1. Original and Transformed State Vectors

The original nonlinear system is defined by the state vector **x**, representing the position and velocity of the spacecraft in the rotating reference frame.

- **Original State Vector (x):** $x = [x, y, vx, vy]^T$ where x, y are the position coordinates and vx, vy are the velocity components.

Through feedback linearization, we define a diffeomorphic mapping **z = Φ(x)** to transform the state into a new set of coordinates where the dynamics are linear.

- **Transformed State Vector (z):** $z = [z_1, z_2, z_3, z_4]^T$
- **State Transformation (Φ(x)):** The transformation is constructed from the output $h = [x, y]^T$ and its successive Lie derivatives, resulting in the following mapping:

- In matrix form, the transformation is: $z = \Phi(x) = [x, y, vx, vy]^T$

  (Note: In this specific case, the transformation is the identity, as the relative degree of each output is 2, summing to the system order of 4.)

2. Original and Linearized Input Vectors

The original physical control inputs are thrust accelerations.

- **Original Input Vector (u):** $u = [u_x, u_y]^T$

A new, virtual input vector **v** is introduced to command the linearized system. The original input **u** is calculated from **v** using a nonlinear feedback law $u = \alpha(x) + \beta(x)v$, which cancels the system's natural dynamics.

- **Linearized Input Vector (v):** $v = [v_1, v_2]^T$ where v_1 and v_2 are the command inputs to the linear system.

3. Continuous-Time Linear System

In the new coordinates (**z**, **v**), the system dynamics are exactly linear and are described by the state-space equation:

$$\dot{z} = A z + B v$$

where the state matrix **A** and input matrix **B** are:

- **State Matrix (A):**

- • **Input Matrix (B):**

-

4. Discrete-Time Linear System

For digital implementation and the Jump-N test, the continuous-time system is discretized with a sample time $T_s = 0.01$ s using a Zero-Order Hold (ZOH). The resulting discrete-time state-space equation is:

```
z[k+1] = A_d z[k] + B_d v[k]
```

The numerical values for the discrete matrices are:

- **Discrete State Matrix (A_d):**

- - **Discrete Input Matrix (B_d):**

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5. Jump-N Prediction Formula

Using the discrete system, the state after N steps, assuming a constant input v, can be predicted directly using the closed-form Jump-N formula:

$$z[N] = A_d^N z[0] + (I - A_d^N)(I - A_d)^{-1} B_d v$$

This formula, using the matrices **A_d** and **B_d** defined above, provides a direct way to compute the system's future state and serves as a basis for validating the linearization against a full nonlinear simulation.