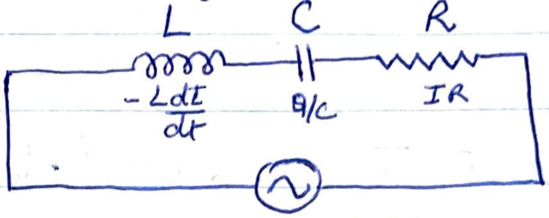
# Application of Resonance in LCR ckt




$E = E_0 \sin pt$

Applied alternating voltage.

Let $I$ be the current in the circuit at any instant and $q$ be the charge on the plates of the capacitor then potential drop across various components will satify the eqn:

$$L\frac{dI}{dt} + IR + \frac{q}{C} = E = E_0 \sin pt \quad \text{--- (1)}$$

Since $I = \frac{dq}{dt}$ we get:

$$\frac{d^2q}{dt^2} + \left(\frac{R}{L}\right)\frac{dq}{dt} + \frac{q}{LC} = \left(\frac{E_0}{L}\right)\sin pt$$

This is similar to forced oscillator eqn. where $x$ replaced by $q$, etc.

Steady state soln. can be obtained by same substitutions:

$$q = \frac{E_0/L}{\sqrt{\left(\frac{1}{LC} - p^2\right)^2 + \left(\frac{pR}{L}\right)^2}} \sin(pt - \phi)$$

$$\phi = \tan^{-1} \frac{pR/L}{\frac{1}{LC} - p^2}$$

Let current $I = I_0 \sin(pt - \phi)$

$$\frac{dI}{dt} = I_0 p \cos(pt - \phi)$$

and $$Q = \int I\,dt = -\frac{I_0}{p}\cos(pt - \phi)$$

Put these values in eqn ①

$$L I_0 p \cos(pt-\phi) + R I_0 \sin(pt-\phi) - \frac{I_0}{pC}\cos(pt-\phi) = E_0 \sin pt$$

$$I_0 \left[R \sin(pt-\phi) + \left(Lp - \frac{1}{Cp}\right)\cos(pt-\phi)\right] = E_0 \sin pt$$

Put $R = a\cos\phi$ $\quad Lp - \frac{1}{Cp} = a\sin\phi$ —— ②

$$I_0 a \left[\cos\phi \sin(pt-\phi) + \sin\phi \cos(pt-\phi)\right]$$
$$= I_0 a \sin[(pt-\phi)+\phi] = E_0 \sin pt$$

Just like forced oscillation, square & add eqns ② :-

$$a = \sqrt{R^2 + \left(Lp - \frac{1}{Cp}\right)^2} \qquad \tan\phi = \frac{\left(Lp - \frac{1}{Cp}\right)}{R}$$

Putting these in above eqn.

$$I_0\sqrt{R^2+\left(Lp-\frac{1}{Cp}\right)^2}\,\sin pt = E_0 \sin pt$$

$$I_0\sqrt{R^2+\left(Lp-\frac{1}{Cp}\right)^2} = E_0$$

$$I_0 = \frac{E_0}{\sqrt{R^2+\left(Lp-\frac{1}{Cp}\right)^2}}$$

$$I = I_0 \sin(pt-\phi) = \downarrow \sin(pt-\phi)$$

$\sqrt{R^2+\left(Lp-\frac{1}{Cp}\right)^2}$ is equivalent to the effective resistance of LCR ckt.

It is k/a the impedance, $Z$.

It has two terms :-

1) Freq. independant ohmic $R$.
2) " dependant reactance ($X$)

$$X = Lp - \frac{1}{Cp}$$

This also has two parts, inductive reactance ($X_L$) & capacitive reactance ($X_C$)

$$X = X_L + X_C$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_C)^2}$$

Peak value of current:

$$I_0 = \frac{E_0}{Z}$$

$$I = \frac{E_0}{Z} \sin(pt - \phi)$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

<u>Case I</u>:

$X_L = X_C$, then $Z = R$ and current is max. Cond. of Resonance.

$$I_0 = \frac{E_0}{R}$$

$I_0 \to \infty$ as $R \to 0$

and no phase diff b/w current and emf.

$$X_L = X_C \Rightarrow Lp = \frac{1}{Cp}$$

$$p^2 = \frac{1}{LC} \Rightarrow p = \frac{1}{\sqrt{LC}}$$

~~Ⓡ~~ Resonant freq $= \frac{p}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

Case - II -

$X_L > X_C$ then net reactance is inductive, and $\tan\phi$ is positive. Current lags behind emf.

Case III

$X_L < L_C$; then net reactance is capacitive, $\tan\phi$ is negative current leads the emf.