

CS274A: Probabilistic Learning

## HOMEWORK 6: EM AND GUASSIAN MIXTURES

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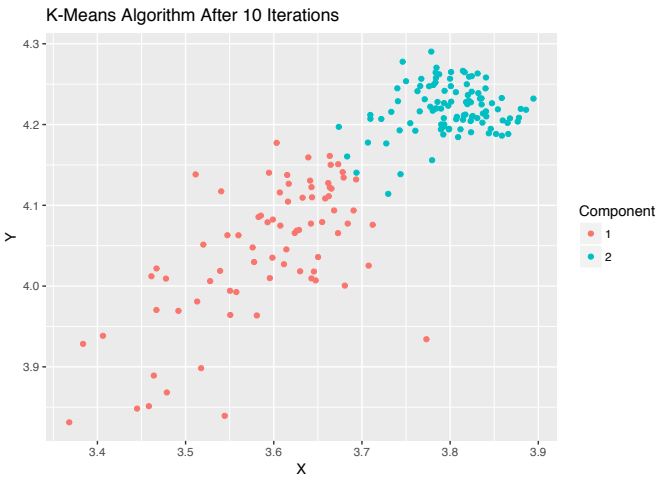
17 March 2017

# K-Means Algorithm

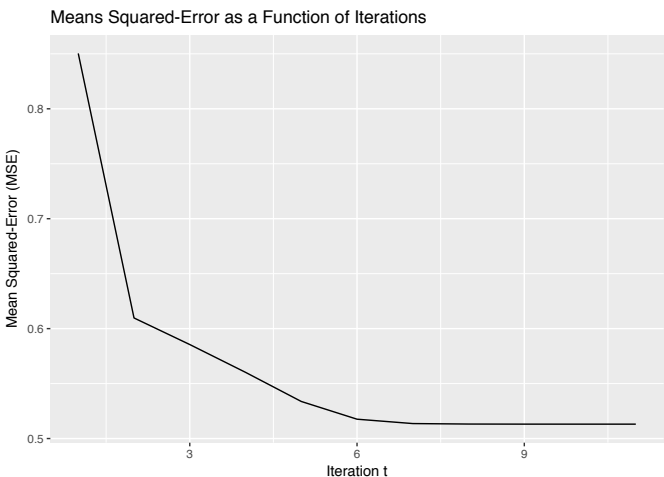
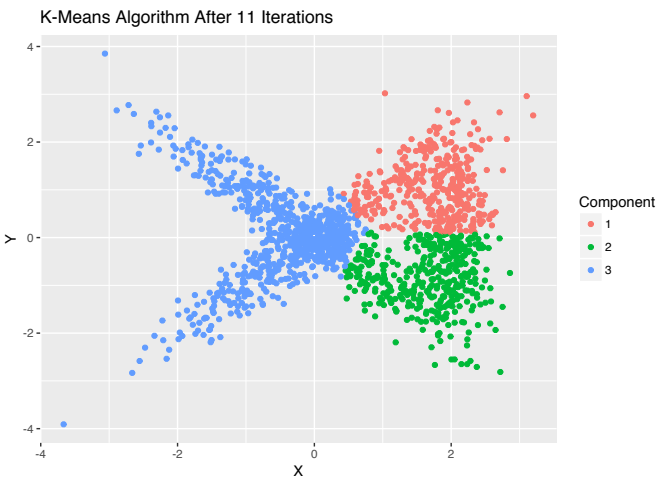
SCATTERPLOTS OF FINAL CLASSIFICATIONS

PLOT OF MEAN SQUARED-ERROR OVER TIME

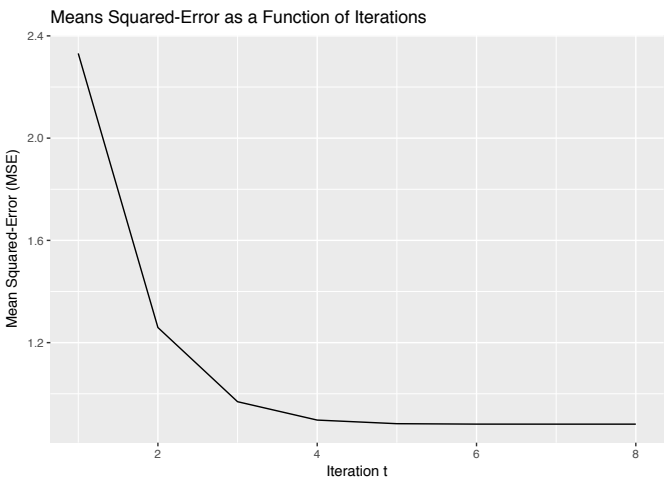
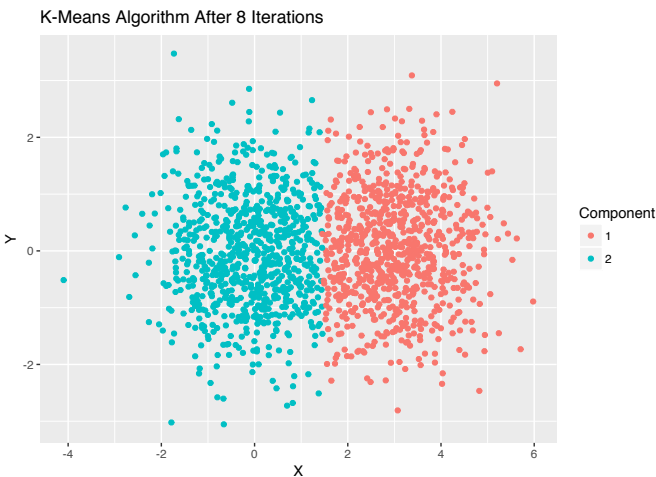
Data Set 3



Data Set 2



Data Set 1



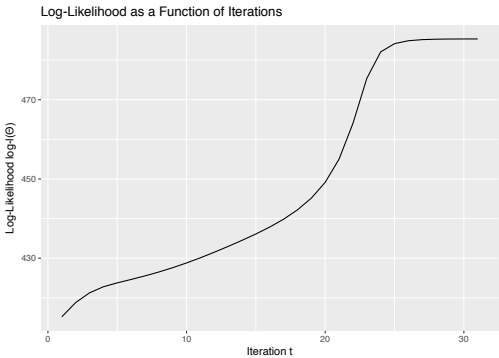
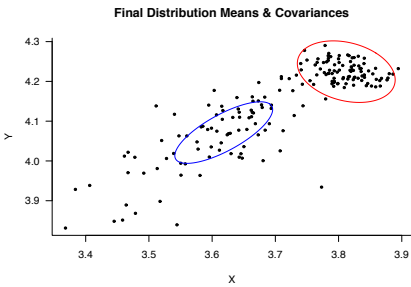
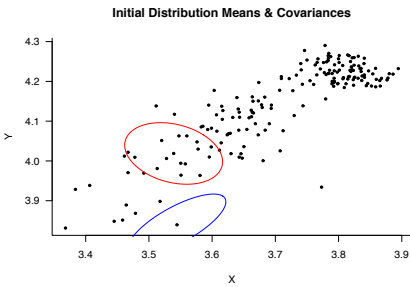
# E-M Algorithm

INITIAL DIST'N PARAMETERS

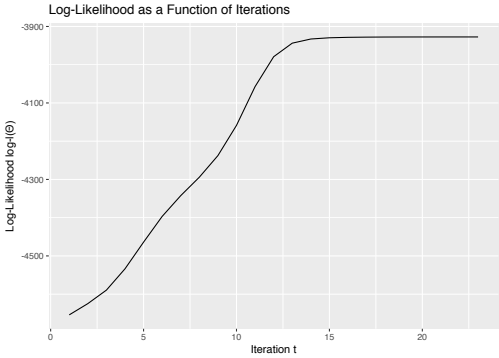
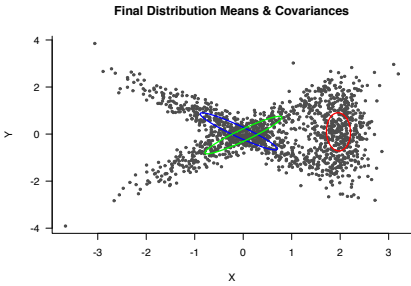
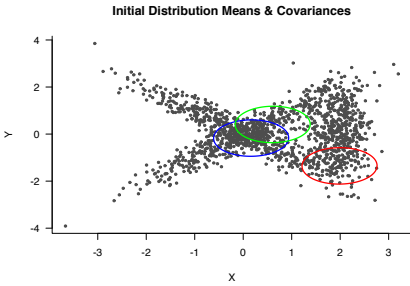
FINAL DIST'N PARAMETERS

PLOT OF LOG-LIKELIHOOD OVER TIME

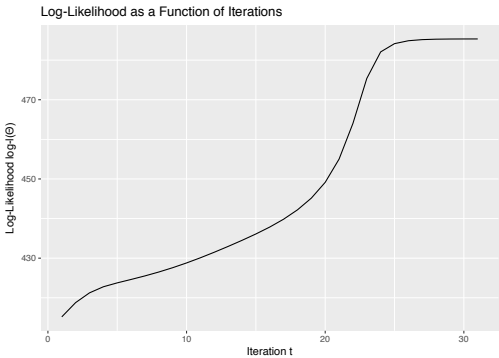
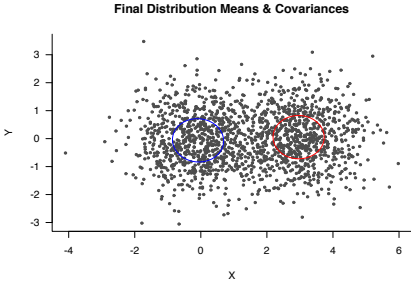
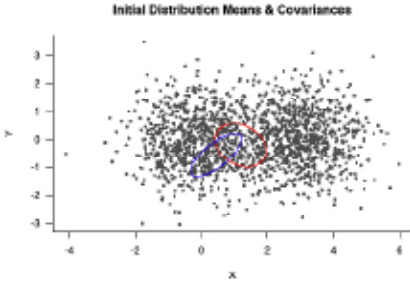
Data Set 3



Data Set 2



Data Set 1



# Comments on the Differences Between K-Means & E-M

## for Data Set 3

Both K-means and E-M algorithm solutions ‘break up’ the epidemiological data in roughly the same place. Both identify two clusters corresponding to patients with high red blood-cell volume and mean hemoglobin concentrations (in the upper right) that are unlikely to have iron deficient anemia. How they differ is in the treatment of border-line cases. K-means must make a sharp partition, whereas E-M can encode graded uncertainty about correct classification in the two groups.

## for Data Set 2

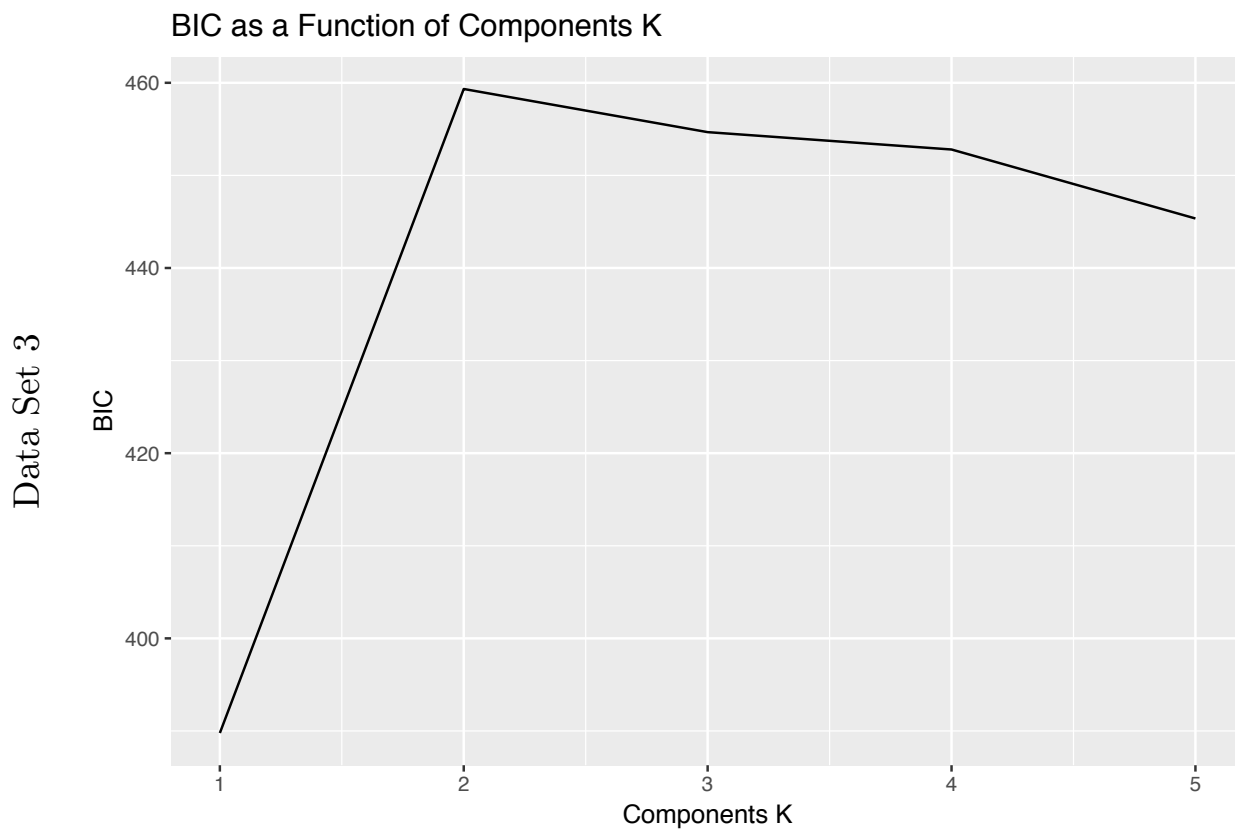
Here, K-means displays its limitations. Limited by the measure of Euclidian distance as its loss rule, it fails to detect the three underlying Gaussian distribution. The E-M solution, on the other hand, accounts for covariances, and so consistently converges on roughly the true means and covariances of the three component (Gaussian) distributions. As a tradeoff, the E-M algorithm (in my implementation) took more iterations to converge to a solution.

## for Data Set 1

Here, both solutions are modest ones. The significant overlap of the underlying component (Gaussian) distributions makes it so that the membership of many points in the penumbra of the two distributions is irreducibly uncertain. The K-means solution divides the data space nearly in half, while the E-M solution posits (roughly) accurate Gaussians at the two epicenters of the data.

# Automatically Selecting $K$ for Gaussian Mixture Clustering

BIC AS A FUNCTION OF NUMBER OF COMPONENTS  $K$



## Comments

The BIC criterion for data set 3 nicely picks out 2 (Gaussian) components as the optimal number. The BIC makes it so that, as  $K$  increases, diminishing gains from increased likelihood are overwhelmed by the penalty for the increased number of parameters. Naturally, this has the potential to deter selection of overly complex models, and so avoid overfitting the data.