Physics

first lecture

Relatiuity

Special theory of Ret..
(1905)
(inertial frame)

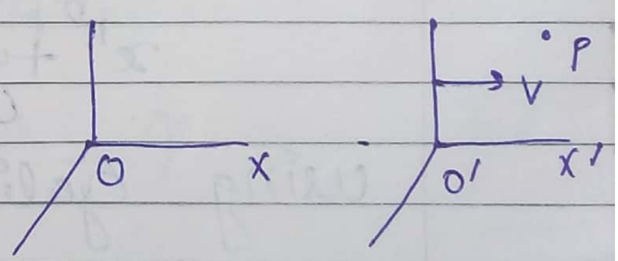
(i) all the laws of physics are applicable in inertial frame

(ii) c is same in all inertial frame

general theory of Rel.
(Accelerated frame)
(Eg. photon, gravity, Black hole and Red shift)

Transformation eqⁿ – if frame of refrence is changed the eqⁿ of its coordinate also changes. The changed eqⁿ are called transformed equation.

– galilean transformation –



$O \Rightarrow p(x, y, z, t)$

$O' = p(x', y', z', t')$

$x' = x - vt$

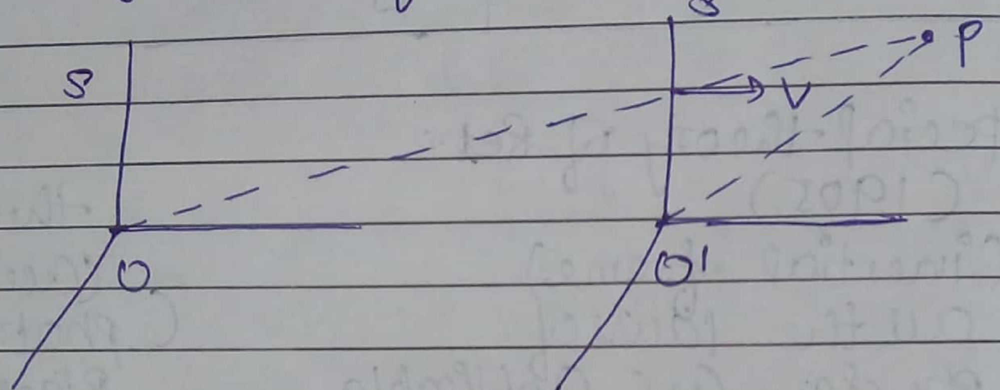
$y' = y$

$z' = z$

$t = t'$

Page :
Date: / /

lorentz transformation –



for S

$$t = \frac{OP}{c} = \frac{(x^2+y^2+z^2)^{1/2}}{c}$$

$$x^2+y^2+z^2 = c^2t^2 \quad \text{—— (1)}$$

for S'

$$t' = \frac{O'P}{c} = \frac{(x'^2+y'^2+z'^2)^{1/2}}{c}$$

$$x'^2+y'^2+z'^2 = c^2t'^2 \quad \text{—— (2)}$$

using galilean transformation

$$x' = x - vt \qquad y=y' \quad z=z' \quad t=t'$$

On putting in (2)

$$(x-vt)^2 + y^2 + z^2 = c^2t^2$$

$$x^2 - 2xvt + v^2t^2 + y^2 + z^2 = c^2t^2 \quad \text{—— (3)}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \frac{1}{\sqrt{1-v^2/c^2}} = \gamma$$

Page : γ, γ

Date: / /

Suppose $x' = \alpha(x - vt)$ $\quad z' = z$

$y' = y$ $\quad t' = \alpha'(t + fx)$

put this in (2)

$$x^2(\alpha^2 - c^2\alpha'^2 f^2) - 2xt(v\alpha^2 + c^2\alpha'^2 f) + y^2 + z^2 = c^2t^2\left(\alpha'^2 - \frac{\alpha^2 v^2}{c^2}\right) \quad (5)$$

Compare (5) and (1)

$$\alpha^2 - c^2\alpha'^2 f^2 = 1$$

$$\alpha'^2 - \frac{\alpha^2 v^2}{c^2} = 1$$

$$v\alpha^2 + c^2\alpha'^2 f = 0$$

On solving these eq^n.

$$\alpha = \alpha' = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma$$

$$f = -\frac{v}{c^2}$$

conclusion:- $x' = \gamma(x - vt)$ $\quad y' = y$

$z' = z$ $\quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$

for x, y, z replace v by $-v$

$$x = \gamma(x' + vt') \qquad z' = z$$

$$y' = y \qquad t = \gamma\left(t' + \frac{vx}{c^2}\right)$$

At low velocity lorentz transformation convert into gallilean transformation

Space time interval –

$O \Rightarrow (x_1, y_1, z_1, t_1) \qquad (x_2, y_2, z_2, t_2)$

$O' \Rightarrow (x_1', y_1', z_1', t_1') \qquad (x_2', y_2', z_2', t_2')$

$O \Rightarrow$ Space interval – $\left((x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right)^{1/2}$

~~time~~ $O' \Rightarrow \left((x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2\right)^{1/2}$

Space time interval

$O \rightarrow \quad S_{12} = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2\right]^{1/2}$

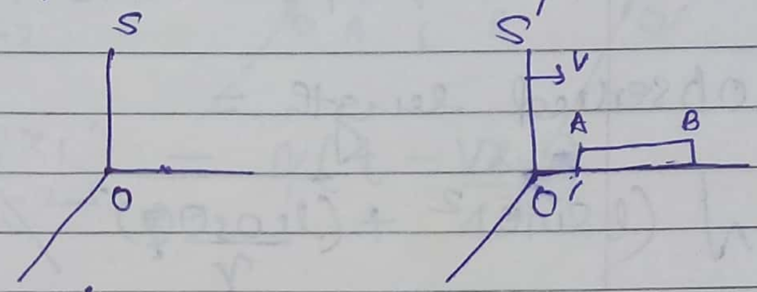
$O' \rightarrow \quad S_{12}' = \left[(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2 - c^2(t_2' - t_1')^2\right]^{1/2}$

* Space-time interval are invarient of lorentz transformation

* Space and time interval if taken alone then they are varient

Concequence of lorentz transformation

(i) length Contraction -



$O' \rightarrow \quad l_0 = x_2' - x_1'$ (proper length)

$O \rightarrow \quad l = x_2 - x_1$ (observed length or relativistic length)

$$l_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt)$$

$$l_0 = \gamma(x_2 - x_1) = \gamma l$$

$$l = \frac{l_0}{\gamma} = l_0\sqrt{1 - v^2/c^2}$$

$l < l_0$

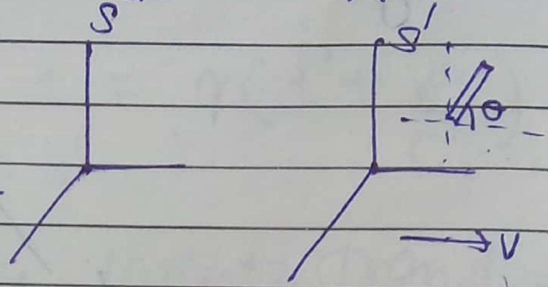
* length contraction is independent of dirn of velocity

* length contraction is always in dirn of relative motion in above case it is along x-axis

Page :
Date: / /

For cube of volume l_0^3

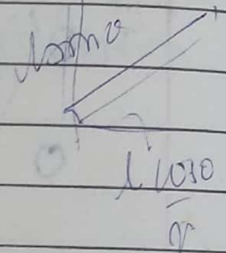
Observed volume $= l_0^3 \sqrt{1 - v^2/c^2}$



Observed length =

$$\sqrt{(l \sin\theta)^2 + \left(\frac{l \cos\theta}{\gamma}\right)^2}$$

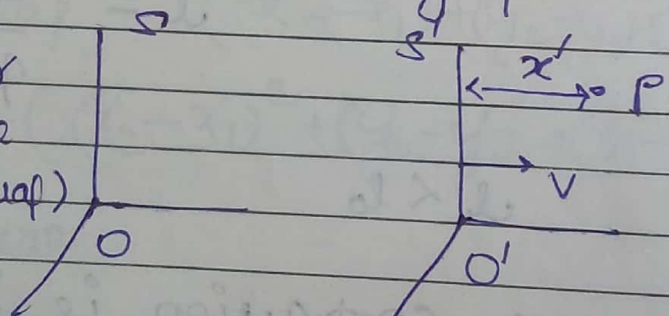
$$= \sqrt{(l \sin\theta)^2 + \left(l \cos\theta \sqrt{1 - v^2/c^2}\right)^2}$$



(II) Time Dilation:-

Let two events are taking place at P after some time gap

$O' \Rightarrow \Delta t_0 = t_2' - t_1'$ (proper time interval)



$O \Rightarrow \Delta t = t_2 - t_1$ (observed or relativistic)

$$\Delta t = \gamma\left(t_2' + \frac{v x'}{c^2}\right) - \gamma\left(t_1' + \frac{v x'}{c^2}\right)$$

$$\Delta t = \gamma(t_2' - t_1') = \gamma \Delta t_0$$

(time dilation) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$

time dilation is independent of dir^n of velocity.

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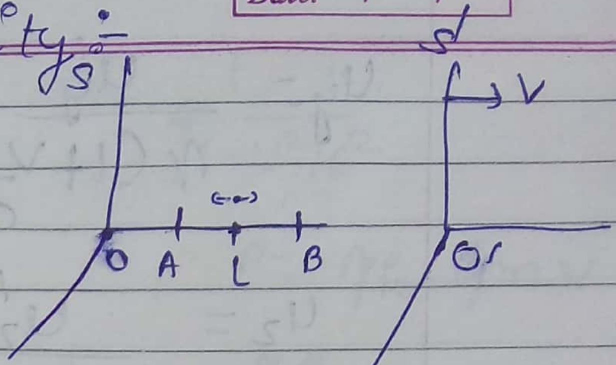
$\Delta t = \frac{\Delta t_0}{e} \gamma$

Page :
Date: / /

(III) failure of simultaneity :-



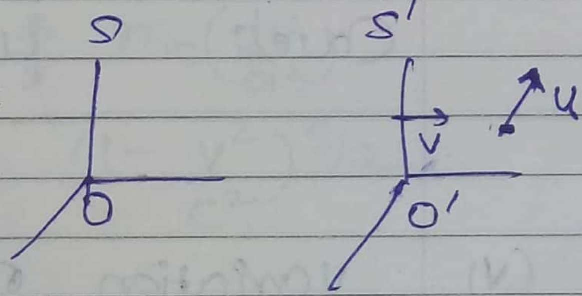
$O \Rightarrow \Delta t = t - t = 0$

$O' \Rightarrow \Delta t' = t_1' - t_2'$

$$\Delta t' = \gamma\left(t - \frac{v x_1}{c^2}\right) - \gamma\left(t - \frac{v x_2}{c^2}\right)$$

$$\Delta t' = \frac{v\gamma}{c^2}(x_2 - x_1) \neq 0$$

(IV) velocity addition :-



$O \rightarrow u$
$u_x = dx/dt$
$u_y = dy/dt$
$u_z = dz/dt$

$$x = \gamma(x' + vt') \qquad y = y'$$

$$t = \gamma\left(t' + \frac{v x'}{c^2}\right) \qquad z = z'$$

$$u_x = \frac{\gamma(dx' + v\,dt')}{\gamma\left(dt' + \frac{v\,dx'}{c^2}\right)}$$

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}$$

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Page :
Date: / /

$$u_y = \frac{u_y'}{\gamma\left(1+\frac{v u_x'}{c^2}\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1+\frac{v u_x'}{c^2}\right)}$$

for inverse transformation replace
$v \rightarrow -v$

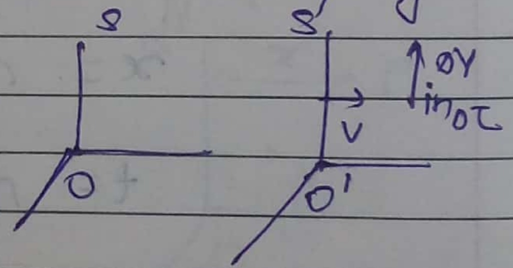
Case: $u_x' << c$ or $v << c$

$$u_x = u_x' + v$$

(V) variation of mass with velocity:

$O' \Rightarrow v_y' = \frac{\Delta Y}{\Delta \tau}$

$p_y' = m_0 \frac{\Delta Y}{\Delta \tau}$



$O \Rightarrow v_y = \frac{\Delta Y}{\Delta t}$

$m_0 \rightarrow$ proper
$m \rightarrow$ observed mass

$$P_y = m \frac{\Delta Y}{\Delta t}$$

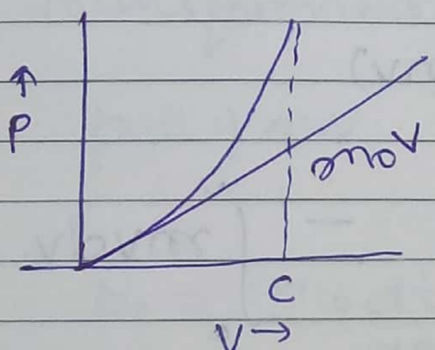
$P_y = P_y'$ (conservation of momentum)

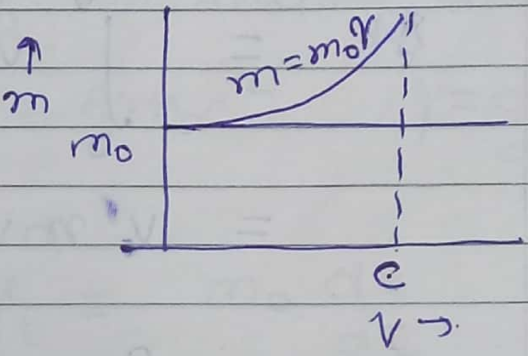
$$m_0 \frac{\Delta Y}{\Delta \tau} = m \frac{\Delta Y}{\Delta t}$$

Page :
Date: / /

$$\frac{m}{m_0} = \frac{\Delta t}{\Delta \tau} = \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$m = \gamma m_0 \quad , \quad p = \gamma p_0 = \gamma m_0 v$$





$$F = \frac{dp}{dt} = \frac{m_0\left(\frac{dv}{dt}\right)}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

The idea of infinite mass:-

(i) we require infinite force for this
(ii) infinite gravitational pull
(iii) Zero volume and infinite mass together are not possible

So, there is no infinite mass

Page:
Date: / /

Equivalence of mass and energy-

$$\Delta K = \int F\,dr = \int \frac{d(mv)}{dt}\,dr$$

$$\Delta K = \int_0^v v\,d(mv)$$

$$= v\,mv\Big|_0^v - \int_0^v mv\,dv$$

$$= mv^2 - \underbrace{\int_0^v \frac{m_0\,v\,dv}{\sqrt{1-v^2/c^2}}}_{I}$$

let $1 - \frac{v^2}{c^2} = x$

$$-\frac{1}{c^2}.2v\,dv = dx$$

$$I = -\frac{m_0 c^2}{2}\int_1^x x^{-1/2}\,dx$$

$$I = m_0 c^2 \left(x^{1/2} - 1\right)$$

$$\Delta E_k = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} + m_0 c^2\left(\left(1-\frac{v^2}{c^2}\right)^{1/2} - 1\right)$$

$$\Delta E_K = mc^2 - m_0 c^2$$

$$\Delta E_K = (m - m_0)c^2 = \Delta m c^2$$

Total ~~mass~~ Energy

$$mc^2 = \underbrace{E_K}_{K.E} + \underbrace{m_0c^2}_{\text{Rest mass energy}}$$

Transformation of momentum and energy

$$p = \gamma m_0 v \qquad E = \gamma m_0 c^2 \qquad \gamma = \frac{dt}{d\tau} = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$p_x = m_0 \frac{dt}{d\tau}\frac{dx}{dt} = m_0 \frac{dx}{d\tau}$$

$$p_y = m_0 \frac{dt}{d\tau}\frac{dy}{dt} = m_0 \frac{dy}{d\tau}$$

$$p_z = m_0 \frac{dz}{d\tau}$$

$$\frac{E}{c^2} = m_0 \frac{dt}{d\tau}$$

$$p'_x = \gamma\left(p_x - \frac{vE}{c^2}\right)$$

Analogue to Lorentz transformation

$$p'_y = p_y$$

$$p'_z = p_z$$

$$\frac{E'}{c^2} = \gamma\left(\frac{E}{c^2} - \frac{v p_x}{c^2}\right)$$

$$E' = \gamma(E - v p_x)$$

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for two collision

$$p_{1x} + p_{2x} = p_{3x} + p_{4x}$$

$$p'_{1x} + p'_{2x} = p'_{3x} + p'_{4x}$$

Page :

Date: / /

Relation b/w relativistic momentum and energy:

$$p = \gamma m_0 v \qquad E = \gamma m_0 c^2$$

$$p^2 = \gamma^2 m_0^2 v^2$$

$$p^2 = m_0^2 (\gamma^2 c^2 - c^2)$$

$$p^2 c^2 = m_0^2 c^4 (\gamma^2 - 1)$$

$$p^2 c^2 = \gamma^2 m_0^2 c^4 - m_0^2 c^4$$

$$p^2 c^2 = E^2 - m_0^2 c^4$$

$$\gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\gamma^2 c^2 - \gamma^2 v^2 = c^2$$

$$\gamma^2 v^2 = \gamma^2 c^2 - c^2$$

$$\boxed{E^2 = p^2 c^2 + m_0^2 c^4}$$

Que Show that above eqn is invariant of lorentz transformation

$$E^2 = p^2 c^2 + m_0^2 c^4$$

$$E'^2 = p'^2 c^2 + m_0^2 c^4$$

use lorentz transformation for E'^2 ...
and make it as $E^2 = + \ldots$

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$$\frac{P}{E} = \frac{V}{c^2}$$

The particle withe zero rest mass

$$E = pc$$

has velocity equal to c

$$\frac{P}{E} = \frac{V}{c^2} \Rightarrow \frac{1}{c} = \frac{V}{c^2} \Rightarrow \boxed{V = c}$$

(for proton)

general theory of relativity :-

(i) gravitational mass and inertial mass are equivalent

(ii) photon & gravity

$$h\nu = mc^2 \Rightarrow m = \frac{h\nu}{c^2}$$

H

$$h\nu' = h\nu + mgH$$

(iii) mass on a Star

m, R

$$h\nu - \frac{GMm}{R}$$

$$h\nu' = h\nu - \frac{GMm}{R}$$

$$h\nu' = h\nu - \frac{GM}{R}\frac{h\nu}{c^2} \Rightarrow \nu' = \nu\left(1 - \frac{GM}{Rc^2}\right)$$

Page :
Date: / /

$$\frac{\nu'}{\nu} - 1 = -\frac{Gm}{Rc^2}$$

$$\frac{\nu' - \nu}{\nu} = -\frac{Gm}{Rc^2}$$

$$\boxed{\frac{\Delta\nu}{\nu} = \frac{Gm}{Rc^2}}$$

as ν' is decreasing λ is inreasing and shifting towards red

this is gravitational red shift due to effect of gravitational pull.

$$\frac{Gm}{c^2R} \geq 1$$ (Condition of black hole)

Page :
Date: / /

Oscillation

Simple

restoring force

$$m\frac{d^2y}{dt^2} = -\mu y$$

$$\frac{d^2y}{dt^2} + \frac{\mu}{m}y = 0$$

$$\because \omega_0^2 = \frac{\mu}{m}$$

$$\frac{d^2y}{dt^2} + \omega_0^2 y = 0$$

$\omega_0 \rightarrow$ natural frequency

Damped/Resistive

two forces acts

Restoring force $= -\mu y$

Damping force $= -\gamma \frac{dy}{dt}$

$$D.F = -\gamma v$$

$$m\frac{d^2y}{dt^2} = -\mu y - \gamma\frac{dy}{dt}$$

$$\boxed{\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega_0^2 y = 0}$$

$$2k = \frac{\gamma}{m}$$

forced/ maintained

forced oscillation –

three forces acts

(i) Restoring force $= -\mu y$

(ii) Damping force $= -\gamma\frac{dy}{dt}$

(iii) periodic apply force $= F\sin pt$

$$m\frac{d^2y}{dt^2} = \mu y - \gamma\frac{dy}{dt} + F\sin pt$$

$$\boxed{\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega_0^2 y = f\sin pt}$$

$$f = \frac{F}{m}$$

$$2k = \frac{r}{m} = \frac{1}{\tau}$$

Page :
Date: / /

for Damped oscillation
on neglecting restoring force

$$\frac{dv}{dt} = -2kv = -\frac{v}{\tau}$$

$$\frac{dv}{v} = -\frac{dt}{\tau}$$

on integrating

$$\int \frac{dv}{v} = -\int \frac{dt}{\tau}$$

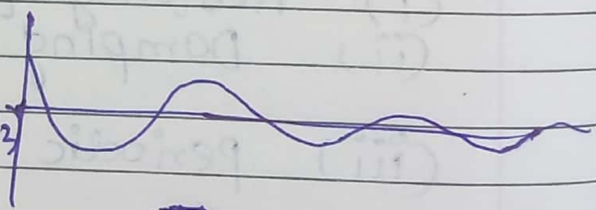
$$v = v_0 e^{-t/\tau}$$

for kinetic Energy $E = E_0 e^{-2t/\tau}$

$\tau \rightarrow$ relexatation time

$$a_n = a_0 e^{-kt}$$

$$a_{n+1} = a_0 e^{-k(t+T/2)}$$



$$\frac{a_n}{a_{n+1}} = \cancel{e^{-kT/2}}\ e^{kT/2} = \lambda$$

λ is logarithmic decrement

Page :
Date: / /

Q → Quality of oscillation.

$$Q = 2\pi \frac{\text{Energy}}{\text{Energy loss per period}}$$

$$Q = 2\pi \frac{E}{PT} = \frac{2\pi E}{E/\tau} = 2\pi\tau$$

$$Q = \frac{2\pi}{2k} = \frac{\pi}{k}$$

Quality of Oscillation will be high when there is less damping

⇒

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega_0^2 y = 0$$

$y = e^{\alpha t}$ (say)

$$\frac{dy}{dt} = \alpha e^{\alpha t}$$

$$\frac{d^2y}{dt^2} = \alpha^2 e^{\alpha t}$$

$$(\alpha^2 + 2k\alpha + \omega_0^2) e^{\alpha t} = 0$$

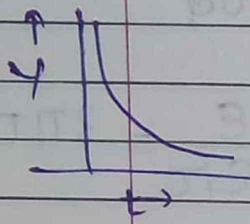
$$\alpha = -k \pm \sqrt{k^2 - \omega_0^2}$$

exact soln will depend upon relative value of k and ω_0

Page:
Date: / /

$$y = Ae^{\left(-k+\sqrt{k^2-\omega_0^2}\right)t} + Be^{\left(-k-\sqrt{k^2-\omega_0^2}\right)t}$$

Case I :- Over damping



$k > \omega_0 \Rightarrow \sqrt{k^2-\omega^2}$ = tve but less than k

exponent will be negative. this motion is called dead beat motion

Case II :- $k \simeq \omega_0$ $\sqrt{k^2-\omega_0^2} = h$ (a very small quantity)

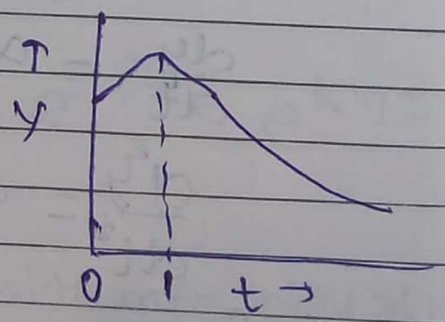
$$y = e^{-kt}\left[Ae^{ht} + Be^{-ht}\right]$$

$$y = e^{-kt}\left[A(1+ht+\cdots) + B(1-ht+\cdots)\right]$$

$$y = e^{-kt}\left[(A+B) + (A-B)ht\right]$$

let $A+B = P$ $(A-B)h = Q$

$$y = e^{-kt}(P+Qt)$$



Case [I] :- $k < \omega_0$ (Case of underdamping)

$$\sqrt{k^2 - \omega_0^2} = j\beta$$

$$y = e^{-kt}\left[Ae^{j\beta t} + Be^{-j\beta t}\right]$$

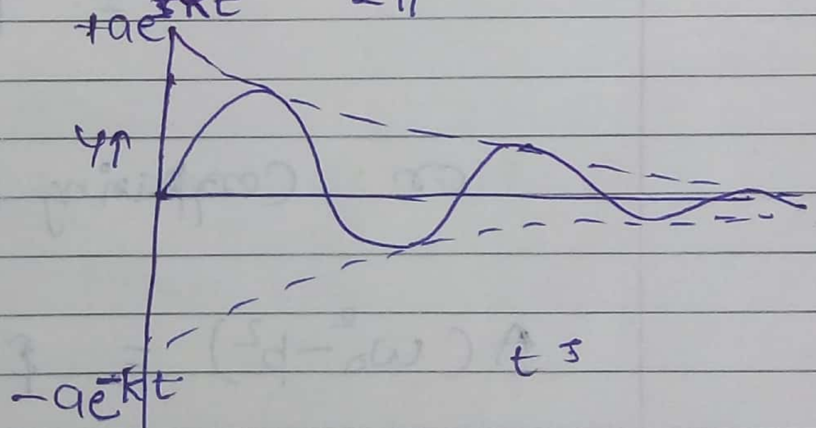
$$y = e^{-kt}\left[A(\cos\beta t + j\sin\beta t) + B(\cos\beta t - j\sin\beta t)\right]$$

$$y = e^{-kt}\left[\underbrace{(A+B)}_{a\sin\phi}\cos\beta t + \underbrace{j(A-B)}_{a\cos\phi}\sin\beta t\right]$$

$$y = ae^{-kt}\left[\sin(\beta t + \phi)\right]$$

$$\beta = \sqrt{\omega_0^2 - k^2}$$

$$\text{frequency} = \frac{\beta}{2\pi}$$



Forced Oscillation :-

$$y = A\sin(pt - \theta)$$

$$\frac{dy}{dt} = Ap\cos(pt - \theta)$$

$$\frac{d^2y}{dt^2} = -Ap^2\sin(pt - \theta)$$

Page :
Date: / /

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + \omega_0^2 y = f\sin pt$$

$$-Ap^2\sin(pt-\theta) + 2kAp\cos(pt-\theta) + \omega_0^2 A\sin(pt-\theta)$$

$$= f\sin\{(pt-\theta)+\theta\}$$

$$-Ap^2\sin(pt-\theta) + 2kAp\cos(pt-\theta) + \omega_0^2 A\sin(pt-\theta)$$

$$= f\sin(pt-\theta)\cos\theta + f\cos(pt-\theta)\sin\theta$$

$$(\omega_0^2 A - Ap^2)\sin(pt-\theta) + 2kAp\cos(pt-\theta)$$

$$= f\cos\theta\sin(pt-\theta) + f\cos(pt-\theta)\sin\theta$$

on comparing

$$A(\omega_0^2 - p^2) = f\cos\theta \quad -(1)$$

$$2kAp = f\sin\theta \quad -(2)$$

from (1) & (2)

$$A = \frac{f}{\sqrt{(\omega_0^2-p^2)^2 + 4k^2p^2}}$$

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For better tuning we require sharp resonance curve

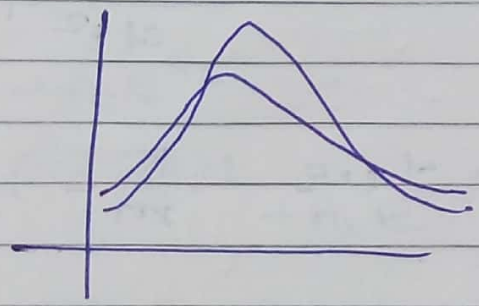
Page :

Date: / /

$$\tan\theta = \frac{2kp}{\omega_0^2 - p^2}$$

for maximum amplitude

$$p = \sqrt{\omega_0^2 - 2k^2}$$



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Oscillations

Page:
Date: / /

Damped oscillation :

$ma = -kx - bv$ where $v = \frac{dx}{dt}$

$ma + kx + bv = 0$

$$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x + \left(\frac{b}{m}\right)v = 0$$

where $\frac{k}{m} = \omega_0^2$ $\quad \frac{b}{m} = 2\gamma$

$\gamma \rightarrow$ damping constant
$\omega_0 \rightarrow$ natural frequency of oscillator

So, eq^n will be

$$\frac{d^2x}{dt^2} + 2\gamma \underset{(dx/dt)}{\underset{\downarrow}{v}} + \omega_0^2 x = 0 \quad \text{--- (1)}$$

Solution of damped harmonic oscillator

$$\left.\begin{aligned} x &= Ae^{\alpha t} \\ \frac{dx}{dt} &= A\alpha e^{\alpha t} \\ \frac{d^2x}{dt^2} &= A\alpha^2 e^{\alpha t} \end{aligned}\right\} \text{--- (2)}$$

from (1) and (2)

$$A\alpha^2 e^{\alpha t} + 2\gamma A\alpha e^{\alpha t} + \omega_0^2 A e^{\alpha t} = 0$$

$$Ae^{\alpha t}(\alpha^2 + 2\gamma\alpha + \omega_0^2) = 0$$

or $\quad \alpha^2 + 2\alpha\gamma + \omega_0^2 = 0$

Page :
Date: / /

$$\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

thus α has two values

$$\alpha_1 = -\gamma + \sqrt{\gamma^2 - \omega_0^2}$$

$$\alpha_2 = -\gamma - \sqrt{\gamma^2 - \omega_0^2}$$

Thus, solution of the second order differential eqn (1) is

$$x = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

$$x = A_1 e^{(-\gamma + \sqrt{\gamma^2 - \omega_0^2})t} + A_2 e^{(-\gamma - \sqrt{\gamma^2 - \omega_0^2})t}$$

Case I: when $\gamma^2 > \omega_0^2$ i.e heavily damped, over damped or dead beat case.

In this type of oscillation, displacement of oscillation die out exponentially with time. It is non-oscillatory.

Examples: (i) pendulum oscillating in highly viscous medium.
(ii) moving coil galvanometer shunted with very low resistance

Case II: $\gamma^2 = \omega_0^2$ (critical damping)
here we assume that

$$\sqrt{\gamma^2 - \omega_0^2} = \beta = \text{very very small.}$$

$$x = A_1 e^{(-\gamma + \beta)t} + A_2 e^{(-\gamma - \beta)t}$$

$$x = e^{-\gamma t}\left[A_1 e^{\beta t} + A_2 e^{-\beta t}\right]$$

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by expanding $e^{\beta t}$ and $e^{-\beta t}$ and neglecting higher ~~oder~~ order terms

$$x = e^{-\gamma t}\left[A_1(1+\beta t) + A_2(1-\beta t)\right]$$

$$x = e^{-\gamma t}\left[A_1 + A_2 + \beta t(A_1 - A_2)\right]$$

$$A_1 + A_2 = A \qquad (A_1 - A_2)\beta = B$$

$$x = e^{-\gamma t}\left[A + Bt\right]$$

In this type of oscillation, the displacement of the oscillator increases and then return back quickly to its equillibrium position.

It is non-periodic and non-oscillatory

* It is used to design pointer type instruments like galvanometer in which the pointer moves to the correct position and returns back to zero position in a very short time.

Case III :- $\gamma^2 < \omega_0^2$ (under damping condition, low damping)

$$\sqrt{\gamma^2 - \omega_0^2} = \sqrt{\omega_0^2 - \gamma^2}\, j$$

$$x = A_1 e^{\left(-\gamma + j\sqrt{\omega_0^2 - \gamma^2}\right)t} + A_2 e^{\left(-\gamma - j\sqrt{\omega_0^2 - \gamma^2}\right)t}$$

~~$x = e^{-\gamma t}[A_1$~~

Let $\sqrt{\omega_0^2 - \gamma^2} = \omega$

$$x = e^{-\gamma t}\left[A_1 e^{j\omega t} + A_2 e^{-j\omega t}\right]$$

Page :
Date: / /

$$x = e^{-\gamma t}[A_1\cos\omega t + jA_1\sin\omega t + A_2\cos\omega t - jA_2\sin\omega t]$$

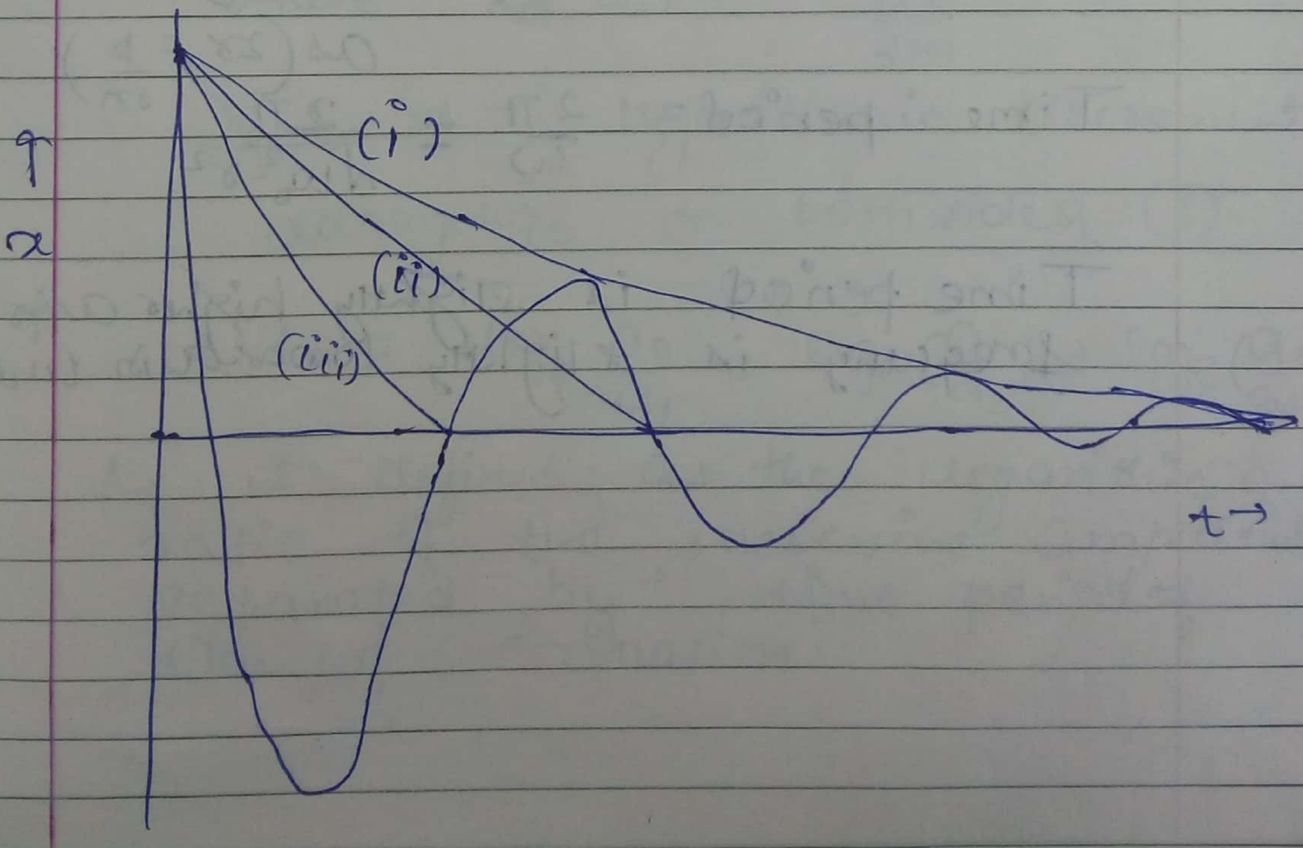
$$x = e^{-\gamma t}[\cos\omega t(A_1+A_2) + j\sin\omega t(A_1-A_2)]$$

$A_1 + A_2 = a\sin\phi \qquad j(A_1 - A_2) = a\cos\phi$

$$x = ae^{-\gamma t}[\sin\phi\cos\omega t + \cos\phi\sin\omega t]$$

$$\boxed{x = ae^{-\gamma t}\sin(\omega t+\phi)}$$

It is non-periodic by is somehow oscillatory whenever it is not clearly mentioned that the oscillation is whether ~~over~~ over, under or critical damped we will assume it to be under damped.



$$2\gamma = \frac{b}{m}$$

damping const

$$x = ae^{-\gamma t} \sin(\omega t + \phi)$$

$ae^{-\gamma t} \longrightarrow$ amplitude

$e^{-\gamma t} \rightarrow$ damping factor

mean life time (τ_m) :- the time in which amplitude to oscillation reduces to $\frac{1}{e}$ of its initial value

$$ae^{-\gamma t} = \frac{1}{e} a$$

$$e^{-\gamma t} = \frac{1}{e} = e^{-1}$$

$$\Rightarrow \quad \gamma t = 1$$

$$\tau_m = t = \frac{1}{\gamma} = \frac{2m}{b}$$

as $\left(2\gamma = \frac{b}{m}\right)$

$$\text{Time period} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \gamma^2}}$$

Time period is slightly higher as in damped frequency is slightly lower as in undamped

$T = \frac{1}{\nu}$ $\omega = 2\pi\nu$

Page :
Date: / /

logarithmic Decrement :

$$x = ae^{-\gamma t} \sin(\omega t + \phi)$$

$$\phi = \pi/2$$

$$x = ae^{-\gamma t} \cos \omega t$$

amplitude at t =	0	T	2T	3T	4T
	a_0	a_1	a_2	a_3	a_4

$a_1 = a_0 e^{-\gamma T}$ $a_2 = a_0 e^{-2\gamma T}$

$a_3 = a_0 e^{-3\gamma T}$ $a_4 = a_0 e^{-4\gamma T}$

$$\frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = \dots = e^{\lambda_d} \quad -(1)$$

where $\lambda_d = \gamma T = \frac{bT}{2m}$.

$\lambda_d \rightarrow$ logarithmic decrement

taking $\log_e$ on both side of (1)

damped

ed

$$\lambda_d = \log_e \frac{a_0}{a_1} = \log_e\left(\frac{a_1}{a_2}\right) = \log_e\left(\frac{a_2}{a_3}\right)$$

λ_d is defined as the logarithmic of ratio of two successive amplitudes separated by time period of damped oscillation

$\omega = \sqrt{\omega_0^2 - \gamma^2}$

Here damping is weak

$\gamma^2 << \omega_0^2 \Rightarrow \omega = \sqrt{\omega_0^2 - \gamma^2} = \omega_0$

Page :
Date: / /

power Dissipation :-

$$x = a e^{-\gamma t} \sin(\omega_0 t + \phi)$$

Let $\phi = 0$

$$x = a e^{-\gamma t} \sin(\omega_0 t)$$

$$v = \frac{dx}{dt} = a\gamma e^{-\gamma t} \sin\omega_0 t + a e^{-\gamma t} \omega_0 \cos\omega_0 t$$

$$\therefore KE = \frac{1}{2} m v^2$$

$$KE = \frac{1}{2} m a^2 e^{-2\gamma t}\left[\omega_0^2 \cos^2\omega_0 t + \gamma^2 \sin^2\omega_0 t - 2\gamma\omega_0 \sin\omega_0 t \cos\omega_0 t\right]$$

$$KE_{av} = \frac{\int_0^T \frac{1}{2} m a^2 e^{-2\gamma t}\left[\omega_0^2 \cos^2\omega_0 t + \gamma^2 \sin^2\omega_0 t - \gamma\omega_0 \sin 2\omega_0 t\right]}{T}$$

$$KE_{av} = \frac{1}{2} m a^2 e^{-2\gamma t}\left[\frac{1}{2}\omega_0^2 + \frac{\gamma^2}{2}\right]$$

$$KE_{av} = \frac{1}{4} m a^2 e^{-2\gamma t}(\omega_0^2 + \gamma^2)$$

for low damping $\omega_0^2 >> \gamma^2$

$\omega_0^2 + \gamma^2 \approx \omega_0^2$

$$KE_{av} = \frac{1}{4} m a^2 \omega_0^2 e^{-2\gamma t}$$

Potential energy of oscillator

$$U = \frac{1}{2} k x^2 = \frac{1}{2} k (a e^{-\gamma t} \sin\omega_0 t)^2$$

$$U = \frac{1}{2} m\omega_0^2 a^2 e^{-2\gamma t} \sin^2\omega_0 t$$

$$U_{av} = \frac{1}{2} m\omega_0^2 a^2 e^{-2\gamma t} \frac{\int_0^T \sin^2\omega_0 t\, dt}{T}$$

Page:
Date: / /

$U_{av} = \cancel{\frac{1}{2}} \frac{1}{4} m\omega_0^2 a^2 e^{-2\gamma t}$

Total average energy,

$E = K_{av} + U_{av}$

$E = \frac{1}{2} m\omega_0^2 a^2 e^{-2\gamma t}$

at $t=0$ $\quad E_0 = \frac{1}{2} m\omega_0^2 a^2$, same as

undamped oscillator

at any time $\quad E = E_0 e^{-2\gamma t}$

loss $\Delta E = \cancel{E_0 t}\ E_0 - E$

the loss of energy is due to the work done against the damping force and appears in the form of heat

Relaxation time: the time in which Energy reduces to $\frac{1}{e}$ of its initial value.

$$E_0 e^{-2\gamma t} = \frac{1}{e} E_0$$

$$e^{-2\gamma t} = e^{-1}$$

$$2\gamma t = 1$$

$$\tau = t = \frac{1}{2\gamma}$$

power, $\quad P = 2\gamma E = \frac{E}{\tau}$

Page:
Date: / /

Quality factor :- define quality of oscillation : lesser is damping better is the quality of oscillation

$$Q = 2\pi \left(\frac{\text{energy stored}}{\text{energy lost per period}} \right) = \frac{2\pi E}{PT}$$

$$Q = \frac{E\omega}{P} = \frac{E\omega}{E/\tau} = \omega\tau$$

for an undamped oscillation τ is infinite so, Q is also infinite

forced Oscillation :-

$$m\frac{d^2x}{dt^2} + Kx + bv = f_0 \sin pt$$

$$\frac{d^2x}{dt^2} + \left(\frac{K}{m}\right)x + \frac{b}{m}v = \left(\frac{F_0}{m}\right)\sin pt$$

$$\frac{K}{m} \downarrow \omega_0^2 \qquad \frac{b}{m} \downarrow 2r \qquad \frac{F_0}{m} \downarrow f_0$$

$$\frac{d^2x}{dt^2} + 2rv + \omega_0^2 x = f_0 \sin pt \quad -(1)$$

Page :
Date : / /

after attaining a steady state
oscillator starts oscillating with the frequency of driven force (P)

$$x = A\sin(pt-\phi)$$
$$v = \frac{dx}{dt} = pA\cos(pt-\phi)$$
$$\frac{d^2x}{dt^2} = -p^2A\sin(pt-\phi)$$

Put this in 1

$$-p^2A\sin(pt-\phi) + 2rpA\cos(pt-\phi) + \omega_0^2 A\sin(\omega t-\phi) = f_0 \sin(pt-\phi+\phi)$$

$$A(\omega_0^2-p^2)\sin(pt-\phi) + 2rpA\cos(pt-\phi) = f_0 \sin(pt-\phi)\cos\phi + f_0\cos(pt-\phi)\sin\phi$$

If this eq^n is to be satisfied for all values of t, then the coefficient of $\sin(pt-\phi)$ and $\cos(pt-\phi)$ on the two side must be equal.

$$A(\omega_0^2-p^2) = f_0\cos\phi$$
$$2rpA = f_0\sin\phi$$

squaring and adding, we get

$$A^2\{(\omega_0^2-p^2)^2+4r^2p^2\} = f_0^2$$

$$A = \frac{f_0}{\sqrt{(\omega_0^2-p^2)+4r^2p^2}}$$

$$\tan\phi = \frac{2rp}{\omega_0^2-p^2}$$

P.T.O

Page :
Date: / /

Thus,

$$x = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4r^2p^2}} \sin(pt - \phi)$$

Case I At very low Driving frequency.
$(p << \omega_0)$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4r^2p^2}} = \frac{f_0}{\omega_0^2} = \frac{F_0/m}{K/m}$$

$$A = \frac{F_0}{K}$$

At low Driving force

A ~~is~~ ~~indipe~~ depends only on K

Case II: At very high Driving frequency
$(p >> \omega_0)$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4r^2p^2}} = \frac{f_0}{p^2} = \frac{F_0/m}{p^2}$$

$$A = \frac{F_0}{mp^2}$$

At high Driving frequency,

A depends on m, Driving frequency (p)

Page :
Date : / /

Amplitude resonance :- the driving frequency at which the amplitude of the driven oscillator is maximum is known as resonant frequency. And the phenomenon is known as amplitude resonance.

for this to happen,

$$\frac{d}{dp}\left((\omega_0^2 - p^2)^2 + 4r^2p^2\right) = 0$$

$$2(\omega_0^2 - p^2)(-2p) + 8r^2p = 0$$

$$4(\omega_0^2 - p^2)p = 8r^2 p$$

$$\omega_0^2 - p^2 = 2r^2$$

$$p^2 = \omega_0^2 - 2r^2$$

$$p = \sqrt{\omega_0^2 - 2r^2}$$

A_{max} (maximum amplitude) $= \dfrac{f_0}{2r\sqrt{(\omega_0^2 - r^2)}}$

or

$$A_{max} = \frac{f_0}{2r(p^2 + r^2)^{1/2}}$$

in ideal case when there is no damping. $(r = 0)$

~~Amax~~ $\boxed{p = \omega_0}$

phase difference

$$\tan\phi = \frac{2\gamma p}{\omega_0^2 - p^2}$$

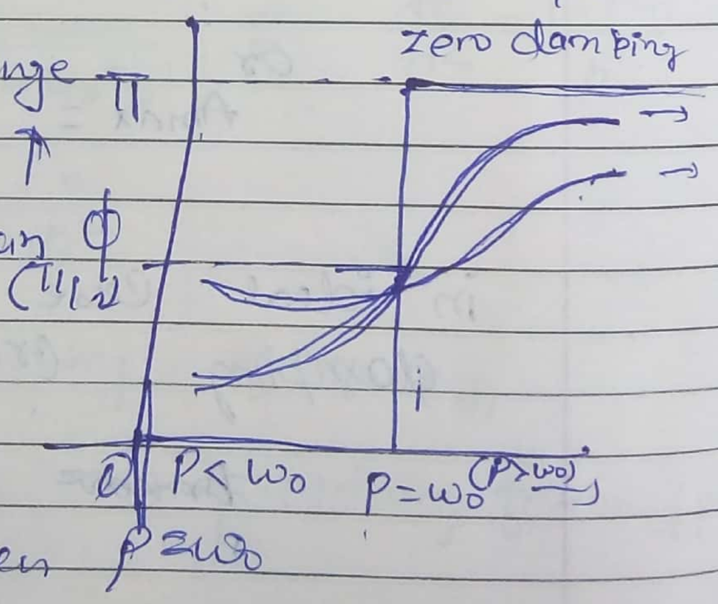
(i) when $p << \omega_0$
$\tan\phi$ is small positive and the driven oscillator is nearly in phase with driving force (0)

(ii) At $p >> \omega_0$
$\tan\phi$ is small negative and the driven oscillator is out of phase with the driving force (π)

(iii) $p = \omega_0$ i.e resonance condition

$\tan\phi = \infty \Rightarrow \phi = \pi/2$ at resonance, displacement lags behind the driving force by $\pi/2$

(a) the rate of change of phase angle is rapid when the damping is lower than when it is high



(b) all curve passes through $\theta = \pi/2$ when $p = \omega_0$

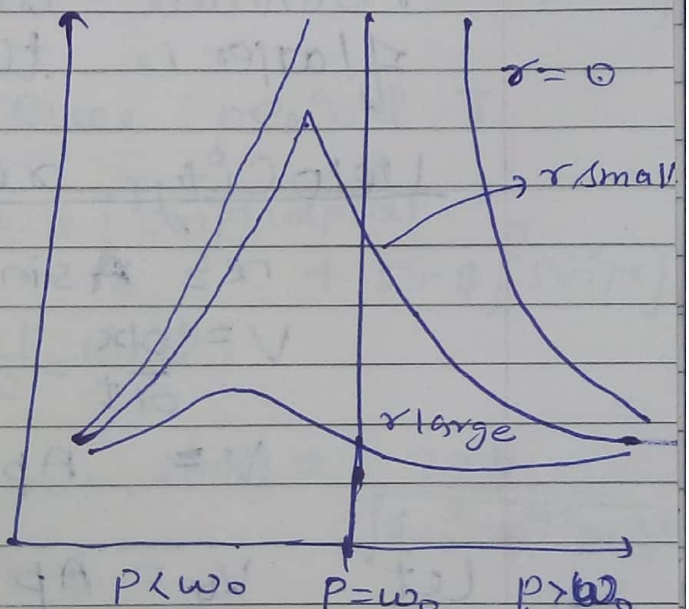
resonance $p = \omega_0$
amplitude resonance $p = \sqrt{\omega_0^2 - 2r^2}$

Page :
Date: / /

Sharpness of resonance :-

It is the measure of fall of amplitude for small change in frequency of driving force

greater is the extend of fall of amplitude for small change in p. greater is the sharpness of resonance



(i) at low driving frequency, for all values of damping the amplitude is nearly same.
As p increases amplitude increases ~~till resonance~~

low (ii) the extent of increase or the peak value depends on the damping present.
high

damping. (iii) when there is no damping ($r=0$) the amplitude becomes infinite at resonance.

(iv) As the damping increases, the peak of curve moves towards the left. i.e the frequency p of the driving force for which amplitude is maximum decreases

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Page :
Date: / /

with
(v) Increasing of damping, peak moves downward i.e. the maximum amplitude falls at resonance.

Smaller the damping greater is the fall of amplitude for small p change.

⇒ Smaller the damping, sharper the resonance and larger the damping flatter is the resonance

Velocity resonance:

$$x = A\sin(pt-\phi)$$

$$V = \frac{dx}{dt} = Ap\cos(pt-\phi)$$

$$V = Ap\sin(pt-\phi+\pi/2)$$

Let $V_0 = Ap$ this is velocity amplitude

$$V = V_0\sin(pt-\phi+\pi/2)$$

for velocity resonance

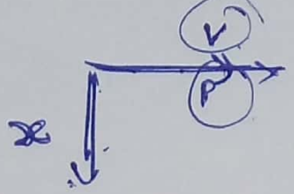
$V_0 \rightarrow V_{max}$ (max amplitude)

this will take place at $p = \omega_0$

i.e at resonance

$$V_0 = \frac{f_0}{2r} = f_0\tau$$

$\tau = \frac{1}{2r}$



Page :
Date: / /

as x laggs p by $\pi/2$
and v leads x by $\pi/2$

so, v and p are in phase

power absorption by forced oscillator.

$$P = Fv = (F_0 \sin pt)\, Ap\cos(pt-\phi)$$

$$P = ApF_0\left[\sin pt \cos pt \cos\phi + \sin^2 pt \sin\phi\right]$$

Average power over period T.

$$P_{av} = \frac{ApF_0}{T}\left[\cos\phi \cancel{\int_0^T \sin pt \cos pt\, dt} + \sin\phi \int_0^T \sin^2 pt\right]$$

$$P_{av} = ApF_0\left(\frac{1}{2}\sin\phi\right)$$

$$\tan\phi = \frac{2rp}{\omega_0^2 - p^2} \Rightarrow \sin\phi = \frac{2rp}{\sqrt{(\omega_0^2-p^2)^2 + 4r^2p^2}}$$

$$f_0 = \frac{F_0}{m} \Rightarrow F_0 = mf_0$$

$$P_{av} = \frac{mAp\, f_0\,(2rp)}{2\sqrt{(\omega_0^2-p^2)^2+4r^2p^2}} = mA^2p^2r$$

as $V_0 = Ap$

$\tau = \frac{1}{2r}$ $\quad P_{av} = mV_0^2 r = \frac{mV_0^2}{2\tau}$

Page :
Date: / /

Power Dissipation by Driven Oscillator :

The power absorbed from the driving force is dissipated in doing work against the damping force $-bv$. The rate of doing work or instantaneous power p' against damping force is

$$p' = bv \cdot v = bv^2$$

$$p' = 2\gamma m \cdot p^2 \cdot \frac{f_0^2}{(\omega_0^2-\gamma^2)^2+4\gamma^2 p^2} \cos^2(pt-\phi)$$

$$P'_{av} = \frac{2m\gamma p^2 f_0^2}{((\omega_0^2-p^2)^2+4\gamma^2 p^2)T} \int_0^T \cos^2(pt-\phi)$$

$$P'_{av} = \frac{\cancel{2} m\gamma p^2 f_0^2}{\cancel{2}[(\omega_0^2-p^2)^4+4\gamma^2 p^2]} = \frac{m\gamma p^2 f_0^2}{(\omega_0^2-p^2)^2+4\gamma^2 p^2}$$

$$P'_{av} = m\gamma v_0^2$$

same as av power absorbed.

maximum power absorbed :

$$P_{av} = \frac{m\gamma f_0^2 p^2}{(\omega_0^2-p^2)^2+4\gamma^2 p^2}$$

for $P_{av} \to$ max
denominator $\to$ min
for this to happen
$\omega_0 = p$

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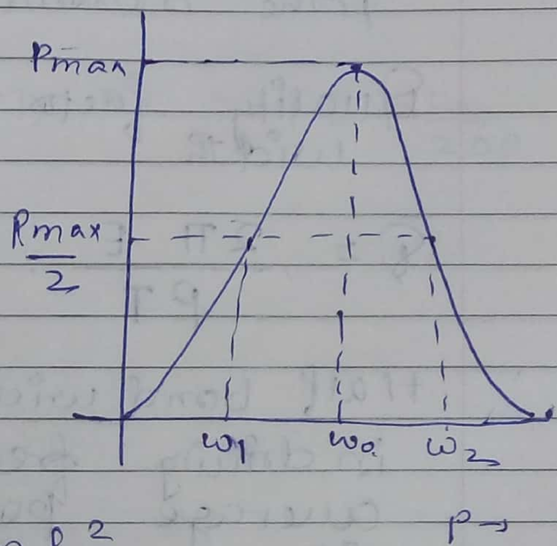
this is the condition of velocity resonance

$$P_{av}(max) = \frac{m r f_0^2 p^2}{4r^2 p^2} = \frac{m f_0^2}{4r} = \frac{m^2 f_0^2}{2b}$$

Band width of resonance Curve:

Bandwidth $= \omega_2 - \omega_1$



$$P_{av}(max) = \frac{m f_0^2}{4r}$$

$$P_{av} = \frac{m f_0^2 r p^2}{(\omega_0^2 - p^2)^2 + 4r^2 p^2}$$

$$\frac{m f_0^2 r p^2}{(\omega_0^2 - p^2)^2 + 4r^2 p^2} = \frac{1}{2} \cdot \frac{m f_0^2}{4r}$$

$$(\omega_0^2 - p^2)^2 + 4r^2 p^2 = 8r^2 p^2$$

$$(\omega_0^2 - p^2)^2 = 4r^2 p^2$$

$$\omega_0^2 - p^2 = \pm 2rp$$

$$\omega_0 - p = \pm \frac{2rp}{\omega_0 + p}$$

If $\omega_2 - \omega_1$ is the band width of the oscillator then

$$\omega_2 - p = \frac{2rp}{\omega_0 + p} \quad \text{and} \quad \omega_1 - p = \frac{-2rp}{\omega_0 + p}$$

$$2\gamma = \frac{b}{m} = \frac{1}{\tau}$$

Page:
Date: / /

$$\therefore \quad \omega_2 - \omega_1 = \frac{4\gamma p}{\omega_0 + p} = \frac{4\gamma}{\frac{\omega_0}{p} + 1} = \frac{4\gamma}{2}$$

$$\omega_2 - \omega_1 = 2\gamma = \frac{1}{\tau}$$

Thus smaller the band width, sharper is the resonance and vise versa.

Quality factor in terms of half-band width

$$Q = \frac{2\pi E}{PT} \quad \text{--- (1)}$$

Half band width (Δp) :- The change in driving frequency for which average power absorbed fall from its max value to half of its max value.

$$\Delta p = \gamma = \frac{1}{2\tau}$$

$$\text{Energy} = KE + PE = \frac{1}{2} m p^2 A^2 \cos^2(pt - \phi) + \frac{1}{2} m \omega_0^2 A^2 \sin^2(pt - \phi)$$

$$E_{av} = \frac{1}{2} m p^2 A^2 \frac{\int_0^T \cos^2(pt - \phi)}{T} + \frac{1}{2} m \omega_0^2 A^2 \frac{\int_0^T \sin^2(pt - \phi)}{T}$$

$$E_{av} = \frac{1}{4} m p^2 A^2 + \frac{1}{4} m \omega_0^2 A^2 = \frac{1}{4} m A^2 (p^2 + \omega_0^2)$$

$$Q = \frac{2\pi \left(\frac{1}{4} m A^2 (p^2 + \omega_0^2)\right)}{\left(\frac{m p^2 A^2}{2\tau}\right) T}$$

$\omega_2 - \omega_1 = \frac{1}{\tau}$ $\quad$ $\omega_2 - \omega_1 = \frac{1}{\tau}$ $\quad$ $Q = 2\pi \frac{E}{P\tau}$

$Q = \frac{\omega_0}{2\Delta P}$ $\quad$ $E = \frac{1}{2}\frac{m v_0^2}{\tau}$ $\quad$ $Q = \frac{\omega_0}{2\Delta P}$

Page: Date: / /

$Q = \frac{\omega_0}{\text{bandwidth}}$

$Q = \frac{1}{2}\frac{(P^2+\omega_0^2)}{P}\cdot\tau = \frac{1}{2}\left(P+\frac{\omega_0^2}{P}\right)\cdot\tau$

$Q = \frac{1}{2}\left(1+\frac{\omega_0^2}{P^2}\right)\cdot P\tau$

$Q = \frac{\omega_0}{2\Delta P}$ at resonance

$P = \omega_0$

$Q = \frac{1}{2}(1+1)\cdot P\tau = P\tau = \frac{\omega_0}{2\Delta P}$

Page :
Date: / /

~~Waves~~

Diffraction :-

fresnel diffraction
(i) at finite distance
(ii) spherical / cylindrical
(iii) eg Zone plate / straight edge

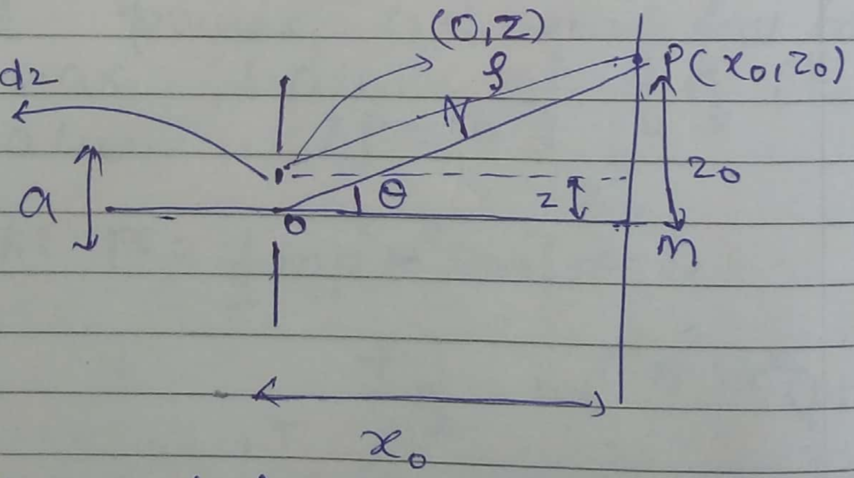
fraunhoffer's
(i) at ∞ infinite
(ii) plane wavefront

We have to discuss fraunhoffers only

topics (i) single slit
(ii) N slit
(iii) RP & grating

Single slit :-

elemental wavefront of width dz



$$dy = K\,dz \sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right)$$

$$y = K\int_{-a/2}^{a/2} \sin\left(2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right)\right) dz \quad \text{---} (1)$$

$$r^2 = x_0^2 + z_0^2$$
$$\rho^2 = x_0^2 + (z_0 - z)^2 \quad \text{In } \triangle OMP$$

Page :
Date: / /

On solving

$$\rho^2 = r^2\left(1 - \frac{2zz_0}{r^2} + \frac{z^2}{r^2}\right)$$

$$\rho^2 = r^2\left(1 - \frac{2zz_0}{r^2}\right) \qquad \frac{z^2}{r^2} \simeq 0$$

$$\rho = r - z\sin\theta \quad \text{—— (2)}$$

from (1) and (2)

$$y = \int_{-a/2}^{a/2} \sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right) dz$$

$$y = \frac{-k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{z\sin\theta}{\lambda}\right)\right]_{-a/2}^{a/2}$$

Or, $$y = \frac{-k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{a\sin\theta}{2\lambda}\right) - \cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} - \frac{a\sin\theta}{2\lambda}\right)\right]$$

$$y = \frac{-k\lambda}{\cancel{2}\pi\sin\theta}\left[\cancel{2}\times\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right).\sin 2\pi\left(-\frac{a\sin\theta}{2\lambda}\right)\right]$$

$$y = \frac{+k\lambda}{\pi\sin\theta}\left[\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right) \sin \pi\left(\frac{a\sin\theta}{\lambda}\right)\right]$$

let $\alpha = \frac{\pi a\sin\theta}{\lambda}$ amplitude

$$y = ka\left[\frac{\sin\alpha}{\alpha}\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)\right]$$

$a\sin\theta \rightarrow$ path diff

Page :

Date: / /

$$I = \frac{k^2 a^2 \sin^2\alpha}{\alpha^2} = I_0 \frac{\sin^2\alpha}{\alpha^2}$$

Case I : Central maxima/principal maxima

$$\theta = 0 \quad , \alpha = 0$$

$$\lim_{\alpha \to 0} \frac{\sin^2\alpha}{\alpha^2} = 1$$

$$I = I_0 = k^2 a^2$$

$$\therefore \boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2}} \rightarrow \text{for all maxima}$$

Case II : for secondary maxima

$$a\sin\theta = (2n+1)\frac{\lambda}{2}$$

$$\alpha = \frac{\pi}{\lambda}(2n+1)\frac{\lambda}{2} = (2n+1)\frac{\pi}{2}$$

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$\downarrow$ first secondary maxima.

(first secondary maxima) $I_1 = I_0 \left(\frac{\sin(\frac{3\pi}{2})}{3\pi/2}\right)^2 \simeq \frac{I_0}{22}$

(second secondary maxima) $I_2 \simeq \frac{I_0}{61}$

$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

Page :
Date: / /

Case III : minima

$$a\sin\theta = n\lambda$$

$$\alpha = \frac{\pi n \lambda}{\lambda} = n\pi$$

$$\alpha = \pi, 2\pi, 3\pi, \ldots$$

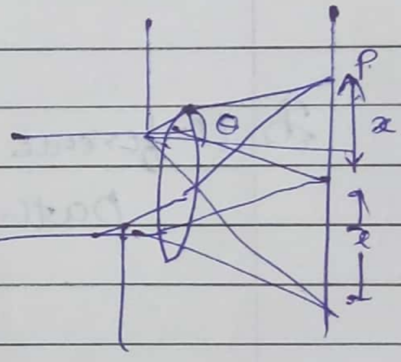
* for secondary maxima (n=0) is not taken as for (n=0) secondary maxima is at $\frac{\pi}{2}$ i.e first secondary maxima comes before first minima (π) which is not possible

→ $\sin\theta = \frac{x}{f}$ is less is very close to slit

$$\sin\theta = \frac{\lambda}{a}$$

$$\frac{x}{f} = \frac{\lambda}{a}$$

$$x = \frac{f\lambda}{a}.$$



width of central maxima

$$2x = \frac{2f\lambda}{a}$$

for n slit

$$y = \int_{-a/2}^{a/2} dy + \int_{d-a/2}^{d+d/2} + \cdots + \int_{(N-1)d-a/2}^{(N-1)d+a/2} dy$$

$$y = \left(ka \frac{\sin\alpha}{\alpha} \frac{\sin N\beta}{\sin\beta}\right) \delta.$$

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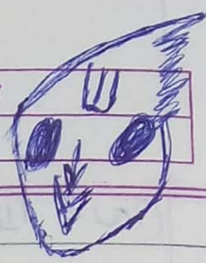
Page :
Date:

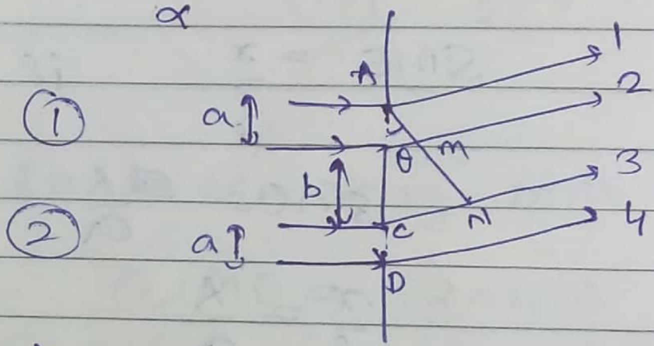


$$\alpha = \frac{\pi a \sin\theta}{\lambda}$$

$$\beta = \frac{\pi d \sin\theta}{\lambda}$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

	Amplitude	
One slit	$k a \frac{\sin\alpha}{\alpha}$	$I_1 = I_0$
two slit	$2 k a \frac{\sin\alpha}{\alpha}$	$I_2 = 2^2 I_0$
3 slit	$3 k a \frac{\sin\alpha}{\alpha}$	$I_3 = 3^2 I_0$



Interference

path diff ⇒

$(a+b)\sin\theta_n$

for maxima

$(a+b)\sin\theta_n = n\lambda$

for minima

$(a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2}$

(Separation b/w Central maxima) : $\sin\theta_2 - \sin\theta_1 = \frac{\lambda}{a+b}$

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Page :
Date: / /

Diffraction pattern.

$$a \sin\theta = n\lambda$$

Separation of Central maxima

$$\sin\theta_2 - \sin\theta_1 = \frac{\lambda}{a}$$

Condition for interference maxima and diffraction minima to coincide.

Diff minima $\Rightarrow$ $a \sin\theta = p\lambda$

Intf -- maxima $\Rightarrow$ $(a+b)\sin\theta = n\lambda$

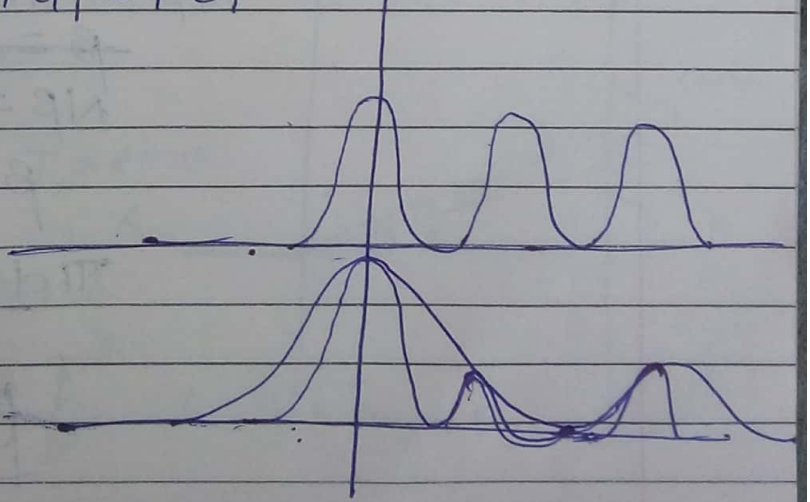
$$\boxed{\frac{a+b}{a} = \frac{n}{p}}$$

Case I :- $a = b$

missing maxima $n = 2p$

$p = 1, 2, 3, 4, \ldots$

$n = 2, 4, 6, 8, \ldots$



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Important * missing order of maxima

Page:
Date: / /

Case I : $b = 2a$, $n = 3p$

$p = 1, 2, 3, \ldots$

missing maxima $n = 3, 6, 9, \ldots$

Case II : $a + b = a$ i.e $b = 0$

$n = p$

missing $n = 1, 2, 3, \ldots$

$p = 1, 2, 3, \ldots$

for the slits will acts as single slit of 2a width

$\Rightarrow$ $I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$

minima - $N\beta = \pi, 2\pi, \ldots, k\pi$

except $k = 0, N, 2N, \ldots nN$

$k = 0, 1, 2, \ldots N-1, N, N+1$

$nN, nN+1.$

Circled values are not allowed

$\cancel{N\beta = nN\pi}$

$\cancel{\beta = n\pi}$

$N\beta = nN\pi$

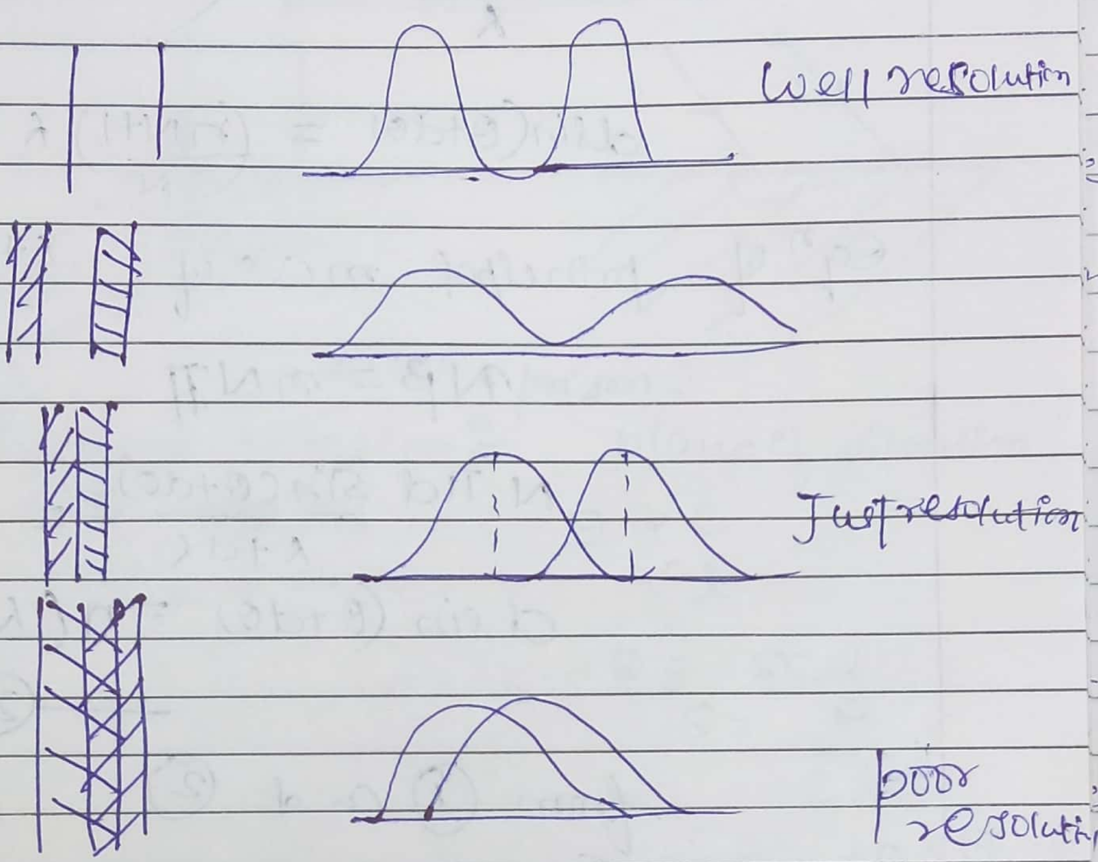
$\beta = n\pi$

$\frac{\pi d \sin\theta}{\lambda} = n\pi$

$\boxed{d \sin\theta = n\lambda}$

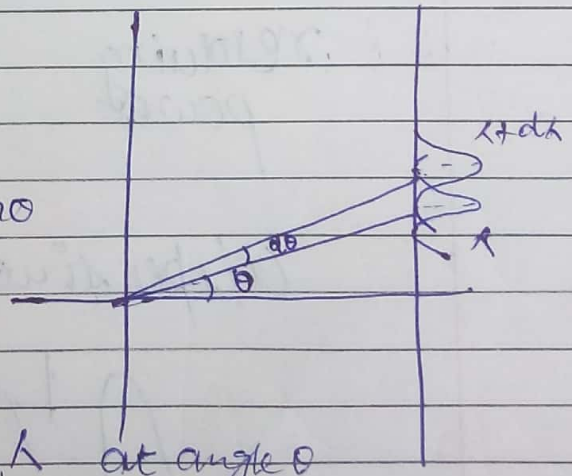
Page :
Date: / /

R.P



$\frac{\lambda}{d\lambda}$

R.P of a grating :-



$N\beta = Nn\pi$

$\beta = n\pi = \frac{\pi d \sin\theta}{\lambda}$

$d\sin\theta = n\lambda$

Condition of n^{th} maxima for λ at angle θ

Condition of mth minima of λ at θ+dθ

$N\beta = (nN+1)\pi$

Page :
Date: / /

$$\frac{N \pi d \sin(\theta + d\theta)}{\lambda} = (nN+1)\pi$$

$$d\sin(\theta + d\theta) = \frac{(nN+1)}{N}\lambda \quad \text{---} \; (1)$$

eqⁿ of principal max of $\lambda + d\lambda$ at $\theta + d\theta$

$$N\beta = nN\pi$$

$$\frac{N \pi d \sin(\theta + d\theta)}{\lambda + d\lambda} = nN\pi$$

$$d\sin(\theta + d\theta) = n(\lambda + d\lambda) \quad \text{---} \; (2)$$

from (1) and (2)

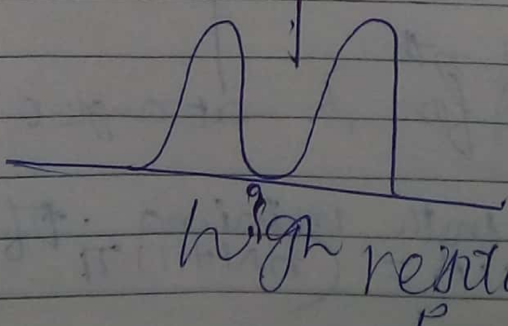
$$\frac{(nN+1)}{N}\lambda = n(\lambda + d\lambda)$$

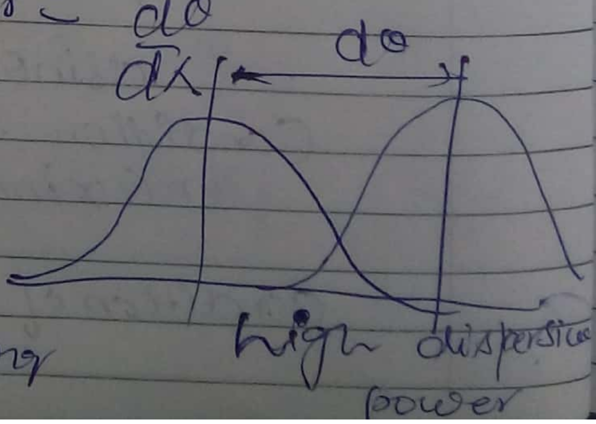
$$nN\lambda + \lambda = nN\lambda + Nn\,d\lambda$$

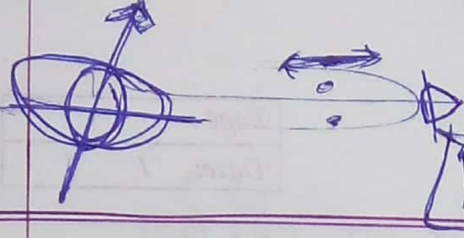
$$\lambda = nN\,d\lambda$$

resolving power $\dfrac{\lambda}{d\lambda} = nN$

dispersive power $= \dfrac{d\theta}{d\lambda}$







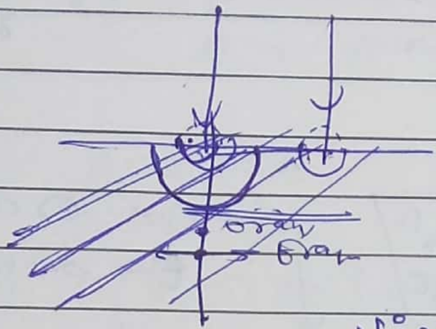
(dx, dy, dz, dt)

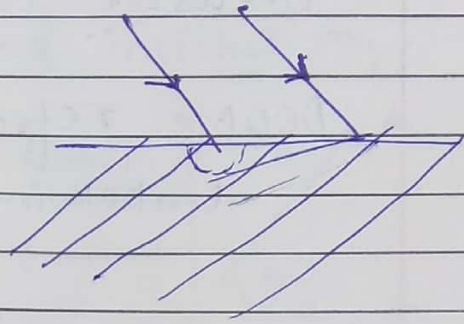
(dx/r, dy, dz, dt)

Page :

Date: / /

polarization





Nicol prism :-

Optical rotation :- plane of vibration gets ~~rotated~~

glsmno = n x

$\theta \propto C$

$\propto l$

$\theta = SCl$

$S = \frac{\theta}{Cl}$

$S = \frac{10 \times \theta}{Cl}$

cm

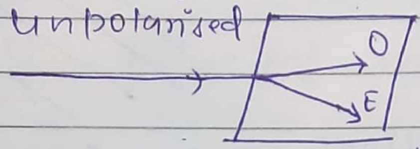
Polarisation

Malus law → $I = I_0 \cos^2\theta$

Brewster law → $i = \tan^{-1}(\mu)$

$r + i = \pi/2$.

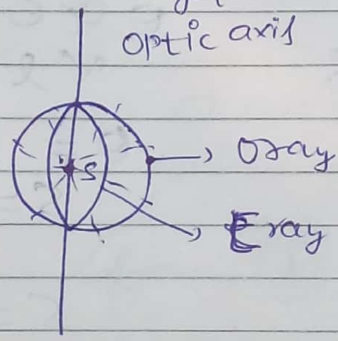
Double refraction



O → ordinary ray
E → extra ordinary

follows law of reflection and refraction

Positive crystal

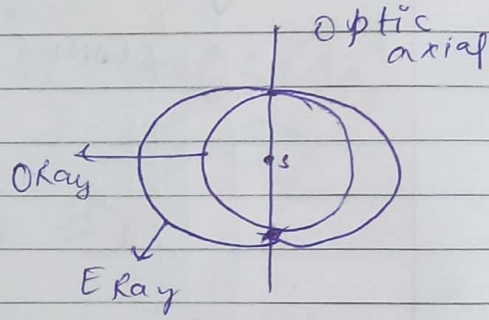


$V_o > V_E$

$\mu_E > \mu_o$

Quartz

negative crystal



$V_E > V_o$

$\mu_E < \mu_o$

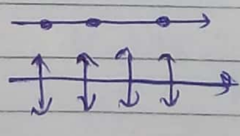
Calcite.

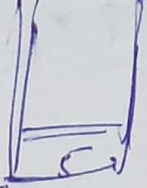
along optic axis $V_o = V_E$ i.e no double refraction takes place

O and E ray are polarised with their plane of vibrations perpendicular to each other.

O →

E →





$n * f(n-1)$

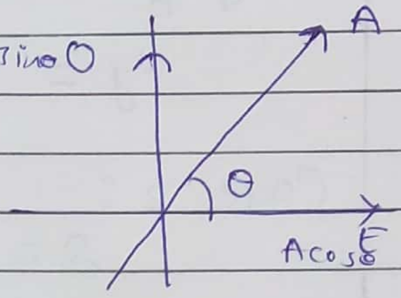
Page: Date: / /

Circular and elliptical polarised light

E ray : $x = A\cos\theta \sin(\omega t + \delta)$

o ray : $y = A\sin\theta \sin\omega t$

$A \rightarrow$ Amplitude of incident ray



put $A\cos\theta = a$

$A\sin\theta = b$

$x = a\sin(\omega t + \delta)$

$y = b\sin\omega t$

$\frac{y}{b} = \sin\omega t$

$\cos\omega t = \sqrt{1 - \frac{y^2}{b^2}}$

$\frac{x}{a} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$

$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1-\frac{y^2}{b^2}}\sin\delta$

$\frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - y^2/b^2}\sin\delta$

Squaring both sides

$\frac{x^2}{a^2} + \frac{y^2\cos^2\delta}{b^2} - \frac{2xy\cos\delta}{ab} = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy\cos\delta}{ab} = \sin^2\delta$

Special cases :

case (i) : linear polarization

$\delta = 2n\pi$ $\quad n \rightarrow 0, 1, 2 \ldots$

$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$

P.T.O

$$\left(\frac{x}{a}-\frac{y}{b}\right)^2=0$$

$$\pm\left(\frac{x}{a}-\frac{y}{b}\right)=0$$

$$y=\pm\frac{b}{a}x$$ linear polarised light.

Case 2: Elliptical polarisation

$$\delta=(2n-1)\frac{\pi}{2}$$ $n \to 1, 2, 3, \ldots$

$\cos\delta=0$ $\sin\delta=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

elliptically polarised light

Case 3: Circular polarization:-

$$\delta=(2n-1)\frac{\pi}{2}$$ $n \to 1, 2, 3, \ldots$

$\theta=45$ ie $a=b$

$$x^2+y^2=a^2$$

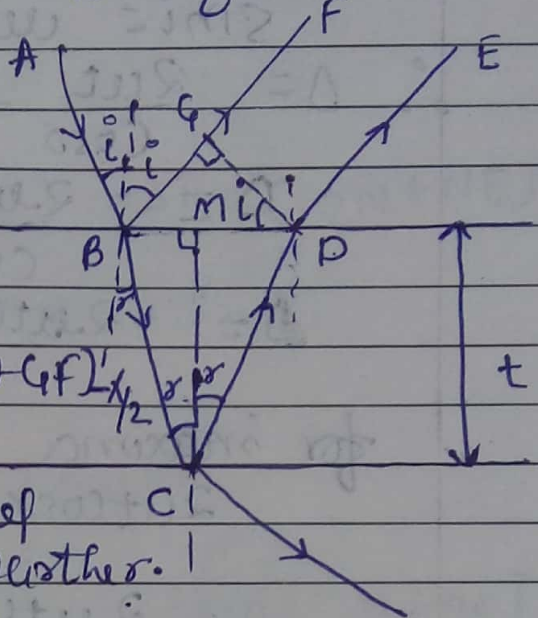
Quater and half wave plate

Optics

Page :
Date: / /

Interference due to thin film :-

(i) due to reflected rays



Optical path diff b/w ray BCDE and BF

$\Delta = \mu(BC+CD)+DE-(BG+GF)-\lambda/2$

let GF and DE travel same distance further. then $GF = DE$

$\Delta = \mu(BC+CD) - BG - \lambda/2$ —— 1

In Δ BCD

$BC = CD = \frac{t}{\cos r}$ —— 2

also in Δ BGD

$\frac{BG}{BD} = \sin i$

$BG = \sin i\,(BD) = \sin i\,(BM+MD)$

as $BM = MD$

$BG = \sin i \times 2BM = 2\sin i \cdot t\tan r$

—— 3

from 1, 2 & 3

$\Delta = \mu\left(\frac{2t}{\cos r}\right) - \frac{t\sin i \cdot 2\sin r}{\cos r}$

Page :
Date: / /

By Snell's law

$\sin i = \mu \sin r$

$\therefore \Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} - \frac{\lambda}{2}$

$\Delta = \frac{2\mu t}{\cos r}[1 - \sin^2 r] - \frac{\lambda}{2}$

$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$

for maxima

$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

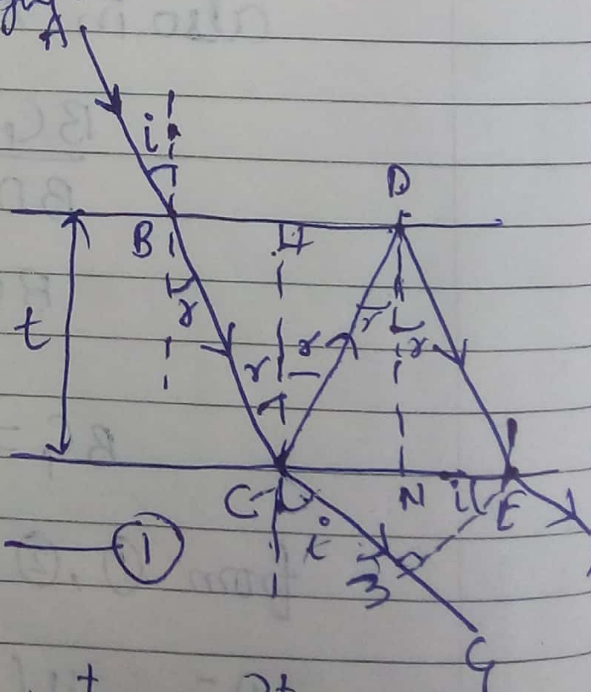
for minima

$2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$2\mu t \cos r = n\lambda + \lambda = \lambda(n+1)$

$2\mu t \cos r = n\lambda$

(ii) Due to transmitted light—



Optical path diff b/w

$\Delta = \mu[CD + DE] - Cm$

as $MG = EF$

so they cancel out

$\Delta = \mu(CD + DE) - Cm$ —— (1)

$CD + DE = \frac{t}{\cos r} + \frac{t}{\cos r} = \frac{2t}{\cos r}$ —— (2)

$2\mu t\cos r = n\lambda$

Page :
Date: / /

In $\triangle$ CME

$$\frac{CM}{CE} = \sin i$$

$$CM = \sin i\,(CE) = \sin i\,(CN+NE)$$

$$CM = \sin i \cdot 2t\tan r \quad -③$$

from ①, ② & ③

$$\Delta = \frac{2\mu t}{\cos r} - 2\sin i\, t\tan r$$

[$\sin i = \mu \sin r$]

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t\sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}(1-\sin^2 r)$$

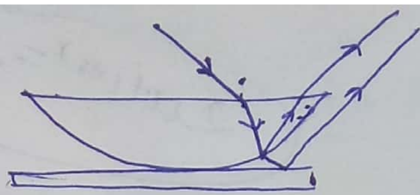
$$\Delta = 2\mu t\cos r$$

for maxima

$$2\mu t\cos r = n\lambda$$

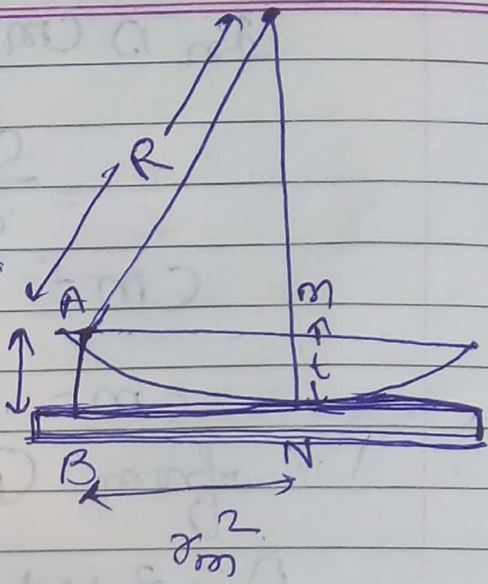
for minima

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}.$$



Page :
Date:

Newtons Rings:-



$$R^2 = (R-t)^2 + r_m^2$$

$$R^2 = R^2 + t^2 - 2Rt + r_m^2$$

$$r_m^2 = 2Rt - t^2$$

as $R >> t$

$$2Rt >> t^2$$

$$r_m^2 \simeq 2Rt$$

for dark fringes

$$2t - \frac{\lambda}{2} = \cancel{n\lambda}\ (2n+1)\frac{\lambda}{2}$$

$$2t = n\lambda + \lambda = (n+1)\lambda$$

put $n+1 = m$

$$2t = m\lambda$$

so, $$r_m^2 = 2R \times \frac{m\lambda}{2} = m\lambda R$$

$$d_m^2 = 4m\lambda R$$

This is for dark fringes due to reflection.

Here path diff $= 2\mu t\cos r - \frac{\lambda}{2}$

$\mu = 1$ $\cos r = 1$

so, $\Delta x = 2t - \frac{\lambda}{2}$

newtons of ~~Dark~~ fringes for transmitted light :-

Bright fringes, $2t = n\lambda$

$r_m^2 = 2Rt = n\lambda R$

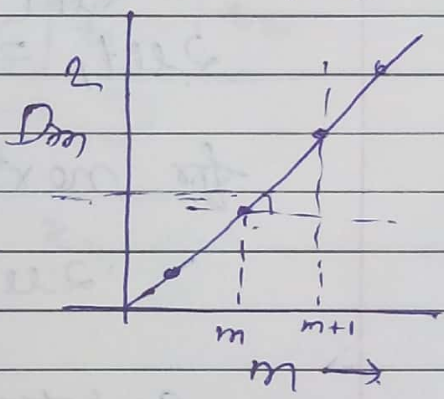
for dark fringes,

$$2t = (2n+1)\frac{\lambda}{2}$$

$$r_m^2 = 2Rt = \frac{(2n+1)\lambda R}{2}$$

Determination of wave length of light :-

$D_m^2 = 4m\lambda R$



$D_{m+1}^2 = 4(m+1)\lambda R$

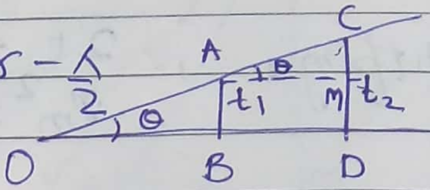
On subtracting these.

$D_{m+1}^2 - D_m^2 = 4\lambda R$

$$\Rightarrow \lambda = \frac{D_{m+1}^2 - D_m^2}{4R} = \frac{\text{slope}}{4R}$$

Interference ~~Intro~~ through wedged shaped film:

path diff $(\Delta) = 2\mu t \cos r - \frac{\lambda}{2}$



for minima

$$2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \Rightarrow$$

$$2\mu t \cos r = n\lambda$$

for normal incidence $\cos r = 1$

$$2\mu t = n\lambda$$

$$2\mu t_1 = n\lambda \quad \text{——— (1)}$$

for next dark fringe at $t = t_2$

$$2\mu t_2 = (n+1)\lambda \quad \text{— (2)}$$

subtract (2) & (1)

$$2\mu(t_2 - t_1) = \lambda$$

$$\cancel{2\mu(Am)} \quad 2\mu(mC) = \lambda$$

$$2\mu(BD \tan\theta) = \lambda$$

BD is seperation b/w fringes i.e β

$$2\mu\beta \tan\theta = \lambda$$

$$\beta = \frac{\lambda}{2\mu \tan\theta}$$

for small θ

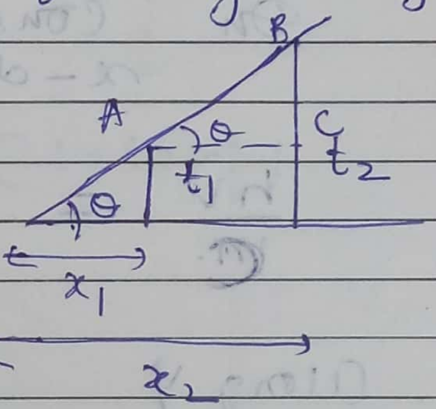
$$\beta = \frac{\lambda}{2\mu\theta}$$

Page:
Date: / /

features of interference through wedge film

(i) At apex fringes are dark
(ii) fringes are straight and parallel
(iii) They are equidistant
(iv) They are localised.
(v) fringes are of equal thickness.

determination of wedge angle θ:-



at t_1

$$2\mu t_1 = n\lambda$$

at t_2 for $(n+N)$ fringe

$$2\mu t_2 = (n+N)\lambda$$

$$2\mu(t_2-t_1) = N\lambda$$

In △ABC $t_2 - t_1 = (x_2-x_1)\tan\theta$

$$2\mu(x_2-x_1)\tan\theta = N\lambda$$

$$\tan\theta = \frac{N\lambda}{2\mu(x_2-x_1)}$$

for small angle θ

$$\theta = \frac{N\lambda}{2\mu(x_2-x_1)}$$

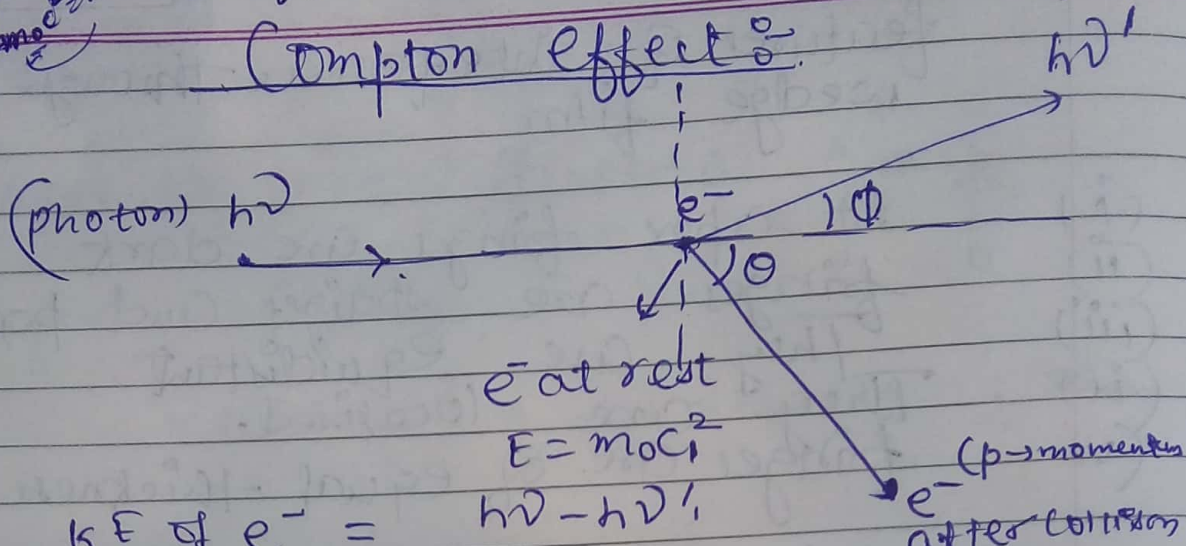
$m c^2 = KE$ $E^2 = p^2 c^2 + m_0^2 c^4$

energy of photon after collision

~~Diffraction~~

Date: /

Compton effect:



KE of e^- = $h\nu - h\nu'$

On conserving momentum in x-dirn.

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + p\cos\theta$$

$$p\cos\theta = \frac{h\nu}{c} - \frac{h\nu'}{c}\cos\phi \quad \text{—①}$$

along Y.

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta$$

$$p\sin\theta = \frac{h\nu'}{c}\sin\phi \quad \text{—②}$$

multiply ① and ② by c and then square and add

$$c^2p^2 = (h\nu - h\nu'\cos\phi)^2 + (h\nu'\sin\phi)^2$$

$$c^2p^2 = (h\nu)^2 + (h\nu')^2 - 2h^2\nu\nu'\cos\phi \quad \text{—③}$$

also $E^2 = p^2c^2 + m_0^2c^4$

$$(K + m_0c^2)^2 = p^2c^2 + m_0^2c^4$$

$$K^2 + 2m_0c^2 = p^2c^2 \quad \text{—④}$$

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Compton ← partial absorption of energy
Photoelectric → Total absorption of energy
Raman effect → re-emitted

Page:
Date: / /

from (3) and (4)

$$K^2 + 2m_0c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi.$$

$$(h\nu - h\nu')^2 + 2m_0c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi$$

$$-2h\nu h\nu' + 2m_0c^2 = -2h\nu h\nu\cos\phi$$

$$2m_0c^2 = 2h\nu h\nu' - 2h\nu h\nu'\cos\phi$$

$$m_0c^2 = h\nu h\nu' - h\nu h\nu'\cos\phi$$

$$m_0c^2 = h^2\nu\nu'(1-\cos\phi)$$

$\vdots$

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)$$

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Page:
Date: / /

LASER → Light Amplification (by) Stimulation (of) emission (of) radiation

LASER

E_2 ———————— N_2

E_1 ———————— N_1

$N \to$ no. of e^-

$N_2 = e^{-E_2/kT}$

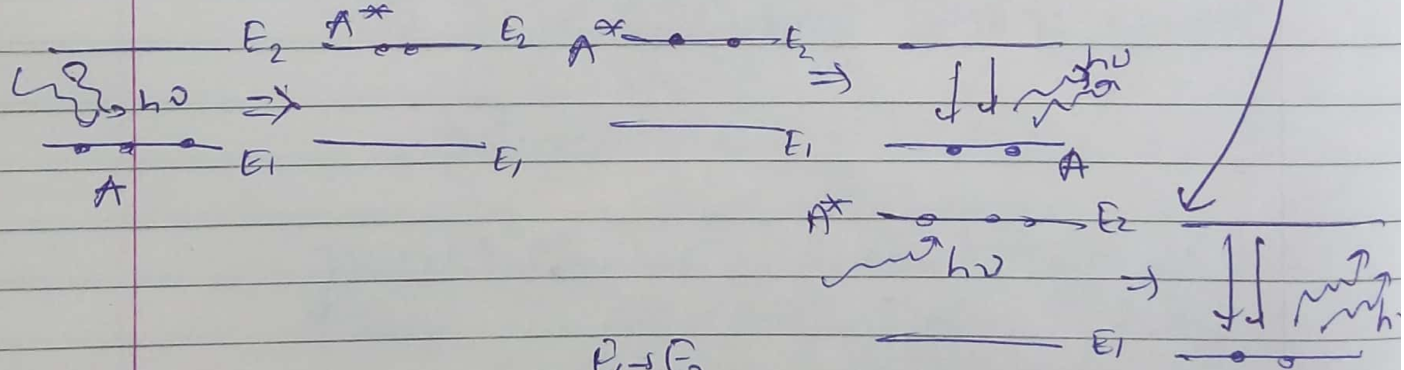
$N_1 = e^{-E_1/kT}$

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

$T \to$ temp effect of T is very small.

LASER

- Absorbtion (induced): $h\nu + A \to A^*$
- spontaneous emission: $A^* \to A + h\nu$
- Stimulated emission: $h\nu + A^* \to A + 2h\nu$



$E_1 \to E_2$

(A) Absorbtion (induced) $(1 \to 2) \Rightarrow P_{12} \propto \rho(\nu)$

probability of excitation $(1\to 2)$ $P_{12} = \rho(\nu) B_{12}$

B_{12} const of proportanality

$\rho(\nu)$ photon density

Page:
Date: / /

Rate of absorption = $P_{12} \times N_1$

$$R_{Ab} = B_{12}\, f(\nu)\, N_1 \quad \text{---} \quad (1)$$

(B) Spontaneous emission:- $(2 \rightarrow 1)$ $(E_2 \rightarrow E_1)$ $P_{21} = A_{21}$

R_{sp} = Rate of emission = $P_{21} \times N_2 = A_{21} N_2$ --- (2)

it is from higher to lower not vise versa

$A_{12} (1 \rightarrow 2) = 0$

(C) stimulated emission:-

$$P_{21} = B_{21}\, f(\nu)$$

R_{st} = Rate of stimulated emission $= B_{21}\, f(\nu) N_2$ --- (3)

At equillibrium:-

$$R_{ab} = R_{sp} + R_{st}$$

$$B_{12}\, f(\nu)\, N_1 = A_{21} N_2 + B_{21}\, f(\nu)\, N_2$$

$$f(\nu) = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

Divide Num & Deno by $N_2 B_{12}$

$$f(\nu) = \frac{\frac{A_{21}}{B_{12}}}{\left(\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}\right)}$$

from boltzman eqn

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/KT}$$

$$E_2 - E_1 = h\nu$$

$\mu = \frac{\lambda}{\lambda'}$ $\left(\mu = \frac{c}{v}\right)$

Page :
Date: / /

$$f(\nu) = \frac{A_{21}/B_{12}}{\left(e^{h\nu/kT} - \frac{B_{21}}{B_{12}}\right)} \quad \text{--- (A)}$$

by plancks law, μ = refractive index of material

$$f(\nu) = \frac{8\pi h \nu^3 \mu^3}{c^3 (e^{h\nu/kT} - 1)} \quad \text{--- (B)}$$

On comparing (A) & (B)

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h \nu^3 \mu^3}{c^3}$$

$$\frac{B_{21}}{B_{12}} = 1$$

Lasering action $\propto \frac{B_{21}}{A_{21}} \propto \frac{1}{\nu^3}$

Lasering action is not possible at higher energy

$$R = \frac{R(\text{stimulated})}{R(\text{spontaneous})} = \frac{B_{21}\, f(\nu)\, N_2}{A_{21} N_2}$$

$$R = \frac{B_{21}}{A_{21}} \cdot f(\nu)$$

$$R = \frac{c^3}{8\pi h \mu^3 \nu^3}\left[\frac{8\pi h \mu^3 \nu^3}{c^3 (e^{h\nu/kT} - 1)}\right]$$

$$R = \frac{1}{e^{h\nu/kT} - 1}$$

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Einstein coefficient :

B_{12} A_{21} A_{12} B_{21}

$\downarrow$

0

Page :

Date: / /

$$R = \frac{\text{stimulated}}{\text{Absorbtion}} = \frac{B_{21} N_2 \rho(\nu)}{B_{12} \rho(\nu) N_1}$$

$$R = \frac{N_2}{N_1}$$

population inversion : $\left(N_2 > \frac{N_1}{2} \right)$

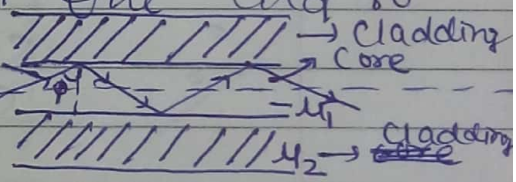
$$N_2 > \frac{N_1}{2}$$

Optical fibre

propagation of light signal through optical fibre :-

Optical fibre is a special thread like fibre made up of glass and quartz. it has two layers, core and cladding. Core has higher refractive index than cladding.

The propagation of light signal through optical fibre take place due to total internal reflection, when light signal travels from core of higher refractive index to the cladding of lower refractive index incident at a surface at an angle greater than critical angle then it is totally internally reflected and transmitted from one end to other end. ($\phi > \phi_c$) ($\mu_1 > \mu_2$)



Acceptance angle:-

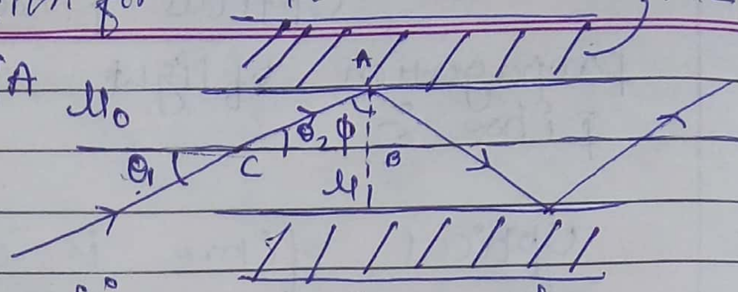
~~The maximum value of angle of incidence at core surface for which the ray incident at core – cladding interface incident a~~

~~The maximum~~ As the angle of incidence at core surface increases, the angle of incidence at core – cladding surface decreases, so, the maximum value of angle of incidence at core surface for which the phenomenon of TIR takes place is called acceptance angle

Page:
Date: / /

Derive expression for acceptance angle & NA

NA → Numerical aperture



Let the refractive indices of outer medium, core and cladding are μ_0, μ_1 and μ_2 respectively

At core - ~~core~~ medium interface, using Snell's law,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_1}{\mu_0}$$

in Δ ABC

$$\phi + \theta_2 = 90^\circ$$

$$\theta_2 = 90^\circ - \phi$$

$$\therefore \quad \frac{\sin\theta_1}{\sin(90^\circ - \phi)} = \frac{\mu_1}{\mu_0}$$

$$\frac{\sin\theta_1}{\cos\phi} = \frac{\mu_1}{\mu_0} \quad \text{——— (1)}$$

~~for~~, the limiting case of TIR

$$\phi = \phi_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right) = \text{critical angle}$$

$$\cos\phi = \cos\phi_c = \sqrt{1 - \left(\frac{\mu_2}{\mu_1}\right)^2}$$

also, $\theta_1 = \theta_a$ = angle of acceptance

$$\mu_0 \sin\theta_a = \sqrt{\mu_1^2 - \mu_2^2}$$

where $NA = \mu_0 \sin\theta_a = \sqrt{\mu_1^2 - \mu_2^2}$

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Page :
Date: / /

if outer medium is air then $\mu_0 = 1$

$$\sin\theta_a = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\sin\theta_a = \sqrt{(\mu_1 - \mu_2)(\mu_1 + \mu_2)}$$

usually $\mu_1 + \mu_2 \simeq 2\mu_1$

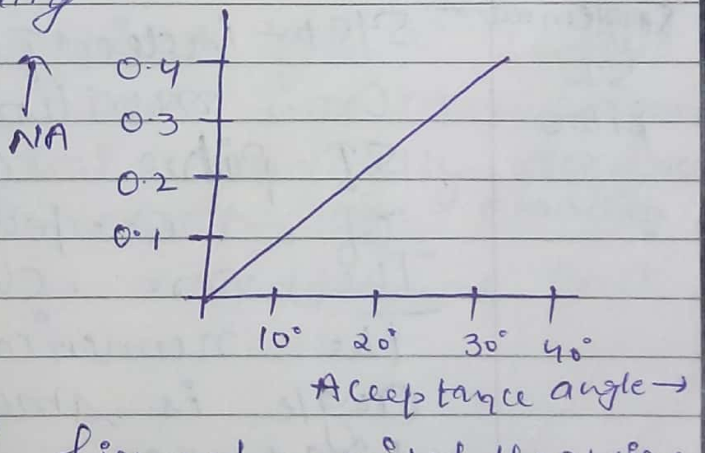
$$\sin\theta_a = \sqrt{2\mu_1(\mu_1 - \mu_2)} = \sqrt{2\mu_1^2 \frac{(\mu_1 - \mu_2)}{\mu_1}}$$

$$\sin\theta_a = \sqrt{2\mu_1^2 \Delta}$$

fraction change in refractive index $\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$

NA is a measure of light gathering ability of a fibre

It is independent of fibre core diameter and depends upon refractive indices of core and cladding



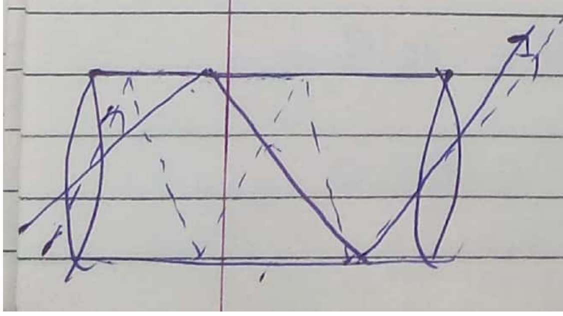
NA is also called figure of merit of the optical fibre

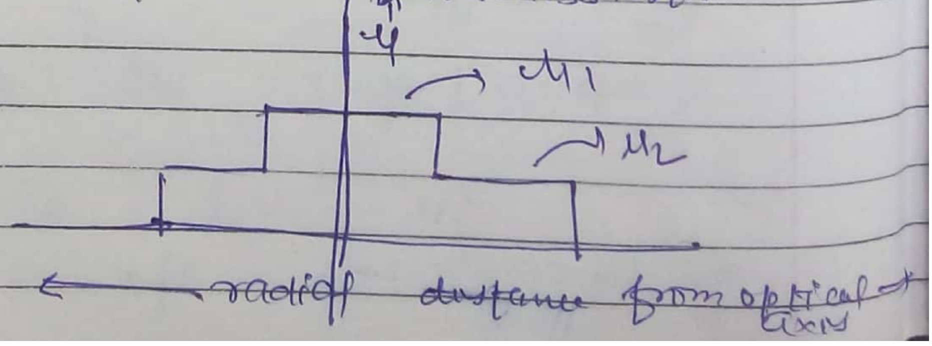
Page :
Date: / /

Types of Optical Fibres
→ Step index optical Fibre
→ Graded Index Optical Fibre

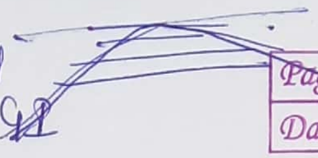
(I) Step index optical fibre :- In this type of fibre the core is homogeneous with constant refractive index μ_1 and the cladding has also a constant refractive index μ_2 ($\mu_1 > \mu_2$). There is an abrupt change of refractive index at the core-cladding interface, hence it is called step index optical fibre.

Single mode SI Fibre * Step-index fibre can be single mode or multimode. Single mode SI fibre allows only one mode of wavelength to propagate through it. The core diameter is very small. The numerical aperture and acceptance angle is small, making insertion of light ray difficult. However these fibres are most efficient and are used for high speed, large band width, long distance transmission.





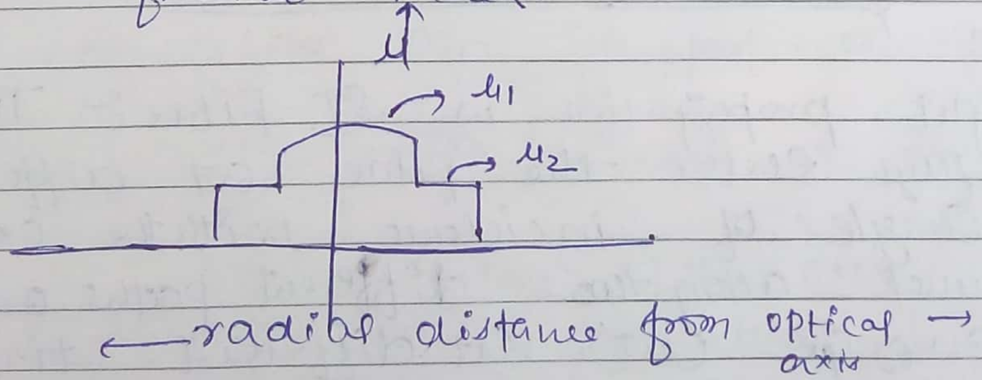
* rays travel straight in SI
* rays travels in the form of round curves in GI



Page:
Date: / /

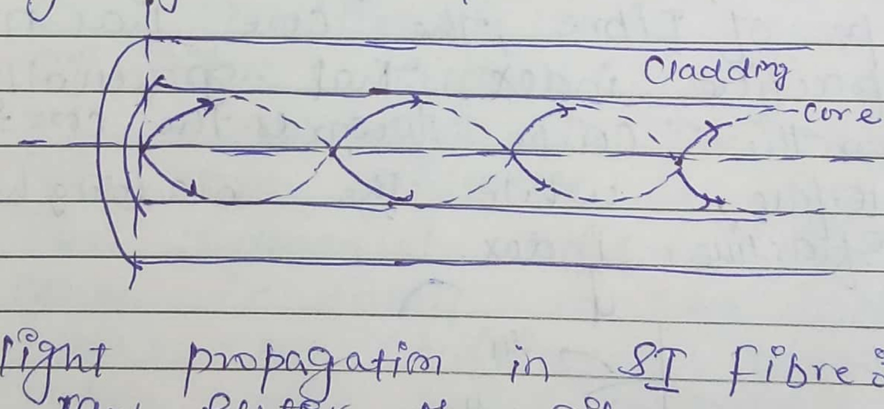
Multimode SI Fibre :- The multimode SI fibre has large diameter. The marginal rays take longer path for propagation than the axial rays and get delayed. This time delay is known as modal dispersion. Causes distortion in the pulse. Due to this short light pulse gets brodened, decreasing the transmission speed. Also there may be interaction between axial and marginal rays causing mode mixing. These factors make these fibres less efficient. However these fibres are cheap and have high numerical aperture and are used for short distance.

(II) Graded - index optical fibre :- In this type of fibre, the core has non-uniform refractive index that gradually decrease from the centre towards the core - cladding interface, while the cladding has const refractive index.

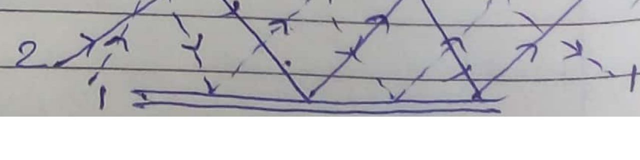


Page :
Date: / /

Light propagation in GRIN Fibre :- The ray goes from a region of higher refractive index to a region of lower refractive index, it is bent away from the normal progressively till the condition of TIR is met and the ray travels towards the core axis again being continuously refracted. The process repeats itself again and again. In these fibres, though the rays making large angles with the axis traverse a longer path, they do so in a region of lower refractive index and hence, at a higher speed of propagation. Consequently, all rays have the same optical path and reach the other end at same time giving a better transmission.



light propagation in SI fibre :- The rays enter the fibre at different angles of incidence with the axis, travel along two different paths and emerge out at different times. A input signal or pulse gets widened as it propagates through the fibre.



Page:
Date: / /

SI Fibre	GRIN Fibre
(i) light propagates in the form of straight lines	(i) light propagates in the form of circular curves.
(ii) It has more dispersion	(ii) It has less dispersion
(iii) It has low data transmission capabilities	(iii) It has large data transmission capabilities
(iv) refractive index of core is homogeneous and fall abruptly at core-cladding interface	(iv) refractive index of core is non-uniform that gradually decreases till core-cladding interface
(v) manufacturing of fibre is easy and are cheaper than GRIN fibre	(v) manufacturing of fibre is difficult owing to the problems in controlling the refractive index variation. and also are costly than SI fibre
(vi) acceptance angle is large as compare to GRIN fibre.	(vi) small acceptance angle than SI fibre

multi mode

Normalized frequency (V - number) of a fibre:-

V- number determines the number of modes of a fibre can support or propagate.

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2\pi a}{\lambda}\cdot(NA)$$

$a \rightarrow$ radius of the core

$\lambda \rightarrow$ the wavelength in (free space)

The maximum no. of mode for SI fibre

$$N_m = \frac{V^2}{2}$$

The maximum no. of modes for GRIN fibre

$$N_m = V^2/4$$

* ~~for single mode V < 2.405~~

Page:
Date: / /

Attenuation in optical Fibre :- The loss of strength of optical signal during transmission is known as attenuation. $(\alpha) = \frac{10}{L} \log \frac{P_i}{P_o}$

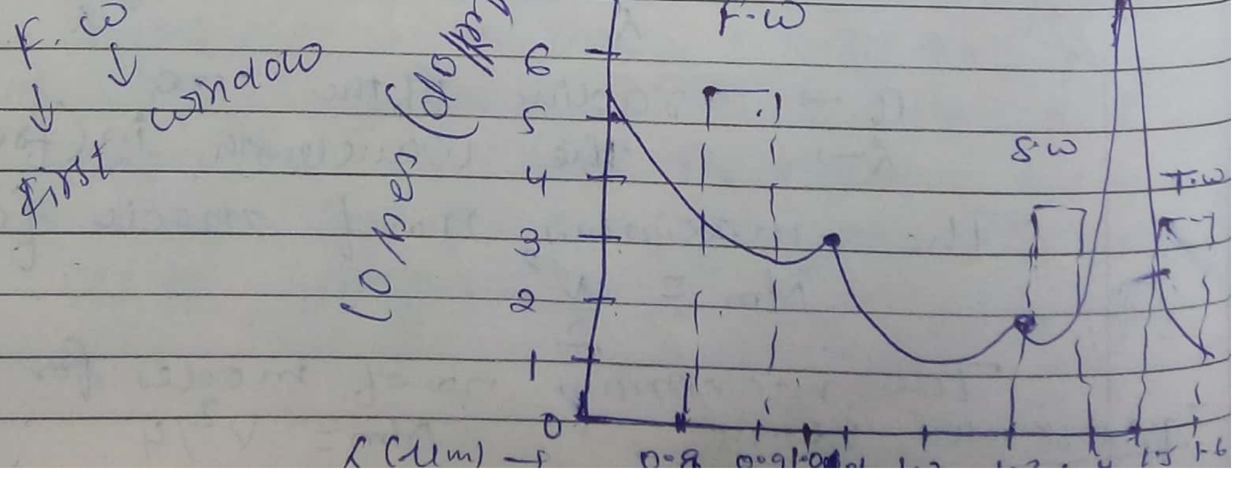
L → length of fibre in km P_i = input P_o = output unit: dB

Causes of attenuation:-

(i) Absorption of signal
(ii) Scattering of signal
(iii) Geometrical effects

(I) Absorption of signal :- The loss of optical signal takes place due to its absorption by ~~by~~ the glass material of fibre.

The loss due to pure glass is not much and can be neglected but Due to the impurities present, there is a ~~suffi~~ significant lose. This is called extrinsic absorption lose. This is mainly due to two types of impurities, first is transition metal like Cu, Zn, Co, Ni etc and second is OH^-. They absorb the signal very effectively.



Page :
Date: / /

window

Optical window or transmission:- the range of wavelength for which the attenuation due to absorption is minimum is called optical window or transmission window.

) Scattering loss :- During fibre manufacture, despite all precautions, localized microscopic variation in density and doping impurities cannot be removed completely due to which local variation in refractive index set in. These variations act as small scattering centres embedded in otherwise homogenous medium. The size of these scattering centres are often smaller than the wavelength. A beam of light propagating through the fibre suffers losses due to Rayleigh scattering. Since, for rayleigh scattering $\propto \frac{1}{\lambda^4}$, Rayleigh scattering set a lower limit on wavelength that can be transmitted through a glass fibre to nearly 0.8 μm. Below this wavelength, scattering loss is appreciably high.

Loss due to geometrical effect: Bending of fibres, during manufacture and/or installation causes loss of power. micro-bending is caused by small cracks in the glass. macrobending occurs when the cracks extend to a large distance along the length

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$6 \times 10 = 60$

Page :
Date: / /

of the fibre. Due to microbending mode, Coupling can Occur while macrobanding obstructs mode propagation

Dispersion in Optical fibre:-

Broadening of pulse during its propagation through an optical fibre is referred as dispersion.

Three types of dispersion:-

(i) Intermodal Dispersion:- Intermodal dispersion is present in multimode fibres. A ray of light follows a zigzag path inside the fibre core. When a number of modes are available in a fibre, the axial and marginal rays travel at different speeds. The marginal rays take longer path than the axial rays. Therefore, the axial rays reach at the output earlier than the marginal rays. This causes a spread in the pulse or a dispersion is produced. This type of dispersion does not depend on the spectral width of the source and even ~~the~~ a pulse from a pure monochromatic source shows intermodal dispersion.

time delay is Δt.

$$\Delta = \frac{\mu_1 - \mu_2}{\mu_1}$$

Page:
Date: / /

$$\Delta t \text{ for SI fibre} = \frac{L}{c} \frac{\Delta}{1-\Delta} \mu_1$$

$$\Delta t \text{ for GRIN fibre} = \frac{\mu_1 L \Delta^2}{c}$$

μ_1 of core

Why GRIN fibres have lower intermodal dispersion then SI fibre?

Ans: This is because light travel from the centre towards cladding, taking a zigzag path, through regions of decreasing refractive index. We know, phase velocity $v_p = \frac{c}{\mu}$, so the phase velocity increases with distance and is maximum at outer edge. Reverse phenomenon occurs as the ray travels towards the core. Thus, there is an alternate rise and fall of group velocity with a higher average phase velocity in higher modes than that in lower modes which propagates along the core axis, which results in lower intermodal dispersion of GRIN fibre.

(ii) Material dispersion:- Light of different ~~velocity~~ wavelength have different velocities in any medium. Even a short pulse of light is not strictly monochromatic but centred about certain wavelength so, it consists of more than one wavelength. Light of shorter wavelength travels slower than those of longer wavelength

Page :
Date: / /

This causes dispersion at the output end and even a short pulse of light gets broadened as it moves through an optical fibre. This type of dispersion is called material dispersion.

$$D_m = \frac{\lambda(\Delta\lambda)}{c} L \frac{d^2\mu}{d\lambda^2}$$

$\Delta\lambda \rightarrow$ spectral width

$\lambda \rightarrow$ peak wavelength

$\mu \rightarrow$ refractive index of core

(iii) wave guide dispersion:- wave guide dispersion arises due to the guiding properties of the fibre. Like ~~the~~ material dispersion, wave guide dispersion is also wavelength dependent

The total dispersion is equal to the root mean square of all the dispersions.

mmf $\rightarrow$ all dispersion take place

smf $\rightarrow$ wave guide ~~&~~ / material takes place

Dispersion limits the bandwidth of the fibre

$$\text{Bandwidth } (MHz, Km) = \frac{310}{\text{Dispersion } (ns/Km)}$$

maximum Bit rate:

$$B_{max} = \frac{1}{5 \times \text{Dispersion}}$$

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Page :
Date: / /

Advantages of Optical Communication

(i) Extremly large Bandwidth
(ii) Smaller diameter and light weight
(iii) No emission of signal
(iv) High tolerance to environmental factor
(v) Immunity to inductive interference – Electrical isolation
(vi) Strong and durable

Application of Optical fibres:-

(i) Optical communication
(ii) medical application – Endoscopy.
(iii) millitary application – Optical sensors are mounted on the missiles to collect video information which is transmitted to ground control, from where further command is sent to the missile.
(iv) Computer networking :- wifi cable etc LAN WAN
(v) Industrial Application :- It is used in fabrication of the fibro scope, which is used to examine welds, nozzles and combustion chambers inside jet aircraft engines.

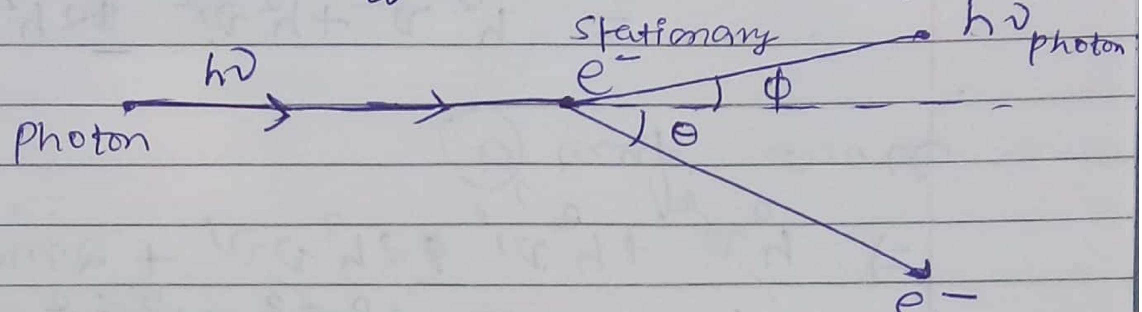
$KE = \Delta mc^2 = (m - m_0)c^2$

Page :
Date: / /

Compton effect :- partial absorption of energy

photoelectric effect → complete absorption

raman effect - $2e^-$ emitted.



$KE = KE$ of $e^- = h\nu - h\nu'$ —— (a) after collision

By conservation of momentum in x-dirⁿ

$$\frac{h\nu}{c} = \frac{h\nu'\cos\phi}{c} + p\cos\theta$$

p → momentum of e^- after collision

$$p\cos\theta = \frac{h\nu}{c} - \frac{h\nu'\cos\phi}{c} \quad \text{—— (1)}$$

along y axis or dirⁿ.

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta$$

$$p\sin\theta = \frac{h\nu'}{c}\sin\phi \quad \text{—— (2)}$$

first multiply by c then Squaring and adding

$$c^2p^2 = h^2\nu'^2\sin^2\phi + h^2\nu^2 + h^2\nu'^2\cos^2\phi - 2h^2\nu\nu'\cos\phi$$

but $E^2 = p^2c^2 + m_0^2c^4$ and $E = KE + m_0c^2$

$$(KE + m_0c^2)^2 - m_0^2c^4 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\phi$$

~~now from (a)~~

$\cancel{h\nu - h\nu'}$

$$KE^2 + m_0^2c^4 + 2(KE)m_0c^2 - m_0^2c^4$$
$$= h^2\nu^2 + h^2\nu'^2 \pm 2h^2\nu\nu'\cos\phi$$

now from (a)

$$\Rightarrow h^2\nu^2 + h^2\nu'^2 \pm 2h^2\nu\nu' + 2m_0c^2(h\nu - h\nu')$$
$$= h^2\nu^2 + h^2\nu'^2 \pm 2h^2\nu\nu'\cos\phi$$

$$\cancel{h^2\nu\nu' + h^2\nu\nu'\cos\phi = 2m_0c^2(h\nu - h\nu')}$$

~~put $\nu = \frac{c}{\lambda}$ $\nu' = \frac{c}{\lambda'}$~~

$$2h^2\nu\nu'(1-\cos\phi) = 2m_0c^2(\nu-\nu')$$

$$h\nu\nu'(1-\cos\phi) = m_0c^2(\quad)$$

$$\frac{hc^2}{\lambda\lambda'}(1-\cos\phi) = m_0c^2\cdot\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right)$$

$$\frac{h}{m_0\lambda\lambda'}(1-\cos\phi) = \frac{c(\lambda'-\lambda)}{\lambda\lambda'}$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\phi)}$$

Page :
Date: / /

Imp topics left

Fresnel's explanation for calcite and quartz

Huygen's construction

Diffraction :

- fresnal
- fraunhaffer.

fresnal	fraunhaffer
image at finite distance	both image and object at infinite distance
whether spherical or cylindrical wavefront	plane wave front

Zone plate / straight edge.

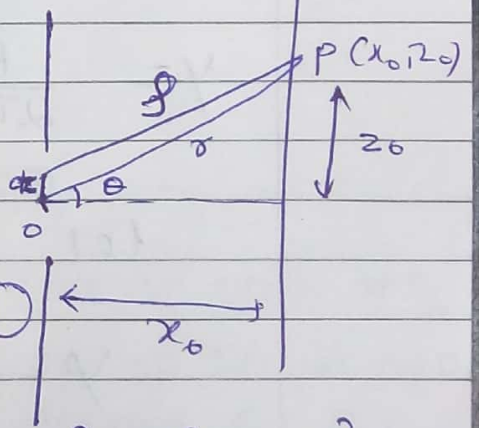
(1) Single slit diffraction :

for minima
$a\sin\theta = n\lambda$.

Displacement :

Calculus method :-

Consider an element wavefront of width dz at z



placement

$$dy = k\,dz\,\sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right) \quad a$$

$$\int dy = \int_{-a/2}^{a/2} k\,\sin 2\pi\left(\frac{t}{T} - \frac{\rho}{\lambda}\right) dz \quad \text{—(1)}$$

$$r^2 = x_0^2 + z_0^2$$

$$\rho^2 = x_0^2 + (z_0 - z)^2$$

$$\rho^2 = x_0^2 + z_0^2 + z^2 - 2zz_0$$

$$\rho^2 = r^2 + z^2 - 2zz_0$$

P.T.O

Page :
Date: / /

$$\rho^2 = r^2\left(1 + \frac{z^2}{r^2} - \frac{2zz_0}{r^2}\right)$$

since, $\frac{z^2}{r^2} \to 0$

$$\rho^2 = r^2\left(1 - \frac{2zz_0}{r^2}\right)$$

$$\rho = r\left(1 - \frac{zz_0}{r^2}\right) = r - z\left(\frac{z_0}{r}\right)$$

$$\rho = r - z\sin\theta \quad \text{--- (2)}$$

from (1) and (2)

$$y = k\int_{-a/2}^{a/2} \sin 2\pi\left(\frac{t}{T} - \left(\frac{r - z\sin\theta}{\lambda}\right)\right) dz$$

$$y = \frac{-k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T} - \left(\frac{r - z\sin\theta}{\lambda}\right)\right)\right]_{-a/2}^{a/2}$$

$$y = \frac{-k\lambda}{2\pi\sin\theta}\left[\cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{a\sin\theta}{2\lambda}\right) - \cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} - \frac{a\sin\theta}{2\lambda}\right)\right]$$

$$y = \frac{k\lambda}{2\pi\sin\theta}\left[2\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right) \cdot \sin 2\pi\left(\frac{a\sin\theta}{2\lambda}\right)\right]$$

let $\alpha = \frac{\pi a\sin\theta}{\lambda}$

$$y = \underbrace{\frac{ka\sin\alpha}{\alpha}}_{\text{Amplitude}} \sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$$

$$\therefore I = (A)^2 = \frac{k^2a^2\sin^2\alpha}{\alpha^2}$$

Page :
Date: / /

Case I : Central maxima / principal maxima

$\theta = 0 \Rightarrow \alpha = 0$

$\lim_{\alpha \to 0} \frac{\sin \alpha}{\alpha} = 1$

$I = k^2 a^2 = I_0$ → Intensity of central maxima

Case 2 : for Ist secondary maxima

$a \sin \theta = (2n+1)\frac{\lambda}{2}$

$\alpha = (2n+1)\frac{\pi}{2}$

$a/2$

$-a/2$

for Ist secondary maxima $\alpha = 3\pi/2$

$I_1 = I_0 \left(\frac{\sin 3\pi/2}{3\pi/2}\right)^2 = \frac{4I_0}{9\pi^2} = \frac{I_0}{22}$

for second secondary maxima $\alpha = 5\pi/2$

$I_2 = I_0 \left(\frac{\sin 5\pi/2}{5\pi/2}\right)^2 = \frac{4I_0}{25\pi^2}$

$= \frac{I_0}{61}$

Case III : minima

$a \sin \theta = n\lambda$

$\alpha = n\pi \Rightarrow \pi, 2\pi, - - - -$

Ques Ist secondary maximuma donot occur at $\pi/2$ because ~~first~~ Central maxima Occurs at 0 and first minima occur at π. so, Ist secondary maxima can't appear before first minima

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$y = \frac{n \lambda D}{d}$

Page: Date: / /

for Double slit or N-slit

$$y = \int_{-a/2}^{a/2} dy + \int_{d-a/2}^{d+a/2} dy$$

for N-slit

$$y = \int_{-a/2}^{a/2} dy + \int_{d-a/2}^{d+a/2} dy + \cdots \int_{(n-1)d-a/2}^{(n-1)d+a/2} dy$$

Amplitude of $y_{/r}$ $= k a \frac{\sin\alpha}{\alpha} \frac{\sin N\beta}{\sin\beta}$

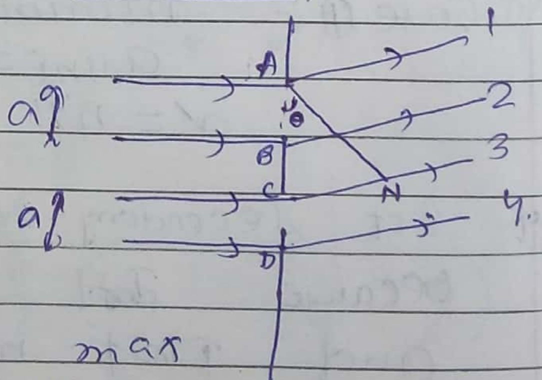
~~$A_r = k$~~ where $\beta = \frac{\pi d \sin\theta}{\lambda}$

$\alpha = \frac{\pi a \sin\theta}{\lambda}$

$$I_r = k^2 a^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\boxed{I_0 = k^2 a^2}$$

Double slit :-



$\sin\theta = \frac{CN}{a+b}$

for interference max

$(a+b) \sin\theta_n = n\lambda$ (max)

$(a+b) \sin\theta_n = (2n+1)\frac{\lambda}{2}$ (min)

Angular seperation b/w two consequetive maxima or minima

$$\sin\theta_2 - \sin\theta_1 = \frac{\lambda}{a+b}$$

for diffraction

$a\sin\theta = n\lambda$ (min)

Angular separation $= \frac{\lambda}{a}$

Condition for missing order,

If at some angle maxima of interference and minima of diffraction occurs together then,

$(a+b)\sin\theta = n\lambda$ $\qquad$ $a\sin\theta = p\lambda$

$$\boxed{\frac{a+b}{a} = \frac{n}{p}}$$

Case (I) : $a = b$

for $p = 1, 2, 3, \ldots$

$n = 2, 4, 6, \ldots$

Case (II) : $a+b = a \Rightarrow b = 0$

then $n = p$

i.e all interference maxima will coincide with minima of diffraction pattern so, it will act like a single slit

for n slit (grating condition) :

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

for minima :

$N\beta = 0, \pi, 2\pi, 3\pi, \ldots k\pi$

(except $k = 0, N\pi, 2N\pi, \ldots$)

at points $k = 0, N, 2N, 3N, \ldots nN$ there will be maxima.

Page :
Date: / /

Eq for n^{th} principle maxima.

$$N\beta = nN\pi$$

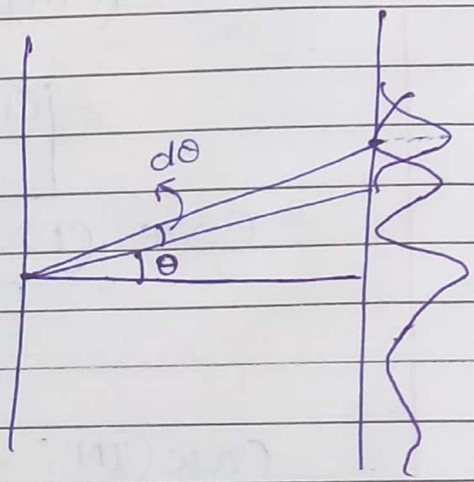
$$\beta = n\pi$$

$$\frac{\pi d \sin\theta}{\lambda} = n\pi$$

$$\boxed{d\sin\theta = n\lambda}$$

This is grating equation.

Resolving power :-



Eq^n of n^{th} principle max for λ at θ

$$N\beta = nN\pi$$

$$\frac{\pi N d \sin\theta}{\lambda} = nN\pi$$

$$d\sin\theta = n\lambda$$

Eq^n of first minima after n^{th} principal maxima,

$$N\beta = (nN+1)\pi$$

$$\frac{N\pi d \sin(\theta+d\theta)}{\lambda} = (nN+1)\pi$$

$$d\sin(\theta+d\theta) = \frac{\lambda}{N}(nN+1) \quad \text{--- (1)}$$

eq^n of principal maxima of wavelength $\lambda+d\lambda$ at $\theta+d\theta$

$$N\beta = nN\pi$$

$$\frac{N\pi d \sin(\theta+d\theta)}{\lambda+d\lambda} = nN\pi$$

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$$d\sin(\theta+d\theta) = n(\lambda+d\lambda) \quad \text{——} ②$$

equate ① and ②

$$\frac{\lambda}{N}(nN+1) = n(\lambda+d\lambda)$$

$$n\,d\lambda = \frac{\lambda}{N}$$

resolving power

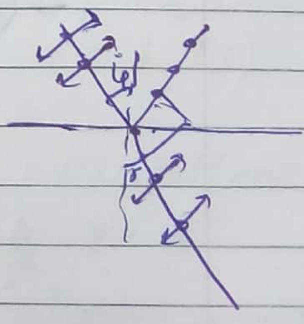
$$\frac{\lambda}{d\lambda} = nN$$

dispersive power :- $\frac{d\theta}{d\lambda} = \frac{nN}{a\sin\theta}$

Polarisation :-

Brewster law :- when a light incident at an angle of incidence called Brewster angle at which reflected and refracted ray are ⊥ to each other, the reflected light is plane polarised. This is brewster law

$$i_p + r = \pi/2$$



$$\frac{\sin i_p}{\sin r} = \mu$$

$$\frac{\sin i_p}{\cos i_p} = \mu$$

$$\tan i_p = \mu$$

$$i_p = \tan^{-1}(\mu)$$

Imp Huygen's Construction for Calcite & Quartz
Fresnel's explanation (+)

Page:
Date: / /

Malus law:- $I = I_0 \cos^2\theta$

$I_0 \to$ max Intensity of polarised light

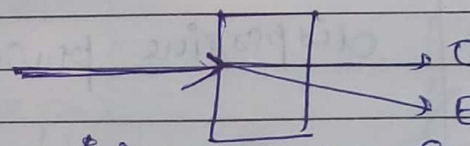
$\theta \to$ angle b/w polariser and analyser

* Double refraction / Birefrigerence

dirn of optic axis

E → Extra ordinary ray

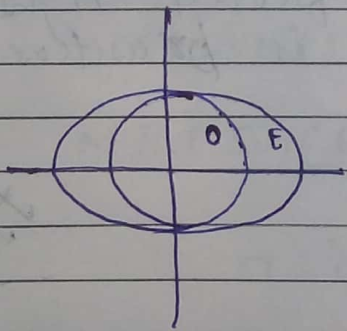
O → ordinary ray → follow laws of Ref & refrn

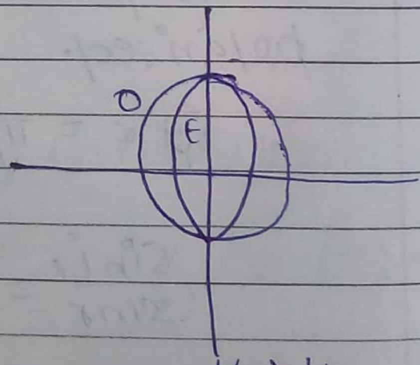


* Specific dirn in which we can't differentiate b/w O-ray and E-ray is called Optic axis.

along this optic axis no double refraction takes place.

Wave front of O-ray and E-ray





$V_E > V_O$

$\mu_E < \mu_O$

such crystals are called negative crystals
eg - calcite

$V_O > V_E$

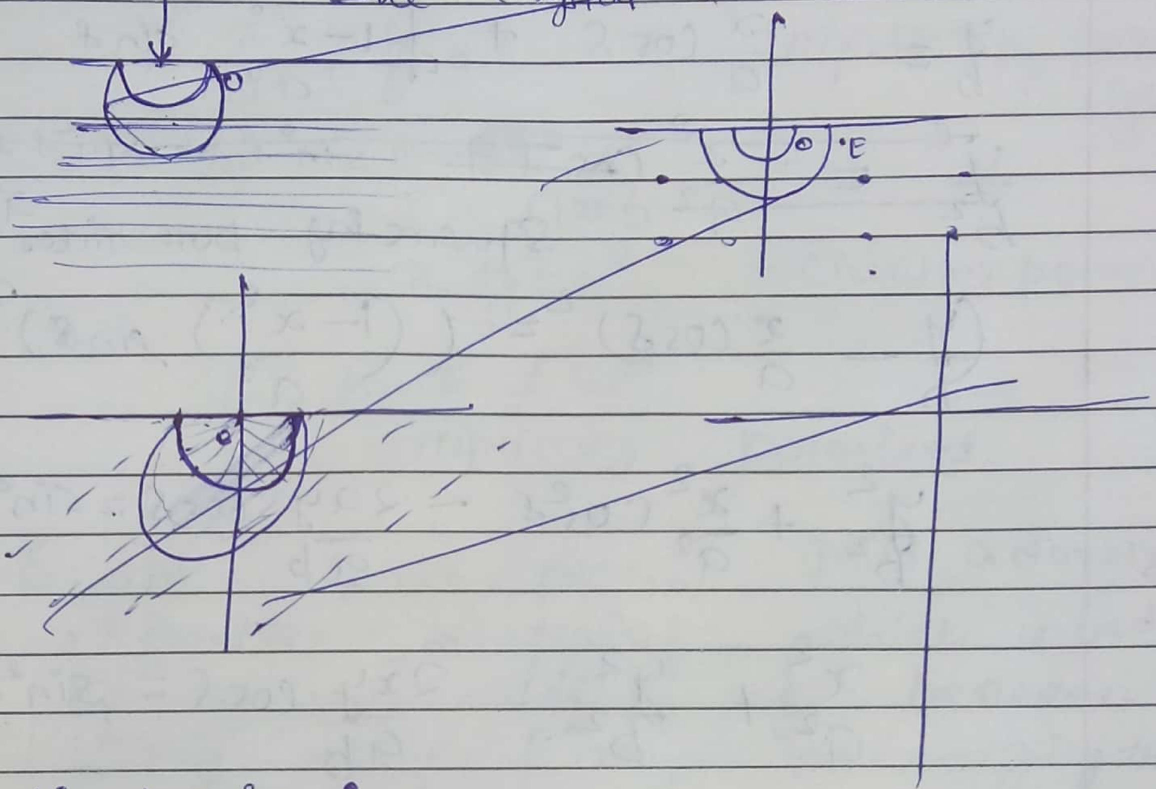
$\mu_O < \mu_E$

Such crystals are +ve crystals
eg. quartz

Page:
Date: / /

Huygen's Construction:-

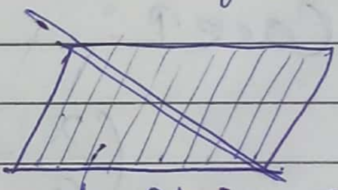
-ve Crystal



Nicol prism:-

this made up of negative crystal.

$\mu_E = 1.48$
$\mu_O = 1.65$



gum used to stick two part is Canada Balsum $\mu = 1.55$

production of circularly, elliptically, and plane polarised light.

$x = a \sin \omega t$
$y = b \sin(\omega t + \delta)$

P.T.O

Page :
Date: / /

$\frac{y}{b} = \sin\omega t \cos\delta + \cos\omega t \sin\delta$

$\frac{y}{b} = \frac{x}{a}\cos\delta + \sqrt{1-\frac{x^2}{a^2}}\,\sin\delta$

~~$\frac{y^2}{b^2} = \frac{x^2}{a^2}\cos^2\delta + \sin^2\delta - \frac{\sin^2\delta\, x^2}{a^2}$~~

~~squareing both sides~~

$\left(\frac{y}{b} - \frac{x}{a}\cos\delta\right)^2 = \left(\left(1-\frac{x^2}{a^2}\right)\sin\delta\right)^2$

$\frac{y^2}{b^2} + \frac{x^2}{a^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta - \frac{\sin^2\delta\, x^2}{a^2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$

this is a equation of ellipse.

Case I : $\delta = 0, 2\pi, \cdots 2n\pi$

$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \Rightarrow x = \frac{a}{b}y$

so, emerging ~~light~~ light will be PPL and angle will be $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

Case II : $\delta = \pi, 3\pi, \cdots (2n+1)\pi$

$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0 \Rightarrow x = -\frac{a}{b}y$

$\theta = \pi - \tan^{-1}\left(\frac{b}{a}\right)$

PPL is generated.

Case

Page :
Date: / /

Case III :- $\delta = \pi/2, 3\pi/2, \ldots (2n+1)\frac{\pi}{2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow$ ellipitically polarised light.

if $\tan\theta = 45°$

$a = b$

$x^2 + y^2 = a^2$ circulary polarised

if $\tan\theta \neq 45°$

ellipitically polarised

Quater wave plate :- It is a doubly refracting material which introduce a phase diff of $\frac{\lambda}{4}$ between O ray and E ray on emerging through the plate

$$t(\mu_0 - \mu_E) = \frac{\lambda}{4}$$

$$t = \frac{\lambda}{4(\mu_0 - \mu_E)}$$

half wave plate :- Doubly refracting material which introduce a phase diff of $\frac{\lambda}{2}$ between O ray and E ray on emerging through the plate

$$t(\mu_0 - \mu_E) = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_0 - \mu_E)}$$

○ or ○ → [Quater plate] plane polarised

↕ [Quater plate] no change

Unknown light pass through

↓

nicol prism

I varies from 0 to maximum 2 times on complete rotation (plane polarised)

I → no change ○ or ↕

→ [Quater wave plate] → [Nicol] →

- I → 0 to max 2 time on complete rotation circularly polarised
- Other

I changes but $I_{min} \neq 0$ elliptical ↕ + ↕

↓ [Quater wave plate] ↓ [Nicol prism] ↓

- I 0 to max 2 times elliptical
- other

Optical rotation :- rotation of plane of vibration

if PPL is passed through optically active sol^n

rotation (θ) $\propto$ concentration

$\propto$ length of tube or distance covered (l)

$$\theta = SCl$$

↳ specific rotation

$$S = \frac{\theta}{Cl}$$

if l is in cm $\quad S = \frac{10\,\theta}{Cl}$

Page :
Date: / /

Fresnel's explanation :-

$$\theta = \frac{\pi d}{\lambda}(\mu_L - \mu_R)$$ for right handed active crystal $(\mu_L > \mu_R)$

$$\text{or } \theta = \frac{\pi d}{T}\left(\frac{1}{v_L} - \frac{1}{v_R}\right)$$

also

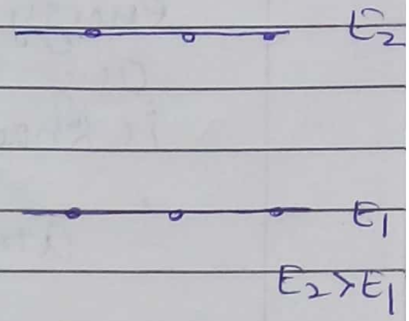
$$\theta = \frac{\pi d}{\lambda}(\mu_R - \mu_L) = \frac{\pi d}{T}\left(\frac{1}{v_R} - \frac{1}{v_L}\right)$$

for left handed crystal $(\mu_L < \mu_R)$

Specific rotation :- The rotation produced when plane polarised light is passed through a ~~column~~ solution of 1cm length and having concentration 1 g/cm³.

LASER

Light amplification by stimulated Emission of radiation

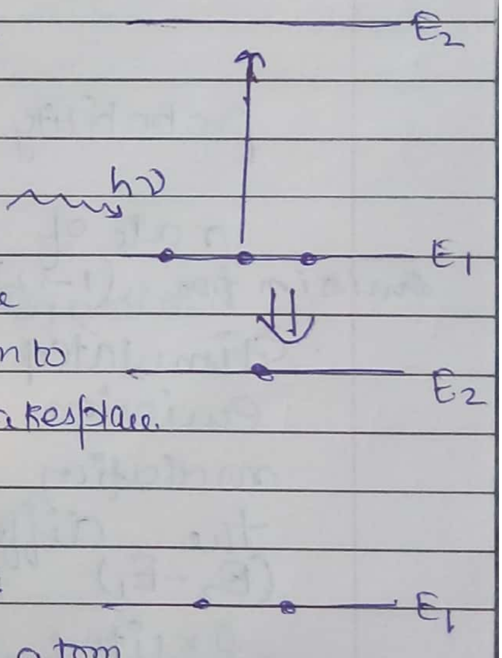


atoms/volume
no. of $\bar{e}$ in E_1 state
$$N_1 = e^{-E_1/KT}$$

atoms/volume
no. of $\bar{e}$ in E_2 state
$$N_2 = e^{-E_2/KT}$$
$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/KT}$$

Absorption:-

atom + photon → atom*



when a radiation of sufficient ν fall on atom., atom absorb the energy and transition to higher energy level takes place.

probability of excitation
$$= \rho B_{12}$$

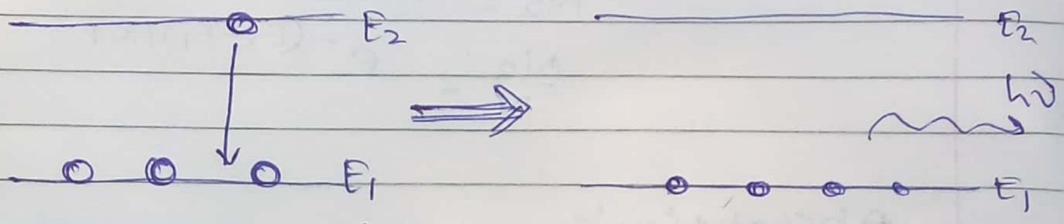
rate of absorption of atom
$$= \rho B_{12} \times N_1$$

ρ → density of radiation
B_{12} → einstein's absorption coefficient
N_1 → No. of atoms in E_1 state

Page:
Date: / /

spontaneous emission:- when an atom undergoes transition to a lower energy state emitting a photon, without any external stimulation, the process is known as spontaneous emission.

atom* ⟶ atom + photon



$(E_2 > E_1)$

probability of spontaneous emission = A_{21}

rate of emission = $A_{21} \times N_2$

emission from $(1 \to 2)$ is not possible so, $A_{12} = 0$

Stimulated emission:- In this type of emission, a photon from the incident radiation, having an energy equal to the difference between the two states $(E_2 - E_1)$ interacts with the atom in the excited state, inducing it to return to the lower energy state

atom* + photon → atom + photon + photon

probability of stimulated emission = $B_{21}\rho$

rate of emission = $B_{21} \times \rho \times N_2$

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Page :
Date: / /

Relation b/w einstein coefficients

At equillibrium.

$$R_{ab} = R_{sp} + R_{st}$$

$$B_{12} \rho N_1 = A_{21} N_2 + B_{21} \rho N_2$$

$$\rho = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Dividing numerator and denomenator by $B_{12} N_2$

$$\rho = \frac{\frac{A_{21}}{B_{12}}}{\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}}$$

from boltzman equation.

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT}$$

and. $E_2 - E_1 = h\nu$

$$\therefore \rho = \frac{A_{21} / B_{12}}{e^{h\nu/kT} - \frac{B_{21}}{B_{12}}} \quad \text{--- (1)}$$

by plank's law,

$$\rho = \frac{8\pi h \nu^3 \mu^3}{c^3 (e^{h\nu/kT} - 1)} \quad \text{--- (2)}$$

on comparing (1) and (2)

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h \nu^3 \mu^3}{c^3}$$

$$\frac{B_{21}}{B_{12}} = 1$$

these two equations are called einstein's equations.

population inversion:- It is a condition when atoms in higher energy state are more than atoms in lower energy state. this is necessary for lasing action.

$N_2 > N_1$ $(E_2 > E_1)$

Waves

$$y = a\sin(\omega t - kx)$$

$\omega = 2\pi\nu$ $k = \frac{2\pi}{\lambda}$

$$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$ progressive wave.

$$V = a\omega\cos(\omega t - kx)$$