General Theory of Relativity

Unit 1

Some preliminaries

- Newton's law of universal gravitation predicts much of what we see within our solar system.

$$\vec{F}_{12} = G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

- Newton predicted force of gravity and its nature, but not about its source of existence.
- Assumptions:
  - action at a distance – force felt at a distance instantaneously without any physical contact
  - Masses are assumed to be exactly the same as Newton's second law of motion. F=ma.
- Both these assumptions are not explained by Newton's theories. But Einstein's theory of general relativity addresses both of these issues

Principle of equivalence

- Mass of a body is defined in two different ways, which are fundamentally different from each other. These are in the following:

    1. The kinematical or inertial mass, m_i
    2. The gravitational mass, m_g

These two masses, although coming from two different concepts come out to be the same.

Principle of equivalence

Einstein argued that the equality of two entirely different concepts of mass, cannot be a mere coincidence, rather it is a pointer to something more basic, which may unify the concepts of inertia and gravitation. He showed successfully that by an extension of the principle of relativity from uniform translatory motion to non-uniform relative motion, the equality of inertial and gravitational masses follows naturally.

Einstein called the extension of the principle of relativity to non-uniform motion as the *Principle of Equivalence* and stated it thus:

'We assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system.'

That is, being at rest on the surface of earth is equivalent to being inside a space ship (far from any source of gravity) that is being accelerated by its engines.

Principle of equivalence

Within a reasonably sized laboratory on Earth, the gravitational field $\vec{g}$ is essentially uniform. The corollary states that any physical experiments performed there have the identical results as those done in a laboratory accelerating at $\vec{a} = \vec{g}$ in deep space, well away from all other masses.

How can these two apparently fundamentally different situations be the same? The answer is that gravitation is not a force between two objects but is the result of each object responding to the effect that the other has on the space-time surrounding it. A uniform gravitational field and a uniform acceleration have exactly the same effect on space-time.

General theory of relativity

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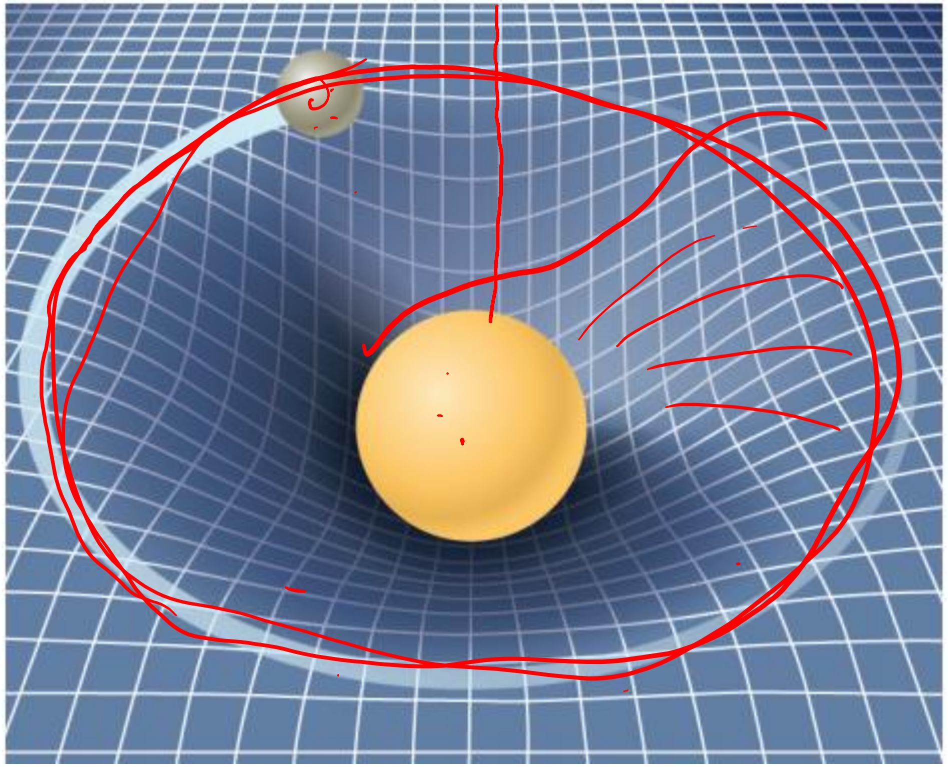
3D

- To explain the origin of gravity (or basically what gravity is).
- The origin is space-time curvature.
- A massive object warps space-time.
- All masses distort the space-time.
- And thus giving rise to force of attraction
- Only empty space is flat.

4 dim



General theory of relativity

- GTR unifies special theory of relativity with Newton's law of universal gravitation.
- According to GTR, the gravitational acceleration is caused by the curvature of space-time, which is produced by the mass-energy and momentum of the matter in space-time. ~~This is mathematically~~ ~~shown in~~ Einstein's field equation.

Fundamental hypotheses and postulates of general relativity

General relativity is based on the following set of fundamental principles, which guided its development. These are in the following:

1. The general principle of relativity—The laws of physics must be the same for all observers (accelerated or not).
2. Physical events are described in a four-dimensional space–time manifold, the metric of which is

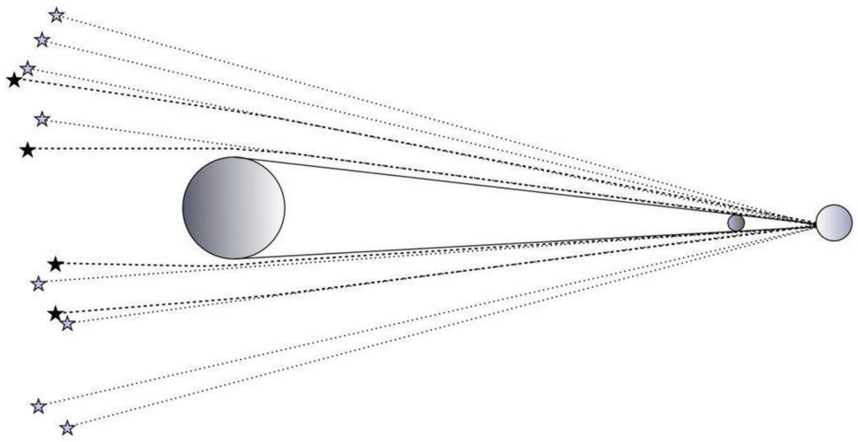
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 t^2 - x^2 - y^2 - z^2$$

3. The inertial motion is geodesic motion.

4. Space-time is curved, giving rise to gravitational force of attraction.

Gravity and Light

- If it is massless then gravity should not work on light. But, light has been seen to bend towards massive objects. Ex. Black holes, around the sun, etc.
- Why does light bend? Because it has mass due to its energy and thus gets affected by the space-time curvature.



During a total eclipse, stars would appear to be in a different position than their actual locations, due to the bending of light from an intervening mass: the Sun.

Gravity and Light

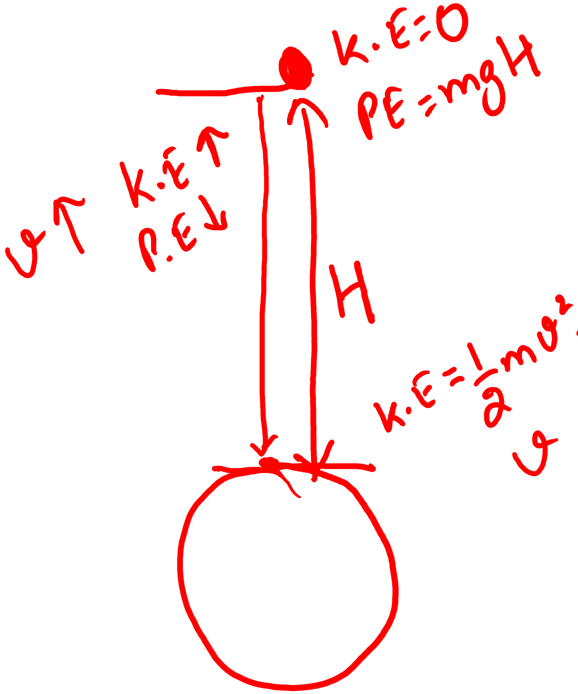
$E = mc^2 = h\nu$

$$\boxed{m = \frac{h\nu}{c^2}}$$

– When an object falls through a dist H, its velocity increases.

But light travels with vel. 'c', so it can't go any faster.

So, the gain in energy is translated into increase in its frequency.



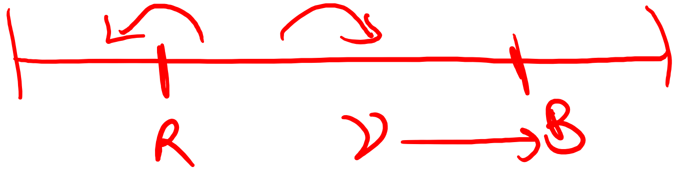
$K.E = \frac{1}{2}mv^2 = mgH$

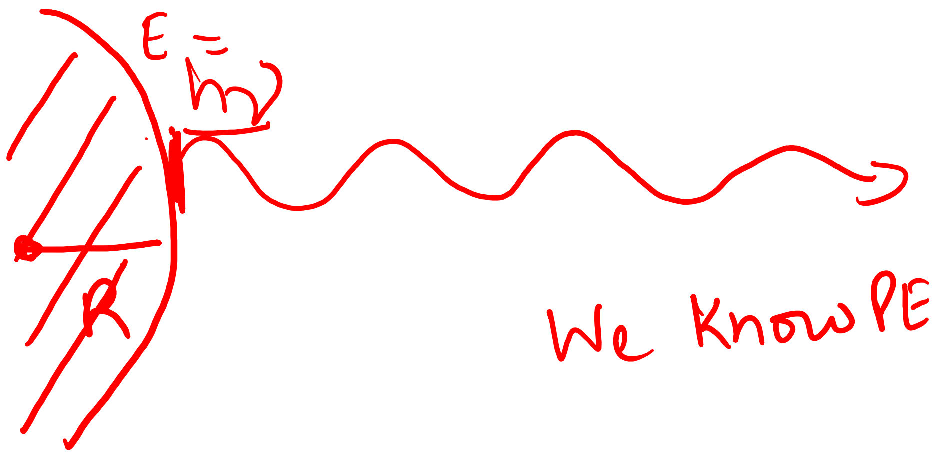
$v = \sqrt{2gH}$

$$E' = h\nu' = h\nu + mgH = h\nu + \frac{h\nu}{c^2}gH$$

$$h\nu' = h\nu\left[1 + \frac{gH}{c^2}\right] \longrightarrow \boxed{\nu' = \nu\left[1 + \frac{gH}{c^2}\right]}$$

Gravitational Red Shift (GRS)





M, radius = R

We know $PE = -\frac{GMm}{R}$

$$PE = -\frac{GM}{R}\cdot\frac{h\nu}{c^2}$$

$$E' = h\nu - \frac{GM}{R}\,\frac{h\nu}{c^2}$$

$$h\nu' = h\nu\left[1 - \frac{GM}{Rc^2}\right]$$

$$\frac{\nu'}{\nu} = 1 - \frac{GM}{Rc^2}$$

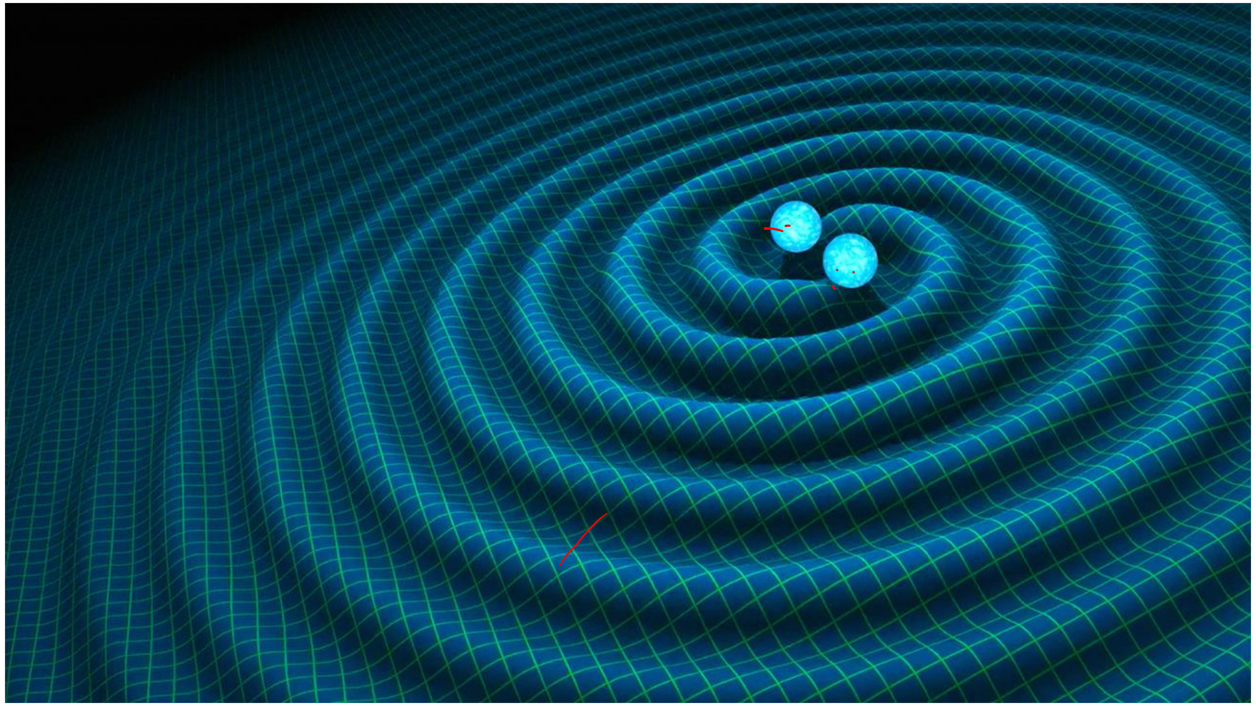
We define GRS as :-

$$\frac{\Delta\nu}{\nu} = \frac{\nu - \nu'}{\nu} = 1 - \frac{\nu'}{\nu}$$

compare

$$\boxed{\frac{\Delta\nu}{\nu} = \frac{GM}{c^2R}}$$

Gravitational Waves



- Extremely weak waves
- Are characterized by frequency/wavelength
- Travel with the velocity of light, $c = \lambda \cdot \nu$
- Results from vibrating/moving massive bodies, which constantly deform curvature of space-time.

Extra slide – only for curiosity

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2
\end{aligned}
$$

where $x^0 = ct,\ x^1 = x,\ x^2 = y,\ x^3 = z$ and $g_{\mu\nu}$ is the metric tensor with constant components

$$
g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
$$