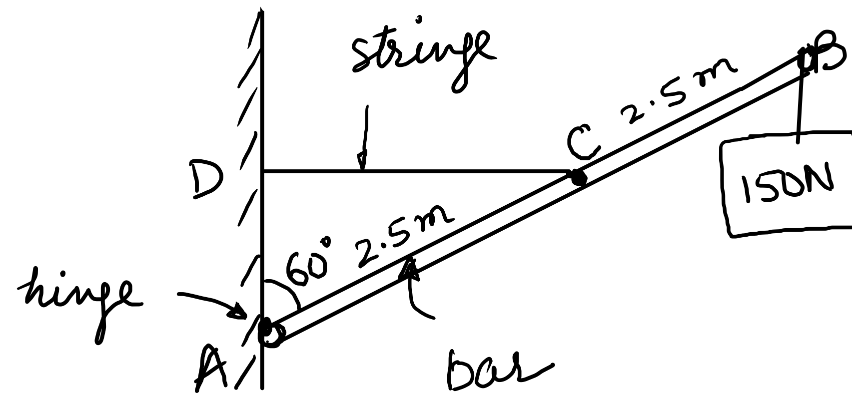
$$= 92.34\,N$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 77.04^\circ$$

(from East towards North)



Q:5 Determine tension in the string and reaction at the hinge.

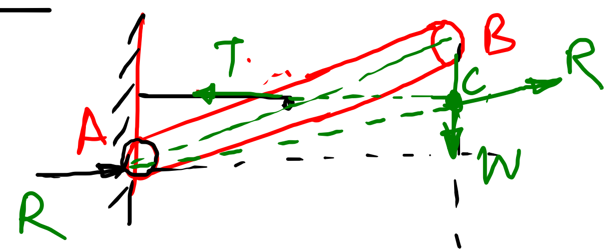


Ans.

Reaction, R = 540.9 N

Tension, T = 519.69 N

Hint:



AB = 5 m

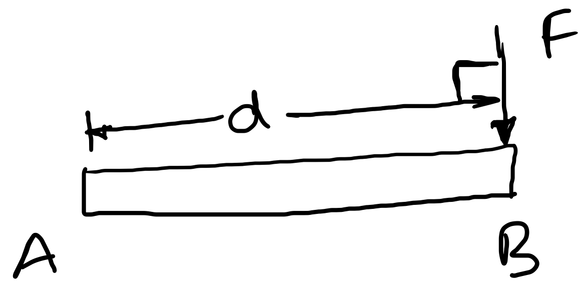
massless bar

Moment of Force

$$\boxed{M_A = F.d}$$

$$\vec{M} = \vec{d} \times \vec{F}$$

$$M = dF \sin 90^\circ$$

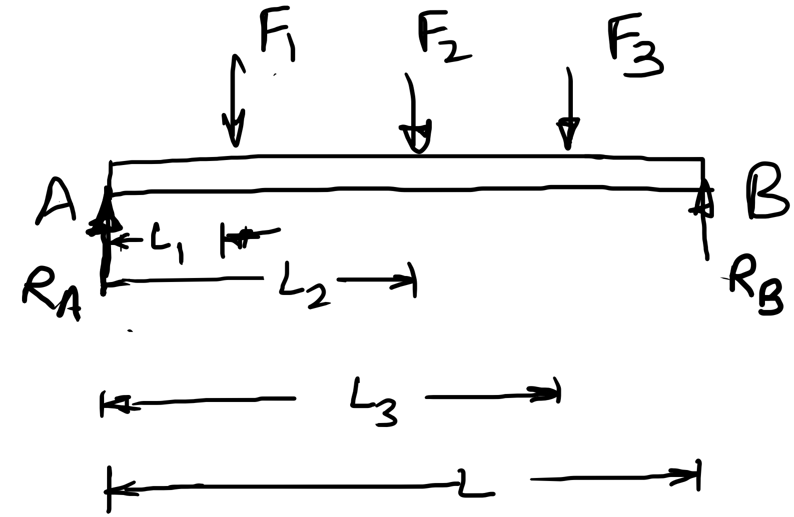


Let, Anti-clockwise moment — positive
clockwise moment — Negative

- Principle of moments:

Σ clockwise moments = Σ Anticlockwise moments

Take moment about point 'A'



$$R_B \times L = F_1 \times L_1 + F_2 \times L_2 + F_3 \times L_3$$

- Varignon's Theorem / Law of moments

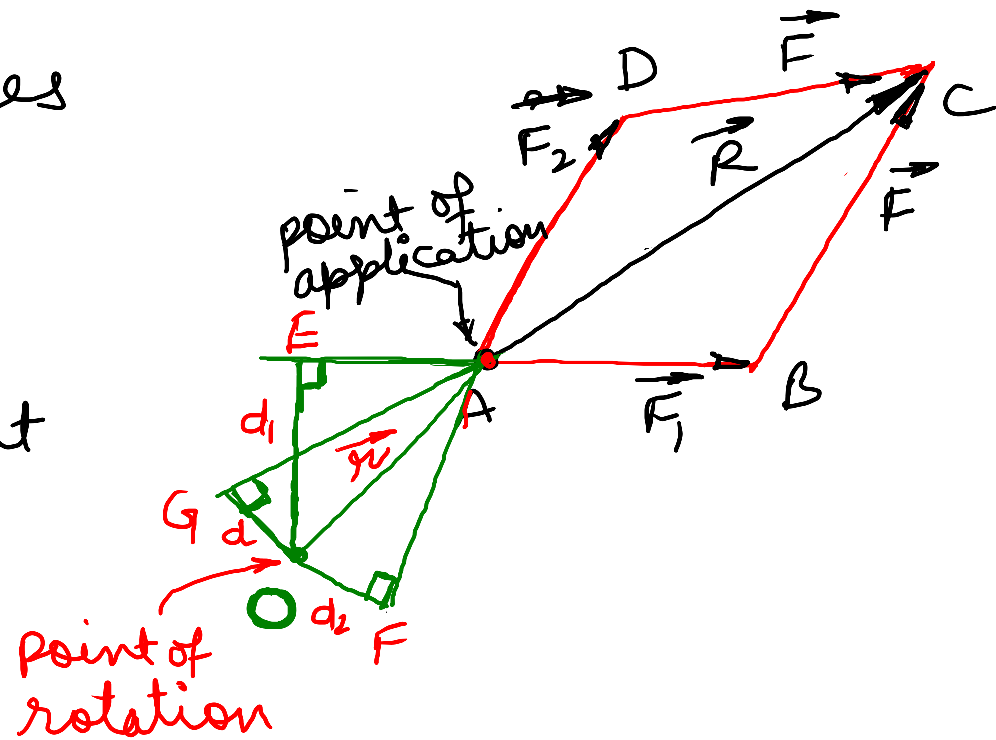
Apply triangle law of forces in Δ ABC

$$\vec{R} = \vec{F_1} + \vec{F_2}$$

Moment of resultant $\vec{R}$ about point 'O'

$$\vec{M_o} = \vec{r} \times \vec{R} = \vec{r} \times (\vec{F_1} + \vec{F_2})$$

$$= \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2}$$



$$\vec{M_0} = \vec{M_1} + \vec{M_2}$$

$$\vec{R} \cdot d = \vec{F_1} d_1 + \vec{F_2} d_2$$

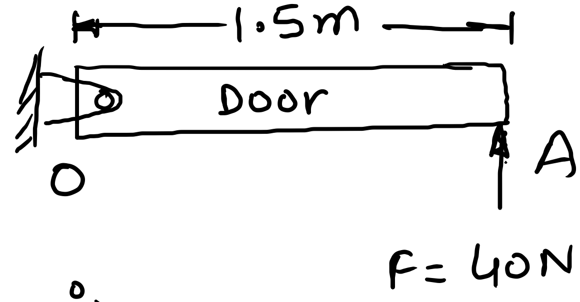
$$\vec{M_0} = R \cdot d$$

$$\vec{M_1} = F_1 \cdot d_1$$

$$\vec{M_2} = F_2 \cdot d_2$$

Q.1 Find moment of force about hinge.

(i) $M_o = 40 \times 1.5 = 60$ N-m (ACW)

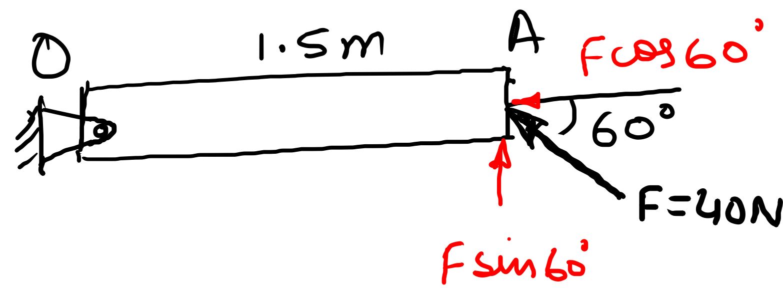


(i)

(ii) $M_o = F\cos 60^\circ \times 0 + F\sin 60^\circ \times 1.5$

$= 0 + 51.96$ Nm

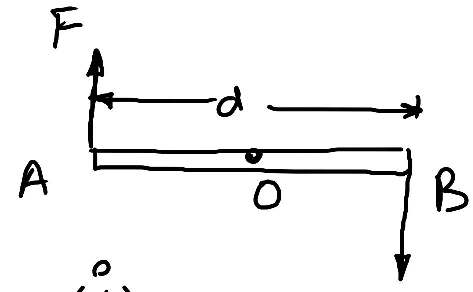
$= 51.96$ N-m (ACW)



(ii)

Couple of forces:

moment due to couple is called as torque.

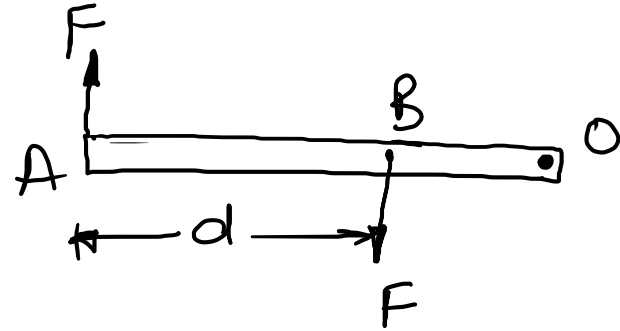


(i)

moment of couple about point 'O'

$$M_O = F(OA) + F(OB)$$

$$= F(OA + OB) = F.d$$



(ii)

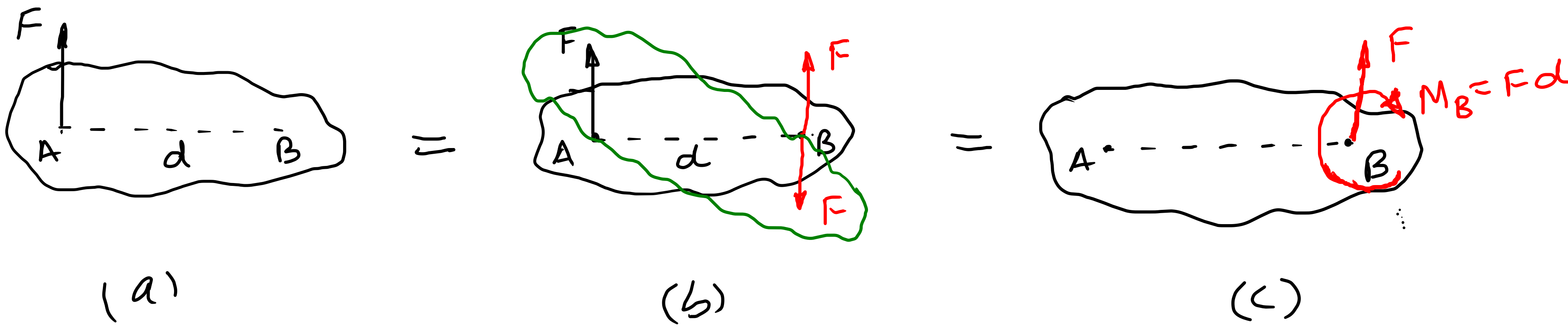
Moment of couple about point 'O'

$$M_O = F(OA) - F(OB)$$

$$= F(OA - OB)$$

$$= F.d$$

Reduce a force-couple sytem to a single force-moment system

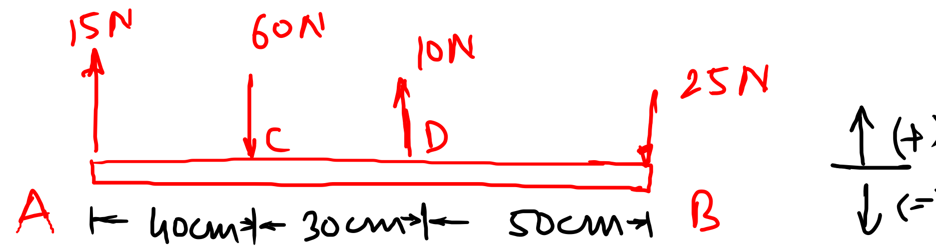


Q:1 Reduce this system to:

(i) A single force

(ii) A single-force-moment system at point 'A'

(iii) A single force-moment system at point 'B'



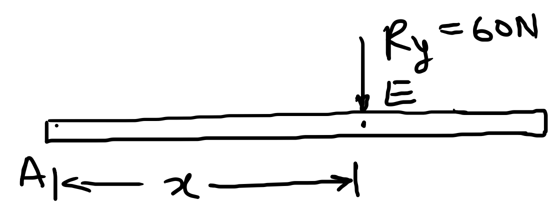
(i)

Sol^n:

(i) A single force system

$R_y = 15 - 60 + 10 - 25 = -60N$

$R_y = 60N$ (vertically downward)



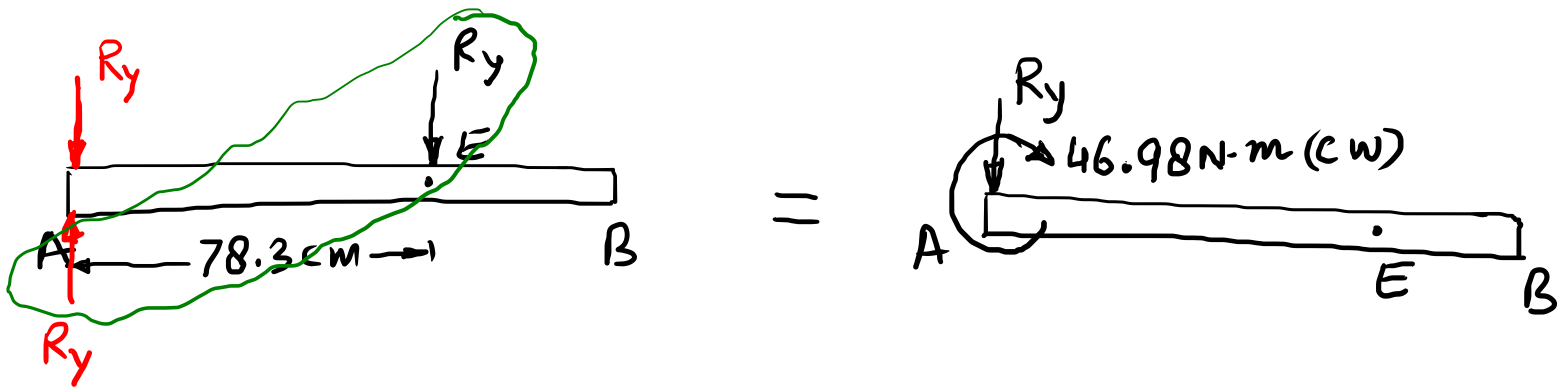
(ii)

Apply varignon's theorem of moment about 'A'

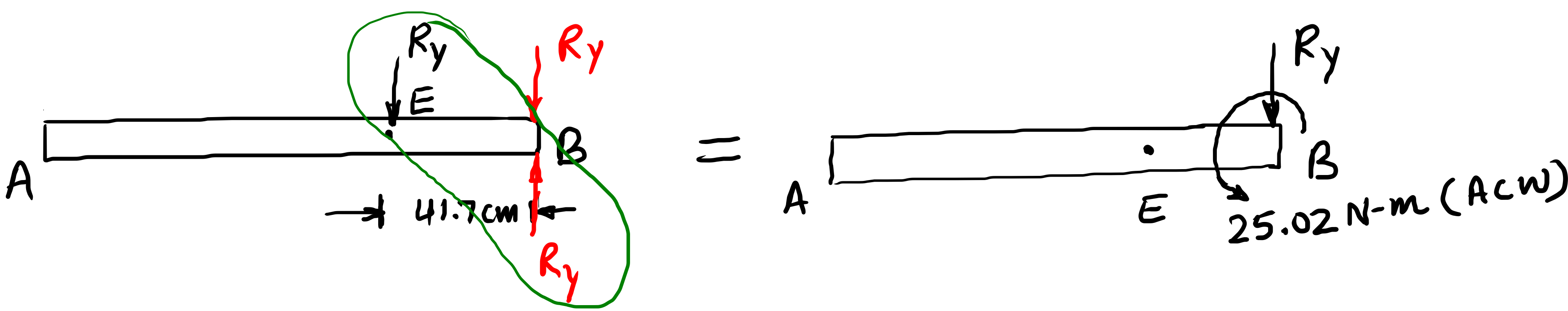
$-R_y . x = 15 \times 0 - 60 \times 40 + 10 \times 70 - 25 \times 120$

$-R_y . x = -4700 \Rightarrow x = \frac{4700}{60} = 78.3 \text{ cm}$

(ii) A single force-moment system at 'A'



(iii) A single force-moment system at 'B'

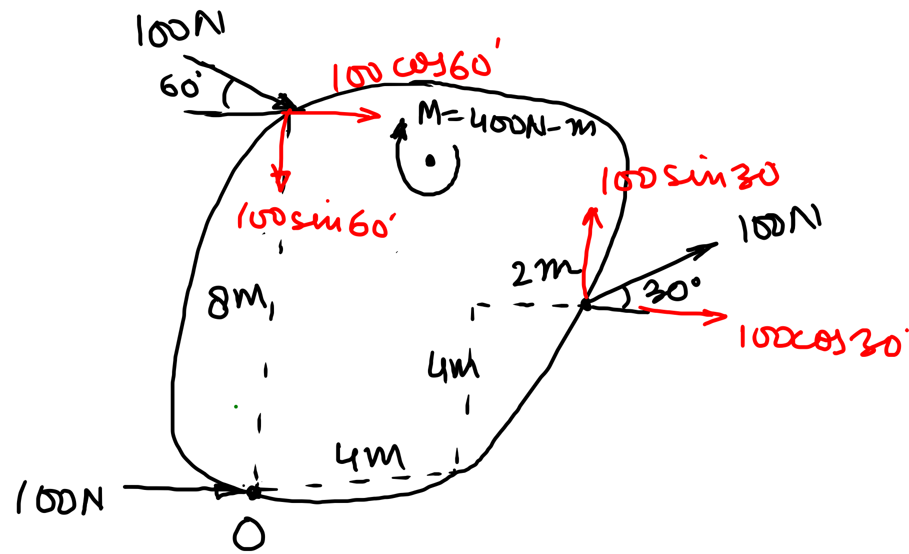


Q: 2 Reduce the force-moment system to a single force.

Soln:

$$\Sigma F_x = 100 + 100\cos 60^\circ + 100\cos 30^\circ$$

$$= 236.6\ N$$



$$\Sigma F_y = -100\sin 60^\circ + 100\sin 30^\circ$$

$$= -36.60\ N$$

Resultant force acting at point 'O'

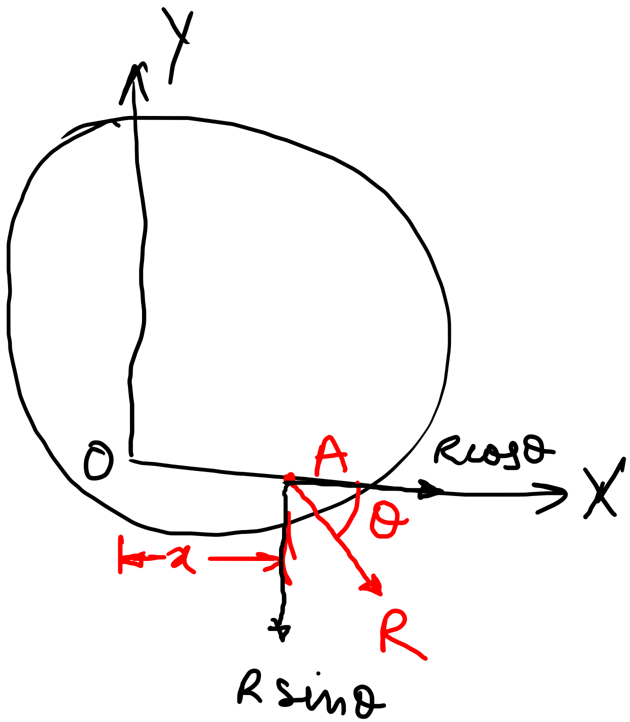
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 239.4\ N$$

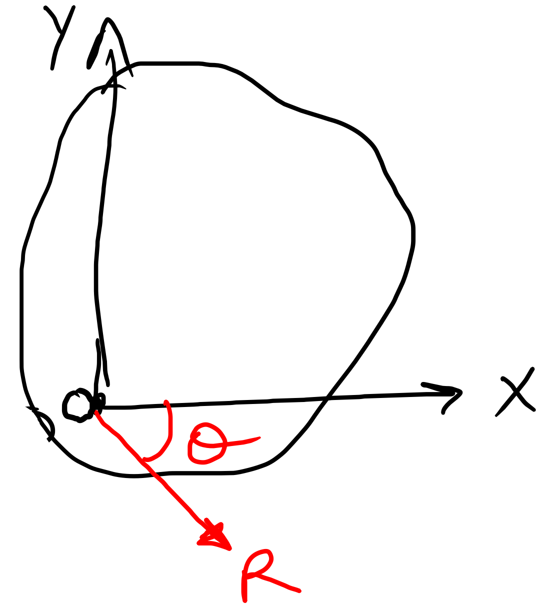
$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = -8.79^\circ$$

$$\Sigma M_o = -400 - 100\cos 60^\circ \times 8 - 100\cos 30^\circ \times 4 + 100\sin 30^\circ \times 6$$

$$= -846.41\ N\text{-}m\ (CW)$$

Point of Application of 'R'





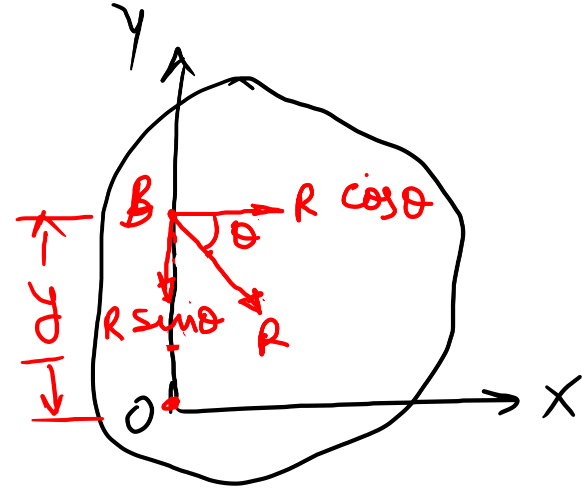
Apply varignon's theorem about 'O'

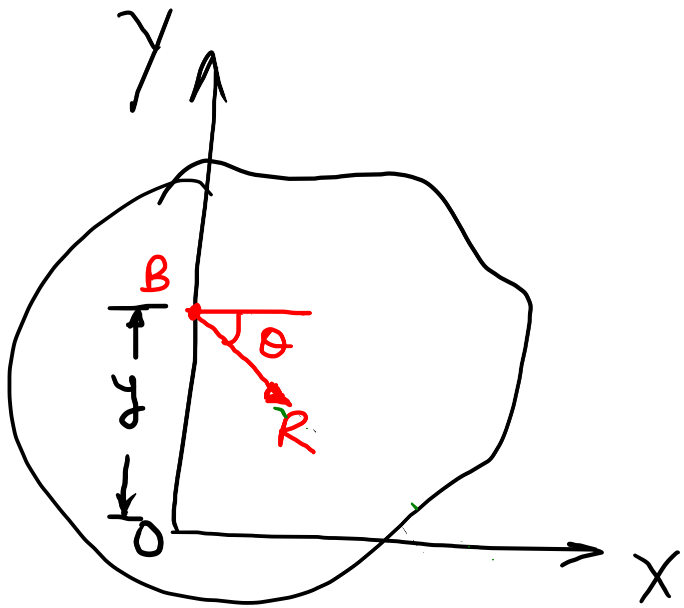
$$-R\sin\theta \cdot x = -846.4\,N$$

$$x = 23.136\,m$$ (which is not possible)

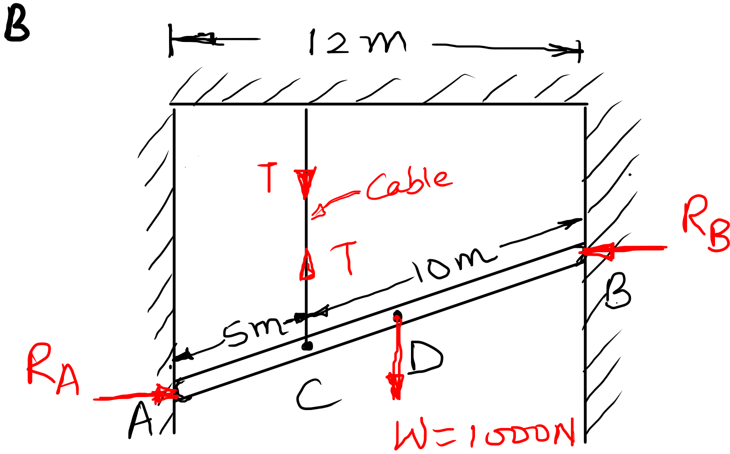
$$-R\cos\theta \cdot y = -846.41 \text{ N-m}$$

$$y = 3.577 \text{ m}$$





Q:3 A rod AB is supported by its smooth ends against the vertical walls by tension in the vertical cable. Determine the reactions at the ends of the rod and tension of the cable. Self weight of AB is 1000N.



Sol^n.

$\Sigma F_x = 0, \quad R_A = R_B$

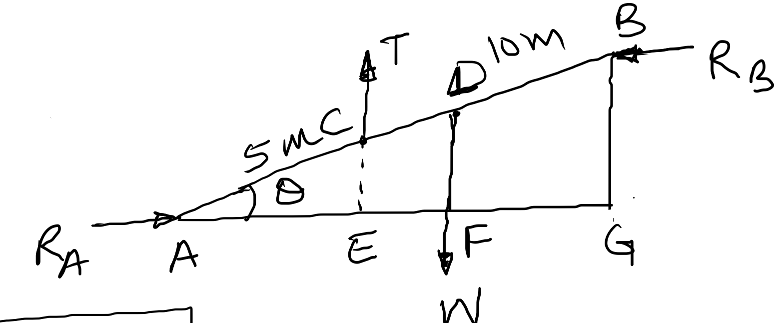
$\Sigma F_y = 0, \quad T = W = 1000\,N$

Take moment about 'A'

$\Sigma M_A = 0, \quad T \times AE - W \times AF + R_B \times BG = 0$

$1000 \times 5\cos\theta - 1000 \times 7.5\cos\theta + R_B \times 9 = 0$

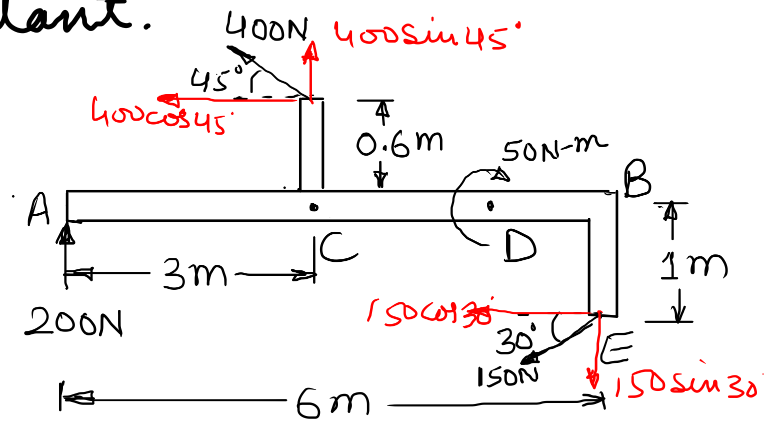
$R_B = 222.22\,N = R_A$



$\boxed{\theta = 36.87^\circ}$

FBD of rod AB

Q: 4 A member is subjected to three forces and a moment as shown in figure. Determine magnitude, direction and line of action of the resultant.



Sol^n. $\Sigma F_x = -400\cos 45^\circ - 150\cos 30^\circ = -412.75\,N$

$\Sigma F_y = 400\sin 45^\circ - 150\sin 30^\circ + 200 = 407.84\,N$

magnitude of $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 580.26\,N$

direction of R, $\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = -44.71^\circ$

Apply varignon's theorem at 'A'

$$R\sin\theta \cdot x = 400\cos 45^\circ \times 0.6 + 400\sin 45^\circ \times 3 - 150\cos 30^\circ \times 1 - 150\sin 30^\circ \times 6 - 50$$

$$x = 0.95\,m$$