ASSIGNMENT-1 AMOGH GARG 2020UCO1688

1) $(D^2+2D+2)y = \sinh x = \frac{e^x - e^{-x}}{2}$

$\Rightarrow$ C.F: $e^{-x}(c_1\cos x + c_2\sin x)$; P.I $= \frac{1}{2(D^2+2D+2)} \times e^x - \frac{1}{2} \times \frac{1}{(D^2+2D+2)} \times e^{-x}$

$= \frac{e^x}{10} - \frac{e^{-x}}{2}$

$\therefore$ CS $= y = e^{-x}(c_1\cos x + c_2\sin x) + \frac{e^x}{10} - \frac{e^{-x}}{2}$

2) $(D^2 - D - 6)y = \frac{e^x(e^{2x}+e^{-2x})}{2} = \frac{1}{2}(e^{3x}+e^{-x})$

$\Rightarrow$ CF : $c_1e^{3x} + c_2e^{-2x}$; P.I $= \frac{1}{2(D^2-D-6)} \times e^{3x} + \frac{e^{-x}}{2(D^2-D-6)} = \frac{xe^{3x}}{10} - \frac{e^{-x}}{8}$

$\therefore$ CS: $c_1e^{3x} + c_2e^{-2x} + \frac{xe^{3x}}{10} - \frac{e^{-x}}{8}$

3) $(D-1)^3y = 16e^{3x}$ $\Rightarrow$ C.F: $(c_1x^2+c_2x+c_3)e^x$

P.I : $\frac{16}{(D-1)^3} \times e^{3x} = 2e^{3x}$; $\therefore$ CS $= y = (c_2x^2+c_2x+c_3)e^x + 2e^{3x}$

4) $(2D^2+3D+4)y = x^2 - 2x$ $\Rightarrow$ C.F : $e^{-\frac{3x}{4}}\left[c_1\cos\frac{\sqrt{23}x}{4} + c_2\sin\frac{\sqrt{23}x}{4}\right]$

P.I : $\frac{1}{(2D^2+3D+4)} \times x^2 - \frac{2}{(2D^2+3D+4)} \times x$ $\Rightarrow \frac{1}{4}\left[1+\left(\frac{2D^2+3D}{4}\right)\right]^{-1} - \frac{2}{4}\left[1+\left(\frac{2D^2+3D}{4}\right)\right]^{-1}x$

$\Rightarrow \frac{1}{4}\left[1 - \frac{2D^2+3D}{4} + \frac{2}{2}\left(\frac{2D^2+3D}{4}\right)^2\right]x^2 - \frac{1}{2}\left[1 - \frac{2D^2+3D}{4}\right]x$

$\Rightarrow$ On solving : $\frac{1}{32}[8x^2 - 28x + 13]$

$\therefore$ CS : $y = e^{-\frac{3x}{4}}\left[c_1\cos\frac{\sqrt{23}x}{4} + c_2\sin\frac{\sqrt{23}x}{4}\right] + \frac{1}{32}[8x^2 - 28x + 13]$

5) $(D^4+4)y = x^4$ $\Rightarrow$ C.F : $e^x(c_1\cos x + c_2\sin x) + e^{-x}(c_3\cos x + c_4\sin x)$

$\hookrightarrow 1\pm i, -1\pm i.$

P.I : $\frac{1}{(D^4+4)} \times x^4 = \frac{1}{4}\left(1+\frac{D^4}{4}\right)^{-1}x^4 = \frac{1}{4}(x^4 - 6)$

C.S : $e^x(c_1\cos x + c_2\sin x) + e^{-x}(c_3\cos x + c_4\sin x) + \frac{1}{4}(x^4 - 6)$

6) $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$ $\Rightarrow$ CF : $c_1e^x + e^x[c_2\cos x + c_3\sin x]$

P.I : $\frac{1}{(D^3-3D^2+4D)} \times e^x + \frac{\cos x}{(D^3-3D^2+4D-2)}$ $\Rightarrow xe^x + \frac{\cos x}{(-D+3+4D-2)}$

$\Rightarrow xe^x + \frac{(3D-1)\cos x}{(-10)}$

$\Rightarrow xe^x + \frac{1}{10}(3\sin x + \cos x)$

7) $(D^2+4) = x^2 + \sin 2x \Rightarrow CF: c_1\cos 2x + c_2 \sin 2x$

$P.I: \frac{1}{4}\left[1+\frac{D^2}{4}\right]^{-1}x^2 + \frac{\sin 2x}{(D^2+4)} = \frac{1}{4}\left[x^2 - \frac{1}{2} - x\cos 2x\right]$

$C.S = y = c_1\cos 2x + c_2\sin 2x + \frac{1}{4}\left[x^2 - \frac{1}{2} - x\cos 2x\right]$

8) $(D^2+1)y = \sin 3x \cos 2x = \frac{1}{2}\left[\sin 5x + \sin x\right]$

$\Rightarrow CF = c_1\cos x + c_2 \sin x$

$P.I: \frac{1}{(D^2+1)} \times \frac{1}{2}\sin 5x + \frac{\sin x}{2(D^2+1)} \Rightarrow \frac{1}{48}\left[-\sin 5x - 12x\cos x\right]$

$C.S: y = c_1\cos x + c_2\sin x + \frac{1}{48}\left[-\sin 5x - 12x\cos x\right]$

9) $(D^2-4D+3)y = 2xe^{3x} + 3e^x\cos 2x$

$P.I: 2\times \frac{e^x}{(D^2-4D+3)} \times x + 3e^x \times \frac{\cos 2x}{(D^2-4D+3)} = \frac{2e^x \times x}{(D^2-2D+4)} + \frac{3e^x \times \cos 2x}{(D^2-2D+4)}$

$\Rightarrow -\frac{3}{8}e^x(\cos 2x + \sin 2x)$ and $CF = c_1e^x + c_2e^{3x}$

$+ e^{3x}\left(\frac{x^2}{2} - \frac{x}{2}\right)$

10) $(D^2-2D+2)y = xe^{3x} + \sin 2x \Rightarrow C.F: c_1e^x + c_2e^{2x} \rightarrow ①$

$P.I: \frac{1}{D^2-3D+2} \times \sin 2x + e^{3x} \times \frac{x}{[(D+3)^2-3(D+3)+2]} \Rightarrow -\frac{\sin 2x}{(3D+2)} + e^{3x} \times \frac{x}{D^2+2+3D}$

$\Rightarrow \frac{-(3D-2)\sin 2x}{9D^2-4} + e^{3x} \times \frac{1}{2}\left[1+\left(\frac{D^2+3D}{2}\right)\right]^{-1} x$

$\Rightarrow \frac{e^{3x}}{4}(2x-3) + \frac{3}{20}\cos(2x) - \frac{1}{20}\sin 2x \rightarrow ②$ $\therefore CS = ① + ②$

11) $(D^2-4D+4)y = 8x^2e^{2x}\sin 2x \Rightarrow C.F: (c_1+c_2x)e^{2x}$

$P.I: \frac{8}{(D^2-4D+4)} \times x^2e^{2x}\sin 2x = 8e^{2x} \times \frac{x^2\sin 2x}{(D+2)^2-4(D+2)+4} = 8e^{2x} \times \frac{1}{D^2} \times x^2\sin 2x$

$\Rightarrow -e^{-2x}\left[4x\cos 2x + (2x^2-3)\sin 2x\right]$

12.) $(D^2-4)y = x\sinh x = \frac{x(e^x - e^{-x})}{2}$ ⇒ CF: $c_1e^{2x}+c_2e^{-2x}$

⇒ P.I: $\frac{e^x}{2}\cdot\frac{x}{(D^2-4)} - \frac{e^{-x}}{2}\times\frac{x}{(D^2-4)}$ ⇒ $\frac{e^x}{2}\times\frac{x}{(D+1)^2-4} - e^{-x}\times\frac{x}{(D-1)^2-4}$

⇒ On solving: CS: $c_1e^{2x}+c_2e^{-2x} - \frac{x}{3}\sinh x - \frac{2}{9}\cosh x$

13) $(D^2+2D+1)y = \frac{e^{-x}}{x^2}$ ⇒ CF: $(c_1+c_2x)e^x$

⇒ P.I: $\left.e^{-x}\times\frac{x^{-2}}{(D^2+2D+1)}\right\}$?

14) $(D^2+2D+2)y = e^{-x}\sec^3 x$ ⇒ CF: $c_1e^{-x}\cos x + c_2e^{-x}\sin x$

P.I: $e^{-x}\times\frac{1}{(D-1)^2+2(D-1)+2}\times\sec^3 x = e^{-x}\times\frac{1}{(D^2+1)}\times\sec^3 x$

⇒ $e^{-x}\times\frac{1}{2i}\left[\frac{1}{(D-i)}-\frac{1}{(D+i)}\right]\sec^3 x$ } using type-6

⇒ $e^{-x}\times\frac{\sin x\cdot\tan x}{2}$ ⇒ CS: $e^{-x}\left(c_1\cos x + c_2\sin x + \frac{\sin x}{\tan x/2}\right)$

15) $(D^2+1)y = x - \cot x$ ⇒ CF: $c_1\cos x + c_2\sin x$

⇒ $\frac{1\times x}{(D^2+1)} - \frac{\cot x}{(D^2+1)}$ ⇒ $(1-D^2)x - \cancel{e^{ix}\int e^{ix}\cot x\,dx}$

⇒ $\cancel{-x\cos^2 x + \sin x\log(\operatorname{cosec} x + \cot x)}$

$\frac{1}{(D-i)}\cot x = e^{ix}\int e^{-ix}\cot x\cdot dx$; $\frac{1}{(D+i)}\cot x = e^{-ix}\int e^{ix}\cot x\cdot dx$

→ Ans) $c_1\cos x + c_2\sin x - x\cos^2 x - \sin x\log(\operatorname{cosec} x - \cot x)$

13.) $(D^2 + 2D + 1)y = \frac{e^{-x}}{x^2}$ $\Rightarrow$ C.F: $(c_1 + c_2 x)e^{-x}$

P.I: $\frac{1}{(D+1)^2} \times \frac{e^{-x}}{x^2} \Rightarrow e^{-x} \times \frac{1}{D^2} \times \frac{1}{x^2} \Rightarrow e^{-x} \frac{1}{D} \int \frac{1}{x^2} \cdot dx$

$\Rightarrow -e^{-x} \int \frac{1}{x} \cdot dx \Rightarrow -e^{-x} \log x$

$\therefore$ CS $= (c_1 + c_2 x)e^{-x} - e^{-x} \log x$

15.) $(D^2+1)y = x - \cot x \Rightarrow$ C.F : $c_1 \cos x + c_2 \sin x$

P.I : $y = \frac{1}{(D^2+1)} \times x - \frac{1}{(D^2+1)} \times \cot x = (1-D^2)x - \frac{1}{(D^2+1)} \times \cot x$

$\Rightarrow \quad x - \frac{1}{(D^2+1)} \times \cot x \Rightarrow \cancel{\frac{1}{(D^2+1)} \cot x} / \neq$

$\frac{1}{(D-ia)} \times \cot x = e^{ix} \int e^{-ix} \cdot \cot x \cdot dx$; $\frac{1}{(D+ia)} \times \cot x = e^{-ix} \int e^{ix} \cdot \cot x \cdot dx$

$\Rightarrow$ P.I $= x - \sin x \ln(\cos 2x - \cot x)$

$\therefore$ CS $= c_1 \cos x + c_2 \sin x + x - \sin x \ln(\cos 2x - \cot x)$