Division of Amplitude

1.11 CHANGE OF PHASE BY REFLECTION AND DIVISION OF AMPLITUDE : STOKE'S LAW

To investigate the phase change in the reflection of light at an interface between two media. Sir G.C. Stoke used the principle of optical reversibility. *The principle states that a light ray, that is reflected or refracted, will retrace its original path, if its direction is reversed, provided there is no absorption of light.*

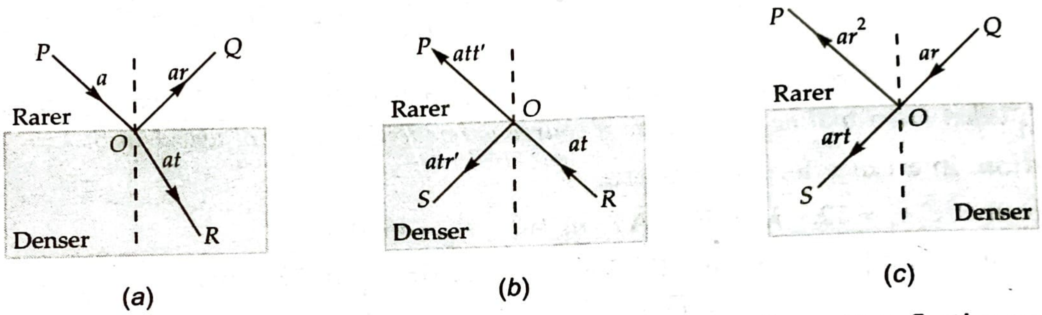

Fig. 1.21 (*a*) Reflection and refraction of light wave from rarer to denser medium ; (*b*) Reflection and refraction of light wave with amplitude fo refracted wave of (*a*) and with incidence angle equal to angle of refraction of (*a*) from denser to rarer medium ; (*c*) Reflection and refraction of light wave with amplitude of reflected wave of (*a*) and with incidence angle equal to angle fo reflection of (*a*) from rarer to denser medium.

Consider a light wave *PO* with amplitude *a* falls on the interface of a denser medium from a rarer medium as shown in Fig. 1.21. Now we can define coefficient of reflection by *r* as

$$r = \frac{\text{Amplitude of reflected wave}}{\text{Amplitude of incident wave}} \quad \text{...(1.45)}$$

and coefficient of refraction by *t* as

$$t = \frac{\text{Ampliude of refracted wave}}{\text{Amplitude of incident wave}} \quad \text{...(1.46)}$$

Therefore, the amplitude of the reflected wave *OQ* is *ar* and that of refracted wave *OR* is *at*.

Now consider the situation when the directions of reflected and refracted waves are reversed. To do so, first it is considered that a light wave of amplitude *at* is allowed to fall an interface from denser to rarer medium along *RO*. Then one has a reflected ray along *OS* with amplitude atr' and a refracted wave with amplitude att' along *OP*, where r' and t' are the coefficient of reflection and coefficient of refraction from denser to rarer medium respectively. Thereafter, it is allowed to fall the light wave of amplitude of *ar* on the interface from rarer to denser medium along *QO*. Now, there is a reflected along *OP* with amplitude ar^2 and a refracted wave with amplitude *art* along *OS*.

Now superposition of these two cases of propagation of light waves gives a light wave with amplitude $(ar^2 + att')$ along *OP* and another one with amplitude $(art + atr')$ along *OS*. The reversal of reflected (with amplitude *ar*) and refracted (with amplitude *at*) light wave must produce a light wave with amplitude along *OP*, and no wave along *OS* because when we have considered propagation of light wave from rarer to denser medium along *PO*, there is no wave along *OS*.

Therefore, $$ar^2 + att' = a \qquad ...(1.47)$$

and $$art + ar't = 0 \qquad ...(1.48)$$

From Eq. (1.47), we have $$tt' = 1 - r^2 \qquad ...(1.49)$$

and from Eq. (1.48), we get $$r' = -r \qquad ...(1.50)$$

The negative sign in Eq. (1.50) indicates a displacement in opposite direction that is equivalent to a phase change of π or a path difference $\frac{\lambda}{2}$. Therefore a phase change of π is associated with reflection occurring at the interface when light propagates from rarer to denser medium. This is known as *Stoke's law of reflection*.

STATEMENT

Stoke's law states that if waves are reflected at a rarer to denser medium interface (for example, air-glass interface), the reflected waves have a phase difference π (or path difference $\frac{\lambda}{2}$) compared to the incident wave.

This also occurs in elastic waves such as sound waves.