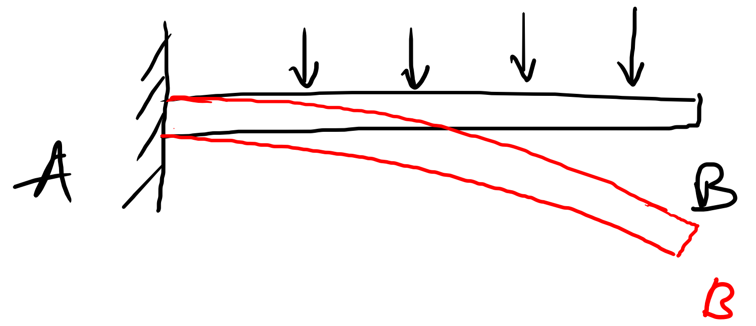
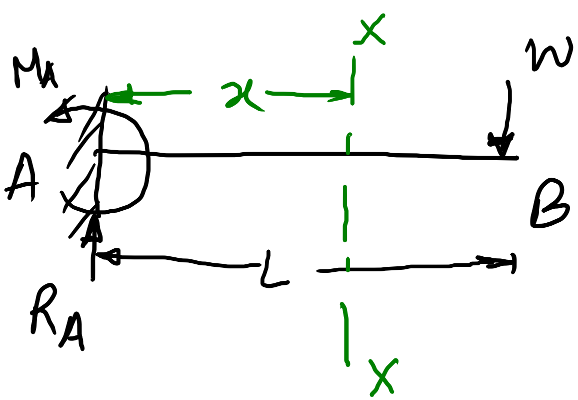
Shear Force and Bending Moment Diagrams

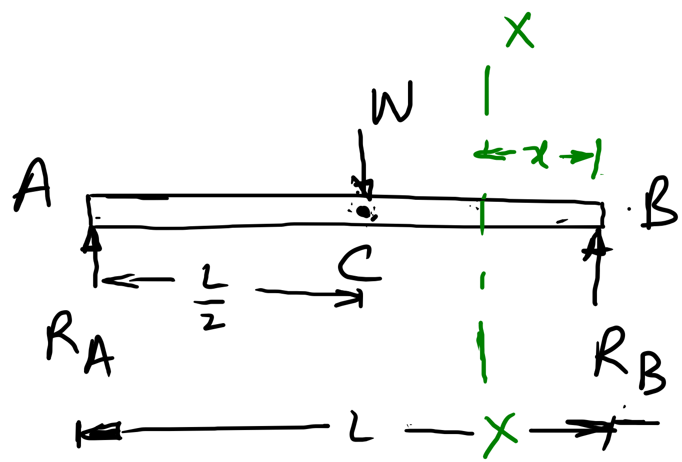
- Beam
- Bending



- Sign conventions

(1.) Shear force



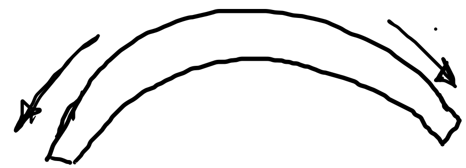


$\begin{bmatrix} \text{Downward Right} \\ \text{upward Left} \end{bmatrix}$ (+)ive

$\begin{bmatrix} \text{upward Right} \\ \text{Downward Left} \end{bmatrix}$ (−)ive

(2.) Bending Moment

(i) Sagging B.M.
(positive B.M.)



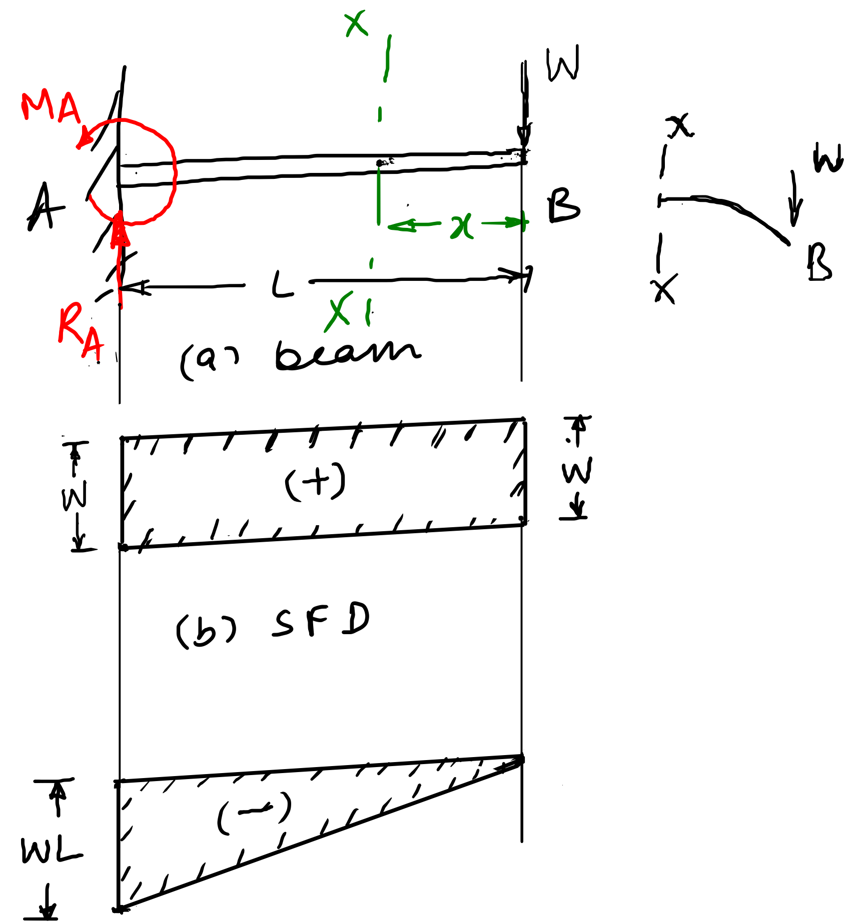
(ii) Hogging B.M.
(Negative B.M.)

① Cantilever beam carrying various types of loads

(1.) Cantilever with a point load

$R_A = W$

$M_A = WL$



(a) beam

(b) SFD

(c) BMD

S.F.

$S_x = +W.$

$S_B = +W$ at $x=0$

$S_A = +W$ at $x=L$

B.M.

$M_x = -W.x$

$M_B = 0$ at $x=0$

$M_A = -W.L$ at $x=L$

(2.) cantilever carrying a U.D.L.

S.F.

$$S_x = +W.x \quad \text{--- (1)}$$

$S_B = 0$, at $x = 0$

$S_A = +WL$ at $x = L$

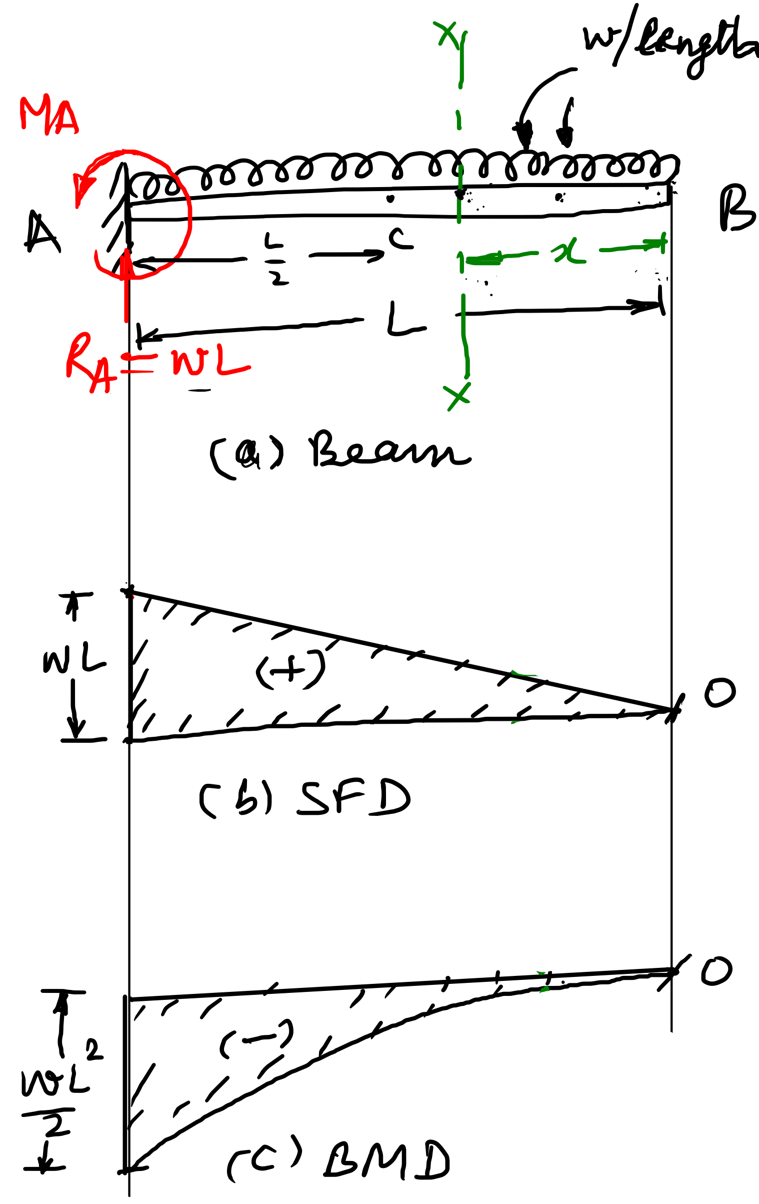
B.M.

$$M_x = -wx \cdot \frac{x}{2} = -\frac{wx^2}{2} \quad \text{--- (2)}$$

$M_B = 0$, at $x = 0$

$M_A = -\frac{wL^2}{2}$, at $x = L$

$M_c = -\frac{W}{2}\left(\frac{L}{2}\right)^2 = -\frac{WL^2}{8}$, at $x = \frac{L}{2}$



(3.) Cantilever with UDL and point load

S.F.

$$S_x = +W + wx \qquad \text{— (1)}$$

$S_B = +W$, at $x = 0$

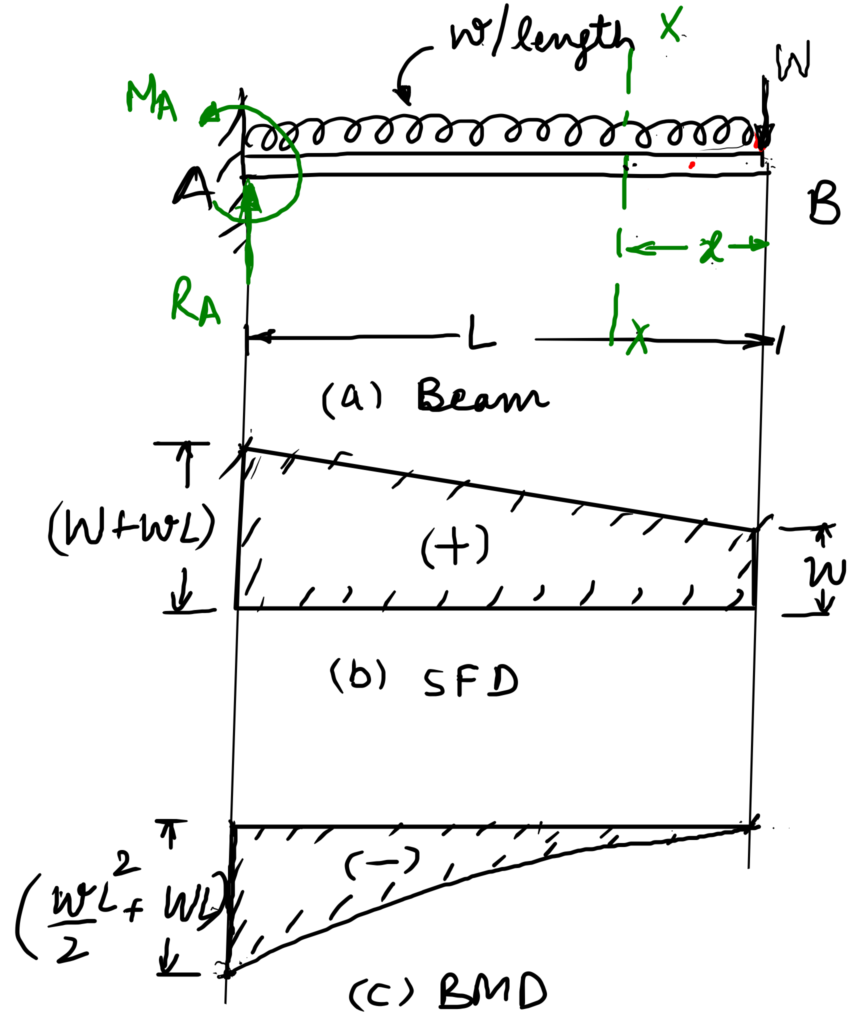
$S_A = +W + wL$, at $x = L$

B.M.

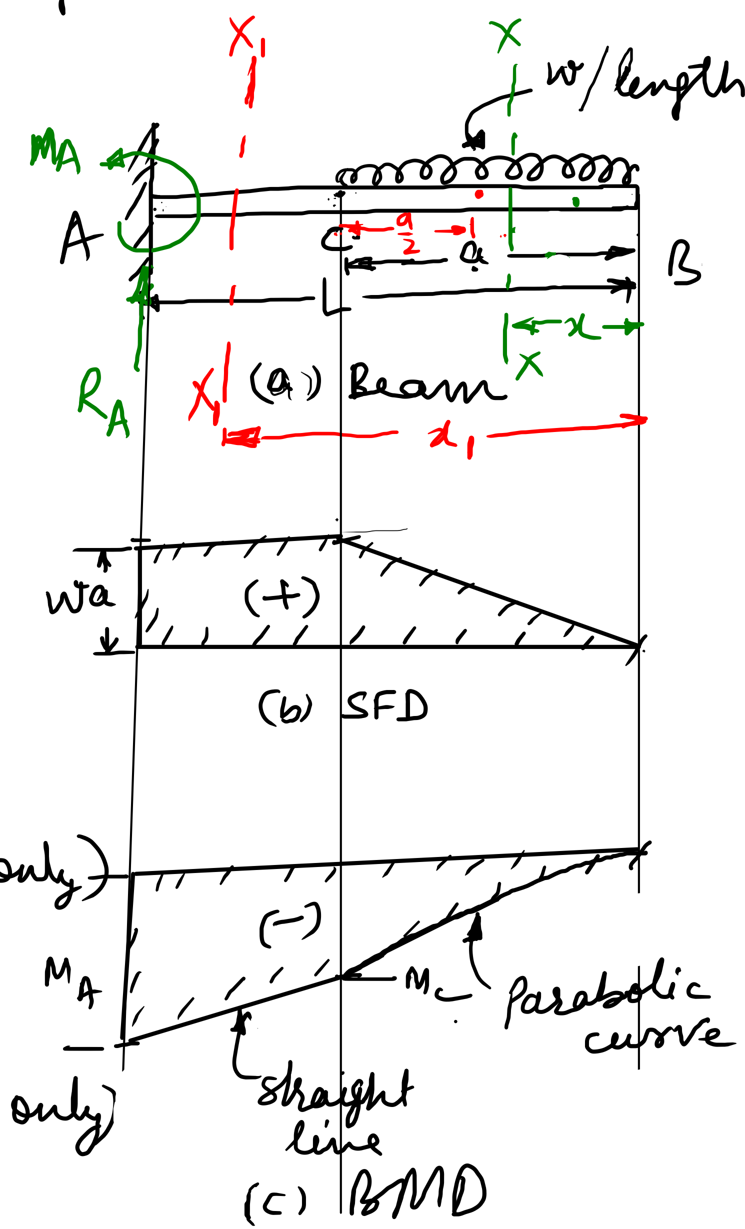
$$M_x = -W.x - w.x.\frac{x}{2} \qquad \text{— (2)}$$

$M_B = 0$, at $x = 0$

$M_A = -WL - \frac{wL^2}{2}$, at $x = L$



(4.) Cantilever carrying UDL partially



S.F.

$S_x = +wx$ — ① (valid for CB only)

$S_B = 0$, at $x = 0$

$S_C = +wa$, at $x = a$

$S_{x_1} = +wa$ (valid for CA only) — ②

$S_C = +wa$

$S_A = +wa$

B.M.

$M_x = -wx \cdot \frac{x}{2} = \frac{-wx^2}{2}$ — ③ (valid for CB only)

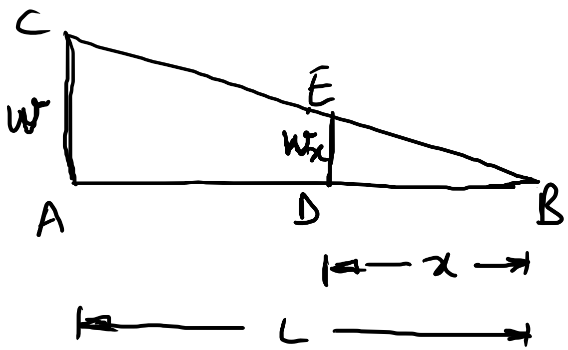
$M_B = 0$, at $x = 0$

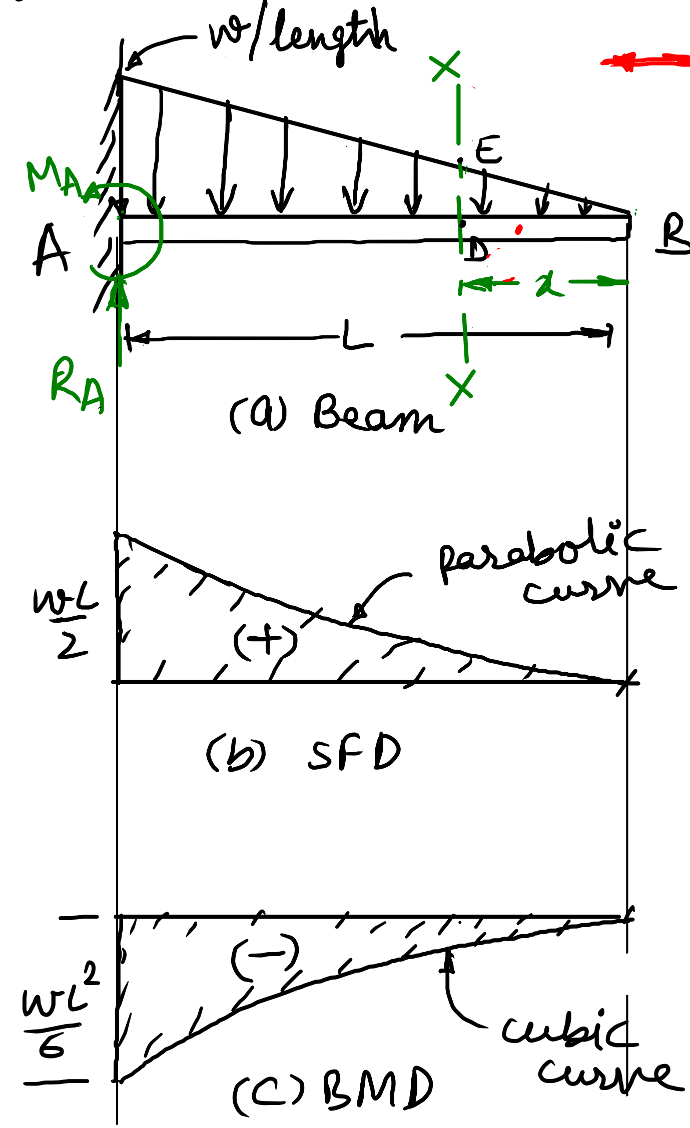
$M_C = -\frac{wa^2}{2}$, at $x = a$

$M_{x_1} = -wa\left(x_1 - \frac{a}{2}\right)$ — ④ (valid for AC only)

$M_A = -wa\left(L - \frac{a}{2}\right)$, at $x_1 = L$

(5.) cantilever carrying U.V.L (zero at free end)





From similar triangles ΔBDE and ΔBAC

$$\frac{BD}{BA} = \frac{DE}{AC} \Rightarrow \frac{x}{L} = \frac{w_x}{w}$$

$$\Rightarrow w_x = \frac{w.x}{L} \quad \text{—(1)}$$

S.F.

$$S_x = +\frac{1}{2}x.w_x = +\frac{1}{2}x.\frac{w.x}{L} = \frac{1}{2}\frac{wx^2}{L} \quad \text{—(2)}$$

$S_B = 0$, at $x = 0$

$S_A = +\frac{wL}{2}$, at $x = L$

B.M.

$$M_x = -\left(\frac{1}{2}x.w_x\right).\frac{x}{3} = -\frac{wx^3}{6L} \quad \text{—(3)}$$

$M_B = 0$, $M_A = -\frac{wL^2}{6}$

(6.) Cantilever carrying UVL (zero at fixed end)

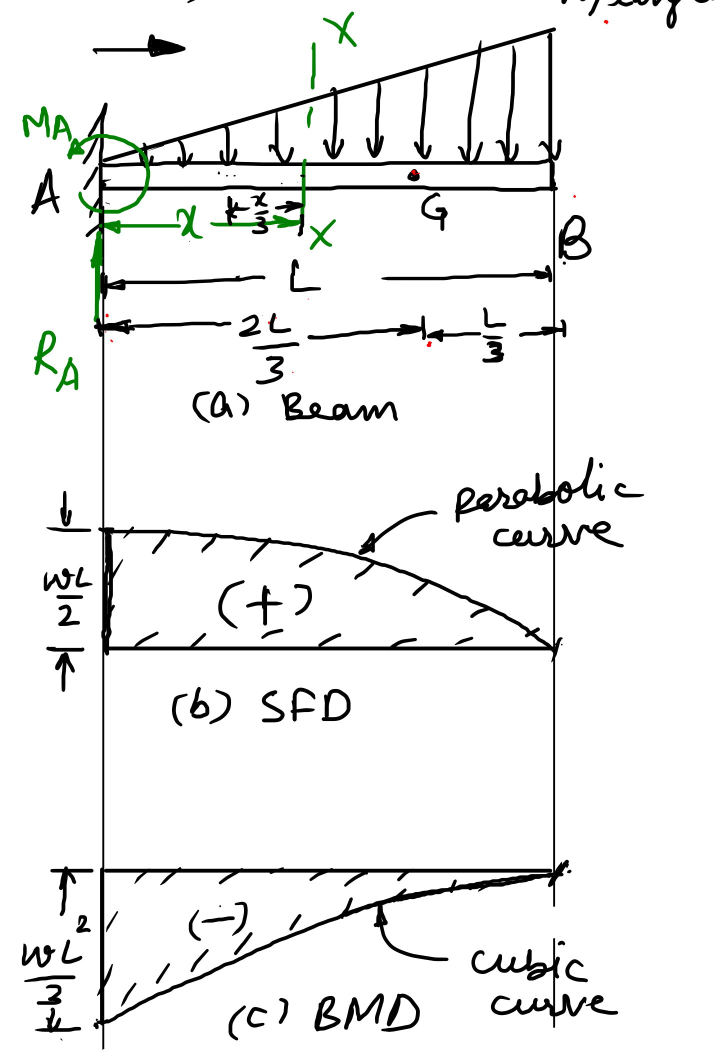
Total load on the beam $= \frac{1}{2} L . w = \frac{wL}{2}$

$$R_A = \frac{wL}{2}$$

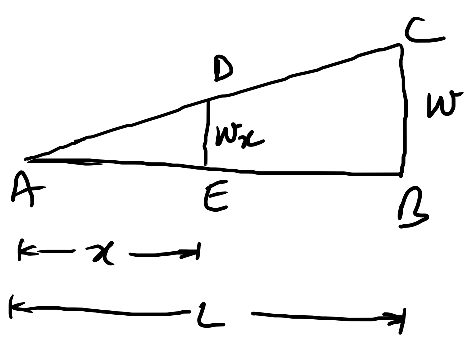
Taking moment about 'A'

$$= \frac{wL}{2} \cdot \frac{2L}{3} = \frac{wL^2}{3} \ (CW)$$

$$M_A = \frac{wL^2}{3} \ (ACW)$$



S.F.



$$S_x = + R_A - \frac{1}{2} x . w_x$$

$$S_x = \frac{wL}{2} - \frac{wx^2}{2L} \quad \text{— (1)}$$

$$\frac{w_x}{w} = \frac{x}{L}$$

$$w_x = \frac{w.x}{L}$$

$S_A = \frac{wL}{2}$, $\quad x = 0$

$S_B = 0$, $\quad x = L$

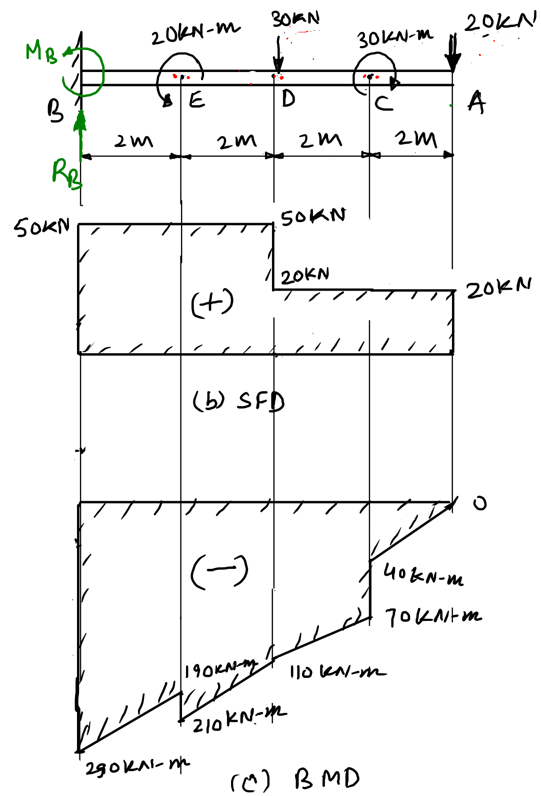
B.M.

$$M_x = - M_A - \left(\frac{1}{2} x . w_x\right) . \frac{x}{3} + R_A \cdot x = \frac{-wL^2}{3} - \frac{wx^3}{6L} + \frac{wLx}{2} \quad \text{— (2)}$$

$M_A = \frac{-wL^2}{3}$ $\quad x = 0$, $\quad M_B = 0$ $\quad x = L$

Q:1 Draw SFD and BMD for the given cantilever beam

Solⁿ: $R_B = 50\,kN$



Taking moment about 'B'

$= -20\times8 - 30\times4 - 30 + 20$

$= -290\ kN\text{-}m$

$\therefore M_B = +290\ kN\text{-}m\ (ACW)$

S.F.

$S_A = +20\,kN$

$S_C = +20\,kN$

$S_D = +20+30 = +50\,kN$

$S_E = +50\,kN$

$S_B = +50\,kN$

B.M.

$M_A = 0$

B.M. at just RHS of $C = -20\times2 = -40\ kN\text{-}m$

B.M. at just LHS of $C = -20\times2 - 30 = -70\ kN\text{-}m$

$M_D = -20\times4 - 30 = -110\ kN\text{-}m$

B.M. at just RHS of $E = -20\times6 - 30 - 30\times2 = -210\ kN\text{-}m$

B.M. at just LHS of $E = -20\times6 - 30 - 30\times2 + 20 = -190\ kN\text{-}m$

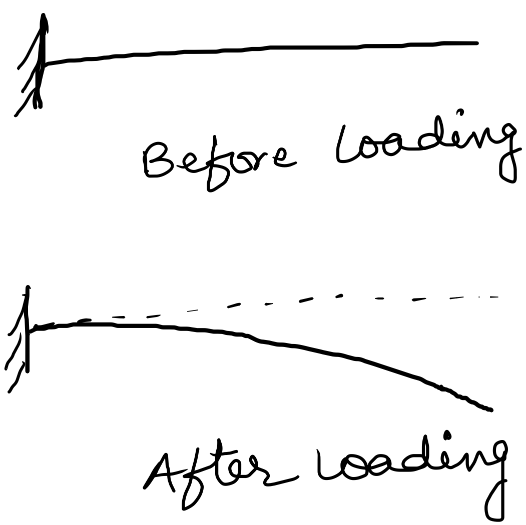
$M_B = -20\times8 - 30 - 30\times4 + 20 = -290\ kN\text{-}m$

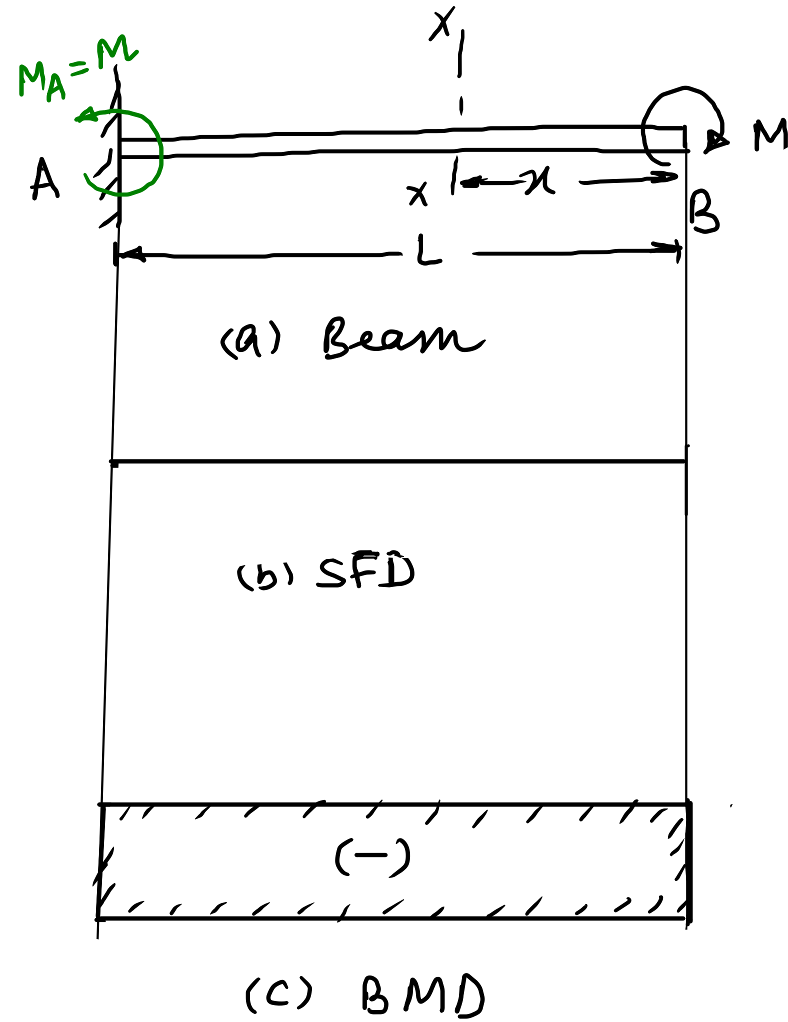
Q:2 Draw SFD and BMD of the given cantilever

Sol^n:

$S_x = 0$

$M_x = -M$





Q:3 Draw SFD and BMD of the given cantilever beam.

S.F.

$S_A = +2\,kN$

$S_C = +2\,kN$

$S_D = +2 + 1\times 2 = +4\,kN$

S.F. at right of 'E' $= +2 + 1\times 2 = +4\,kN$

S.F. at left of 'E' $= +2 + 1\times 2 + 3 = +7\,kN$

$S_B = +2 + 1\times 2 + 3 = +7\,kN$

B.M.

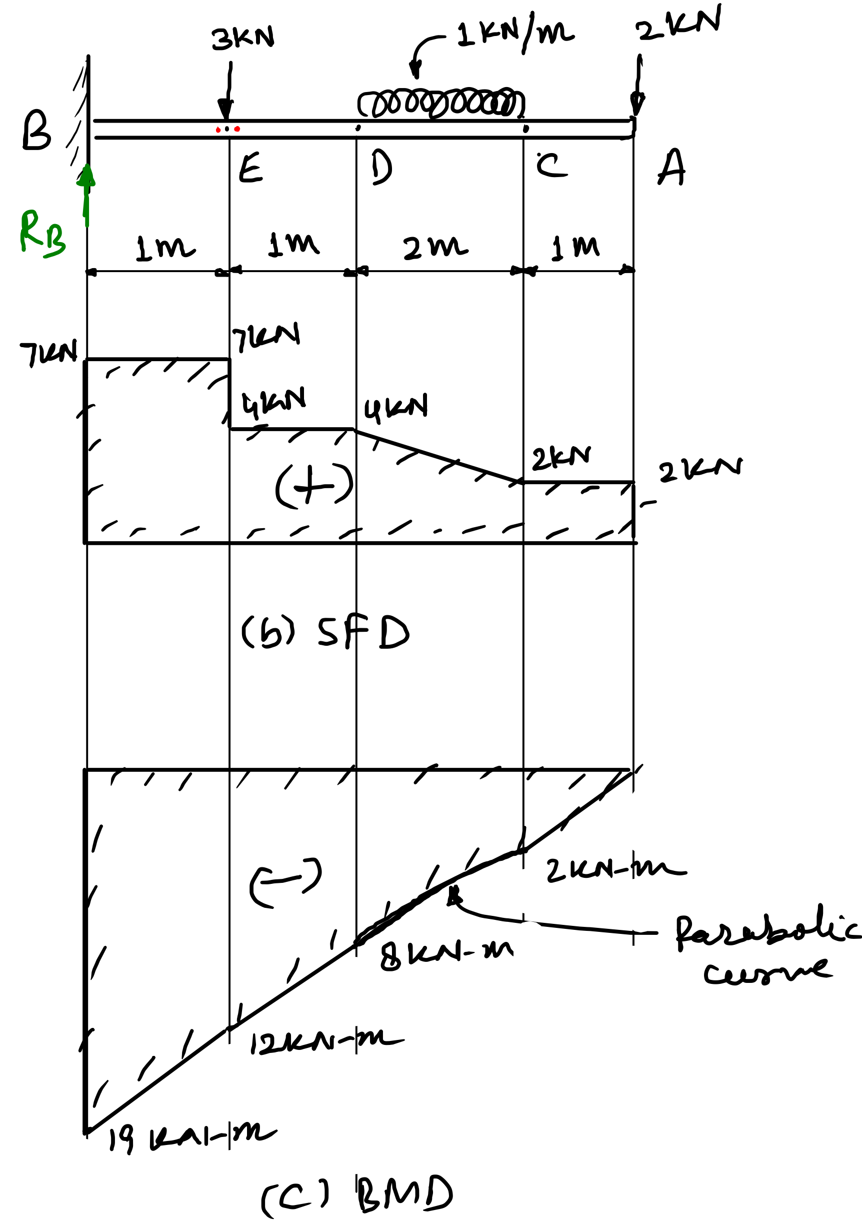
$M_A = 0$

$M_C = -2\times 1 = -2\,kN\text{-}m$

$M_D = -2\times 3 - 1\times 2\times 1 = -8\,kN\text{-}m$

$M_E = -2\times 4 - 1\times 2\times 2 = -12\,kN\text{-}m$

$M_B = -2\times 5 - 1\times 2\times 3 - 3\times 1 = -19\,kN\text{-}m$



	Loading	SFD	BMD
1.	Point	Rectangle	Inclined line
2.	UDL	Inclined line	Parabolic curve
3.	UVL	Parabolic curve	cubic curve

(II) Simply Supported Beam carrying various loads

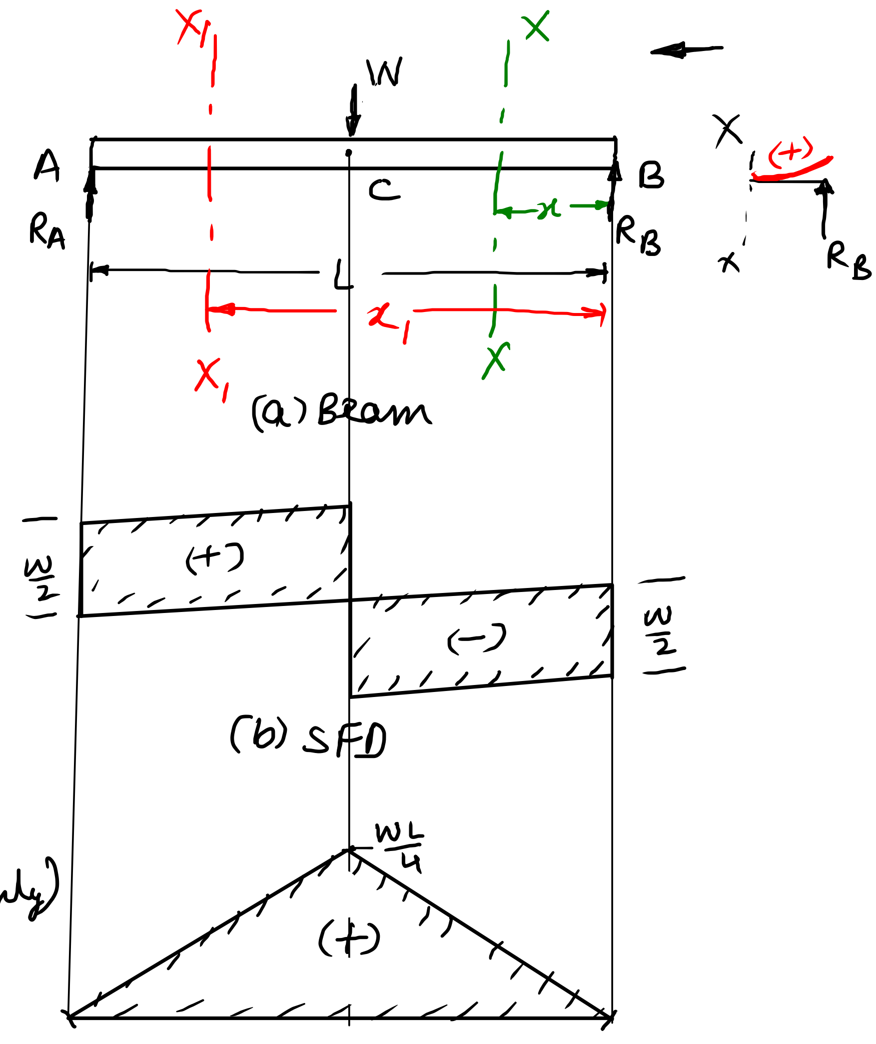
(1.) Simply supported beam with a point load at mid-point

$R_A + R_B = W$ — (1)

$\Sigma M_A = 0$

$R_B \cdot L - W \cdot \frac{L}{2} = 0$ — (2)

$R_B = \frac{W}{2}$, $R_A = \frac{W}{2}$



(a) Beam

(b) SFD

(c) BMD

S.F.

$S_x = -\frac{W}{2}$ — (3) (valid for CB only)

$S_{x_1} = -\frac{W}{2} + W = +\frac{W}{2}$ (valid for CA only) — (4)

B.M.

$M_x = R_B \cdot x = +\frac{W}{2} \cdot x$ — (5) (valid for CB only)

$M_B = 0$, $x = 0$

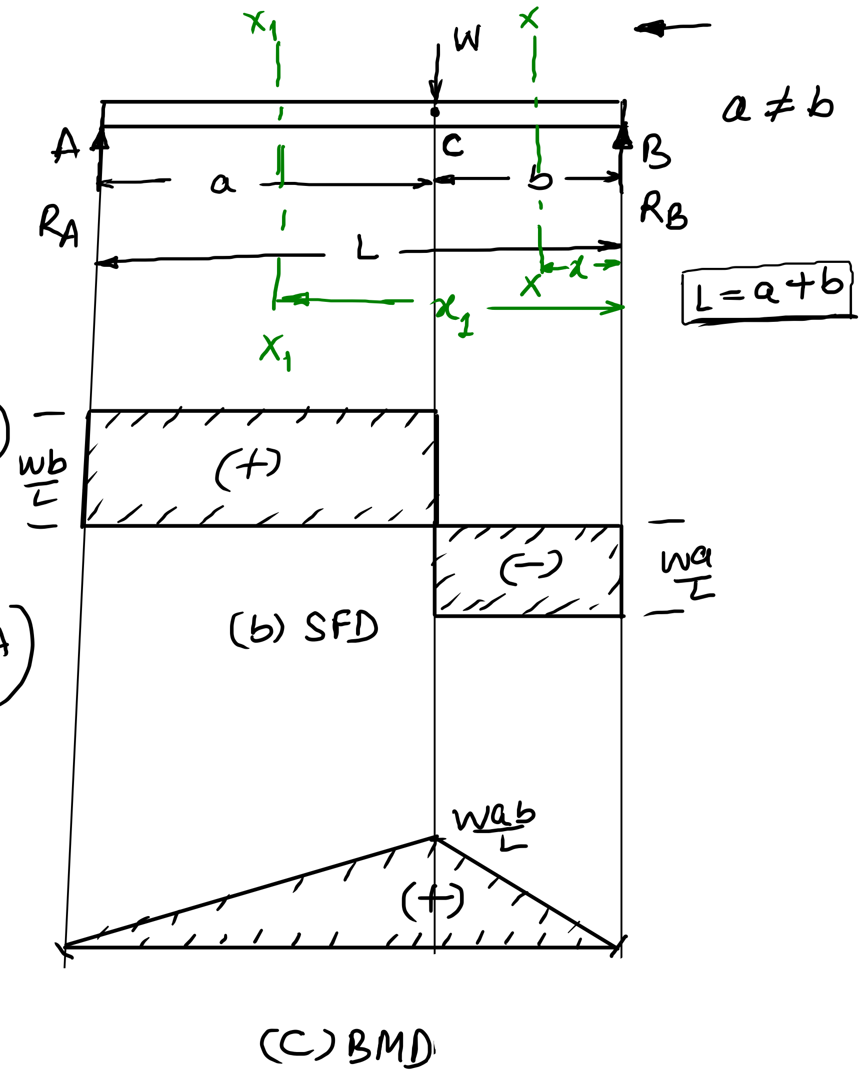
$M_C = \frac{WL}{4}$ $x = \frac{L}{2}$

$M_{x_1} = \frac{W}{2} x_1 - W\left(x_1 - \frac{L}{2}\right)$ — (6) (valid for CA only)

$M_C = \frac{WL}{4}$ $x_1 = \frac{L}{2}$

$M_A = 0$ $x_1 = L$

(2.) Simply supported beam with a point load not at mid-point



$R_A + R_B = W \quad — ①$

$\Sigma M_A = 0, \quad R_B \cdot L - Wa = 0 \quad — ②$

$R_B = \frac{Wa}{L}, \quad R_A = \frac{Wb}{L}$

S.F.

$S_x = -R_B = -\frac{Wa}{L} \quad — ③$ (valid for CB only)

$S_B = -\frac{Wa}{L}, \quad S_C = -\frac{Wa}{L}$

$S_{x_1} = -\frac{Wa}{L} + W = \frac{Wb}{L} \quad — ④$ (valid for CA only)

$S_C = +\frac{Wb}{L}, \quad S_A = +\frac{Wb}{L}$

B.M.

$M_x = +\frac{Wa}{L} \cdot x \quad — ⑤$ (valid for CB only)

$M_B = 0, \quad x = 0,$

$M_C = \frac{Wab}{L}, \quad x = b$

$M_{x_1} = +\frac{Wa}{L} \cdot x_1 - W(x_1 - b) \quad — ⑥$ (Valid for CA only)

$M_C = +\frac{Wab}{L} \quad x_1 = b$

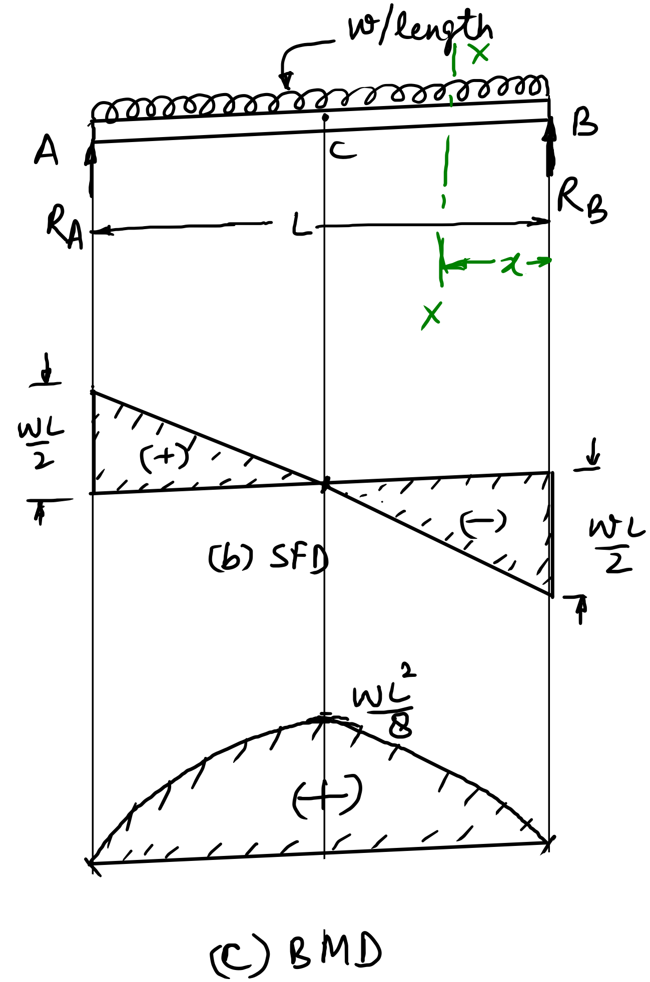
$M_A = 0 \quad x_1 = L$

(3.) Simply supported beam with UDL

$R_A + R_B = wL$

$\Sigma M_A = 0, \quad R_B L - wL \cdot \frac{L}{2} = 0$

$R_B = \frac{wL}{2}, \quad R_A = \frac{wL}{2}$



S.F.

$S_x = -\frac{wL}{2} + wx \quad — ①$

$S_B = -\frac{wL}{2}, \quad x = 0$

$S_A = +\frac{wL}{2}, \quad x = L$

$S_x = 0 \Rightarrow -\frac{wL}{2} + wx = 0$

$\Rightarrow x = \frac{L}{2}$

B.M.

$M_x = +\frac{wL}{2} \cdot x - wx \cdot \frac{x}{2} = \frac{wLx}{2} - \frac{wx^2}{2} \quad — ②$

$M_B = 0, \quad x = 0$

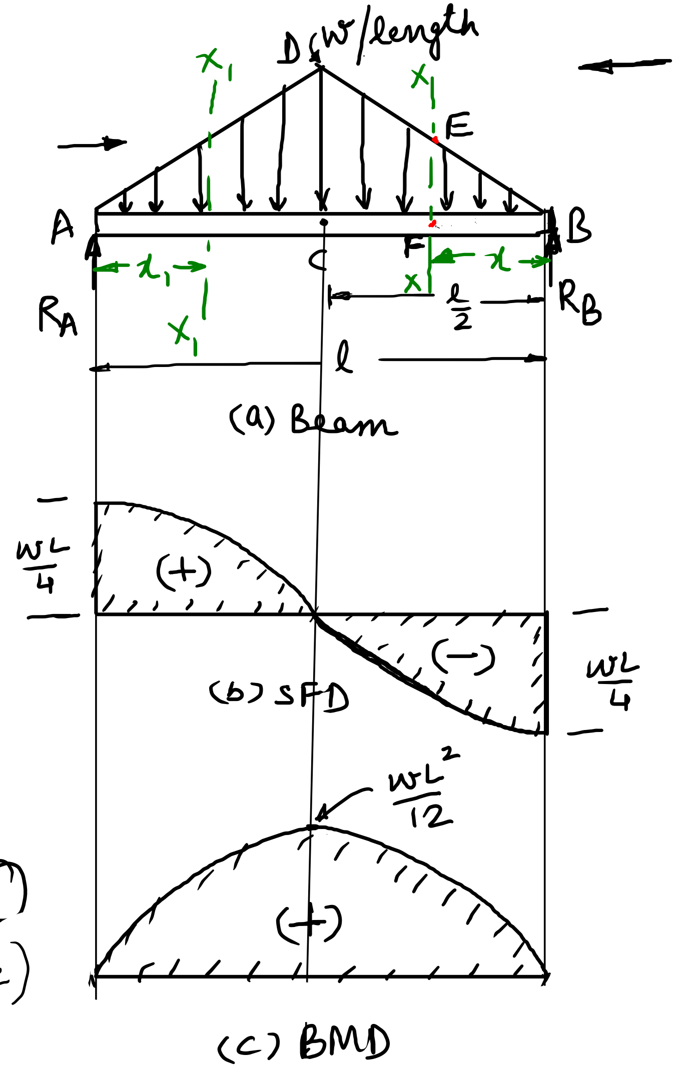
$M_A = 0, \quad x = L$

For max. B.M. $\frac{dM_x}{dx} = 0$

$\frac{wL}{2} - wx = 0 \Rightarrow \boxed{x = \frac{L}{2}}$

$\therefore M_c = \frac{wL^2}{8}, \quad x = \frac{L}{2}$

(4.) Simply supported Beam carrying UVL



Total load on beam = Area of ΔABD

$$= \frac{1}{2} L \times w = \frac{wL}{2}$$

$$R_A = R_B = \frac{wL}{4}$$

From ΔBEF and ΔBCD

$$\frac{EF}{CD} = \frac{BF}{BC}$$

$$\frac{w_x}{w} = \frac{x}{\frac{l}{2}} \Rightarrow w_x = \frac{2wx}{l}$$

S.F.

$$S_x = -R_B + \frac{1}{2} x . w_x = -\frac{wL}{4} + \frac{wx^2}{l} \quad \text{— (1)}$$

(valid for BC only)

$S_B = -\frac{wL}{4}$, at $x = 0$

$S_C = 0$, at $x = \frac{l}{2}$

$$S_{x_1} = +R_A - \frac{1}{2} x_1 . w_{x_1} = \frac{wL}{4} - \frac{wx^2}{l} \quad \text{— (2)}$$

(valid for AC only)

$S_A = +\frac{wL}{4}$, at $x_1 = 0$

$S_C = 0$, at $x_1 = \frac{l}{2}$

$$M_x = \frac{wL}{4}\cdot x - \frac{1}{2}\cdot x\left(\frac{2wx}{L}\right)\cdot\frac{x}{3} = \frac{wLx}{4} - \frac{wx^3}{3L} \quad \text{(valid for BC only)} \quad — (3)$$

$M_B = 0$, at $x = 0$

$M_C = +\frac{wL^2}{12}$, at $x = \frac{L}{2}$

$$\frac{wL^2}{8} - \frac{wL^2}{24} = \frac{wL^2}{12}$$

$$M_{x_1} = \frac{wL}{4}x_1 - \frac{1}{2}x_1\left(\frac{2wx_1}{L}\right)\cdot\frac{x_1}{3} = \frac{wLx_1}{4} - \frac{wx_1^3}{3L} \quad — (4)$$

(valid for AC only)

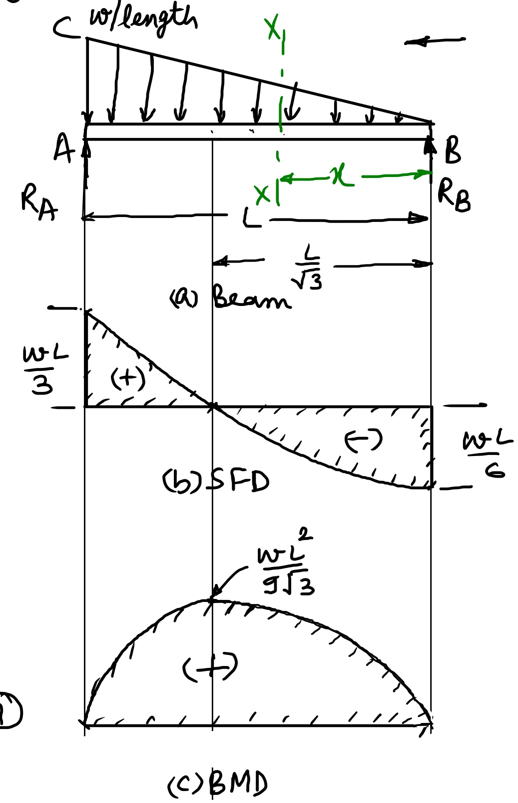
$M_A = 0$, at $x_1 = 0$

$M_C = +\frac{wL^2}{12}$, at $x_1 = \frac{L}{2}$

For max. B.M., $\frac{dM_x}{dx} = 0$, $\frac{wL}{4} - \frac{wx^2}{L} = 0$

$x = \frac{L}{2}$, $M_{max} = \frac{wL^2}{12}$, at $x = \frac{L}{2}$

(5.) Simply supported Beam carrying UVL



$$R_A + R_B = \frac{1}{2} \cdot L \cdot w$$

$$\Sigma M_A = 0, \quad R_B \cdot L - \frac{wL}{2} \cdot \frac{L}{3} = 0$$

$$\Rightarrow R_B = \frac{wL}{6}$$

$$R_A = \frac{wL}{3}$$

$$\frac{w_x}{w} = \frac{x}{L}$$

$$\Rightarrow w_x = \frac{w \cdot x}{L}$$

<u>S.F.</u>

$$S_x = -R_B + \frac{1}{2} \cdot x \cdot w_x = -\frac{wL}{6} + \frac{wx^2}{2L} \quad \text{— (1)}$$

$$S_B = -\frac{wL}{6}, \quad \text{at } x = 0$$

$$S_A = \frac{wL}{3}, \quad \text{at } x = L$$

$$S_x = 0, \quad -\frac{wL}{6} + \frac{wx^2}{2L} = 0 \Rightarrow x = \frac{l}{\sqrt{3}} = 0.577L$$

$$M_x = \frac{wL}{6} \cdot x - \frac{1}{2} x \cdot \left(\frac{w \cdot x}{L}\right) \cdot \frac{x}{3} = \frac{wLx}{6} - \frac{wx^3}{6L} \quad \text{— (2)}$$

$$M_B = 0, \quad \text{at } x = 0$$

$$M_A = 0, \quad \text{at } x = L$$

For max. B.M. $\frac{dM_x}{dx} = 0$, $\quad \frac{wL}{6} - \frac{wx^2}{2L} = 0$

$$x = \frac{L}{\sqrt{3}}$$

$$M_{max} = \frac{wL^2}{9\sqrt{3}}, \text{ at } x = \frac{L}{\sqrt{3}}$$

Point of contra-flexure / Inflexion:

- It is a point at which bending moment is zero and bending moment diagram changes its direction from (+)ive to (−)ive or vice-versa.

Q:1 Draw the SFD and BMD of the given beam.

Sol^n:

$\Sigma F_x = 0, \quad R_{Dx} = 2\sqrt{3}\ kN$

$\Sigma F_y = 0, \quad R_A + R_{Dy} = 1\times2 + 4 + 2 = 8 kN$

$\Sigma M_D = 0, \quad -R_A \times 6 + 1\times2\times5 + 4\times2 - 2\times1 + 2 = 0$

$\Rightarrow R_A = 3 kN$

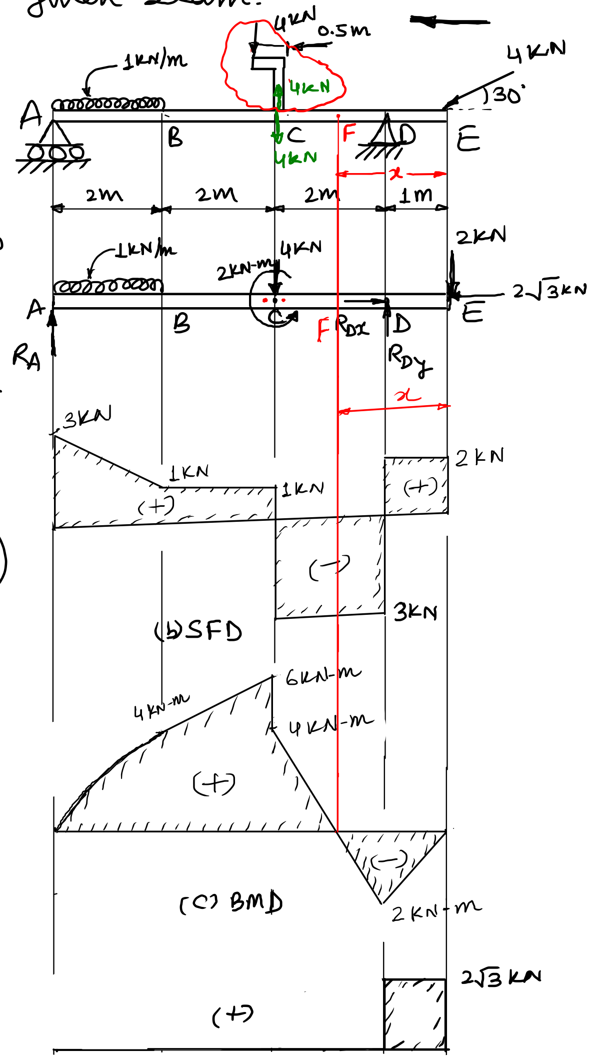
$\therefore R_{Dy} = 5 kN$

Resultant reaction at D, $R_D = \sqrt{R_{Dx}^2 + R_{Dy}^2}$

$= 6.08 kN$

Direction of R_D, $\theta = \tan^{-1}\left(\frac{R_{Dy}}{R_{Dx}}\right)$

$= 55.32°$



(d) Thrust force diagram

S.F.

$S_E = +2 kN$

$S_D = +2 - 5 = -3 kN$

$S_C = +2 - 5 + 4 = +1 kN$

$S_B = +2 - 5 + 4 = +1 kN$

$S_A = +2 - 5 + 4 + 1\times2 = +3 kN$

B.M.

$M_E = 0$

$M_D = -2\times1 = -2 kN\text{-}m$

B.M. at just RHS of C $= -2\times3 + 5\times2 = +4 kN\text{-}m$

B.M. at just LHS of C $= -2\times3 + 5\times2 + 2 = +6 kN\text{-}m$

$M_B = -2\times5 + 5\times4 - 4\times2 + 2 = +4 kN\text{-}m$

$M_A = -2\times7 + 5\times6 - 4\times4 + 2 - 1\times2\times1 = 0$

Point of contraflexure

$M_x = -2x + 5(x-1)$

$M_x = 0 \Rightarrow x = \frac{5}{3} m$

Q:2 Find the magnitude of the clockwise moment M to be applied at C so that the reaction at B will be 30 KN upward and then draw SFD and BMD.

Sol[n]

$R_B = 30\,KN$ (Given)

$R_{Bx} = R_{cx} = 0$

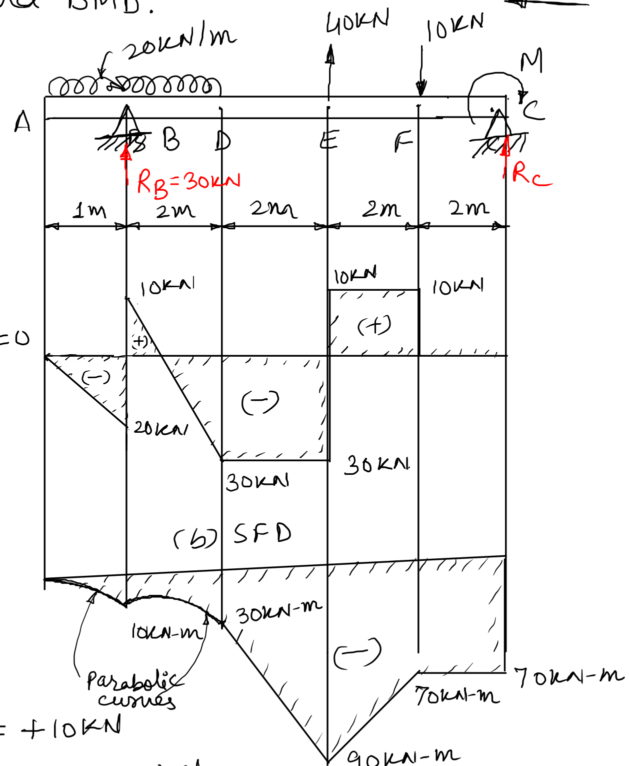
$\Sigma F_y = 0, \quad R_B + R_C + 40 - 10 - 20\times3 = 0$

$\Rightarrow R_C = 0$

$\Sigma M_C = 0,$

$-M + 10\times2 - 40\times4 - 30\times8 + 20\times3\times7.5 = 0$

$\Rightarrow M = 70\,KN\text{-}m$ (CW)



S.F.

$S_C = 0$

$S_F = +10\,KN$

$S_E = +10 - 40 = -30\,KN$

$S_D = +10 - 40 = -30\,KN$

S.F. at just R.H.S. of $B = +10 - 40 + 20\times2 = +10\,KN$

S.F. at just L.H.S. of $B = +10 - 40 + 20\times2 - 30 = -20\,KN$

$S_A = +10 - 40 + 20\times3 - 30 = 0$

B.M.

$M_C = -70\,kN\text{-}m$

$M_F = -70\,kN\text{-}m$

$M_E = -70 - 10\times2 = -90\,kN\text{-}m$

$M_D = -70 - 10\times4 + 40\times2 = -30\,kN\text{-}m$

$M_B = -70 - 10\times6 + 40\times4 - 20\times2\times1 = -10\,kN\text{-}m$

$M_A = -70 - 10\times7 + 40\times5 - 20\times3\times1.5 + 30\times1 = 0$

Q;3 Draw S.F.D. and B.M.D.

Sol^n.

$w \times 8 = 10 \times 4$

$w = 5 kN/m$

S.F.

$S_x = -5x$ (valid for BD only)

$S_B = 0$ at $x = 0$

$S_D = -10 kN$ at $x = 2m$

$S_x = -5x + 10(x-2)$ (valid for CD)

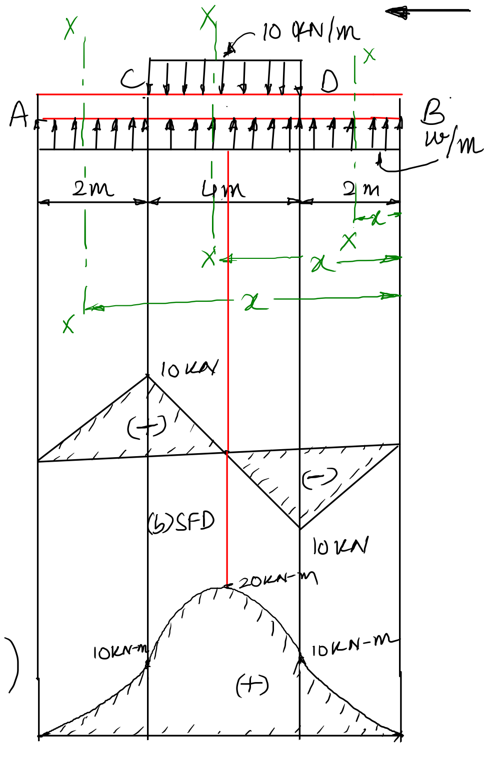
$S_D = -10 kN$, at $x = 2m$

$S_C = +10 kN$ at $x = 6m$

$S_x = -5x + 10 \times 4 = -5x + 40$ (valid for AC)

$S_C = +10 kN$ at $x = 6m$

$S_A = 0$, at $x = 8m$



(c) BMD

B.M.

$M_x = +5x \cdot \frac{x}{2} = \frac{5x^2}{2}$ (valid for BD only)

$M_B = 0$, at $x = 0$

$M_D = 10 kN\text{-}m$ at $x = 2m$

$M_x = +5x \cdot \frac{x}{2} - 10(x-2) \cdot \left(\frac{x-2}{2}\right)$

$= +\frac{5x^2}{2} - 5(x-2)^2$

$M_x = -\frac{5}{2}x^2 + 20x - 20$ (valid for CD)

$M_D = 10 kN\text{-}m$, at $x = 2m$

$M_C = 10 kN\text{-}m$, at $x = 6m$

$M_x = +5x \cdot \frac{x}{2} - 10 \times 4(x-4)$

$= \frac{5x^2}{2} - 40x + 160$ (valid for CA)

$M_C = 10 kN\text{-}m$ at $x = 6m$

$M_A = 0$ at $x = 8m$

max. B.M. is in CD portion of the beam.

For max. B.M. $\frac{dM_x}{dx} = 0$

$\frac{d}{dx}\left(-\frac{5}{2}x^2 + 20x - 20\right) = 0$

$-5x + 20 = 0$

$\Rightarrow x = 4m$

$\therefore M_{max} = -\frac{5}{2}(4)^2 + 20(4) - 20 = 20 kN\text{-}m$